Optimization Problem Formulation

Objective Function

$$\text{minimize} \quad \sum_{u \in U} \sum_{t \in T} s_{u,t}$$

Decision Variables

- $s_{u,t}$: Binary variable indicating power reduction status.
- $y_{u,t}, z_{u,t}$: Binary variables for activation and deactivation of reduction actions.
- $P_{i,\phi,t}, Q_{i,\phi,t}$: Active and reactive power demands at bus i, phase ϕ, and time t.
- $u_{i,t}$: Voltage magnitude squared at bus i, time t.
- $S_{ij,t}$: Apparent power flow on branch (i,j) at time t.

Constraints

A. User Demand Constraints

1. **Active Power Demand:**

$$P_{i,\phi,t} = s_{u,t} P_u^{\text{gtd}} + (1 - s_{u,t}) P_{u,\phi,t}^{\text{fx}}, \quad \forall u \in U, \phi \in \Phi_i, t \in T$$

2. **Reactive Power Demand:**

$$Q_{i,\phi,t} = s_{u,t} Q_u^{\text{gtd}} + (1 - s_{u,t}) Q_{u,\phi,t}^{\text{fx}}, \quad \forall u \in U, \phi \in \Phi_i, t \in T$$

3. **Apparent Power Demand**

$$S_{i,\phi,t} = P_{i,\phi,t} + iQ_{i,\phi,t}$$

B. Network Power Flow Constraints

1. **Power Flow Balance:**

$$\sum_{i \in B^+} \text{diag}(S_{ij,t}) + S_{j,t} = \sum_{k \in B^-} \text{diag}(S_{jk,t}), \quad \forall j \in B, t \in T$$

2. **Voltage Approximation (Ohm's Law):**

$$u_{j,t} = u_{i,t} - S_{ij,t} Z_{ij}^H - Z_{ij} S_{ij,t}^H, \quad \forall (i,j) \in L, t \in T$$

3. **Voltage Limits:**

$$U_{\min}^2 \leq |u_{i,t,\phi}| \leq U_{\max}^2, \quad \forall i \in B, \phi \in \Phi_i, t \in T$$

4. **Thermal Limits:**

$$|S_{ij,t,\phi}| \leq S_{ij}^{\max}, \quad \forall (i,j) \in L, \phi \in \Phi_{ij}, t \in T$$

C. User Comfort Constraints

1. **Maximum Number of Reductions Per Day:**

$$\sum_{t \in T} y_{u,t} \leq \eta_u, \quad \forall u \in U$$

2. **Maximum Reduction Duration:**

$$\sum_{t^* \in T^*(t,\alpha_u)} s_{u,t^*} \leq \alpha_u, \quad \forall u \in U, t \in T$$

3. **Minimum Interval Between Reductions:**

$$\sum_{t' \in T^*(t,\delta_u)} z_{u,t'} \leq 1 - s_{u,t}, \quad \forall u \in U, t \in T$$

4. **Binary Transition Consistency:**

$$s_{u,t} - s_{u,t-1} - y_{u,t} + z_{u,t} = 0, \quad \forall u \in U, t \in T \setminus \{0\}$$

D. Linearization of Power Flow:

1. **Off-Diagonal Approximation:**

$$S_{ij,t} = \gamma \cdot \text{diag}(S_{ij,t}), \quad \forall (i,j) \in L, t \in T$$

$$\gamma = \begin{bmatrix} 1 & \alpha^2 & \alpha \\ \alpha & 1 & \alpha^2 \\ \alpha^2 & \alpha & 1 \end{bmatrix}, \quad \alpha = e^{-i2\pi/3}$$

Optimization Problem Summary

Objective Function:

$$\text{minimize} \quad \sum_{u \in U} \sum_{t \in T} s_{u,t}$$

Subject to:

1. User Demand Constraints (A.1, A.2)
2. Network Power Flow Constraints (B.1–B.4)
3. User Comfort Constraints (C.1–C.4)
4. Linearization of Power Flow (D.1)