維基百科

三角函数积分表

维基百科，自由的百科全书

以下是部分三角函数的积分表（省略积分常数）：

目录

积分只有sin的函数

$$\int \sin cx \, dx = -\frac{1}{c}\cos cx$$

$$\int \sin^n cx \, dx = -\frac{1}{nc}\sin^{n-1} cx \cos cx + \frac{n-1}{n}\int \sin^{n-2} cx \, dx \qquad (\text{其中} n > 0)$$

$$\int \sqrt{1-\sin x}\, dx = \int \sqrt{\operatorname{cvs} x}\, dx = 2\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\sqrt{\operatorname{cvs} x}\left(=2\sqrt{1+\sin x}\right)$$（其中 $\operatorname{cvs} x$ 是 余矢(Coversine)函数(参阅正矢(versine)函数)）

$$\int x\sin cx \, dx = \frac{\sin cx}{c^2} - \frac{x\cos cx}{c}$$

$$\int x^n \sin cx \, dx = -\frac{x^n}{c}\cos cx + \frac{n}{c}\int x^{n-1}\cos cx \, dx \qquad (\text{其中} n > 0)$$

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 \sin^2 \frac{n\pi x}{a} \, dx = \frac{a^3(n^2\pi^2-6)}{24n^2\pi^2} \qquad (\text{其中} n = 2, 4, 6\ldots)$$

$$\int \frac{\sin cx}{x} dx = \sum_{i=0}^{\infty}(-1)^i \frac{(cx)^{2i+1}}{(2i+1)\cdot(2i+1)!}$$

$$\int \frac{\sin cx}{x^n}dx = -\frac{\sin cx}{(n-1)x^{n-1}} + \frac{c}{n-1}\int \frac{\cos cx}{x^{n-1}}dx$$

$$\int \frac{dx}{\sin cx} = \frac{1}{c}\ln\left|\tan\frac{cx}{2}\right|$$

$$\int \frac{dx}{\sin^n cx} = \frac{\cos cx}{c(1-n)\sin^{n-1} cx} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} cx} \qquad (\text{其中} n > 1)$$

$$\int \frac{dx}{1 \pm \sin cx} = \frac{1}{c}\tan\left(\frac{cx}{2} \mp \frac{\pi}{4}\right)$$

$$\int \frac{x\,dx}{1 + \sin cx} = \frac{x}{c}\tan\left(\frac{cx}{2} - \frac{\pi}{4}\right) + \frac{2}{c^2}\ln\left|\cos\left(\frac{cx}{2} - \frac{\pi}{4}\right)\right|$$

$$\int \frac{x\,dx}{1 - \sin cx} = \frac{x}{c}\cot\left(\frac{\pi}{4} - \frac{cx}{2}\right) + \frac{2}{c^2}\ln\left|\sin\left(\frac{\pi}{4} - \frac{cx}{2}\right)\right|$$

$$\int \frac{\sin cx\,dx}{1 \pm \sin cx} = \pm x + \frac{1}{c}\tan\left(\frac{\pi}{4} \mp \frac{cx}{2}\right)$$

$$\int \sin c_1 x \sin c_2 x\,dx = \frac{\sin(c_1 - c_2)x}{2(c_1 - c_2)} - \frac{\sin(c_1 + c_2)x}{2(c_1 + c_2)} \qquad (\text{其中} |c_1| \neq |c_2|)$$

积分只有cos的函数

$$\int \cos cx\,dx = \frac{1}{c}\sin cx$$

$$\int \cos^n cx\,dx = \frac{1}{nc}\cos^{n-1} cx \sin cx + \frac{n-1}{n}\int \cos^{n-2} cx\,dx \qquad (n > 0)$$

$$\int x\cos cx\,dx = \frac{\cos cx}{c^2} + \frac{x\sin cx}{c}$$

$$\int x^n \cos cx\,dx = \frac{x^n \sin cx}{c} - \frac{n}{c}\int x^{n-1}\sin cx\,dx$$

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 \cos^2\frac{n\pi x}{a}\,dx = \frac{a^3(n^2\pi^2 - 6)}{24n^2\pi^2} \qquad (n = 1, 3, 5\ldots)$$

$$\int \frac{\cos cx}{x}dx = \ln|cx| + \sum_{i=1}^{\infty}(-1)^i \frac{(cx)^{2i}}{2i \cdot (2i)!}$$

$$\int \frac{\cos cx}{x^n}dx = -\frac{\cos cx}{(n-1)x^{n-1}} - \frac{c}{n-1}\int \frac{\sin cx}{x^{n-1}}dx \qquad (n \neq 1)$$

$$\int \frac{dx}{\cos cx} = \frac{1}{c}\ln\left|\tan\left(\frac{cx}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)cos^{n-1}cx} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} cx} \qquad (n > 1)$$

$$\int \frac{dx}{1+\cos cx} = \frac{1}{c}\tan\frac{cx}{2}$$

$$\int \frac{dx}{1-\cos cx} = -\frac{1}{c}\cot\frac{cx}{2}$$

$$\int \frac{x\,dx}{1+\cos cx} = \frac{x}{c}\tan\frac{cx}{2} + \frac{2}{c^2}\ln\left|\cos\frac{cx}{2}\right|$$

$$\int \frac{x\,dx}{1-\cos cx} = -\frac{x}{c}\cot\frac{cx}{2} + \frac{2}{c^2}\ln\left|\sin\frac{cx}{2}\right|$$

$$\int \frac{\cos cx\,dx}{1+\cos cx} = x - \frac{1}{c}\tan\frac{cx}{2}$$

$$\int \frac{\cos cx\,dx}{1-\cos cx} = -x - \frac{1}{c}\cot\frac{cx}{2}$$

$$\int \cos c_1 x \cos c_2 x\,dx = \frac{\sin(c_1-c_2)x}{2(c_1-c_2)} + \frac{\sin(c_1+c_2)x}{2(c_1+c_2)} \qquad (|c_1| \neq |c_2|)$$

积分只有tan的函数

$$\int \tan cx\,dx = -\frac{1}{c}\ln|\cos cx| \ = \frac{1}{c}\ln|\sec cx|$$

$$\int \tan^n cx\,dx = \frac{1}{c(n-1)}\tan^{n-1} cx - \int \tan^{n-2} cx\,dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\tan cx + 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{dx}{\tan cx - 1} = -\frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$

$$\int \frac{\tan cx\,dx}{\tan cx + 1} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{\tan cx\,dx}{\tan cx - 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$

积分只有sec的函数

$$\int \sec cx\,dx = \frac{1}{c}\ln|\sec cx + \tan cx|$$

$$\int \sec^2 x \mathrm{d}x = \tan x + C$$

$$\int \sec^n cx\,dx = \frac{\sec^{n-2} cx \tan cx}{c(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2} cx\,dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\sec x + 1} = x - \tan\frac{x}{2}$$

积分只有csc的函数

$$\int \csc cx\, dx = -\frac{1}{c} \ln \left|\csc cx + \cot cx\right|$$

$$\int \csc^2 x \mathrm{d}x = -\cot x + C$$

$$\int \csc^n cx\, dx = -\frac{\csc^{n-2} cx \cot cx}{c(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} cx\, dx \qquad (\text{for } n \neq 1)$$

积分只有cot的函数

$$\int \cot cx\ dx = \frac{1}{c} \ln \left|\sin cx\right|$$

$$\int \cot^n cx\ dx = -\frac{1}{c(n-1)} \cot^{n-1} cx - \int \cot^{n-2} cx\ dx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{1+\cot cx} = \int \frac{\tan cx\ dx}{\tan cx + 1}$$

$$\int \frac{dx}{1-\cot cx} = \int \frac{\tan cx\ dx}{\tan cx - 1}$$

积分只有sin和cos的函数

$$\int \frac{dx}{\cos cx \pm \sin cx} = \frac{1}{c\sqrt{2}} \ln \left|\tan\left(\frac{cx}{2} \pm \frac{\pi}{8}\right)\right|$$

$$\int \frac{dx}{(\cos cx \pm \sin cx)^2} = \frac{1}{2c} \tan\left(cx \mp \frac{\pi}{4}\right)$$

$$\int \frac{dx}{(\cos x + \sin x)^n} = \frac{1}{n-1}\left[\frac{\sin x - \cos x}{(\cos x + \sin x)^{n-1}} - 2(n-2)\int \frac{dx}{(\cos x + \sin x)^{n-2}}\right]$$

$$\int \frac{\cos cx\ dx}{\cos cx + \sin cx} = \frac{x}{2} + \frac{1}{2c} \ln|\sin cx + \cos cx|$$

$$\int \frac{\cos cx\ dx}{\cos cx - \sin cx} = \frac{x}{2} - \frac{1}{2c} \ln|\sin cx - \cos cx|$$

$$\int \frac{\sin cx\ dx}{\cos cx + \sin cx} = \frac{x}{2} - \frac{1}{2c} \ln|\sin cx + \cos cx|$$

$$\int \frac{\sin cx\ dx}{\cos cx - \sin cx} = -\frac{x}{2} - \frac{1}{2c} \ln|\sin cx - \cos cx|$$

$$\int \frac{\cos cx\ dx}{\sin cx(1+\cos cx)} = -\frac{1}{4c} \tan^2 \frac{cx}{2} + \frac{1}{2c} \ln\left|\tan \frac{cx}{2}\right|$$

$$\int \frac{\cos cx\ dx}{\sin cx(1+-\cos cx)} = -\frac{1}{4c} \cot^2 \frac{cx}{2} - \frac{1}{2c} \ln\left|\tan \frac{cx}{2}\right|$$

$$\int \frac{\sin cx\ dx}{\cos cx(1+\sin cx)} = \frac{1}{4c}\cot^2\left(\frac{cx}{2}+\frac{\pi}{4}\right) + \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \frac{\sin cx\ dx}{\cos cx(1-\sin cx)} = \frac{1}{4c}\tan^2\left(\frac{cx}{2}+\frac{\pi}{4}\right) - \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \sin cx\cos cx\ dx = \frac{1}{2c}\sin^2 cx$$

$$\int \sin c_1x\cos c_2x\ dx = -\frac{\cos(c_1+c_2)x}{2(c_1+c_2)} - \frac{\cos(c_1-c_2)x}{2(c_1-c_2)} \qquad (\text{for } |c_1| \neq |c_2|)$$

$$\int \sin^n cx\cos cx\ dx = \frac{1}{c(n+1)}\sin^{n+1} cx \qquad (\text{for } n \neq 1)$$

$$\int \sin cx\cos^n cx\ dx = -\frac{1}{c(n+1)}\cos^{n+1} cx \qquad (\text{for } n \neq 1)$$

$$\int \sin^n cx\cos^m cx\ dx = -\frac{\sin^{n-1} cx\cos^{m+1} cx}{c(n+m)} + \frac{n-1}{n+m}\int \sin^{n-2} cx\cos^m cx\ dx \qquad (\text{for } m,n>0)$$

also:

$$\int \sin^n cx\cos^m cx\ dx = \frac{\sin^{n+1} cx\cos^{m-1} cx}{c(n+m)} + \frac{m-1}{n+m}\int \sin^n cx\cos^{m-2} cx\ dx \qquad (\text{for } m,n>0)$$

$$\int \frac{dx}{\sin cx\cos cx} = \frac{1}{c}\ln|\tan cx|$$

$$\int \frac{dx}{\sin cx\cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} + \int \frac{dx}{\sin cx\cos^{n-2} cx} \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\sin^n cx\cos cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} + \int \frac{dx}{\sin^{n-2} cx\cos cx} \qquad (\text{for } n \neq 1)$$

$$\int \frac{\sin cx\ dx}{\cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} \qquad (\text{for } n \neq 1)$$

$$\int \frac{\sin^2 cx\ dx}{\cos cx} = -\frac{1}{c}\sin cx + \frac{1}{c}\ln\left|\tan\left(\frac{\pi}{4}+\frac{cx}{2}\right)\right|$$

$$\int \frac{\sin^2 cx\ dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)\cos^{n-1} cx} - \frac{1}{n-1}\int \frac{dx}{\cos^{n-2} cx} \qquad (\text{for } n \neq 1)$$

$$\int \frac{\sin^n cx\ dx}{\cos cx} = -\frac{\sin^{n-1} cx}{c(n-1)} + \int \frac{\sin^{n-2} cx\ dx}{\cos cx} \qquad (\text{for } n \neq 1)$$

$$\int \frac{\sin^n cx\ dx}{\cos^m cx} = \frac{\sin^{n+1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-m+2}{m-1}\int \frac{\sin^n cx\ dx}{\cos^{m-2} cx} \qquad (\text{for } m \neq 1)$$

also: $\int \frac{\sin^n cx\ dx}{\cos^m cx} = -\frac{\sin^{n-1} cx}{c(n-m)\cos^{m-1} cx} + \frac{n-1}{n-m}\int \frac{\sin^{n-2} cx\ dx}{\cos^m cx} \qquad (\text{for } m \neq n)$

also: $\int \frac{\sin^n cx\ dx}{\cos^m cx} = \frac{\sin^{n-1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-1}{m-1}\int \frac{\sin^{n-2} cx\ dx}{\cos^{m-2} cx} \qquad (\text{for } m \neq 1)$

$\int \frac{\cos cx\ dx}{\sin^n cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} \qquad (\text{for } n \neq 1)$

$\int \frac{\cos^2 cx\ dx}{\sin cx} = \frac{1}{c}\left(\cos cx + \ln\left|\tan\frac{cx}{2}\right|\right)$

$\int \frac{\cos^2 cx\ dx}{\sin^n cx} = -\frac{1}{n-1}\left(\frac{\cos cx}{c\sin^{n-1} cx)} + \int \frac{dx}{\sin^{n-2} cx}\right) \qquad (\text{for } n \neq 1)$

$\int \frac{\cos^n cx\ dx}{\sin^m cx} = -\frac{\cos^{n+1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-m-2}{m-1}\int \frac{cos^n cx\ dx}{\sin^{m-2} cx} \qquad (\text{for } m \neq 1)$

also: $\int \frac{\cos^n cx\ dx}{\sin^m cx} = \frac{\cos^{n-1} cx}{c(n-m)\sin^{m-1} cx} + \frac{n-1}{n-m}\int \frac{cos^{n-2} cx\ dx}{\sin^m cx} \qquad (\text{for } m \neq n)$

also: $\int \frac{\cos^n cx\ dx}{\sin^m cx} = -\frac{\cos^{n-1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-1}{m-1}\int \frac{cos^{n-2} cx\ dx}{\sin^{m-2} cx} \qquad (\text{for } m \neq 1)$

积分只有sin和tan的函数

$\int \sin cx \tan cx\ dx = \frac{1}{c}(\ln|\sec cx + \tan cx| - \sin cx)$

$\int \frac{\tan^n cx\ dx}{\sin^2 cx} = \frac{1}{c(n-1)}\tan^{n-1}(cx) \qquad (\text{for } n \neq 1)$

积分只有cos和tan的函数

$\int \frac{\tan^n cx\ dx}{\cos^2 cx} = \frac{1}{c(n+1)}\tan^{n+1} cx \qquad (\text{for } n \neq -1)$

积分只有sin和cot的函数

$\int \frac{\cot^n cx\ dx}{\sin^2 cx} = \frac{1}{c(n+1)}\cot^{n+1} cx \qquad (\text{for } n \neq -1)$

积分只有cos和cot的函数

$\int \frac{\cot^n cx\ dx}{\cos^2 cx} = \frac{1}{c(1-n)}\tan^{1-n} cx \qquad (\text{for } n \neq 1)$

积分只有tan和cot的函数

$\int \frac{\tan^m(cx)}{\cot^n(cx)}\ dx = \frac{1}{c(m+n-1)}\tan^{m+n-1}(cx) - \int \frac{\tan^{m-2}(cx)}{\cot^n(cx)}\ dx \qquad (\text{for } m+n \neq 1)$

取自“https://zh.wikipedia.org/w/index.php?title=三角函数积分表&oldid=58500429”

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