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Financial Econometrics Case Study for Cisco Systems, Inc.

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Introduction

- Using GARCH and GAS for volatility forecasting using high-frequency stock data from Cisco Systems, Inc.
- Applying realized kernel methods to improve the models
- Model Performance for Estimation
  - Log Likelihood
  - AIC
  - BIC
  - Our innovation: Markov Chain Monte Carlo
- Model Forecasting accuracy
  - Loss Functions
  - Diebold-Mariano Tests
  - Our innovation: Dynamic Time Warping Distance

Data Set: Cisco Systems, Inc. (CSCO)

	In-sample	Out-of-sample
Period	Jan 5, 2018 - Jan 3, 2023	Jan 4, 2023 - Jan 3, 2025
Number of Records	30,514,903	12,238,721
Skewness	0.52	0.21
Kurtosis	51.93	18.61

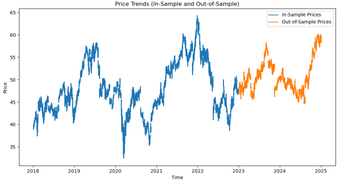
Table: Summary of In-sample and Out-of-sample Datasets

- Positive skewness $\implies$ asymmetry with a fatter right tail
- Positive kurtosis $\implies$ extreme outliers and heavy tails
- Calls for advanced modelling approaches

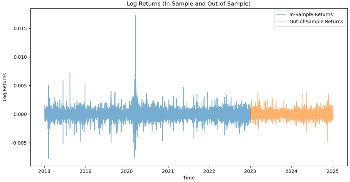
Data Set: Cleaning

- **Data Cleaning Approach (Barndorff-Nielsen et al. (2009))**
  - Restricted to official trading hours (09:30–16:00 EST).
  - Removed anomalous entries such as zero prices and duplicates.
  - Excluded corrected trades
  - Median price of identical time trades, not averages to reduce outliers.

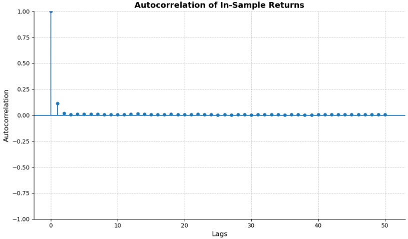
Data Set: Preliminary Analysis



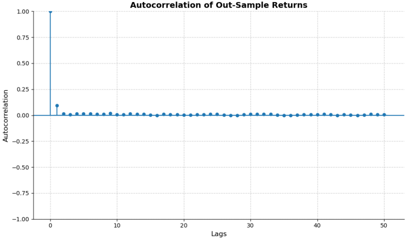
(a)



(b)

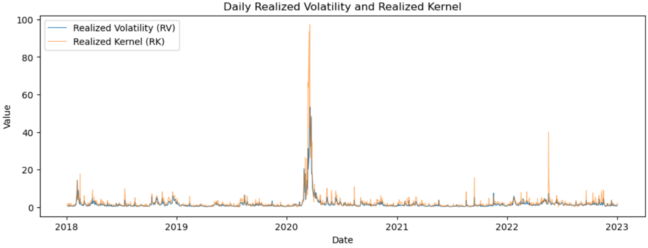


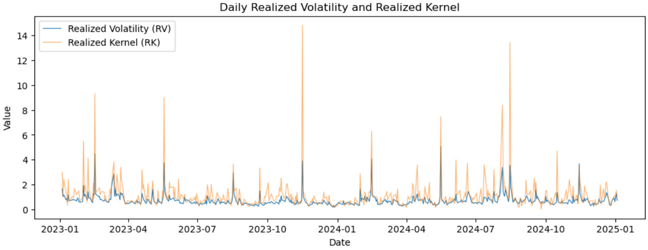
(c)



(d)

Realized Kernels Estimation





Methodology

- Realized Kernel versus Daily Realized Volatility
- Distributions
  - Normal Distributions
  - Student-t Distributions
  - Skewed Student-t Distributions
  - Markov Chain Monte Carlo
- Model Types
  - GAS model
  - Realized GAS model
  - GARCH
  - GJR-GARCH
  - EGARCH
  - Realized GARCH
  - RiskMetrics Model

Methodology: Model Types Explained

GAS Model

The updating equation:

$$f_{t+1} = \omega + \alpha s_t + \beta f_t, \quad \textit{with} \quad s_t = S_t \times \nabla_t \quad \textit{and} \quad \nabla_t = \frac{\partial \ln p(r_t | f_t, \mathcal{F}_t; \theta)}{\partial f_t}$$

In this study, $S_t = 1$ such that $s_t = \nabla_t$ and $f_t = \log(h_t)$ such that

$$\nabla_t = \frac{\partial \ln p(r_t | f_t, \mathcal{F}_t; \theta)}{\partial h_t} \times h_t.$$

Realized GAS is defined as

$$f_{t+1} = \omega + \beta f_t + \alpha\Big(\frac{\nu_1}{2}\Big(\frac{X_t}{exp(f_t)} - 1\Big) + \nabla_t\Big),$$

Methodology: Model Types Explained

GJR-GARCH

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \gamma I_{t-1} \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

$$I_{t-1} = \begin{cases} 1, & \text{if } \epsilon_{t-1} < 0 \text{ (negative shock)} \\ 0, & \text{if } \epsilon_{t-1} \geq 0 \text{ (positive shock)} \end{cases}$$

EGARCH

$$\log(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - \mathbb{E}(|z_{t-1}|)) + \gamma z_{t-1} + \beta \log(\sigma_{t-1}^2),$$

Realized GARCH

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma z_{t-1} + \alpha(|z_{t-1}| - \mathbb{E}(|z_{t-1}|)),$$

$$x_t = \xi + \varphi \log(\sigma_t^2) + \tau(z_t) + u_t,$$

Methodology: Model Types Explained

Bayesian estimation

Metropolis-Hastings

$$\alpha = \min\left(1, \frac{p(\Theta^* \mid \text{data})}{p(\Theta \mid \text{data})}\right)$$

RiskMetrics Model

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2, \quad \text{where} \quad \lambda = 0.94$$

Results: Model Assessment Criteria

Model	Log-likelihood	AIC	BIC
GARCH-Norm	-2423.9562	4853.9125	4869.3219
GARCH-STD	-2317.7402	4643.4803	4664.0263
GARCH-SSTD	-2317.1373	4644.2746	4669.9570
EGARCH-Norm	-2394.3986	4796.7972	4817.3431
EGARCH-STD	-2302.1877	4614.3754	4640.0578
EGARCH-SSTD	-2301.6993	4615.3986	4646.2175
GJR-GARCH-Norm	-2401.7283	4811.4567	4832.0026
GJR-GARCH-STD	-2312.5113	4635.0226	4660.7050
GJR-GARCH-SSTD	-2311.9425	4635.8850	4666.7039
R-GARCH-RK-Norm	-2387.0278	4790.0556	4831.1475
R-GARCH-RK-STD	-2294.4192	**4606.8385***	**4653.0668***
R-GARCH-RK-SSTD	-2309.3148	4638.6297	4689.9945
GAS-Norm	-22220.6382	44447.2764	44462.6859
GAS-STD	-8584.7785	17177.5571	17198.1030
GAS-SSTD	-8582.9529	17175.9058	17201.5883
R-GAS-Norm	-6268.3751	12544.7502	12565.2961
R-GAS-STD	-3323.0813	6656.1627	6681.8451
R-GAS-SSTD	-3323.0242	6658.0484	6688.8673

Results: Model Forecasting (Out-Of-Sample Performance)

- Loss Functions (MSE, RMSE, MAE, MAPE)
  - Best-performing model is the realized kernel GAS model with the normal distribution
  - Among the GARCH models, models with realized kernel are significantly better than traditional GARCH models
- Diebold-Mariano Test
  - R-GAS-Norm significantly outperforms others for MSE and RMSE.
  - GAS-STD and GAS-SSTD lead MAPE but without statistical superiority over R-GAS-Norm.

All the Forecasting Results

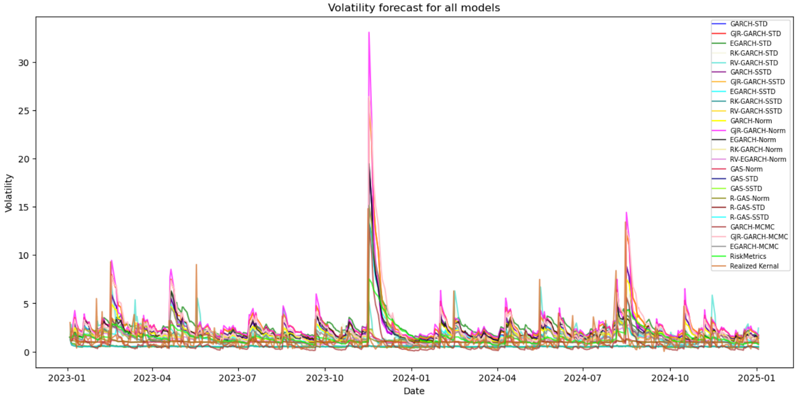


Figure: Forecasting results for all Models

- Some are smoother, others are better at capturing high fluctuations

Two Forecasting Results and a more dynamic measure Dynamic Time Warping Distance

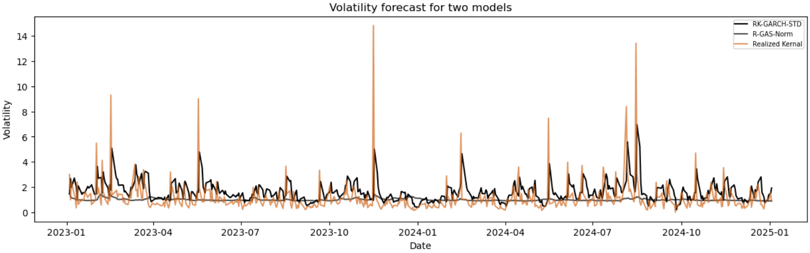


Figure: Forecasting comparison for RK-GARCH-STD and R-GAS-Norm

- DTW distance: complementing traditional metrics
- RK-GARCH-STD is the top model under DTW, aligning well with observed fluctuations in the realized kernel

Model performance

Table: Overall Model Performance

Model	MSE	RMSE	MAE	MAPE	DTW Distance	Score
RK-GARCH-STD	2.135623	1.461377	0.916653	1.148057	18.141753	23.803463
RK-GARCH-SSTD	1.819745	1.348979	0.784579	1.018624	23.235776	28.207703
GARCH-MCMC	2.829283	1.682047	0.836653	0.742975	22.535220	28.626179
RK-GARCH-Norm	1.813989	1.346844	0.792369	1.056644	24.033994	29.043840
R-GAS-Norm	1.677341	1.295122	0.609589	0.704173	27.829693	32.115918
RiskMetrics	2.624715	1.620097	0.919522	1.333152	25.651595	32.149081
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Conclusion and Future Research

- Models with realized kernels outperform. Realized kernel GAS (normal) excels in loss functions, while realized kernel GARCH (Student-t) captures dynamic trends better, with choice depending on prioritizing volatility differences or trend alignment.
- Future research should apply Markov Chain Monte Carlo estimation to GAS and GARCH models with realized kernels and explore more efficient realized kernel computations.

Thank you! Questions?