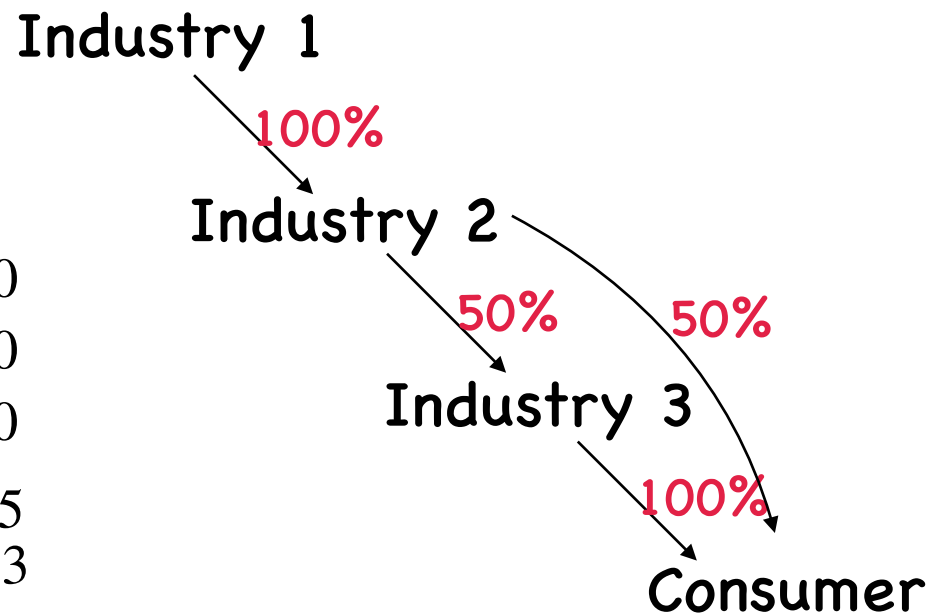


# Upstreamness: an elementary example

- Antras, Chor, Fally and Hillberry (2012)



$$Y_1 = 100$$

$$Y_2 = 200$$

$$Y_3 = 300$$

$$d_{12} = 0.5$$

$$d_{23} = 1/3$$

- the rest of the coefs are zeros...

$$Y_1 = F_1 + d_{11}Y_1 + d_{12}Y_2 + d_{13}Y_3$$

$$Y_2 = F_2 + d_{21}Y_1 + d_{22}Y_2 + d_{23}Y_3$$

$$Y_3 = F_3 + d_{31}Y_1 + d_{32}Y_2 + d_{33}Y_3$$

$d_{12}$  - 1块钱industry 2的产出, 需要industry 1几块钱的产出?

- 这里,  $d_{12} = 0.5$

$$Y_1 = 100 = 0 + 0 \times 100 + 0.5 \times 200 + 0 \times 300$$

$$Y_2 = 200 = 100 + 0 \times 100 + 0 \times 200 + 1/3 \times 300$$

$$Y_3 = 300 = 300 + 0 \times 100 + 0 \times 200 + 0 \times 300$$

According to Equation (2) in the paper

$$Y_i = F_i + d_{i1}F_1 + d_{i2}F_2 + d_{i3}F_3$$

$$\begin{aligned} (j=1, k=1,2,3) &+ d_{i1}d_{11}F_1 + d_{i2}d_{21}F_1 + d_{i3}d_{31}F_1 \\ (j=2, k=1,2,3) &+ d_{i1}d_{12}F_2 + d_{i2}d_{22}F_2 + d_{i3}d_{32}F_2 \\ (j=3, k=1,2,3) &+ d_{i1}d_{13}F_3 + d_{i2}d_{23}F_3 + d_{i3}d_{33}F_3 \end{aligned}$$

$$Y_1 = F_1 + d_{11}F_1 + d_{12}F_2 + d_{13}F_3$$

$$\begin{aligned} &+ d_{11}d_{11}F_1 + d_{12}d_{21}F_1 + d_{13}d_{31}F_1 \\ &+ d_{11}d_{12}F_2 + d_{12}d_{22}F_2 + d_{13}d_{32}F_2 \\ &+ d_{11}d_{13}F_3 + d_{12}d_{23}F_3 + d_{13}d_{33}F_3 \end{aligned}$$

$$Y_1 = 0 + 0 \times 0 + 0.5 \times 100 + 0 \times 300$$

$$\begin{aligned} &+ 0 \times 0 \times 0 + 0.5 \times 0 \times 0 + 0 \times 0 \times 0 \\ &+ 0 \times 0 \times 100 + 0.5 \times 0 \times 100 + 0 \times 0 \times 100 \\ &+ 0 \times 0 \times 300 + 0.5 \times 1/3 \times 300 + 0 \times 0 \times 300 = 100 \end{aligned}$$

$$U_{1i} = 1 \times F_i / Y_i + 2 \times (d_{i1}F_1 + d_{i2}F_2 + d_{i3}F_3) / Y_i$$

$$\begin{aligned} &+ 3 \times (d_{i1}d_{11}F_1 + d_{i2}d_{21}F_1 + d_{i3}d_{31}F_1 \\ &+ d_{i1}d_{12}F_2 + d_{i2}d_{22}F_2 + d_{i3}d_{32}F_2 \\ &+ d_{i1}d_{13}F_3 + d_{i2}d_{23}F_3 + d_{i3}d_{33}F_3) / Y_i \end{aligned}$$

$$U_{11} = 1 \times F_1 / Y_1 + 2 \times (d_{11}F_1 + d_{12}F_2 + d_{13}F_3) / Y_1$$

$$\begin{aligned} &+ 3 \times (d_{11}d_{11}F_1 + d_{12}d_{21}F_1 + d_{13}d_{31}F_1 \\ &+ d_{11}d_{12}F_2 + d_{12}d_{22}F_2 + d_{13}d_{32}F_2 \\ &+ d_{11}d_{13}F_3 + d_{12}d_{23}F_3 + d_{13}d_{33}F_3) / Y_1 \end{aligned}$$

$$U_{11} = 1 \times 0 / 100 + 2 \times (0.5 \times 100) / 100$$

$$\begin{aligned} &+ 3 \times (0 \\ &+ 0 \\ &+ 50) / 100 = 2.5 \end{aligned}$$

So the "upstreamness" of industry 1 in this example is 2.5.

Similarly, the "upstreamness" of industry 2 (i.e.,  $U_{12}$ ) = 1.5, and  $U_{13} = \underline{1}$ .