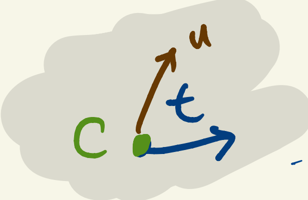
Problem 1: camera frame



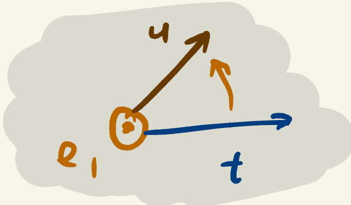
Let c be a point and T be a direction into the scene. Let u be a direction suggesting "up".

- Let $\hat{u} = \dfrac{u}{\|u\|}$ and $\hat{t} = \dfrac{t}{\|t\|}$

These are each unit vectors in the same direction of the given vector.

- Compute a vector perpendicular to both $\hat{t}$ and $\hat{u}$. This unit vector gives us the x-axis direction:

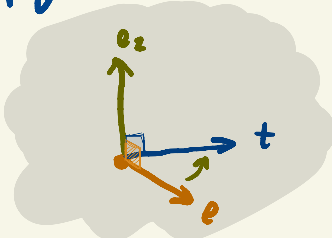
$$e_1 = \hat{t} \times \hat{u}$$



This is shown pointing out of the page above.

- If we then compute

$$e_2 = e_1 \times \hat{t}$$



we get the direction of the y axis as shown above. Notice that we've tilted e_1 slightly to depict that cross product. e_2 is perpendicular to both e_1 and t.

- Finally, we can set $e_3 = e_1 \times e_2$. This should just be

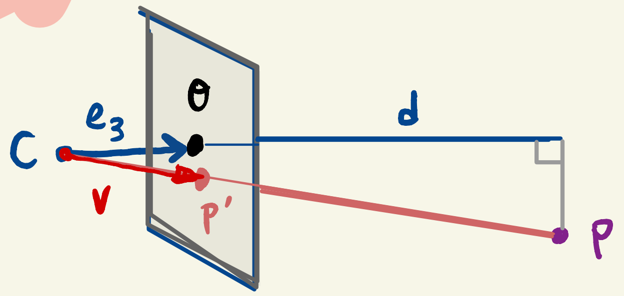
$$e_3 = -\hat{t}$$

And yields a right-handed frame with +z pointing behind the camera. An alternative is to set

$$e_3 = \hat{t}$$

This yields a left-handed frame with +z pointing into the scene.

Problem 2: projection



- To compute the depth, we calculate

$$d = (p - c) \cdot e_3$$

This is just the length of the projection of $\overline{CP}$ onto e_3.

- The projection of p onto that plane is the point p' as shown in the picture. Let $v = p' - C$

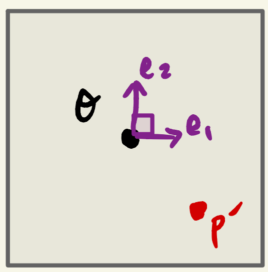
We can compute it with

$$v := \frac{p - C}{d}$$

and so

$$p' := C + \frac{p - C}{d}$$

- Consider the sheet of paper as shown below:

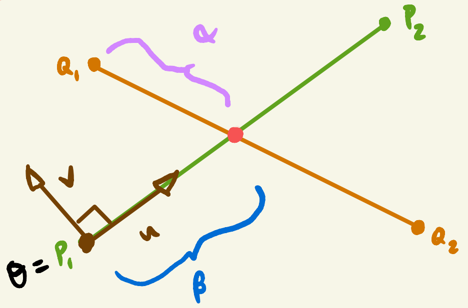


We compute the paper coordinates of p' with

$$x := (p' - \theta) \cdot e_1$$

$$y := (p' - \theta) \cdot e_2$$

Problem 3: segment intersection in 2-D



- let $u = \dfrac{P_2 - P_1}{\|P_2 - P_1\|}$ and $v = u^{\perp}$.

  These directions give us an orthonormal frame along with choice of an origin $\Theta := P_1$.

- We can compute

$$x_1 = (Q_1 - \Theta)\cdot u \qquad y_1 = (Q_1 - \Theta)\cdot v$$

$$x_2 = (Q_2 - \Theta)\cdot u \qquad y_2 = (Q_2 - \Theta)\cdot v$$

- What we need is for

$$\operatorname{sgn}(y_1) \neq \operatorname{sgn}(y_2)$$

  because this places Q_1 and Q_2 on opposite sides of $\overline{P_1P_2}$.

- Consider the ratio

$$\alpha := \frac{y_1}{y_1 - y_2}$$

  It can be shown that the intersection point $I = \overline{Q_1Q_2} \cap \overleftrightarrow{P_1P_2}$ is given by

$$I = (1-\alpha)\, Q_1 + \alpha\, Q_2$$

- We need to check whether I falls on $\overline{P_1P_2}$. One way to do that is to compute

$$\beta = \frac{I - P_1}{\|P_2 - P_1\|}$$

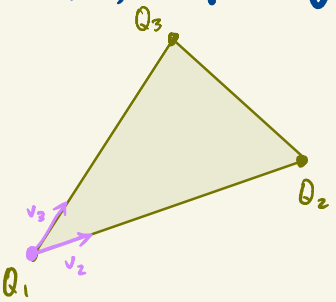
  If $0 < \beta < 1$ then the intersection point $I = (1-\beta)P_1 + \beta P_2$ sits on $\overline{P_1P_2}$

Problem 4: ray-triangle intersection

We have a ray from R along direction d meeting a plane containing $\Delta Q_1 Q_2 Q_3$.

- We can compute $v_2 = \dfrac{Q_2 - Q_1}{\|Q_2 - Q_1\|}$ $\quad v_3 = \dfrac{Q_3 - Q_1}{\|Q_3 - Q_1\|}$

These two unit vectors give us directions from Q_1 to points Q_2 and Q_3, respectively. These are depicted below:

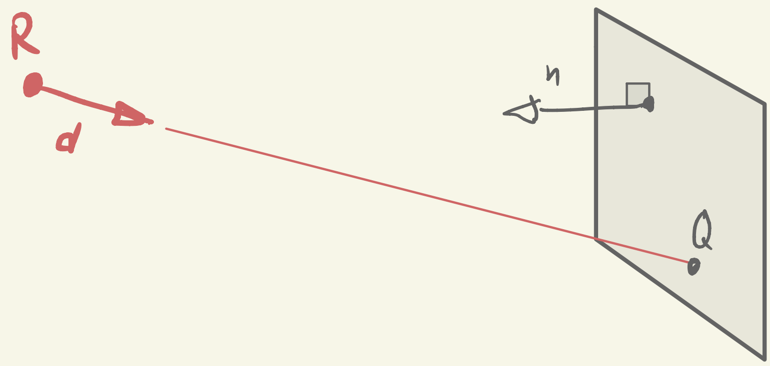


} within the plane containing the triangle's points.

- We can then compute the direction

$$n = v_2 \times v_3$$

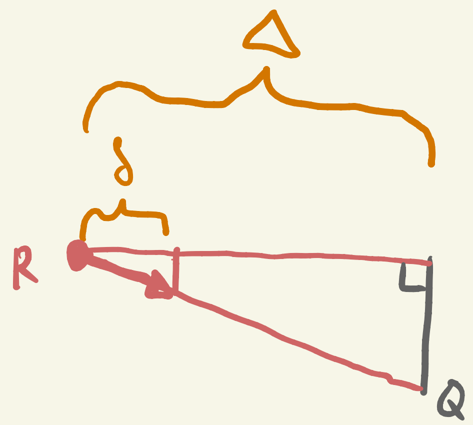
This is a normal to the plane containing $\Delta Q_1 Q_2 Q_3$



} <u>Note:</u> the normal might face the opposite direction.

- We can now compute

$$\Delta = (Q_1 - R) \cdot n$$
$$\delta = d \cdot n$$



This gives us

$$Q = R + \frac{\Delta}{\delta} d$$

by similar triangles

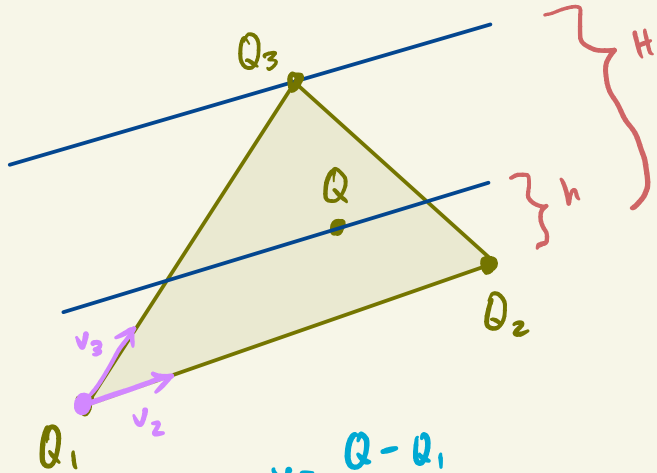
4. ray-triangle intersection, continued.

We've now placed a Q on the plane containing $\triangle Q_1 Q_2 Q_3$

let's now figure out $\alpha_1, \alpha_2, \alpha_3$ s.t.

$$Q = \alpha_1 Q_1 + \alpha_2 Q_2 + \alpha_3 Q_3$$

The trick is to determine the height ratio h/H as shown below, and set $\alpha_3 = \frac{h}{H}$



- We can compute $v = \frac{Q - Q_1}{\|Q - Q_1\|}$

  This gives us a third direction, from Q_1 to Q, which (when Q lies in the triangle) is swept through when we scan around Q_1 from v_2 to v_3.
- We can then compute the heights off $\overline{Q_1 Q_2}$ with

$$h = \| v_2 \times v \| \qquad H = \| v_2 \times v_3 \|$$

  We want $0 \leq h \leq H$ which places Q within the "cone" with corner Q_1 and swept from v_2 to v_3.
- Repeating this with roles of Q_2 & Q_3 reversed yields α_2.

  and then we just set $\alpha_1 = 1 - \alpha_2 - \alpha_3$.