Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
B		2209826

A Water Allocation Plan Between Dams and States

Summary

Five states (AZ, CA, WY, NM, and CO) and two dams (the Glen Canyon dam and the Hoover dam) located in the Colorado River basin are facing a crisis of possible undersupply of water resources in the face of increasing drought. In order to solve this crisis, our team firstly obtained the allocation ratio of general water use and power generation water use by entropy weighting method, based on which an optimal water allocation model was built to obtain the specific optimal allocation scheme. At the same time, we dynamically monitored the sustainability of reservoir operation to guarantee the frequency of this model under the premise of meeting the demand.

First, we construct an **entropy-weighting-based model (EWN)** for the balance of competing interests for general water use and power generation based on the data collected for each state, with reference to the demand establishment criteria, and obtained the weights and proportionality coefficients for general water use and power generation for each state, which led to the criteria we are using to resolve competing interests.

Then, we create an **optimization model for water allocation** by combining the criteria solved in Model 1 and collecting data on the inflow and storage of the two lakes. The objective function is constructed in terms of meeting the needs of each state. Constraints were constructed by the specific conditions of the two reservoirs (including storage volume and monthly inflow). The model was solved by a **particle swarm optimization (PSO) algorithm**, and the final supply of each dam to each state for general water use and power generation was obtained. In addition, the sustainable operation of the reservoirs was dynamically monitored (keeping the reservoir level above the minimum level) in order to allow the two reservoirs to sustainably supply the five states with water for general use and hydroelectric power generation. In summary we obtained the sustainable optimal water allocation scheme.

After that, considering that water resources are often in short supply in arid regions, we propose solutions in terms of both increase source and flow reduction. In particular, we build a **directional weighted network diagram for optimal resource** scheduling based on the distance and transportation difficulty between each state and the dams to find the optimal path between the dams and the states to reduce the consumption of resources on the road, thus achieving the purpose of resource saving.

Finally, we perform a sensitivity test on our model in terms of changing the demand, improving renewable energy technologies, and incorporating water and energy conservation measures. The results show the robustness and practicality of our model.

Keywords: Dam, Water Allocation, EWM, the Optimization Model, PSO, Network Diagram,Floyd Algorithm

Contents

1 Introduction

1.1 Background and Problem Statement

Dams, as an architecture to intercept river flow to raise the water level and regulate runoff, can store water to form reservoirs for water supply, hydropower generation, irrigation, flood control, fishery farming, recreation, etc.. It is important for the production and life of modern society. For Glen Canyon Dam (Lake Powell) and Hoover Dam (Lake Mead), which are located in the more arid western region, the most noteworthy are their functions of water supply and hydroelectric power generation. In the current Colorado River Basin (the basin in which the two dams are located), we are faced with the reality of a growing drought, both internally in terms of the conflict between water resources for hydroelectric power generation and general water use, and externally in terms of water allocation between the two reservoirs and the five states. We should address the following issues:

• We have to do a water allocation plan to meet the needs of the five states.

• Establish a set of criteria to balance the conflict between water for hydropower generation and general water use.

• Our response when water is not enough.

• The impact of the three changes on our model is analyzed to test the sensitivity of our model.

1.2 Our work

The work we have done in this paper is shown as follows:

First, we made model assumptions and symbolic interpretations to prepare for the modeling.

Then, we used the entropy weighting method EWN to create weights for general water use and water use for power generation in each state as a criterion to balance the contradiction between the two.

After that, we modeled a dynamic optimal allocation of water resources based on the criteria established above to ensure that the demand for general water use and hydroelectric power generation is met continuously in the five states under the current drought conditions, and to protect the sustainable operation of the two reservoirs as much as possible.

Finally, we performed a sensitivity analysis of the model under the condition of varying the fixed water demand, while discussing the effects of the latter two conditions on the model.

2 Preparation of the Models

2.1 Assumptions and Justification

We made the following assumptions to help us with our models. Our subsequent analysis is based on these assumptions.

• **Reservoir levels do not include inflows for the month**: Although the states draw their water from the water in the reservoirs, the reservoir levels examined do not include the inflow for the month because we equivalently consider the inflow to the reservoirs to be utilized first, given the consideration of maintaining the reservoir levels.

• **Disregard the difference between electricity consumption and electricity generation**: Because in practice power generation is not fully applied to all aspects, but because the power generation under examination is large, we choose to ignore this part.

• **Inflows from the downstream reservoirs do not include water for power generation from the upstream reservoirs**: Because there are several rivers that flow into Mead Reservoir, and the amount of water used for power generation in Powell Reservoir is small compared to the flow in these rivers, we assume that the downstream reservoir inflows obtained do not include the water used for power generation in the upstream reservoir.

• **The cost of transporting resources from state to state is lower than the cost of transporting them between dams and states**: Considering that the actual use of resources is mainly within the states, it can be assumed that the states have better resource dispatching facilities, i.e., the resources are cheaper to transport.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Parameter Settings

parameter	description
x_i	Sample data
Z	Normalized matrix
e_i	The information entropy
d_i	The weight of index I
η_i	State I's ratio of general to electricity use
a_i	Demand for general water in state I
b_i	Demand for hydroelectric power in state I
$a_i^{'}$	The supply for general water in state I
$b_i^{'}$	The supply for hydroelectric power in state I
Δi	Difference between incoming flow and demand
P	Water level of Lake Powell
M	Water level of Lake Mead
W_{ij}	The weight of the edge connecting nodes i and j

3 Model Overview

To obtain an optimal water allocation plan, we model both the internal allocation of water resources (trade-off between general usage and power generation water) and the external allocation (the supply of general and power generation water from each dam to each state).

We first consider the internal allocation of water resources, i.e., the resolution of conflicts of interest between water for general use and water for power generation. For this purpose, we collect data on the demand for water for general use and water for power generation in each state, and developed a model for balancing the competing relationship between water for general use and water for power generation based on the entropy weight method.

We then develop an optimal allocation model for water resources, combining the results of the first model with the specific conditions of the reservoirs to obtain the supply of each dam to each state for general water use and power generation, thus obtaining an optimal allocation scheme for water resources. In addition, considering the sustainability of reservoir operation, we use the inflow rate as the monthly supply and reservoir storage as a supplement when the supply exceeds the demand, thus enabling dynamic monitoring of the sustainability of reservoir operation and reducing the risk of reservoir drying up.

In response to the short supply of water resources, we consider both open-source and cost-cutting aspects. In particular, we establish a resource optimal scheduling network model to optimize the transportation routes and reduce the loss of resources in the road to achieve the purpose of cutting costs.

Finally, we conduct a sensitivity test on our model in terms of changing the demand, improving renewable energy technologies, and incorporating water and energy saving measures. The results show the stability and practicality of our model.

4 Balance the competition interests of different water availabilities based on EWN

4.1 Model Establishment

We believe that the water allocation between general water use and power generation should be determined by the proportion of general water use and power generation water needed by each of the five states. So we collected and compiled data on the amount of water used for general water use (from dams) and for power generation in each state.

We used the entropy weighting method because it allows us to use the given data to objectively assign weights to general water usage and water used for power generation to find their scale factors. Using this method, the data is able to tell us the weights of the two, rather than being artificially determined, which effectively ensures the objectivity of our model.

▸ Creat a positive matrix

After consideration, we consider both the general water useage and power generation indicators (denoted by a,b, respectively) as intermediate indicators, and we take the average of the sample data (i.e., the corresponding general water use and power generation data for each state) to represent this optimal value x_{best}:

Table 2: Interpretation of intermediate type indicators

Indicator Name	Indicator Features	Indicator code
Intermediate indicators	The closer to a certain value, the better	a,b

$$M = max\{|x_i - x_{best}|\} \tag{1}$$

Get the elements of each matrix:

$$x_i = 1 - \frac{|x_i - x_{best}|}{M} \tag{2}$$

Finally we get an positive matrix:

$$X(4 \times 2) \tag{3}$$

▸ Normalization of the positive matrix

Let the normalized matrix be Z and each element Z_{ij}in Z:

$$Z_{ij} = \frac{x_{ij}}{\sqrt{\sum_1^n x_{ij}^2}} \tag{4}$$

thus obtaining a non-negative matrix: Z

Based on this normalized matrix, we can obtain the probability matrix P. Calculate each element P_{ij} of the probability matrix P:

$$P_{ij} = \frac{Z_{ij}}{\sum_1^n Z_{ij}} \tag{5}$$

A probability matrix is finally obtained: P

▶ Calculate the entropy weight of each index

For the jth indicator, its information entropy e_j is calculated as:

$$e_j = -\frac{\sum_1^n P_{ij} \times \ln P_{ij}}{\ln n}, (j = 1, 2) \tag{6}$$

When $p(x1) = p(x2) = p(x3) = ... = p(xn) = 1/n$, $H(x)$ takes the maximum value, at this time $H(x) = \ln n$,here divide by $\ln n$, can make the value of information entropy always lies in the interval of [0,1].

The entropy values are then converted into weight values.

$$d_j = \frac{1 - e_j}{m - \sum_{j=1}^m e_{ij}}, (j = 1, 2; m = 2) \tag{7}$$

The entropy weights of the two indicators for general water use and water use for power generation can then be obtained as d_1, d_2

In this way we were able to determine the proportional coefficients η of water supply from dams for general water use and power generation in the five states.

$$\eta = \frac{d_2}{d_1} \tag{8}$$

In summary, our conflicting interests in resolving general and generation water use are determined by the different general and generation water needs of the five states, ultimately yielding a ratio factor for each state's respective general and generation water use to be used as the standard for resolving conflicting interests.

4.2 Model solving and results

We used the entropy weighting method(EWN) to calculate the entropy weights for solving for general water use and water use for power generation in the five states to obtain the proportionality factors for each state.

In terms of data, we compiled four quarters of general and generation water demand (converted from total industrial, agricultural, and residential electricity use) for each state from the U.S. Energy Information Administration (EIA) and the United States Bureau of Reclamation (USBR) (converted from total industrial, agricultural, and residential electricity use) demand for each state. The table 3 shows.

Where a denotes general water use and b denotes water for power generation.

Table 3: Four quarters of general and power generation water demand per state

Indicators, season	summer	autumn	spring	winter
a_CA	20	23	17	16
b_CA	0.98	1.21	0.74	0.69
a_AZ	7	9	6	4
b_AZ	0.38	0.52	0.26	0.21
a_WY	6	8	7	3
b_WY	0.28	0.45	0.34	0.19
a_NM	9	11	8	5
b_NM	0.54	0.61	0.42	0.24
a_CO	7	10	7	4
b_CO	0.36	0.59	0.35	0.26

Based on the entropy weighting method and the given data, we obtained the entropy weights and scaling factors for general water use and water use for power generation for each state.

Table 4: Entropy weights and scaling factors for each state

	d_1	d_2	η
AZ	0.6739	0.3261	0.4839
CA	0.5943	0.4057	0.6857
WY	0.7213	0.2787	0.3864
NM	0.8156	0.1844	0.2261
CO	0.6539	0.3461	0.5293

5 optimal allocation of water use between dams and states

5.1 Model Establishment

Given that the water levels of the two lakes are known, how much water is pumped from each lake, whether the water pumped is used for hydroelectric power generation or general water use, how it is distributed to the five states, and how to continuously meet the water and electricity needs of the five states becomes a problem we need to solve.

Therefore, we modeled the optimal allocation of water use (both for power generation and general use) between the two dams and the five states to analyze how the dams and states could achieve a sustainable supply-demand balance under different conditions with known water levels and demands.

► The Objective

We use the symbol m to indicate the degree to which the dam meets the state's general usage and power generation water needs. When the value of m is larger, the water allocation between the dam and the state is more optimal. And the problem about the proportion of each state's water demand for general water use and power generation has been obtained in the entropy method model above. Therefore, we can establish the following equation to obtain the optimal allocation index m_i for state i:

$$m_i = \frac{a_i}{a_i+b_i}({a_{iu}}' + {a_{id}}') + \eta_i \frac{b_i}{a_i+b_i}({b_{iu}}' + {b_{id}}')$$

(i= CA, AZ,WY, NM, CO) (9)

where m_i is the optimal allocation index, a_i, a_i are the demand for general water and water for power generation in state i, respectively; $a_{iu}{}'$, $a_{id}{}'$ are the supply of general water to state i from the two lakes, respectively; $b_{iu}{}'$, $b_{id}{}'$ are the supply of water for power generation to state i from the two dams, respectively.

However, what we need is to maximize the optimal allocation coefficient for the five states combined, and therefore establish the objective function.

$$max(m_{AZ} + m_{CA} + m_{WY} + m_{NM} + m_{CO}) \tag{10}$$

▸ The Constraints

First of all, we believe that the water from the two lakes is fully capable of meeting the demand of the five states for a period of time without considering the inflow, but in order to maintain the sustainability of the water supply from the dams, we will give priority to consuming the inflow of the lake each month, and will only withdraw water from the lake as a supplement when the monthly inflow cannot meet the consumed water (the lake without considering the inflow of the month).

The total water supply from Lake Powell (upstream) to the five states should be less than its inflow, as shown by the following equation.

$$s.t. \sum_i (a_{iu}{}' + b_{iu}{}') \le p \tag{11}$$

where $a_{iu}{}'$, $b_{iu}{}'$ are the supply of Lake Powell to state i for general water use and power generation, respectively, and p is the inflow to Lake Powell for the month.

Since Lake Mead is located downstream, its available water should include the amount of water used by Lake Powell for power generation in addition to the current month's inflow to the lake, because water for power generation does not have a one-time consumption characteristic unlike general water, and water from upstream for power generation will flow downstream.

$$\sum_i (a_{id}{}' + b_{id}{}') \le m + \sum_i b_{iu}{}' \tag{12}$$

where $a_{id}{}'$, $b_{id}{}'$ are the supply of Lake Mead to state i for general water use and power generation, respectively, and m is the inflow to Lake Mead for the month is the total amount of water used for power generation in Lake Powell.

We also believe that the supply of hydroelectric power and general water from each lake to each state should not exceed the demand for hydroelectric power and general water in that state, otherwise there will be an imbalance between supply and demand.

$$a_{iu}{}', a_{id}{}' \le a_i, b_{iu}{}', b_{id}{}' \le b_i \tag{13}$$

In particular, a_i, b_i are the demand for general water and water for power generation in state i, respectively; $a_{iu}{}'$, $a_{id}{}'$ are the supply of general water to state i from the two lakes, respectively; $b_{iu}{}'$, $b_{id}{}'$ are the supply of water for power generation to state i from the two dams, respectively.

In summary, to maximize the water supply allocated by the dams to the five states to meet the needs of the five states while satisfying the constraints, we determined the optimal allocation model for water use between the dams and the states as follows.

$$max(m_{AZ}+m_{CA}+m_{WY}+m_{NM}+m_{CO}) \tag{14}$$

$$s.t.\begin{cases}\sum_i({a_{iu}}'+{b_{iu}}')\leq p\\ \sum_i({a_{id}}'+{b_{id}}')\leq m+\sum_i {b_{iu}}'\\ {a_{iu}}',{a_{id}}'\leq a_i,{b_{iu}}',{b_{id}}'\leq b_i\end{cases} \tag{15}$$

where m_i is the optimal allocation index, a_i, b_i are the demand for general water and water for power generation in state i, respectively; ${a_{iu}}'$, ${a_{id}}'$ are the supply of general water to state i from the two lakes, respectively; ${b_{iu}}'$, ${b_{id}}'$ are the supply of water for power generation to state i from the two dams, respectively; p,m are the inflows to Lake Powell and Lake Mead in that month.

As mentioned earlier, the needs of the five states are first met by the inflow to the lake each month, and when the inflow does not fully meet the needs, we pump the lake to fill the missing water.

We can obtain the total amount of water missing as shown in the following equation.

$$\Delta i=(a_i+b_i)-({a_i}'+{b_i}') \tag{16}$$

where Δi represents the difference between the incoming flow and the demand.

Then we get the amount of water that needs to be taken from each of the two dams separately for each state based on the amount of water stored at a certain water level, so we create the following equation.

$$\Delta i_{Ps}=\frac{P_s(t)}{P_s(t)+M_s(t)}\Delta i \tag{17}$$

$$\Delta i_{Ms}=\frac{M_s(t)}{P_s(t)+M_s(t)}\Delta i \tag{18}$$

Where, Δi_{Ps}, Δi_{Ms} represents the amount of water withdrawn from Lake Powell and Lake Mead by state i. $P_s(t)$, $M_s(t)$ is the amount of water stored in Lake Powell and Lake Mead at P,M level at month t.

5.2 Model solving and results

▶ Calculation–Particle Swarm optimization(PSO)

For the optimal allocation model of water consumption, we used the Particle Swarm optimization algorithm (PSO) to solve, which as an iteration-based optimization algorithm, has a fast convergence rate and is easier to implement. The following is the flowchart of our algorithm.

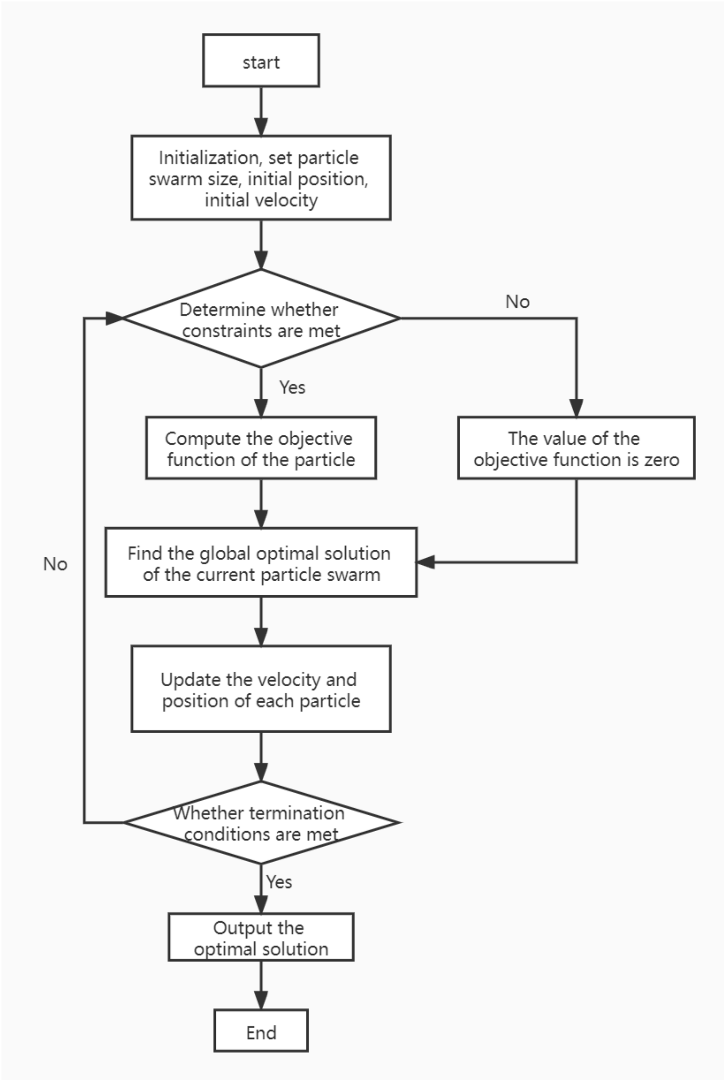

Figure 1: Flow Chart of PSO

In the algorithm, the speed represents the moving speed and the position represents the moving direction, and we obtain the global optimal solution by comparing the corresponding objective function values of the particles. Considering the constraints, we add a penalty function to exclude the particles that do not satisfy the constraints by making their objective function 0.

Finally, we update the velocity and position of the particle with the following equation.

$$V_{id} =_{id} + C_1 random(0,1)(P_{id} + X_{id}) + C_1 random(0,1)(P_{gd} + X_{id}) \tag{19}$$

$$X_{id} = X_{id} + V_{id} \tag{20}$$

where ω is called the inertia factor, C_1 and C_2 is called the acceleration constant, and is generally taken as $C_1 = C_2 \in [0,4]$. Random(0,1) denotes the random number on the interval [0,1]. P_{id} denotes the d-th dimension of the individual extreme value of the i-th variable. P_{gd}denotes the d-th dimension of the global optimal solution.

During the run of the model, based on the convergence of the iterations we plotted the following graph.

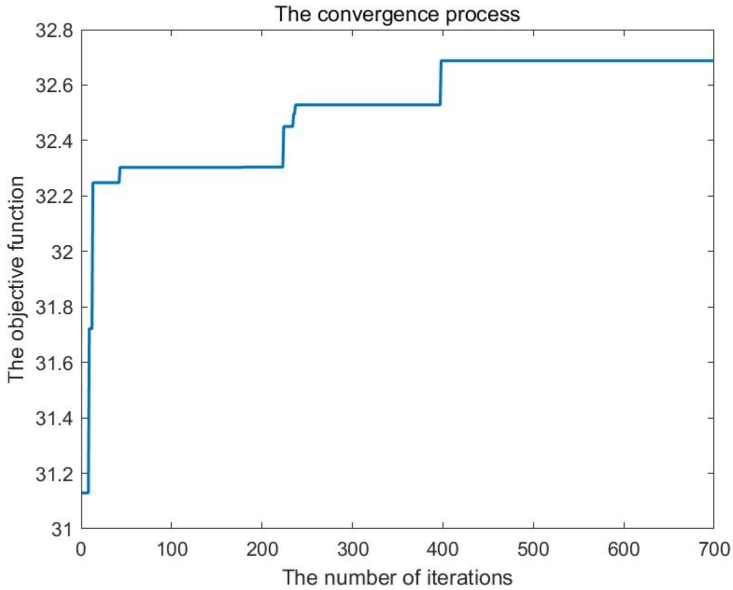

Figure 2: iterative process of PSO

According to the line graph, we see that the objective function stabilizes when the number of iterations reaches about 400, and the value obtained at this time can be approximated as the global optimal solution. The speed at which the iterations stabilize varies with the initial speed, and the accuracy of the iterations also has an impact. When the speed is fast, the accuracy decreases, and vice versa.

▸ Optimal water allocation

First we collected 12 months of inflows for the two lakes for 2021. Then the optimal allocation scheme for January was then obtained by combining the water demands of the five states and using the optimal water allocation model between dams and states, as follows:

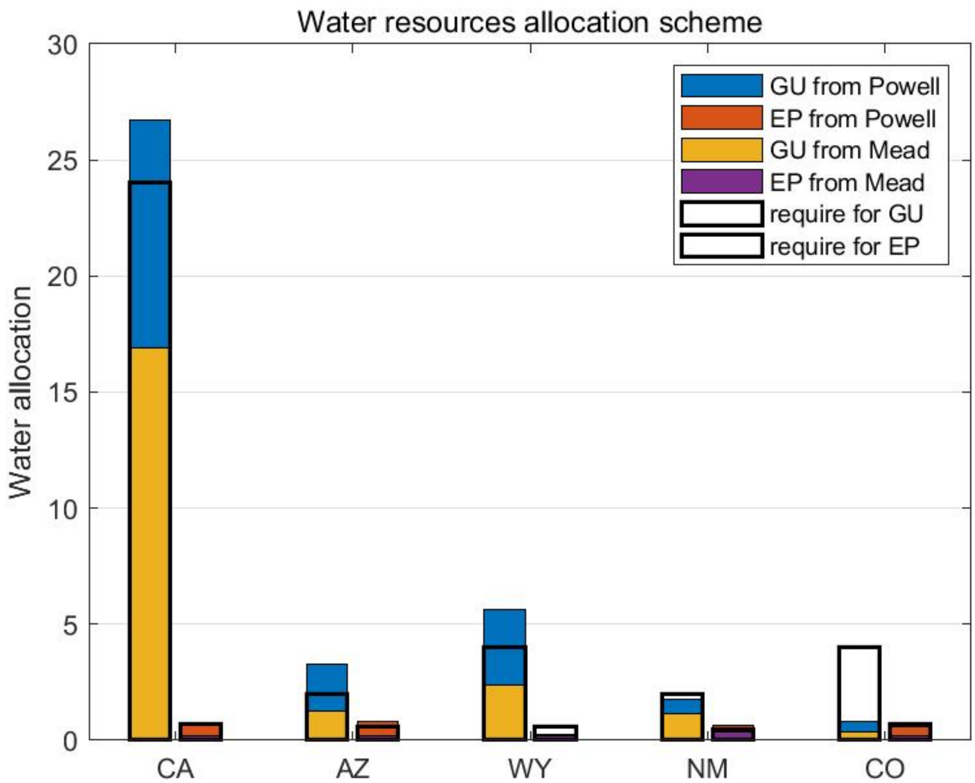

Figure 3: Water Resources Allocation Program

Where GU stands for general water use and EP stands for water for hydropower generation.

As we can see, for most cases the water demand is satisfied, but because the model is built with a focus on maximizing the benefits, in the actual allocation situation, there is a tight allocation of water resources in some states and an over-allocation in some states. At this time, we can get the final optimal solution through appropriate manual adjustment.

▶ Time and amount of water required to meet demand

In the model, we consider that when the inflow of water in the reservoir is not enough to supply water to the five states, the additional demand is to supplement it with the water stored in the reservoir, and at this time, the water level in the reservoir will change over time. Based on the relevant data, and combined with the current situation, we simulated the change of the water level in the reservoir, and the results are shown in the following figure

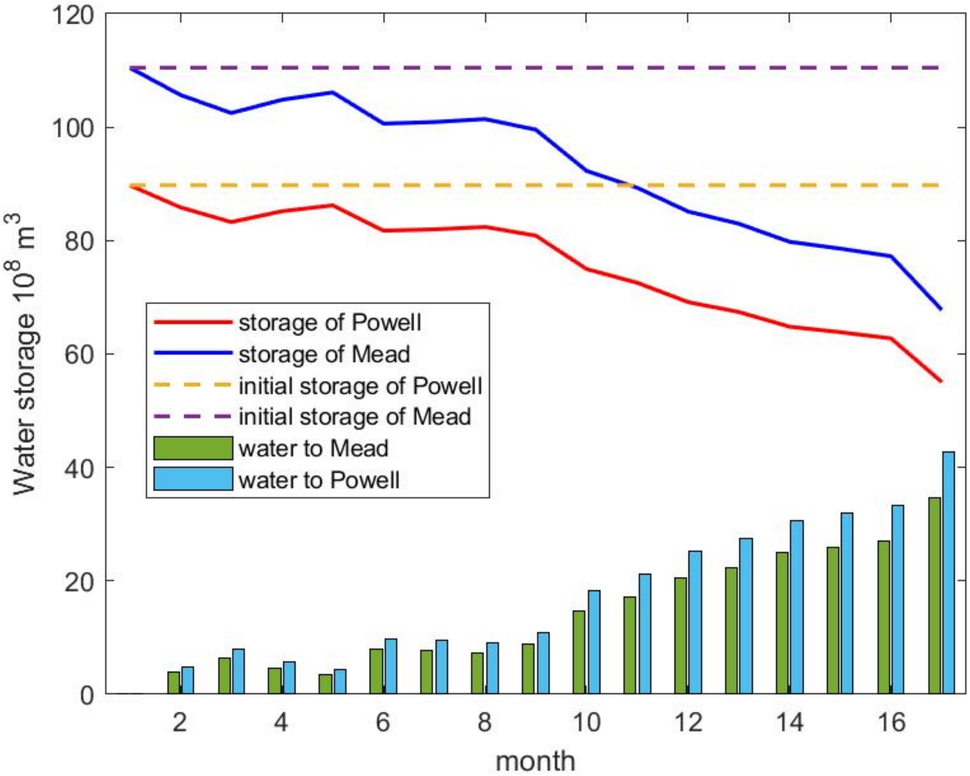

Figure 4: Water level changes in reservoirs

As we can see, with the increase of water flow, the water in the reservoir will gradually return to the original level after about 5 months. However, considering the current situation, both Mead and Powell reservoirs are in the dry season, so the water storage in the reservoirs is still decreasing in the long term, and in order to solve the problem of insufficient water supply in the area, we need to transfer water from other places. In order to maintain the sustainable development of fisheries and tourism industries in the reservoirs, we will maintain the initial water level as the target for water replenishment, at this time, the amount of water that should be replenished each month is shown in the bar graph.

6 Multi-approach to water shortage problem

In order to solve the problem of water shortage, we have proposed solutions mainly from two aspects: source and flow reduction, without considering climate change.

6.1 Throttling - Resource Optimal Scheduling Network Model

In terms of flow saving, we model the optimization of the transportation routes of water resources. In the solution of the first question, we obtained the optimal water resource allocation scheme, i.e., destination, origin and transport volume. However, we neither considered the geographical location of the dam and the state, nor determined the specific transportation route. In fact, the loss of water and power resources during transportation is not negligible. If the transportation route is too long, it will not only cause more resource loss but also increase the transportation cost. Therefore, we will create a resource optimal scheduling network model to solve this problem.

6.1.1 Model Establishment

We created a directed graph of resource transportation routes using five states and two dams as nodes, with the distance between the states and the dams and the difficulty of transportation as the weights of the edges.

▶ **Node**

We first assume that the nodes V for the five states and the two dams are as follows:

$$V = \{v_{CA}, v_{AZ}, v_{WY}, v_{NM}, v_{CO}, v_P, v_M\} \tag{21}$$

▶ **Weighted directed graph**

We build a weighted directed graph based on the known weights of the nodes and the edges (the edges are determined by the distance between the nodes and the transport difficulty)

$$G = (V, E) \tag{22}$$

where V denotes a node and E denotes an edge.

▶ **The shortest route from the dam to the states**

We use $w_i j$ to denote the weights of the edges $E(v_i, v_j)$ and $E(v_j, v_i)$ of node i and node j. Thus we obtain the shortest circuit between each node as follows.

$$W_P = min_{pP} \tag{23}$$

P is the distance from v_j to v_i

6.1.2 Model solving and results

Since we need the shortest circuit of any two points in the network graph, the Warshall-Floyd algorithm is used in the solution. Suppose when v_i and v_j are not adjacent,

$$W_{ij} = +\infty \tag{24}$$

Warshall-Floyd algorithm flow chart:

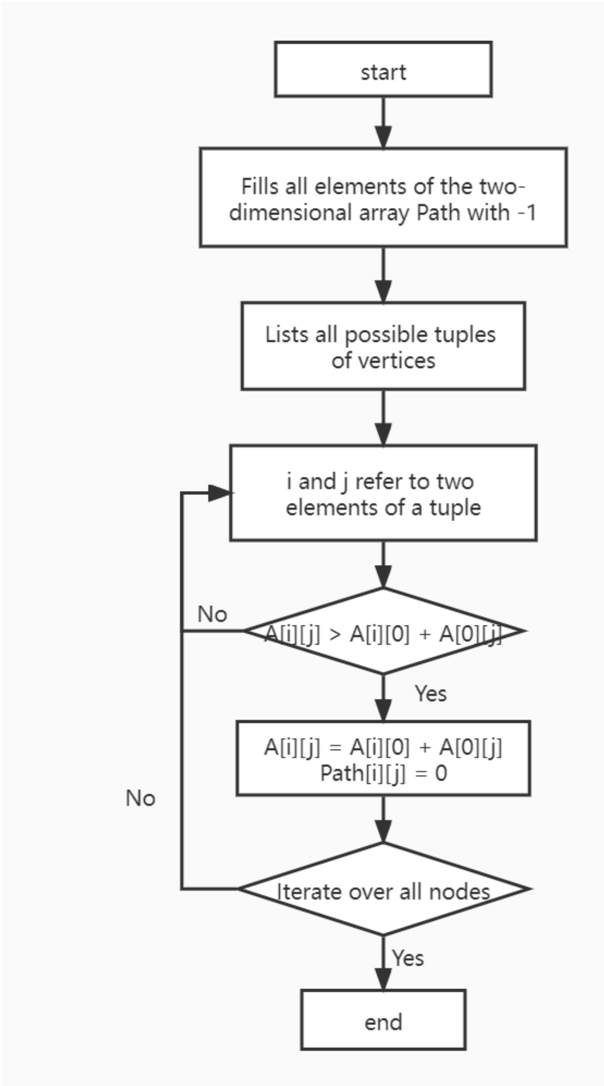

Figure 5: Flow Chart of PSO

Because the weights of the edges are positively correlated with the distance between the dams and the states, we obtained the distance relationships between the two dams and the five states based on Google Maps as shown in the following table:

Table 5: Table of distance relationship between dams and states

	AZ	**CA**	**Wy**	**NM**	**CO**	**Powell**	**Mead**
AZ	0	756	963	503	822	279	16
CA	756	0	1087	1141	1166	830	536
WY	963	1087	0	492	114	699	850
NM	503	1141	492	0	392	438	672
C0	822	1166	114	392	0	577	833
Powell	279	830	699	438	577	0	309
Mead	270	536	850	672	833	309	0

From the above distance data between each point, and taking into account that the transportation difficulty between states is less than that between dams and states, the final weights of the sides can be obtained, and by substituting the weights into the calculation, the shortest path of the resource transportation route is obtained as shown in the following table:

Table 6: The shortest path from the dam to each state

	AZ	**CA**	**Wy**	**NM**	**CO**
Powell	1116	1872	2052	1619	1938
Mead	1080	1836	2016	1583	1902

As an example, we can plot the shortest route from Lake Powell to California, as shown below:

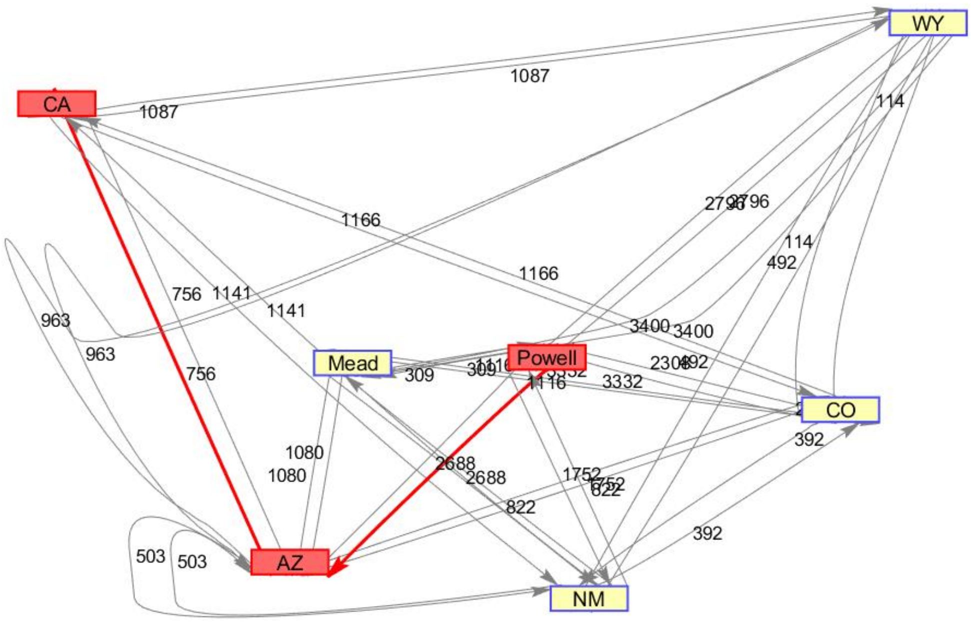

Figure 6: Shortest route from Lake Powell to CA

From the above figure, we can see that the shortest transportation route from Lake Powell to California will pass through Arizona. Similarly we can get the shortest route from any dam to any state.

6.2 Open Source

Considering that general water use is more dependent on reservoirs than electricity is on reservoirs, we believe that the share of other types of power generation, such as thermal power generation, solar power generation, and wind power generation, can be promoted. In the case of California, for example, the climate analysis shows that the state has sufficient sunshine conditions, so solar power generation can be promoted.

By doing so, we can ensure the supply of electricity and make more water available in reservoirs for agriculture, industry, and domestic use. In this way, we can alleviate the problem of insufficient power supply due to water shortage.

7 Sensitivity test - our model under different conditions

7.1 The impact of changes in water and electricity demand on the results

Under condition one, we know that the state's demand for water and electricity is no longer a fixed value, but a variable value. We then divided the variable demand into three cases: below the initial fixed demand, equal to the fixed demand, and above the fixed demand. Based on these three cases, we obtained the new resource allocation by changing the fixed value of demand in solving problem 1 and then reallocating it and presenting it in the form of a graph. The results are as follows.

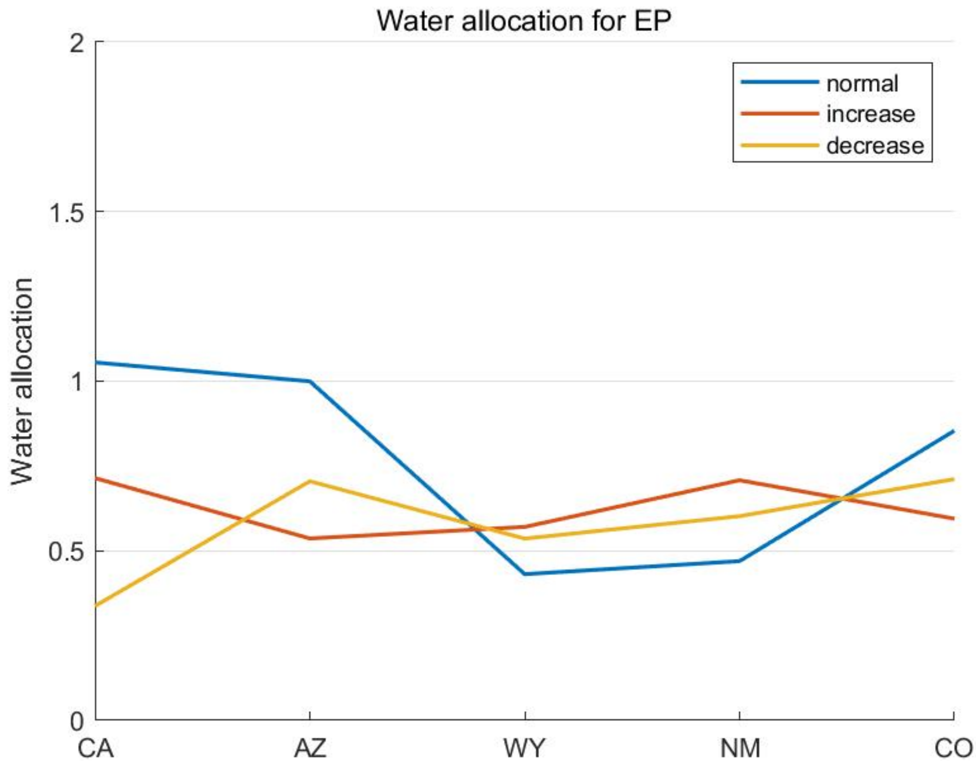

Figure 7: Allocation of water for power generation in five states

According to the results of the above graph considering the actual situation, the general industry shrinkage or growth will rarely show all consistent changes, so the growth scenario implies that three states show growth while the other two are unchanged or declining; the decline scenario is the same.

As can be seen by the results, the final allocation scenario is generally consistent with the proposed conditions. Generally speaking, three states show an increase in water allocation, while two states remain more or less the same or decrease. On the other hand, from the stability of the model, if we only look at the change of the proportion, we can see that the most for the final allocation scheme does not appear particularly large changes, so it can be seen that the model has good stability.

7.2 Increase in the proportion of renewable energy technologies

Under the second condition, the proportion of renewable energy use increases, i.e., the amount of electricity generated by other means increases, the proportion of hydroelectric power generation decreases, and the consumption of water used for power generation decreases, then

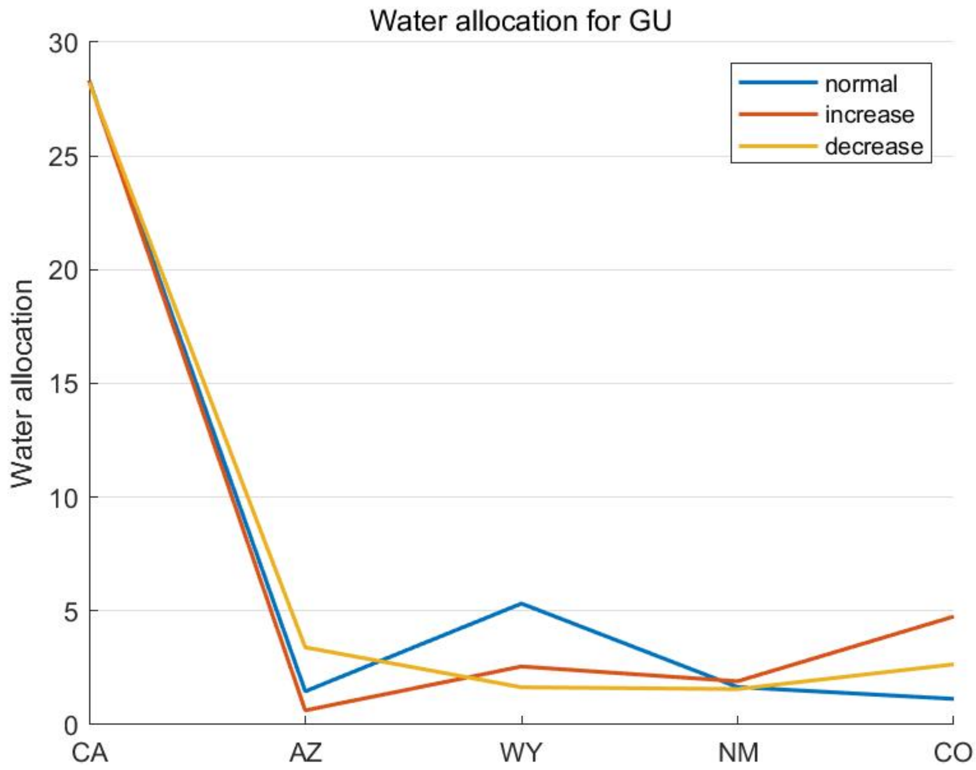

Figure 8: General water allocation in five states

the amount of water that can be allocated to general water use increases while the supply remains the same.

At this point, it is clear that the amount of water available for domestic, agricultural and industrial use can be increased to a certain extent, which is a boost to the overall economy. Therefore, our model indicates that it is a boost for the economy.

7.3 Additional water and energy saving measures

Under the third condition, we set additional water and energy saving measures, which are reflected in our model as a reduction in overall consumption, and thus a reduction in fixed demand.

Thus, our model reflects the reduction of overall water consumption, the conservation and rational use of energy and environment for the whole planet.

8 Model Evaluation

8.1 Strength

• In this model, we fully considered the demand of each state for general water and water for power generation, and proposed the allocation ratio for each state by the entropy weight method. Based on this proportion, we used the model to give allocation schemes that meet the demand in general, and in the robustness test, we obtained that the model gives solutions that are generally consistent with the accurate data even when the data are not so accurate.

• Considering the geographical location between states, we used the shortest-circuit model of graph theory for resource scheduling between reservoirs and five states to find the scheduling solution with the lowest scheduling cost. Further, for the sudden resource shortage situation caused by the possible unexpected disasters in each state, a model of resource coordination between states and states is given.

• With the introduction of the renewable energy impact, we have made dynamic adjustments to the state demand ratios to bring them closer to reality.

• For the data sources, we combined data from recent years for Lake Powell and Lake Mead with current actual conditions, and obtained the final data through simulation, which has a high reliability. For water use, we fit the data through the population and economic development of each state, which also has a high reliability.

8.2 Weakness

• Since the data we collected were only the sum of residential, agricultural and industrial water use, we only considered the sum of the three in terms of general water use and did not analyze them one by one.

• The time period involved in our solution process is within a few months, and we do not perform a prediction solution for several years or even a decade.

• because of the dry climate of these two reservoirs all year round, we did not consider the reservoir flooding caused by the small probability of continuous heavy rainfall during the analysis.

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MEMO

To: editors from *Drought and Thirst* magazine
From: Team 2209826
Date: February 22th, 2022
Subject: A Water Allocation Plan Between Dams and States

Dear the editors from *Drought and Thirst* magazine:

It's our honor to receive your invitation of writing an article for the Drought and Thirst magazine, which provide our team with an opportunity to show our achievements in dynamic water resource allocation issues between five statesincluding CA, AZ, NM, WY,CO and two dams (the Glen Canyon dam and the Hoover dam) by our models.

Since our water resources optimal allocation model is based on the proportional criteria for general water use and water for power generation, we first assigned weights to each state for general water use and water for hydroelectric power generation by collecting data on the demand for general water use and water for hydroelectric power generation in five states, and thus obtained the demand criteria for both in each state. Based on this demand standard based on realistic data, we were able to scientifically allocate water supplies for general water use and hydropower generation according to demand under the water stress conditions in the Southwest, which solved the problem of structural imbalance in the internal allocation of water resources.

Based on the results of the model solution, we found that the demand for general water use is greater than the demand for hydroelectric power generation in all states. This is because for the arid southwest, the main source of general water use is reservoirs, and regarding the source of electricity, there are many other suitable power generation methods such as wind power and tidal power in addition to hydroelectric power. Therefore the five states are more dependent on reservoirs for general water use than on hydroelectric power generation.

After solving the problem of standard setting, we developed a water resource optimal allocation model to achieve the optimal allocation of water resources between states and reservoirs. In addition to considering the demand problem between states, we considered more about the sustainable operation of the reservoirs. In order to avoid the situation that the reservoir water level is too low to meet the demand, we use the monthly inflow as the water supply for the month. Only when the inflow cannot meet the demand, we call on the reservoir's water storage (not including the inflow of the month). In this way, we can effectively monitor the sustainability of the current reservoir operation and get the water shortfall of the reservoir at different time periods.

Our team believes that we need to think in terms of both open source and Throttling.

- **Throttling** Throttling: In terms of cutting costs, we start from optimizing the transportation routes of resources, and build an optimal resource scheduling network diagram based on the distance and difficulty of transportation between the states and the dams, to find the optimal path between the dams and the states, to reduce the consumption of resources on the road, so as to achieve the purpose of resource cutting.

- **Open source** In terms of open source, we need to increase the proportion of other renewable energy technologies, such as wind power, solar power, tidal power, and sewage purification, to relieve the pressure on reservoir water resources.

We also conducted sensitivity tests on our model in terms of changing the demand, improving renewable energy technologies, and incorporating water and energy saving measures. The results show the stability and practicality of our model.

We hope our findings will be helpful to you, and if you have suggestions you would like to provide, we very much look forward to hearing from you.

Appendices

```
clear
clc

cof = [0.8739    0.8739  0.3261  0.3261...
0.5943   0.5943  0.4057  0.4057...
0.7213   0.7213  0.2787  0.2787  ...
0.8156   0.8156  0.1844  0.1844...
0.6539   0.6539  0.3461  0.3461];

a = [1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0];
b = [0 1 -1 1 0 1 -1 1 0 1 -1 1 0 1 -1 1 0 1 -1 1];
re = [17 17 0.7 0.7 3 3 0.6 0.6 4 4 0.6 0.6 2 2 0.5 0.5 4.5 4.5 0.7 0.7];
P = 18.139970195;
Lp = 0;
M = 21.46730000;
Lm = 0;
fun = @(x)((cof * x));
cons1 = @(x,P)(a * x <= P - Lp);
cons2 = @(x,M)(b * x <= M - Lm);
cons3 = @(x)(x> 0);

%% Set population Parameters
sizepop = 2000;                            % Initial population number
dim = 20;                                  % The spatial dimension
ger = 700;                                 % Maximum iteration
% xlimit_max = 25*ones(dim,1);             % Set position parameter limits
xlimit_max = re';
xlimit_min = zeros(dim,1);
vlimit_max = 0.001*ones(dim,1);            % Set speed limit
vlimit_min = zeros(dim,1);
c_1 = 0.1;                                 % Inertia weight
c_2 = 0.5;                                 % Self-learning factor
c_3 = 0.5;                                 % Group learning factor

%% Generating initial population
% First, the initial population position is generated randomly
% Then the initial population velocity is generated randomly
% It then initializes the individual historical best position, as well a
% Then initialize the population history best position, and the populati
for i=1:dim
    for j=1:sizepop
        pop_x(i,j) = xlimit_min(i)+(xlimit_max(i) - xlimit_min(i))*rand;
% The location of the initial population
```

```
        pop_v(i,j) = vlimit_min(i)+(vlimit_max(i) - vlimit_min(i))*rand;
% The velocity of the initial population
    end
end
gbest = pop_x;                                          % The historical best posit
for j=1:sizepop
    if cons1(pop_x(:,j),P)
        if cons2(pop_x(:,j),M)
            if cons3(pop_x(:,j))
                fitness_gbest(j) = fun(pop_x(:,j));
% The historical best position of each individual
            else
                fitness_gbest(j) = 0;
            end
        else
            fitness_gbest(j) = 0;
        end
    else
        fitness_gbest(j) = 0;
    end
end
zbest = pop_x(:,1);                                     % The historical best locat
fitness_zbest = fitness_gbest(1);                       % Historical optimum fitnes
for j=1:sizepop
    if fitness_gbest(j) > fitness_zbest
        zbest = pop_x(:,j);
        fitness_zbest=fitness_gbest(j);
    end
end


%% Particle swarm iteration
%    Update speed and perform speed boundary processing
%    Updates the position and bounds the position
%    Adaptive mutation is performed
%    To judge the constraints and calculate the fitness of each individua
%    The new fitness was compared with the individual's historical best f
%    The best fitness in individual history was compared with the best fi
%    Loop again or end

iter = 1;
record = zeros(ger, 1);
while iter <= ger
    for j=1:sizepop
        %    Update speed and perform speed boundary processing
        pop_v(:,j)= c_1 * pop_v(:,j) + c_2*rand*(gbest(:,j)-pop_x(:,j))+c
        for i=1:dim
            if  pop_v(i,j) > vlimit_max(i)
                pop_v(i,j) = vlimit_max(i);
            end
```

```
            if  pop_v(i,j) < vlimit_min(i)
                pop_v(i,j) = vlimit_min(i);
            end
        end

        %   Updates the position and bounds the position
        pop_x(:,j) = pop_x(:,j) + pop_v(:,j);
        for i=1:dim
            if  pop_x(i,j) > xlimit_max(i)
                pop_x(i,j) = xlimit_max(i);
            end
            if  pop_x(i,j) < xlimit_min(i)
                pop_x(i,j) = xlimit_min(i);
            end
        end

        %   Adaptive mutation is performed
        if rand > 0.85
            i=ceil(dim*rand);
            pop_x(i,j)=xlimit_min(i) + (xlimit_max(i) - xlimit_min(i)) *
        end

        %   To judge the constraints and calculate the fitness of each i
        if cons1(pop_x(:,j),P)
            if cons2(pop_x(:,j),M)
                if cons3(pop_x(:,j))
                    fitness_pop(j) = fun(pop_x(:,j));
% Fitness of the current individual
                else
                     fitness_pop(j) = 0;
                end
            else
                fitness_pop(j) = 0;
            end
        else
            fitness_pop(j) = 0;
        end

        %   The best fitness in individual history was compared with the
        if fitness_pop(j) > fitness_gbest(j)
            gbest(:,j) = pop_x(:,j);                % Update group history
            fitness_gbest(j) = fitness_pop(j);      % Update population hi
        end

        %   The best fitness in individual history was compared with the
        if fitness_gbest(j) > fitness_zbest
            zbest = gbest(:,j);                     % Update group history
            fitness_zbest=fitness_gbest(j);         % Update population hi
        end
    end
```

```
    record(iter) = fitness_zbest;%Maximum  record

    iter = iter+1;

end
%%

plot(record,'Linewidth',1.5); title('The␣convergence␣process')
ylabel('The␣objective␣function')
xlabel('The␣number␣of␣iterations')
disp(['The␣optimal␣ value ',num2str(fitness_zbest)]);
disp('State␣ variable ');
disp(zbest);
for i = 1:10
    c(i) = zbest(2*i) + zbest(2*i-1);
end
for i = 1:5
    C(i,1) = zbest(4*i-2) + zbest(4*i-3);
    C(i,2) = zbest(4*i) + zbest(4*i-1);
end
r = [17 0.7 3 0.6 4 0.6 2 0.5 4.5 0.7];
for i  = 1:10
    if c(i) <= r(i)
        delter_c(i) = r(i) - c(i);
        Delter(i) = 0;
    else
        delter_c(i) = 0;
        Delter(i) = c(i) - r(i);
    end
end
for i = 1:5
    u(i,1) = zbest(4*i-3);
    u(i,2) = zbest(4*i-1);
    d(i,1) = zbest(4*i-2);
    d(i,2) = zbest(4*i);
end

if P + M < 33
    rem = sum(delter_c);
    Rem = sum(Delter);
else
    rem = 33 -(P+M);
end

figure
bar((d+u))
hold on
bar((d))
bar([24,0.7;2,0.6;4,0.6;2,0.5;4,0.7],'FaceColor','none','EdgeColor','k','
```

```
legend('GU from Powell','EP from Powell','GU from Mead','EP from Mead','r
set(gca, 'XTickLabel',{'CA','AZ','WY','NM','CO'})
set(gca,'ylim',[0,30],'ytick',[0:5:30]);
set(gca, 'Ygrid','on');
ylabel('Water allocation')
title('Water resources allocation scheme')
```