# EE 160: Introduction to Control
# Project

Prof. Yang Wang

Deadline for the initial report: Dec.11, Deadline for final report: Jan.03

## 1 Introduction

The EE160 Project is about the modeling, analysis, design and simulation of a control system of your choice (including but not limited to mechanical, circuit, thermodynamic system). Here, you have two options:

1. Pick at least one of the application examples (with given physical models) from the appendix of this project sheet; or
2. Propose a control problem of your own choice. If you want to go for this second option, please contact prof.Wang or one of the TAs. If you have a good idea, we will accept this as a project topic, too, as long as the project is related to the methods that are discussed in the lecture.

**You should form your team of no more than 2 members and submit your initial report before Dec.11, independent of whether you go for one of the default project or come up with your own project, you will need to complete a final report ($\geq 4$ pages) and a presentation, and your presentation should consist of several slides as well as on-site simulation display for $15-20$ minutes per group.** The goal of this project is to practice how to use control method such as root locus method, bode analysis to analyze your system and design a feedback controller for a practical application. Moreover, you will learn how to analyze the performance of the the closed-loop system, then show the effectiveness of your design

## 2 Main requirements on the initial report

The main tasks of the initial report are as follows:

1. **You should clarify your team member and the control problem you choose.**
2. *Modeling* By the given
$$\dot{x}(t) = f(t, x(t), u(t)). \tag{1}$$
Explain what $x, u$, and $f$ are in your application and *define your preliminary control goal.* Calculate the transfer function and find the time domain solution of the system.

## 3 Main requirements on the final report

1. Model the system in both Time and Frequency domain.
2. Analyze the performance of the open-loop system.
3. Design a feedback controller or use the controller given in each appendix and analyze the performance of the closed-loop system.
4. Analyze the stability of the system and show the performance with simulation. Verify the robustness, sensitivity, relative stablity and other performance of the system.

## 4 Some specifications for the report

Write a short report (preferably in Latex) containing the following sections:

1. *Title and Authors* (find a good title + name of the author)
2. *Introduction* (describe the problem that you want to solve and cite relevant literature)
3. *Problem Formulation* (introduce a suitable mathematical notation to define the problem that you are trying to solve)
4. *Open-loop performance analysis* (use the method in class to describe the performance)
5. *Controller Design and closed-loop performance analysis* (explain how you design your controller and why the controller is able to achieve your control objective.)
6. *Numerical Results* (plot/visualize and explain your numerical results)
7. *Conclusion* (analyze and summarize the highlights of your results)

Other requirements:

1. The length should be at least 4 single-column pages with 10pt font. Be brief.
2. **The amount of contribution/work of both students involved should be clearly stated**.
3. No plagiarism or self-plagiarism. The student is never allowed to reuse his/her own published papers as the final project.

## 5 Presentation

Every group will get a $15 - 20$ mins time slot to present your project. The presentation should consist of at least 5 slides, preferably in pdf format. In order to "pass", you will need to:

1. prepare clean and efficient slides, contains
   - the project title and the name of all authors
   - introducing/explaining the problem that you are solving
   - introducing your design logic for the controller
   - the numerical results showing the effectiveness of your control system
   - one conclusion slide summarizing and assessing your results

   Don't use small fonts.
2. present your work freely in English or Chinese and by using your own words.
3. answer questions by the Lecturer, TAs, or other people from the audience.

# Appendix

This appendix collects a few suggestions for possible project topics and some hints on how to derive a differential equation model. For each proposal, we have listed a few basic control objectives and related simulation requirements. If you propose your own project, please contact the lecturer to discuss about task details.

## A Hydraulic servo systems (Supervisor: Qinxiao Ma)

Hydraulic servo systems play a significant role in industrial applications, such as manipulators, various vehicle components/subsystems, by virtue of their large power-to-weight ratios, high-response and high-load capability[1, 2]. However, high-performance control of hydraulic systems is challenging due to the inherent nonlinear behaviors and modeling uncertainties (e.g., uncertain parameters and unmodeled nonlinearities).

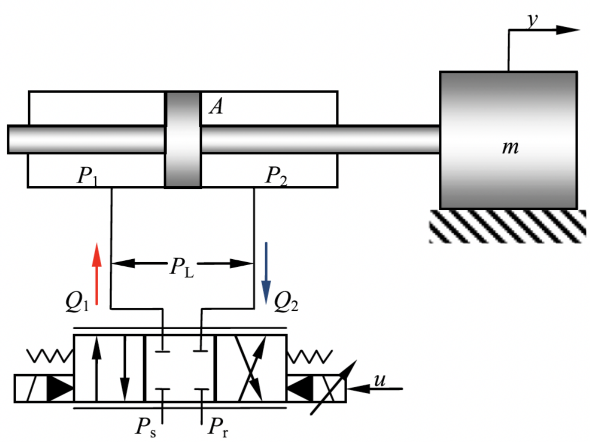



Figure 1: Schematic diagram of the double-rod hydraulic system.

The considered hydraulic servo system is depicted in Fig. 1. As shown, an inertia load is driven by a servovalve controlled double-rod hydraulic cylinder. The force balance equation of the inertia load can be described by

$$m\ddot{y} = P_L A - B\dot{y} - A_f S_f(\dot{y}) + f(t)$$

where $y$ and $m$ represent the displacement and the mass of the load, respectively; $P_L = P_1 - P_2$ is the load pressure of the cylinder, in which $P_1$ and $P_2$ are the pressures inside the two chambers of the cylinder; $P_s$ is the supplied pressure of the fluid, and $P_r$ is the return pressure; $A$ is the effective ram area of the cylinder; $B$ represents the combined coefficient of the modeled damping and viscous friction on the load and the cylinder rod; $A_f S_f$ represents the approximated nonlinear Coulomb friction, in which $A_f$ is the Coulomb friction amplitude and $S_f$ is a known shape function; $f(t)$ represents other disturbances like unmodeled nonlinear frictions and external disturbances. Considering the compressibility of the oil, the pressure dynamics of the cylinder are given by

$$\begin{aligned} \tfrac{V_1}{\beta_e}\dot{P}_1 &= -A\dot{y} - C_t P_L + q_1(t) + Q_1 \\ \tfrac{V_2}{\beta_e}\dot{P}_2 &= A\dot{y} + C_t P_L - q_2(t) - Q_2 \end{aligned}$$

where $V_1 = V_{01} + Ay$ and $V_2 = V_{02} - Ay$ are the volumes of the cylinder chambers, in which $V_{01}$ and $V_{02}$ are the original control volumes of the two chambers; $\beta_e$ is the effective oil bulk modulus; $C_t$ is the coefficient of the total internal leakage of the cylinder due to pressure; $Q_1$ is the supplied flow rate of the forward chamber, and $Q_2$ is the return flow rate of the return chamber; $q_1(t)$ and $q_2(t)$ are the modeling errors in the dynamics of $P_1$ and $P_2$, respectively.

Since a high-response servovalve whose dynamics are much faster than the remaining parts of the system is used in this paper, the valve dynamics are neglected. Hence, we assume that the control input $u$ is directly proportional to the valve spool displacement $x_v$, hence $Q_1$ and $Q_2$ can be modeled by

$$\begin{aligned} Q_1 &= k_u u\left[s(u)\sqrt{P_s - P_1} + s(-u)\sqrt{P_1 - P_r}\right] \\ Q_2 &= k_u u\left[s(u)\sqrt{P_2 - P_r} + s(-u)\sqrt{P_s - P_2}\right] \end{aligned}$$

where $k_u$ is the total flow gain with respect to the control input $u$; and the function $s(u)$ is defined as

$$s(u) = \begin{cases} 1, & \text{if } u \geq 0 \\ 0, & \text{if } u < 0 \end{cases}$$

Define the state variables as $x = [x_1, x_2, x_3]^T = [y, \dot{y}, P_L]^T$, then the system can be expressed in a state-space form as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ m\dot{x}_2 &= Ax_3 - Bx_2 - A_f S_f(x_2) + f(t) \\ \dot{x}_3 &= \beta_e k_u f_1 u - \beta_e f_2 + \beta_e C_t f_3 + q(t) \end{aligned}$$

where $f_1 = \left[s(u)\sqrt{P_s - P_1} + s(-u)\sqrt{P_1 - P_r}\right]/V_1 + \left[s(u)\sqrt{P_2 - P_r} + s(-u)\sqrt{P_s - P_2}\right]/V_2$, $f_2 = (1/V_1 + 1/V_2)\,Ax_2$, $f_3 = (1/V_1 + 1/V_2)\,x_3$, and $q(t) = \beta_e\,(q_1/V_1 + q_2/V_2)$.

To simplify the problem, let us treated the state $x_3$ as an input signal that can be arbitrarily designed by the user, that is, only consider the subsystem of $x_1$ and $x_2$, $x = [x_1, x_2]^T$ namely

$$\begin{aligned} \dot{x}_1 &= x_2 \\ m\dot{x}_2 &= Ax_3 - Bx_2 - A_f S_f(x_2) + f(t) \end{aligned}$$

when $S_f(x_2) = (2/\pi)\arctan(900x_2)$, the linearization process is

$$(x_{ref}^{\mathrm{T}}, u_{ref}) = \left( \begin{bmatrix} x_1 ref \\ x_2 ref \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, u_{ref} = 0 \right) \tag{2}$$

$$y_{ref} = x_{1ref} \tag{3}$$

$$z = x - x_{ref} \tag{4}$$

$$v = u - u_{ref} \tag{5}$$

$$Y = y - y_{ref} \tag{6}$$

$$A_1 = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\right|_{z_1=0, z_2=0, v=0} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{m} - A_f\frac{1800}{\pi m} \end{bmatrix} \tag{7}$$

$$B_1 = \left.\begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}\right|_{x_1 = x_{1ref}, x_2 = x_{2ref}, u = u_{ref}} = \begin{bmatrix} 0 \\ \frac{A}{m} \end{bmatrix} \tag{8}$$

$$b = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t) \tag{9}$$

So the linear approximation at steady states is

$$\dot{z} = f(z, v) = A_1 z + B_1 v + b \tag{10}$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{m} - A_f\frac{1800}{\pi m} \end{bmatrix} z + \begin{bmatrix} 0 \\ \frac{A}{m} \end{bmatrix} v + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t) \tag{11}$$

$$Y = Cz \tag{12}$$

| [h] Parameter | Quantity |
|---|---|
| A | $905 * 10^{-6}$ |
| B | 2000 |
| $A_f$ | 100 |
| $m$ | 30 |
| $f$ | 0 |
| $A_1$ | $\begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{m} - A_f \frac{1800}{\pi m} \end{bmatrix}$ |
| $B_1$ | $\begin{bmatrix} 0 \\ \frac{A}{m} \end{bmatrix}$ |
| $C_1$ | $\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ |
| $C$ | $\begin{bmatrix} 1 & 0 \end{bmatrix}$ |

Table 1: Parameter table

The overall goal is to have the inertia load to track any smooth desired position trajectory as closely as possible, while the specific design task of this topic are listed as follows:

1. Convert the state space of the linearized system into transfer function.
2. Neglect the input noise $f(t)$. Design a feedback PD control law. By using at least one of the methods that has been introduces in the lecture. Use MATLAB to draw the open-loop diagrams of the system, draw bode diagram of the system and analyze the performance in frequency domain(P.O.,P.M.) of the system. Apply root locus(You can choose a proper $\frac{K_p}{K_D}$) to analyze the performance of the closed-loop system, such as stability and convergence speed.
3. Design a compensator to improve the performance of the closed-loop system in **Problem 2**. This requires you to sketch the block diagram of your design. Moreover, achieve a deadbeat response to a step command by tunning your parameters both in your controller and your compensator. Use Matlab to justify your conclusion. Draw the bode diagram and compare it with the original system.
4. Analyze the performance and robustness of your closed-loop system with respect to the input noise, that is, $f(t)$ is some additive norm-bounded noise.

**References**

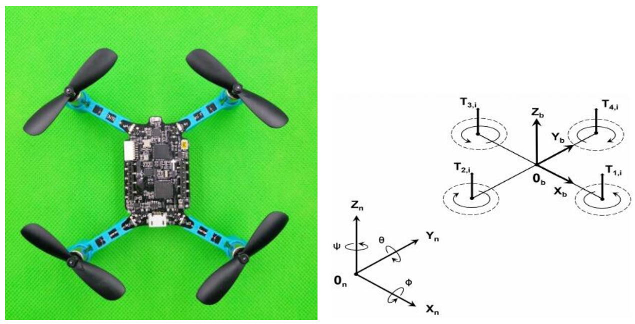

1 Jianyong Yao and Wenxiang Deng, Active Disturbance Rejection Adaptive Control of Hydraulic Servo Systems,IEEE transactions on Industrial Electronics, Vol. 64, No. 10, October 2017.

2 H. E. Merritt, Hydraulic Control Systems. New York, NY, USA: Wiley, 1967.

# B Quadrotor Control (Supervisor: Jiangkun Xu)

## B.1 Introduction

A quadrotor is a mainly flying robot that composed of motor, electric control, battery, propeller, frame, remote control (manual control) and flight control. The dynamics of quadrotor mainly includes position dynamics and attitude dynamics. For position dynamics, we have three states $p_x, p_y, p_z$ representing the three-dimensional coordinates of the quadrotor in the world coordinate system, and three other states $v_x, v_y, v_z$ representing the velocities in each direction. And for attitude dynamics, we have 3 more states for angular velocity of the quadrotor which are $w_x, w_y, w_z$ and $\phi, \theta, \psi$ are the additional 3 states for the Euler angle.



(a) Quadrotor (b) Coordinate system

Figure 2: Quadrotor.

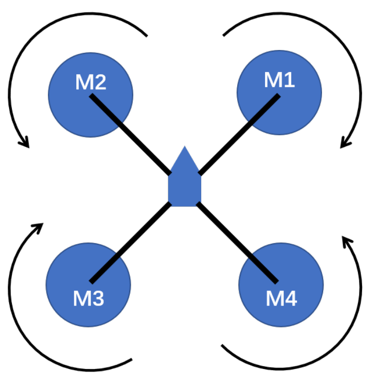



Figure 3: Quadrotor mode.

## B.2 Simplified Model

To simplify the model, this topic only discusses the tilt control of the quadrotor along one of the supporting axes, and does not consider the altitude, rotation, and tilt of the quadrotor along the other axis, nor the disturbance caused by the rotor on the other axis. The control objective is regulate the quadrotor bank angle to zero degrees (steady, wings level) and maintain the wings-level orientation in the presence of unpredictable external disturbances.

For our purposes, a simplified dynamic model is required for the quadrotor design process. A simplified model might consist of a transfer function describing the input-output relationship between the Motor voltage and the quadrotor bank angle. By making the simplifying assumptions and linearizing about the steady, wings-level flight condition, we can obtain a transfer function model describing the bank angle output, $C(s)$, to the motor voltage input, $U(s)$

$$G(s) = \frac{C(s)}{U(s)} = \frac{2s + 0.1}{s(s^2 + 0.1s + 4)}. \tag{13}$$

where $U(s)$ is the output of the select key controller/compensator $G_c(s)$ to be designed. The system configuration is shown in Figure 4.

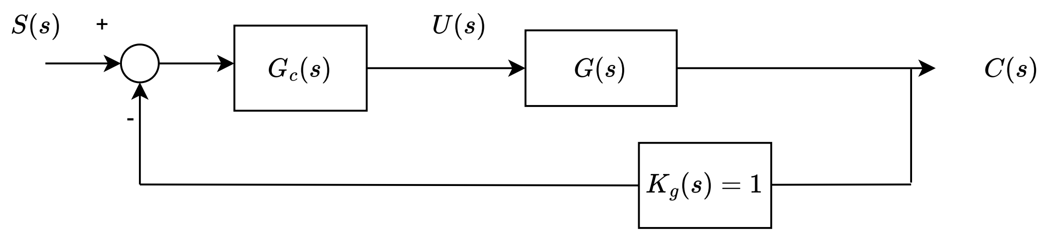



Figure 4: Quadrotor attitude control system.

**Q1. The controller we select for this question is a proportional controller $G_c(s) = K$. Sketch and analyze the root locus for open-loop system. Choose an appropriate $K$, sketch and analyze the unit step response, Bode plot and Nyquist plot.**

**Q2. We want to make the overshoot and the peak time are as small as possible and want to eliminate the oscillation mode. Try to design an appropriate compensator $G_c(s)$ such that the closed-loop system has a pair of dominant poles whose location can be chosen by yourself properly. Then we consider there exists a disturbance $n(t) = t$ added between $G_c(s)$ and $G(s)$, verify whether the controller can eliminate the influence of the disturbance $n$, if not, redesign your controller to satisfy the above requirements at the same time. Write the form of $G_c(s)$ and give the closed-loop transfer function. Describe the root locus of the compensated system, and use matlab to test the transient response characteristics of the system.**

## B.3 Complete Model

Up to now, you've controlled one of the tilt angles of the quodrator, now we can give a complete model.

The position dynamics model of quadrotor is given as follows:

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + RT_B$$

$$\Rightarrow m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \begin{bmatrix} c_\theta c_\psi & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ c_\theta s_\psi & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}$$

where $p = [x, y, z]^\top \in \mathbb{R}^3$ is the position of the quadrotor, $R$ is the rotation matrix from the body frame to the inertial frame, $\phi, \theta, \psi$ represent the attitude angles roll, pitch, and yaw, respectively ($c$ and $s$ represent *cos* and *sin*). $f$ is the total thrust on the quadrotor. $m$ is the mass of the quadrotor and $g$ is acceleration of gravity.

The attitude dynamics model of quadrotor is given as follows:

$$I\dot{\omega} + \omega \times (I\omega) = \tau$$

$$\Rightarrow \begin{bmatrix} \dot{\omega_x} \\ \dot{\omega_y} \\ \dot{\omega_z} \end{bmatrix} = \begin{bmatrix} \tau_x I_{xx}^{-1} \\ \tau_y I_{yy}^{-1} \\ \tau_z I_{zz}^{-1} \end{bmatrix} + \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}}\omega_y\omega_z \\ \frac{I_{zz}-I_{xx}}{I_{yy}}\omega_x\omega_z \\ \frac{I_{xx}-I_{yy}}{I_{zz}}\omega_x\omega_y \end{bmatrix}$$

where $\omega = [\omega_x, \omega_y, \omega_z]^\top$ is the angular velocity vector of the quadrotor. $I_{xx}, I_{yy}, I_{zz}$ represent the moment of inertia of the three axes. $\tau_x, \tau_y, \tau_z$ represent the torques generated on three axes.

In order to convert angular velocities $[\dot{\phi}, \dot{\theta}, \dot{\psi}]^\top$ into the angular velocity vector $\omega = [\omega_x, \omega_y, \omega_z]^\top$, we can use the following relation:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & c_\theta s_\phi \\ 0 & -s_\phi & c_\theta c_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

The angular velocity of the each propeller are $\omega_1(t), \omega_2(t), \omega_3(t), \omega_4(t)$ which are also the input of the system. In the x-mode, the torque $\tau_{x,y,z}$ and force $f$ are defined as the following equations:

$$U(t) = \begin{bmatrix} \tau_x(t) \\ \tau_y(t) \\ \tau_z(t) \\ f(t) \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}d \cdot c_T & -\frac{\sqrt{2}}{2}d \cdot c_T & -\frac{\sqrt{2}}{2}d \cdot c_T & \frac{\sqrt{2}}{2}d \cdot c_T \\ \frac{\sqrt{2}}{2}d \cdot c_T & \frac{\sqrt{2}}{2}d \cdot c_T & -\frac{\sqrt{2}}{2}d \cdot c_T & -\frac{\sqrt{2}}{2}d \cdot c_T \\ c_M & -c_M & c_M & -c_M \end{bmatrix} \begin{bmatrix} \omega_1^2(t) \\ \omega_2^2(t) \\ \omega_3^2(t) \\ \omega_4^2(t) \end{bmatrix}$$

where $d$ is shaft length of the quadrotor, $c_T, c_M$ are tension coefficient and torque coefficient. The values of system parameters are summarized in Table 2.

Table 2: Parameters of complete quadrotor.

| Symbol | Value |
|---|---|
| $c_T$ | $1.105 \times 10^{-5}$ |
| $c_M$ | $1.779 \times 10^{-7} \times 2$ |
| $d$ | 0.225 |
| $m$ | 1.4 |
| $g$ | 9.8 |
| $I_{xx}$ | 0.0211 |
| $I_{yy}$ | 0.0219 |
| $I_{zz}$ | 0.0366 |
| $J_{RP}$ | 0.0001287 |

**Q3. Try to find a proper control law to drive quadrotor is to hover at a fixed position under all sorts of initial conditions.**

## C Active Suspension Control (Supervisor: Yang Wang)

As an automobile moves along the road, the vertical displacements at the tires act as the motion excitation to the automobile suspension system. Figure 5 is a schematic diagram of a simplified automobile suspension

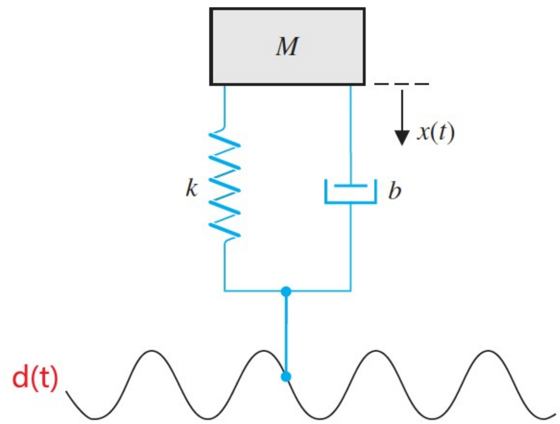



Figure 5: simplified automobile suspension system

system, for which we assume the input is sinusoidal.

**Q1. Determine the transfer function X(s)/D(s), and sketch the Bode plot when M = 1 kg, b = 4 N s/m, and k = 18 N/m.**

One of the beneficial applications of an automotive control system is the active control of the suspension system. One feedback control system uses a shock absorber consisting of a cylinder filled with a compressible fluid that provides both spring and damping forces. The cylinder has a plunger activated by a gear motor, a displacement-measuring sensor, and a piston. Spring force is generated by piston displacement, which compresses the fluid. During piston displacement, the pressure imbalance across the piston is used to control damping. The plunger varies the internal volume of the cylinder. This system is shown in Figure 6 Active suspension systems for modern automobiles provide a comfortable firm ride. The design of an active suspension system adjusts the valves of the shock absorber so that the ride fits the conditions. A small electric motor, as shown in Figure 7, changes the valve settings.

**Q2. Specify the performance for a step command in terms of a settling time (with a 2 % criterion). Then select a design value for k and the parameter q in order to satisfy the performance specifications. Upon completion of your design, predict the resulting percent overshoot for a step input, and verify it using simulation.**

Assuming there is a proportional relation between valve position and the value of b. Now embedded the valve position control system into the overall active suspension control system, where b now becomes adjustable.

**Q3. Try to design the appropriate control law r(t) so that the vehicle accommodates (a) a large bump at high speeds and (b) a small bump at low speeds.**

**References**

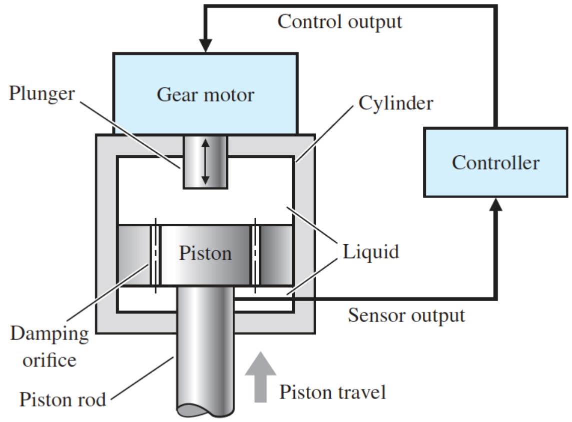



Figure 6: Laser Model

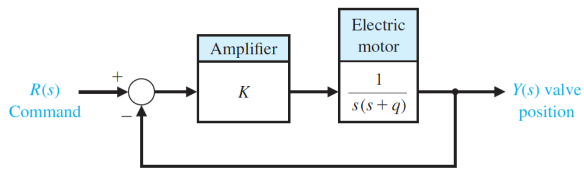



Figure 7: Laser Model

1 A. Titli, S. Roukieh and E. Dayre, Three control approaches for the design of car semi-active suspension (optimal control, variable structure control, fuzzy control), Proceedings of 32nd IEEE Conference on Decision and Control, 1993.

2 Ashley S. Putting a suspension through its paces[J]. Mechanical Engineering-CIME, 1993.

3 Hrovat D. Applications of optimal control to advanced automotive suspension design[J]. 1993.

# D DC-DC Ćuk Converter (Supervisor: Qinxiao Ma)

## D.1 Background

The DC-DC Ćuk converter is one of the most widely studied power converters. Its goal is to invert the polarity of the input voltage and to step-up or step-down its absolute value. Its main application is in regulated DC power supplies, where an output voltage with negative polarity (with respect to the common terminal of the input voltage) is desired. A significant advantage of this converter over other inverting topologies, such as the buck-boost and flyback, is the use of inductors in the input and output loops (see Figure 8), which induces very little input and output current ripple. Related information can be found there [Reference 1 Chaper 8.2 Page 181]2008Nonlinear

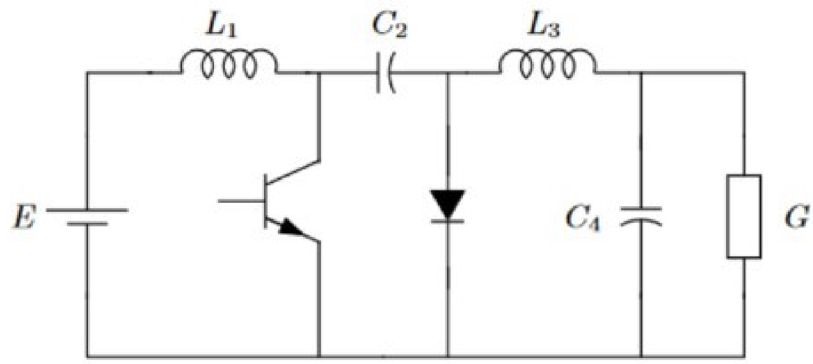



Figure 8: DC–DC Ćuk Converter circuit.

## D.2 Problem Formulation

The simplified model of the DC–DC Ćuk converter shown in Figure 8.4 is given by the equations

$$\begin{aligned} L_1\dot{i}_1 &= -(1-u)v_2 + E \\ C_2\dot{v}_2 &= (1-u)i_1 + ui_3, \\ L_3\dot{i}_3 &= -uv_2 - v_4 \\ C_4\dot{v}_4 &= i_3 - Gv_4 \end{aligned}$$

where $i_1$ and $i_3$ describe the currents in the inductances $L_1$ and $L_3$, respectively; $v_2$ and $v_4$ are the voltages across the capacitors $C_2$ and $C_4$, respectively. ( $L_1, C_2, L_3$ and $C_4$ are also used for the values of the capacitances and of the inductances.) Finally, $G$ denotes the load admittance, $E$ the input voltage and $u \in (0,1)$ is a continuous control signal, which represents the duty ratio of the transistor switch.

It must be noted that, because of physical considerations, the state vector $x = [i_1, v_2, i_3, v_4]^\top$ is constrained in the set

$$\mathcal{D} = \mathbb{R}_{>0} \times \mathbb{R}_{>0} \times \mathbb{R}_{<0} \times \mathbb{R}_{<0}$$

As a result, in what follows we implicitly assume that $x(t) \in D$ for all $t$. The control goal is to regulate the voltage across the load (i.e., the capacitor voltage $v_4$ ) to a constant value $V_d$.

Note that, by setting $u$ to a constant value $\bar{u}$, the equilibria $(\bar{i}_1, \bar{v}_2, \bar{i}_3, \bar{v}_4)$ of system (8.8) verify the following relations

$$\bar{i}_1 = -\frac{\bar{u}}{1-\bar{u}^2}\bar{i}_3, \quad \bar{v}_2 = \frac{E}{1-\bar{u}}, \quad \bar{i}_3 = G\bar{v}_4, \quad \bar{v}_4 = -\frac{\bar{u}}{1-\bar{u}}E.$$

As a result, setting $\bar{v}_4 = -V_d$ yields the control input

$$u^* = \frac{V_d}{V_d + E}$$

and the desired operating equilibrium

$$i_1^* = \frac{GV_d^2}{E}, \quad v_2^* = V_d + E, \quad i_3^* = -GV_d, \quad v_4^* = -V_d.$$

In what follows we show that the considered control goal is achievable with a bounded control signal $u(t) \in (0,1)$ and with partial state and parameter information, namely we assume that the only measured states are $v_2$ and $i_3$ and the parameters $E$ and $G$ are unknown. The linearized model is

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 0 & \frac{E}{(V_d+E)L_1} & 0 & 0 \\ \frac{E}{(V_d+E)C_2} & 0 & \frac{V_d}{(V_d+E)C_2} & 0 \\ 0 & -\frac{V_d}{(V_d+E)L_3} & 0 & -\frac{1}{L_3} \\ 0 & 0 & \frac{1}{C_4} & -\frac{G}{C_4} \end{bmatrix} \\
B_1 &= \begin{bmatrix} \frac{V_d+E}{L_1} \\ -\frac{GV_d^2}{C_2 E} - \frac{GV_d}{C_2} \\ -\frac{V_d+E}{L_3} \\ 0 \end{bmatrix} \\
\dot{x} &= A_1(x - x^*) + B_1(u - u^*) \\
y &= v_4 \\
C &= [0, 0, 0, 1]
\end{aligned}
$$

Define $z = x - x^*$,$w = u - u^*$,$Y = y - y^* = y - v_4^*$, the state space model:

$$
\begin{aligned}
\therefore \dot{z} &= A_1 z + B_1 w \\
y &= Cz
\end{aligned}
$$

## D.3 Requirements

The goal of the project is to design a feedback controller for the DC-DC Ćuk circuit using the methods introduced in the lecture. Analyse this system and design a compensator to achieve a better performance. The main tasks of the project are as follows:

1. Convert the linearized state space model to transfer function.
2. Design a feedback P control law. By using at least one of the methods that has been introduces in the lecture. Use MATLAB to draw the open-loop diagrams of the system, draw bode diagram of the system and analyze the performance in frequency domain(P.O.,P.M.) of the system. Apply root locus to analyze the performance of the closed-loop system, such as stability and convergence speed.
3. Design a compensator to improve the performance of the closed-loop system. This requires you to sketch the block diagram of your design. Moreover, the overshoot of the system in the step response is required to be half of that of the closed-loop system in **Problem 2** in the step response. Use Matlab to justify your conclusion. Draw the bode diagram and compare it with the original system.

### References


1 Astolfi A, Karagiannis D, Ortega R. Nonlinear and adaptive control with applications[M]. London: Springer, 2008.

# E Single-link Flexible-joint Robot (Supervisor: Jiangkun Xu)

## E.1 Background

The problem of position control is a critical issue, for the rigid manipulators and rigid manipulators in industries. In modern robot systems, flexibility becomes very important to meet the special needs of industrial automation. To date, control engineers have been working on the development of a mathematical model and control of flexible structures.

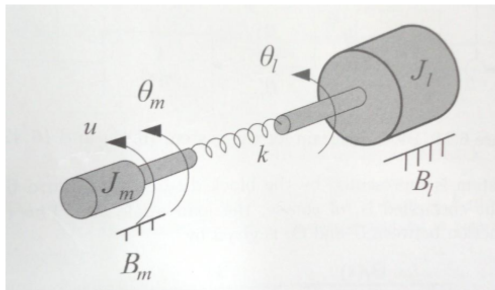



Figure 9: Single-link flexible-joint robot

## E.2 Model

A mathematical model for single-link flexible-joint robots can be obtained easily from Lagrange equations. As shown Fig. 9, the system has two degree of freedoms and the joint which is mounted to the shaft moves according to rorate direction of the motor. In the Fig. 9, $\theta_m$ is motor angle and $\theta_l$ is load angle. The values and symbols of system parameters are summarized in Table 3.

Hint: $k$ belongs to the internal structure of the system and forms the open-loop transfer function together with $G_1(s)$ and $G_2(s)$.

Table 3: Parameters of flexible joint robot manipulator.

| Symbol | Description | Value |
|---|---|---|
| $J_m$ | *motor inertia* | $0.05kg \cdot m^2$ |
| $J_l$ | *load inertia* | $0.5kg \cdot m^2$ |
| $B_m$ | *motor damping* | $0.01kg \cdot m^2$ |
| $B_l$ | *load damping* | $0.01kg \cdot m^2$ |
| $k$ | *coefficient of rotational elasticity* | $6N \cdot m \cdot rad^{-1}$ |

The simplified system can be expressed as follows

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) = 0 \tag{14}$$

$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k(\theta_l - \theta_m) = u \tag{15}$$

where $u$ is the motor torque.

## E.3 Requirements

1. Solve the response of $\theta_l$ when the input $r$ is a unit step signal under the initial condition $\theta_l = 0.5, \dot{\theta}_l = \theta_m = \dot{\theta}_m = 0$.

2. Write the state space model of the system in a compact form (i.e., identify the matrices $A, B, C, D$) and analyze the controllability and the observability, calculate the solution of the state differential equation under the initial condition in question 1. Then convert the state space model to transfer function and analyze whether there is zero-pole cancellation.

3. Consider the following two cases: 1) $\theta_m$ is unavailable, we can only use $\theta_l$ as system output for feedback; 2) $\theta_l$ is unavailable, we can only use $\theta_m$ as system output for feedback (see Figure 10 and Figure 11). Design PD controllers for each two cases. $\frac{K_P}{K_D} = 2$ is fixed. Use MATLAB to draw the root locus, and analyze the performance (stability and rapidity) of the closed-loop system with the change of the root locus gain. Draw the bode diagram of the system, and use the frequency-domain analysis method to calculate phase margin and gain margin. Select a group of good poles for each case and draw the response. In addition, draw the response for a group of unstable poles for the second case. The values of stable and unstable poles should be indicated in the report.

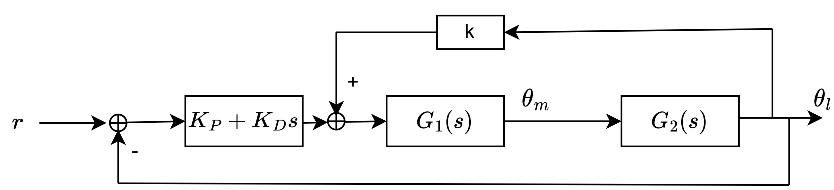



Figure 10: $\theta_l$ as feedback.

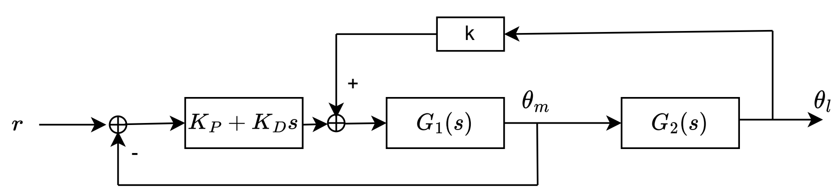



Figure 11: $\theta_m$ as feedback.

4. Using the frequency domain analysis method, choose a compensator (phase lag/phase lead) scheme according to the requirements that the settling time is less than $5s$, and make the overshoot of the closed-loop system of the step response as small as you can. Theoretical analysis and calculation are required, and comparison with simulation results is required. Draw the bode diagram of the compensated system, compare it with the bode diagram of the uncompensated system, judge whether the compensator meets the performance index requirements, if not, analyze the reasons for mismatches.