CSCI 385: Written Part of Project 3

Due: Thursday, March 10, 2022

Several parts of the *Walk Through It* program rely on geometric calculations to perform ray casting and hidden line removal. Below we ask you to work out those calculations on paper so as to prepare for the coding of them. In doing so, I strongly encourage you to take a "coordinate-free" approach. This has you work to calculate with point and vector operations, rather than directly work with their coordinate data.

I'll share solutions of these next week.

This is a "low impact" assignment to get you thinking about the project right away. Make an attempt and hand in what you figure out.

1. Suppose we are given a center point C for projection, and a direction t into the scene that we are projecting. Suppose also we are given an additional direction u that represents a general upward direction for orienting the projection. You can assume that there is no nonzero scalar s for which $u = st$. The two vectors are not parallel.

   Give an orthonormal frame centered at C and with basis directions e_1, e_2, e_3. These correspond to the x, y, z directions for a coordinate system for specifying the projection. It should have the property that $u \cdot e_1 = 0$.

   In devising this, tell me whether your frame is left- or right-handed.

2. Imagine a plane $\mathcal{P}$ that contains the point $\mathcal{O} = C + e_3$ and has the normal e_3. Now imagine a scene point P where $(P - C) \cdot t \neq 0$ Compute coordinates x, y. These correspond to the coordinates where a projected point P' lives within the plane $\mathcal{P}$. In particular, we want that $P' = \mathcal{O} + xe_1 + ye_2$.

   For Program 3, this calculation gives us points P' to draw in the PDF based on objects' vertex locations P. We will also need to track the depth of points due to that projection. This is used for ray casting to determine whether points are hidden by faces, or are visible. Compute the depth information d for the projection of point P.

3. Consider two non-parallel line segments $\overline{P_1 P_2}$ and $\overline{Q_1 Q_2}$ that sit in Euclidean 2-space. Assume they intesect, and not at their endpoints. Find their intersection point R. In doing so, you can assume for any vector v that $v^{\perp}$ is a vector perpendicular to v and whose direction is rotated 90 degrees (counter-clockwise) from the direction of v.

   What are the conditions that these two segments intersect in this way?

4. Let ray r emanate from a point C in a (unit) direction d in Euclidean 3-space. Let $Q_1, Q_2, and Q_3$ be the vertex locations of a trianglular facet $\mathcal{T}$. Suppose r intersects $\mathcal{T}$. Find the point of intersection Q.

   What are the conditions that r intersect $\mathcal{T}$ in general?