

## Limitations of Legacy Fault Location Method

### Requirements of Legacy Fault Location Method:

- Traveling Wave Based Methods
- Synchronization Measurements among terminals
- Extremely high sampling rate (typically MHz)
- Artificial Intelligence Based Methods
- A large number of high-quality fault data
- Fundamental Frequency Phasor Based Methods
- Availability of PMUs (e.g. synchronized voltage magnitude and phase measurements)

To get rid of the dependence on synchronization measurements

Retain the advantages of sparse estimation

### Proposed method

- Model the sparse estimation problem base on voltage magnitude.
- Adjust FISTA algorithm to solve for fault location.

### Advantages

- Avoid the complexity of PMU installation.
- Only a limited number of bus voltage magnitude is needed.

## Review of The Existing Method

### Equivalent Injection Current

$\Delta \mathbf{V} \in \mathbb{C}^{3N \times 1}$ , which is the voltage changes of buses of the entire power network caused by the fault current at the fault location, can be equivalently represented by the injection current at the terminal buses of the faulted line even when the distributed parameter line model is fully considered.

With  $\mathbf{Z}_{bus} \in \mathbb{C}^{3N \times 3N}$ , the sparse injection current  $\Delta \tilde{\mathbf{I}} \in \mathbb{C}^{3N \times 1}$  can be calculated by  $\Delta \mathbf{V} = \mathbf{Z}_{bus} \cdot \Delta \tilde{\mathbf{I}}$

### Original Sparse Estimation Problem

Assume the voltage phasors of only  $m(m \ll N)$  buses are measured. By extracting the corresponding rows, we have  $(\Delta \mathbf{V})_m = (\mathbf{Z}_{bus})_m \cdot \Delta \tilde{\mathbf{I}}$  and the corresponding optimization problem is

$$\beta \in \argmin_{\beta} \frac{1}{2} \|(\Delta \mathbf{V})_m - (\mathbf{Z}_{bus,dis})_m \cdot \Delta \tilde{\mathbf{I}}\|_2^2 + \lambda \|\Delta \tilde{\mathbf{I}}\|_1$$

### Relationship between Injection Current and Fault Location

With  $P = \sqrt{(R + j\omega L) \cdot j\omega C}$ , where  $R, L, C$  is series resistance, series inductance and shunt capacitance per unit length, respectively. Then the fault location satisfies the following equation

$$\frac{\Delta \tilde{I}_i}{\Delta \tilde{I}_j} = \frac{\sinh(P \cdot (l_{ij} - l_{if}))}{\sinh(P \cdot l_{if})}$$

## Proposed Method

### Obtain the Voltage Magnitude Measurements $|(\Delta \mathbf{V})_m|$

These measurements only need to calculate phasors at the local end and output phasor magnitudes, and do not require time synchronization.

### Improved Optimization Problem

Take the absolute values on both sides, we have  $|(\Delta \mathbf{V})_m| = |(\mathbf{Z}_{bus,dis})_m \cdot \Delta \tilde{\mathbf{I}}|$ , which can be rewritten as  $|(\Delta \mathbf{V})_m|^2 = \text{re}((\Delta \mathbf{V})_m)^2 + \text{im}((\Delta \mathbf{V})_m)^2$ . Define

$$\mathbf{A} = [\text{re}((\mathbf{Z}_{bus,dis})_m) \quad -\text{im}((\mathbf{Z}_{bus,dis})_m)] \in \mathbb{R}^{3m \times 6N}, \mathbf{B} = [\text{im}((\mathbf{Z}_{bus,dis})_m) \quad \text{re}((\mathbf{Z}_{bus,dis})_m)] \in \mathbb{R}^{3m \times 6N}$$

$\mathbf{C} = |(\Delta \mathbf{V})_m|^2 \in \mathbb{R}^{3m \times 1}$ ,  $\mathbf{x} = [\text{re}(\Delta \tilde{\mathbf{I}})^T \quad \text{im}(\Delta \tilde{\mathbf{I}})^T]^T \in \mathbb{R}^{6N \times 1}$ . The original equation can be changed into  $(\mathbf{A} \cdot \mathbf{x}) \odot (\mathbf{A} \cdot \mathbf{x}) + (\mathbf{B} \cdot \mathbf{x}) \odot (\mathbf{B} \cdot \mathbf{x}) = \mathbf{C}$ . The corresponding optimization problem can be constructed as

$$\mathbf{x} \in \argmin_{\mathbf{x}} \frac{1}{2} \|(\mathbf{A} \cdot \mathbf{x}) \odot (\mathbf{A} \cdot \mathbf{x}) + (\mathbf{B} \cdot \mathbf{x}) \odot (\mathbf{B} \cdot \mathbf{x}) - \mathbf{C}\|_2^2 + \lambda \|\mathbf{x}\|_1$$

### Adjust FISTA Algorithm to Estimate $\Delta \tilde{\mathbf{I}}$

Let  $f(\mathbf{x}) = (\mathbf{A} \cdot \mathbf{x}) \odot (\mathbf{A} \cdot \mathbf{x}) + (\mathbf{B} \cdot \mathbf{x}) \odot (\mathbf{B} \cdot \mathbf{x}) - \mathbf{C}$ , we can get  $\nabla f(\mathbf{x}) = 2 \cdot \text{diag}(\mathbf{A} \cdot \mathbf{x}) \cdot \mathbf{A} + 2 \cdot \text{diag}(\mathbf{B} \cdot \mathbf{x}) \cdot \mathbf{B}$ , with hyperparameters  $L_0, \lambda$  and  $t^1 = 1$ . The  $v^{\text{th}}$  step iteration of FISTA can be calculated as

$$t^{v+1} = 1/2 \cdot [1 + \sqrt{1 + 4(t^v)^2}]$$

$$\mathbf{y}^{v+1} = \mathbf{x}^v + (t^v - 1)/t^{v+1} (\mathbf{x}^v - \mathbf{x}^{v-1})$$

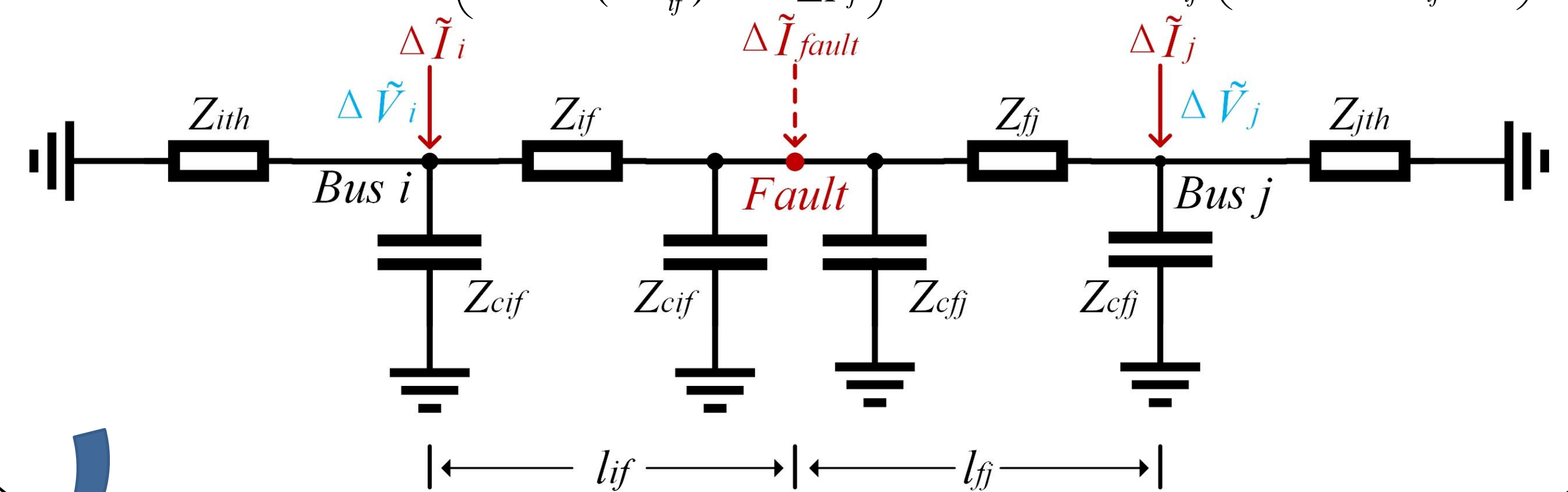
$$\mathbf{x}^{v+1} = \text{soft}[1/L_0 \cdot [\nabla f(\mathbf{x}^{v-1})^T \cdot f(\mathbf{y}^{v+1})] + \mathbf{y}^{v+1}, \lambda/L_0]$$

$$\text{soft}[\mathbf{x}, T] = \text{sign}(\mathbf{x}) \odot \max(\|\mathbf{x}\| - T, 0)$$

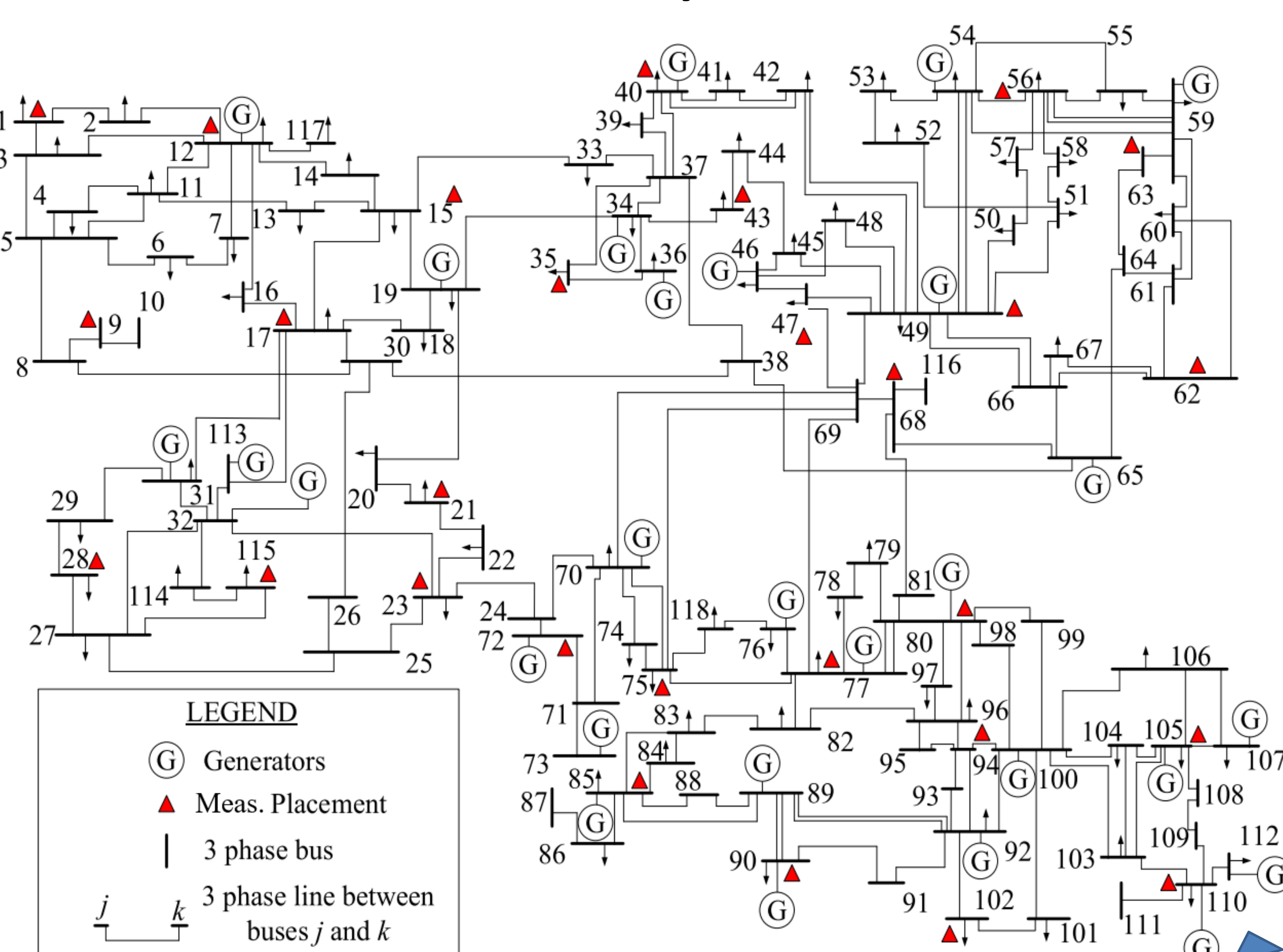
### Locate the Fault by Newton's Method

The fault can be located by solving  $\min_{l_{if}} F(l_{if}) = \left\| \frac{\Delta \tilde{I}_i}{\Delta \tilde{I}_j} - \frac{\sinh(P \cdot (l_{ij} - l_{if}))}{\sinh(P \cdot l_{if})} \right\|_2^2$ . According to Newton Method, the iterative procedure from the  $v^{\text{th}}$  step to the  $(v+1)^{\text{th}}$  step is,

$$l_{if}^{v+1} = l_{if}^v - (H^T H)^{-1} H^T \left( l_{if}^v \right) \left( \frac{\sinh(P \cdot (l_{ij} - l_{if}^v))}{\sinh(P \cdot l_{if}^v)} - \frac{\Delta \tilde{I}_i}{\Delta \tilde{I}_j} \right), \text{ where } H = \frac{\partial}{\partial l_{if}} \left( \frac{\sinh(P \cdot (l_{ij} - l_{if}))}{\sinh(P \cdot l_{if})} \right).$$



## Numerical Experiments



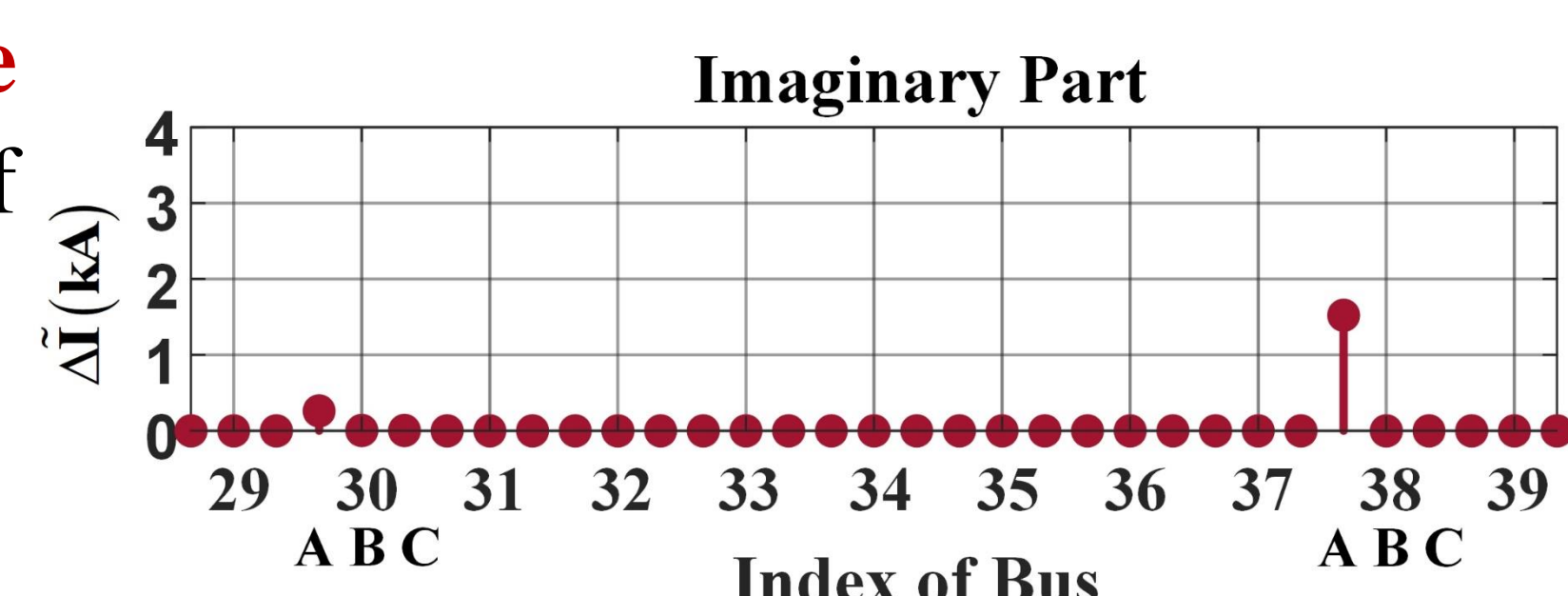
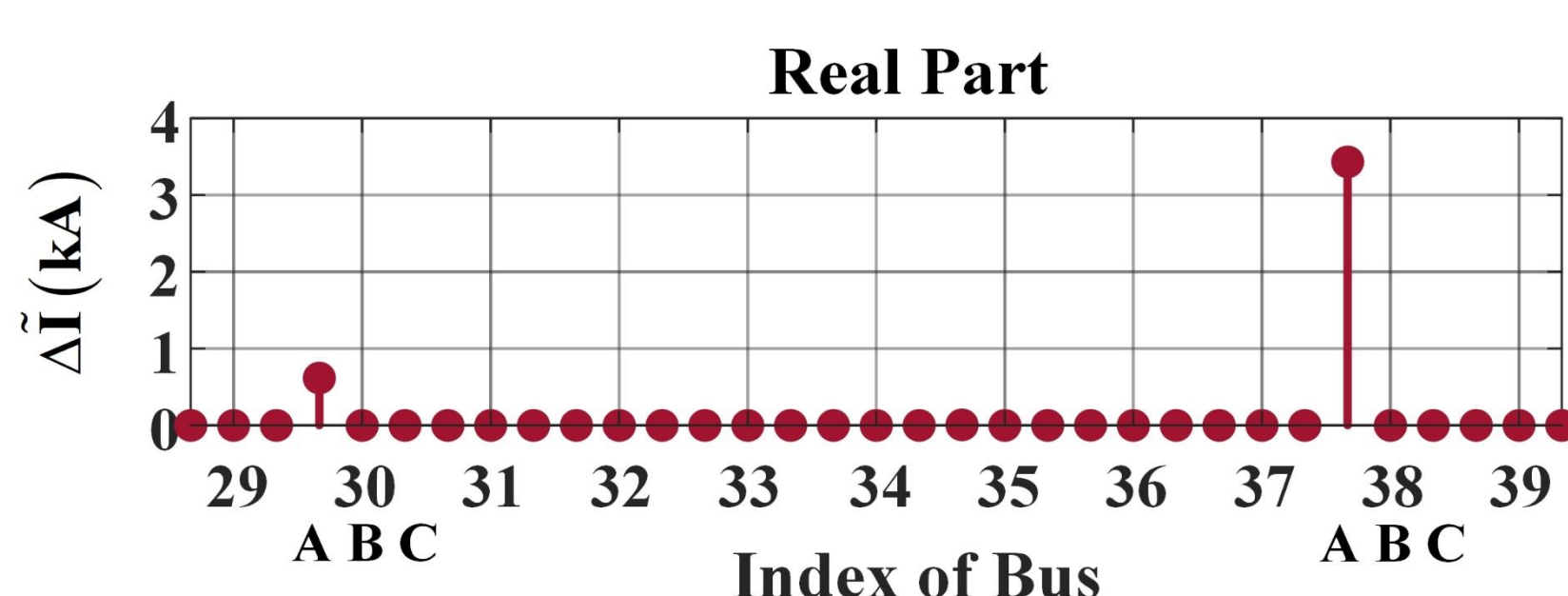
- IEEE118-bus system

- Build in PSCAD

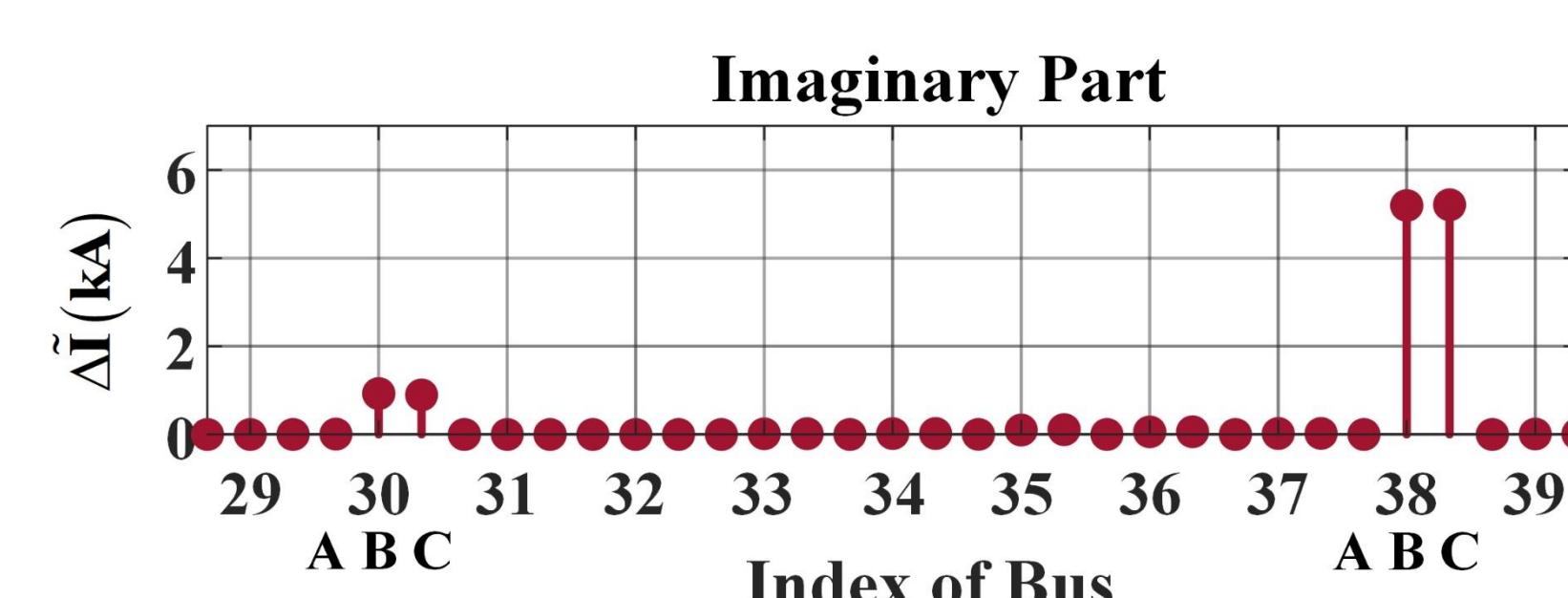
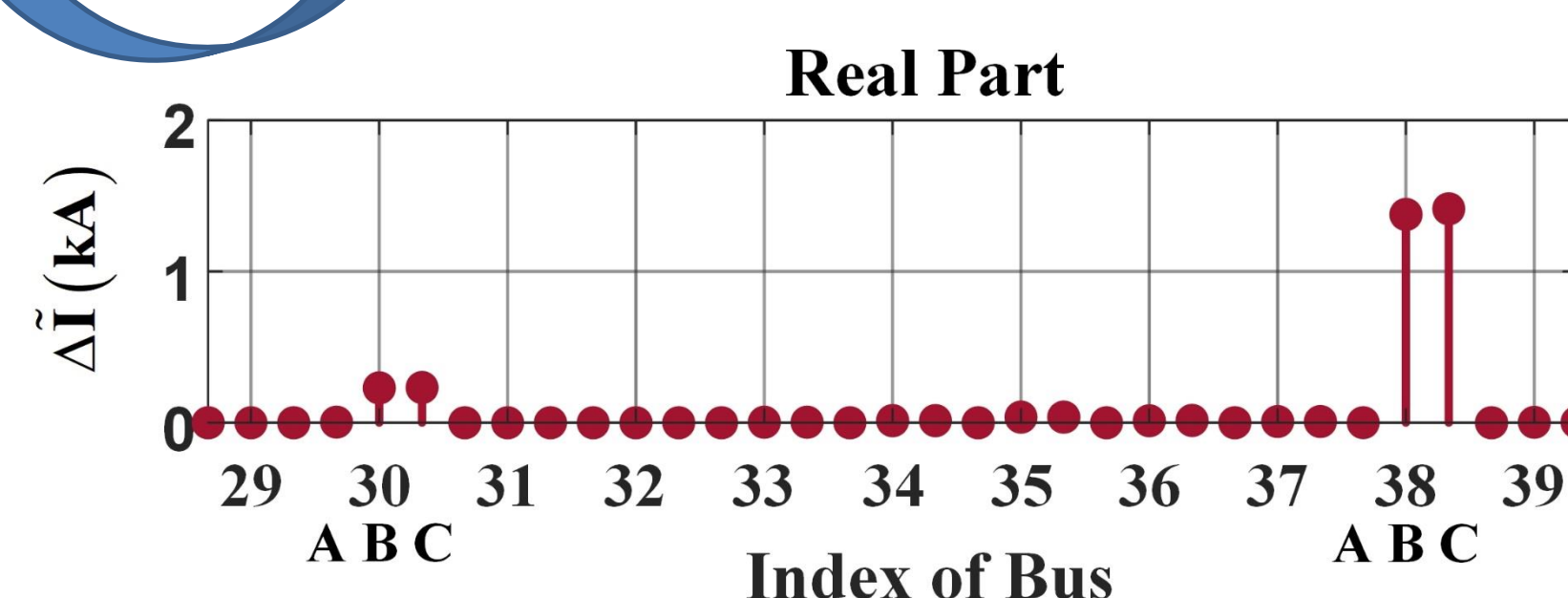
- 28 voltage magnitude measurements out of 118 buses

- Proposed method:

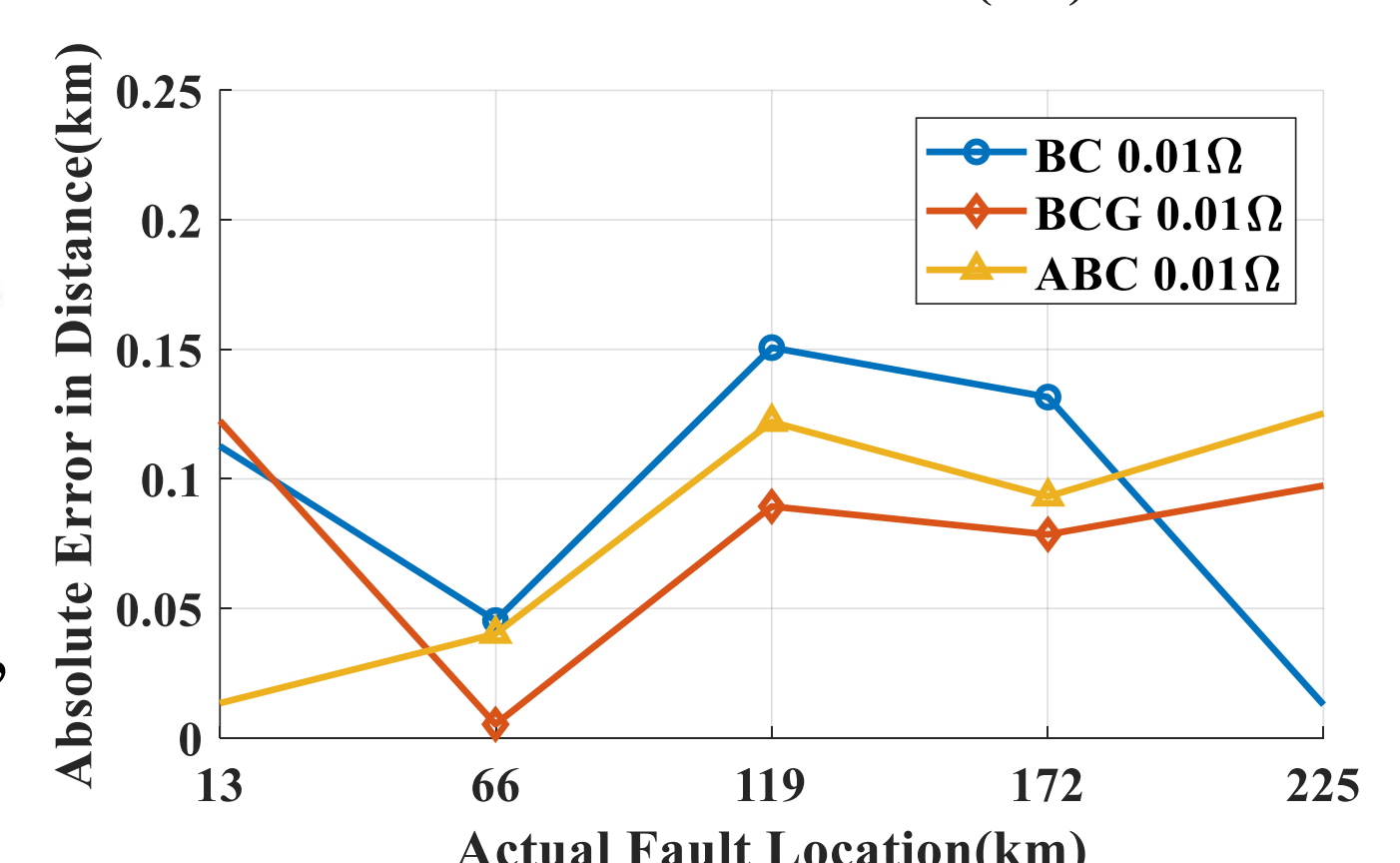
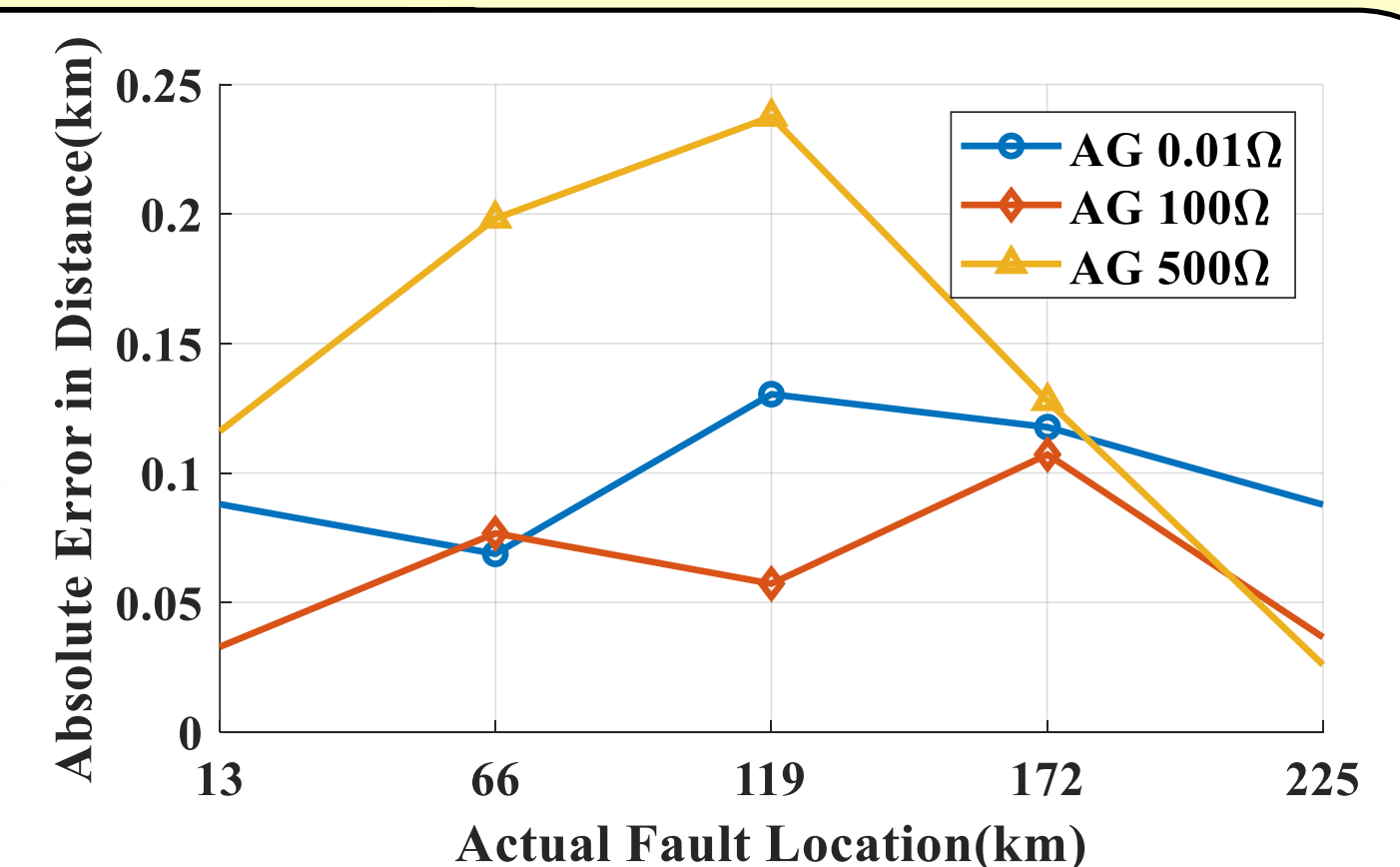
- (1) Identify the faulty line & faulty phases
- (2) Find fault location



Estimated equivalent injection current, 0.01Ω A-G fault, 225 km, line 30-38



Estimated equivalent injection current, 0.01Ω AB-G fault, 225 km, line 30-38



Identify faulty line & phases: 30-38, A-G

Identify faulty line & phase: 30-38, AB-G

Max error: 0.25 km