Predicting Unseen Links Using Learning-based Matrix Completion

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***Abstract*—Researchers have noticed the AS-level Internet topology that can be observed from the current measurement infrastructure is far from complete, which means researchers have to deploy more measurement vantage points (VPs) and conduct measurements for more source/destination pairs to fully understand the whole Internet. Unfortunately, it is known that blindly deploying more points and conducting more measurements to achieve the goal is inefficient, if not infeasible. In this paper, we try to improve the efficiency by predicting where unseen AS links might be located from the observed AS paths to guide the measurements towards a more complete AS-level topology. We formulate the prediction of unseen links as a matrix completion problem. However, the traditional matrix completion methods have limited learning capacities and cannot deal with the complex constraints on the underlying topology. We develop a learning-based matrix completion method specifically for the unseen AS link prediction problem. The method exploits a neural network and utilizes side-information which is carefully chosen from AS attributes based on our understanding on Internet peering practices, therefore our method is able to learn more expressive latent vectors and achieves outstanding prediction performance in our scenario. Experiments performed on a real-world dataset show the prediction results can achieve a high AUC (Area Under the Receiver Operating Characteristic Curve) of 0.834.**

***Index Terms*—AS-level Internet Topology, Link Prediction, Matrix Completion, Side-Information**

I. Introduction

Obtaining an accurate and complete picture of the Internet's inter-domain connectivity structure is important for understanding, operating, and diagnosing the Internet [1]–[3]. Due to the distributed nature of the Internet, such a complete AS-level topology is not readily available. Despite significant efforts in distributedly measuring Internet connectivity, the AS-level topology observed from current measurement data is still far from complete [1], [4], [5], leaving many topology "dark areas". Researchers [6], [7] have estimated that there remains a considerable amount of unseen AS links to be further revealed.

To improve the completeness of AS-level topology, researchers have focused on deploying more VPs and conducting measurements for more source and destination pairs [6], [8], [9]. Each candidate VP location may provide varying degrees of redundant view of the Internet topology compared with known VPs, and the resources that the VP can be used for conducting measurements may be limited. Therefore, blindly selecting VP deployment locations and conducting measurements may consume a lot of resources but not bring equivalent topological completeness improvements. Unlike such opportunistically conducted measurements, researchers [7], [10], [11] recently launched targeted measurements with the help of specific data sources. The new links discovered by each of these works are of the same type due to the limitation of the data source it uses. For example, Augustin et.al [7] use IXP data, so they can only discover unseen AS links at IXPs. Different from their works, we are trying to use multi-source data to help predict general unseen AS links for further efficiently finding them. We believe our algorithm can provide valuable information and it can also be viewed as complementary to the topology measurement community.

The establishment of BGP connections between ASes is affected by complex factors, making the AS link prediction task extremely challenging. We notice that the observed AS paths do not only contain directly observed AS connectivity information but also contain a lot of implicit connectivity information, which provides an opportunity to overcome the challenge. Each observed AS path is generated by routing on the complete AS-level topology, so these paths may reflect a lot of constraints on the complete topology. By integrating a large number of such constraints, we have the potential to obtain Internet-scale implicit connectivity information that has not been observed before.

Learning from a set of observed AS paths to help "fill in" unseen connectivity information between ASes is suitable to be formulated as a matrix completion problem. Traditional matrix completion methods have limited learning capacities to deal with the complex constraints on the underlying topology that are reflected by measured AS paths. Despite researchers have developed several completion methods with stronger learning capabilities by integrating neural networks [12], [13], these methods are developed for recommended systems. They do not incorporate the understanding of Internet peering practices knowledge, thereby cannot perform well in our problem.

In this paper, we develop a side-information assisted GMF-based matrix completion method (SGMF) for Internet-scale unseen link prediction. The method inherits the powerful learning capability of neural networks in the Generalized Matrix Factorization model (GMF) [12] to learn from observed AS paths, and integrates well-designed AS attributes related

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to the inter-domain topology and routing as side-information. As a result, our SGMF method can learn more expressive AS latent vectors that are jointly contributed by multiple sources of data. Experimental results show the SGMF method can achieve a higher average AUC of 0.834, which improves the link prediction performance by about 17% to 44% than other state-of-art completion methods. Moreover, a validation using multiple datasets indicates our predicted AS link results have the potential to bring significant topological completeness improvements of observing at least 64% of all unseen links.

The rest of this article is structured as follows. We review the related work in §II. The problem formulation and feasibility analysis are presented in §III. In §IV, we introduce the side-information assisted GMF-based matrix completion method. Experimental results are presented in §V. And §VI concludes the paper.

II. Related Work

Researchers have attributed the incompleteness of AS-level topology to limited measurement capability [1], [4], [14], which has motivated reinforcing measurement infrastructures to conduct more measurements. DIMES [8] recruits volunteers to install agents to support measurements towards the entire IP prefix space. By means of BitTorrent extensions, Dasu leverages volunteer clients to perform traceroutes randomly to a subset of connected peers [6], [9]. However, due to the lack of effective guidance, these works neither carefully select VP deployment locations nor carefully select source/destination pairs. The problem of inefficiency in blindly conducting more measurements to improve topology completeness has been reported [6], [15], [16]. Shavitt et.al [16] show that the number of new AS links observed by adding DIMES VPs and conducting more measurements decreases as the number of DIMES VPs increases. Moreover, Faggiani et.al [6] find the deployment of measurement infrastructures including DIMES, Portolan and Dasu is not optimal, which causes some VPs to provide a lot of redundant connectivity information.

Some researchers focus on specific types of AS links and explore data sources about them to effectively find more links [1], [7], [10], [11], [17]. For example, the emergence of interconnections at IXPs brings IXP data (including prefixes, geographic locations, list of their members and BGP communities) into play in discovering more AS links [7], [10], [17]. With the increase of interconnections at colocation facilities, researchers [11] begin to use publicly available data about facilities for guiding performing localized measurements. In addition to the above data, observed AS paths can also reveal information about missing links. Jin et.al [1] mine BGP routes to learn characteristics of detour couples with corresponding singleton AS links to predict missing AS links whose endpoints share similarities with detour couples in their training set. Although workable, the above works are limited in that they can only help discover specific types of missing links (e.g. IXP-specific AS links). In contrast, we aim to predict general unseen AS links based on observed AS paths and other multi-source data for guiding the efficient discovery of them. The problem of predicting general unseen links at the router level based on observed paths has been proposed and studied in [18]. Different from our problem, it can be solved well without requiring considering complex inter-domain connection and routing constraints.

Missing data prediction problems have been widely formulated as matrix completion problems (such as traffic matrix completion, performance matrix completion) [19]–[21]. For years, various general matrix completion methods have been developed by researchers, including nuclear norm minimization methods [22], [23], non-convex relaxation methods [24] and matrix factorization methods [25], [26]. Among them, the matrix factorization methods are more prominent in terms of efficiency and scalability. However, these factorization methods, which regards matrix entries as a linear combination of row and column latent vectors, have limited learning capabilities. Recently, the emergence of neural networks makes it possible to enhance the learning capability of the traditional matrix factorization methods [12], [13]. For example, He et.al [12] use neural network architecture to express and generalize the matrix factorization model and gain better performance. But these improved completion methods cannot solve our problems well due to the lack of effective side-information. In this paper, we develop a particular learning-based matrix completion method for unseen AS link prediction by integrating a neural network and multi-source information related to Internet peering practices.

III. Problem and Feasibility Analysis

A. Problem Formulation

From current measurement infrastructures, we can obtain a set of observed AS paths. Each AS path tells us some explicit AS connectivity information, including hop counts between partial AS pairs and directly observed links between partial AS pairs (with hop counts equal to one). However, there remain a large number of unseen AS connectivity information to be shedded light on.

In the work, our problem is to predict unseen AS links given a set of observed AS paths. Specifically, we define an AS hop count matrix $\mathbf{H}_{N\times N}$ to hold the observed AS paths, where N denotes the number of ASes appearing in the paths. The entry of $\mathbf{H}$ in the i^{th} row and j^{th} column $\mathbf{H}_{i,j}$ is the shortest hop distance from AS n_i to AS n_j if there is at least one path through n_i to n_j, and otherwise $\mathbf{H}_{i,j}$ is unknown. By recording these AS paths and their directly observed shortest hop counts between partial AS pairs, $\mathbf{H}$ contains lots of routing constraints on the underlying complete topology. The commonly shared topology and routing constraints make hop distances between ASes normally have strong correlations, i.e. the AS hop count matrix should be low-rank. Therefore, it is possible to apply *Matrix Completion* methods to leverage known entries in $\mathbf{H}$ to recover all unknown hop count entries.

After recovering a complete AS hop count matrix $\mathbf{H}'_{N\times N}$, we now use it to predict unseen AS links. One simple methodology would be to divide the recovered hop counts with a threshold T. We assume that the recovered hop distance between two ASes is inversely proportional to the likelihood of the existence of links between the two ASes. Given T, we predict a complete AS connectivity matrix $\mathbf{A}_{N\times N}$ from the recovered matrix $\mathbf{H}^{'}$, in which $\mathbf{A}_{i,j}=0$ (no link between AS n_i and AS n_j exists) if $\mathbf{H}^{'}_{i,j}$ is larger than T and otherwise, $\mathbf{A}_{i,j}=1$ (a link between AS n_i and AS n_j exists). As a result, the unseen part of the complete AS-level topology can be seen from the predicted matrix $\mathbf{A}$.

B. The Feasibility of AS Hop Count Matrix Completion

According to the theory of matrix completion, only low-rank matrices can be accurately recovered [27]. In this section, we will present the low-rank feature of AS hop count matrix to illustrate the feasibility of matrix completion in our scenario.

Considering that it is hard to correctly estimate the rank of AS hop count matrix with only partial known entries, based on the inter-domain routing model proposed by Gao [28] et.al, we simulate a full AS hop count matrix that can reflect real-world Internet routing. Specifically, we use the CAIDA's AS Relationships dataset (including Internet-scale AS links and corresponding AS relationships) in January 2021 [29] as the basis of the model. Then we model each AS in the dataset as a software router which runs BGP with a configuration of the common routing policies. In terms of import routing policies, we set three standard rules in order: (1) local preference (i.e., prefer announcements from customers, over those from peers, and those from providers), (2) shortest AS path, (3) breaking ties with the smallest ASN. As for export routing policies, we set it as the commonly assumed valley-free routing. In this way, we can infer AS paths between any pair of ASes, and then construct a full AS hop count matrix $\mathbf{S}_{N\times N}$ ($N=70,917$).

To check whether $\mathbf{S}_{N\times N}$ is low-rank, we use singular value decomposition (SVD) [27], [30] to decompose it as $\mathbf{S}=U\Sigma V^T$, where U and V are both $N\times N$ unitary matrix, and Σ is a $N\times N$ diagonal matrix. The diagonal entries of Σ are singular values of the matrix $\mathbf{S}$ arranged in descending order (i.e. $\Sigma=diag(\sigma_1,\sigma_2,\sigma_3,...,\sigma_N),\sigma_1\geq\sigma_2\geq...\geq\sigma_N$). According to PCA (Principal Components Analysis), if top k singular values of $\mathbf{S}$ capture almost all the total variance, i.e. $\sum_{i=1}^{k}\sigma_i^2\approx\sum_{i=1}^{N}\sigma_i^2$, we regard k as the approximate rank of $\mathbf{S}$ [27], [31]. If $k\ll N$, the matrix $\mathbf{S}$ is low-rank. Figure 1 plots the percentage of the total variance ($g(k)=\sum_{i=1}^{k}\sigma_i^2/\sum_{i=1}^{N}\sigma_i^2$) that can be captured by top k singular values. It can be seen that the top 100 singular values can occupy 98.8% variance of the matrix $\mathbf{S}_{N\times N}$ ($N=70,917$), showing the AS hop count matrix is low-rank.

IV. AS Side-Information Assisted GMF-Based Completion Method

The low-rank feature makes it possible to estimate all unknown AS hop count entries given a partially known hop count matrix. If we apply traditional matrix factorization (MF) methods to complete the unknown AS hop count matrix $\mathbf{H}$ with rank R, the basic idea of these methods is to map each row AS n_i to a vector of latent feature $x_i\in\mathbb{R}^{1\times R}$ and each column AS n_j to a vector of latent feature $y_j\in\mathbb{R}^{1\times R}$, so that

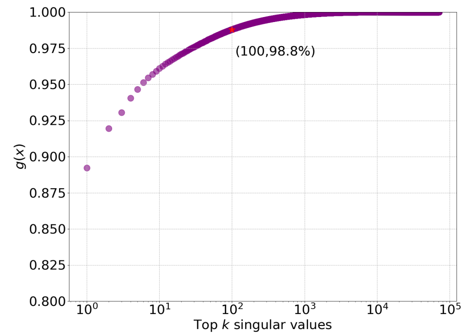

Fig. 1. The percentage of the total variance that can be captured by the top k singular values.

their inner products fit all known matrix entries closely. The specific mathematical expression is as follows:

$$\min_{x_i,y_j}\sum_{(i,j)\in\Omega}(x_iy_j^T-\mathbf{H}_{i,j})^2+\lambda(\|x_i\|^2+\|y_j\|^2) \quad (1)$$

where Ω denotes the set of locations of known AS hop count entries in the matrix and λ is the regularization coefficient that controls the extent of the regularization to avoid over-fitting. After mapping, each unknown entry $\mathbf{H}_{i,j}$ can be calculated by the inner product $x_iy_j^T$ of corresponding latent vectors. The traditional MF methods only regard hop counts as a linear combination of AS latent vectors, while the interactions between latent vectors are in fact more complex than that. To tackle the limited learning capability of the inner product, He et.al [12] develop a GMF method by replacing the inner product with neural network architecture. Although GMF can be applied in our problem to help capture a more complex relationship between AS latent vectors and hop counts, it fails in integrating valuable side-information to learn more expressive AS latent vectors.

In this section, we present our insight into the design of AS side-information and propose the SGMF framework which integrates side-information into the GMF model [12] to enhance prediction performance.

A. AS Side-information

We notice that there are multi-source information related to the inter-domain topology and routing, such as geographical presence, and peering strategies, which are valuable for recovering unknown AS hop counts. Based on our understanding of the nature of Internet interconnection, we carefully select several AS features as side-information as follows.

- *AS tier feature*: AS tier indicates the location of the AS in the Internet hierarchy. In general, ASes in the same or similar tiers are easier to establish interconnections. Therefore, we include a feature that records the type of AS tiers, including clique (with no providers), high tier (customers of clique ASes with a degree larger than 100), low tier (not clique, high tier or stub ASes), and stub tier (with no customers).

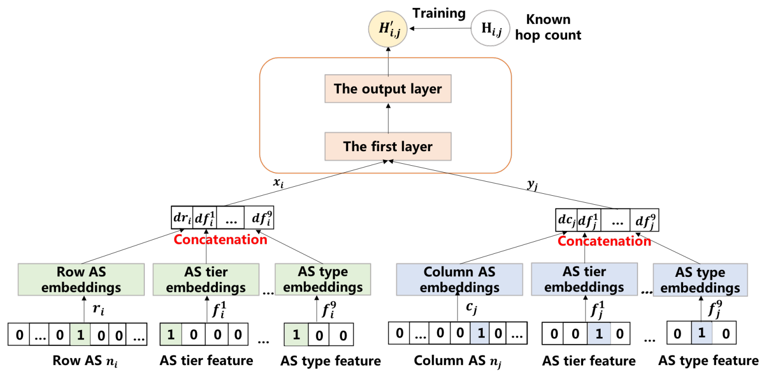

Fig. 2. The framework of SGMF

- *Traffic information features*: The AS traffic information features, i.e. the overall traffic volume feature and traffic ratio feature, are also important factors in the inter-domain interconnection assessment. We use approximate traffic levels (which range from 0-20Mbps to 100+Tbps in 18 distinct bins) as the volume feature and approximate traffic ratios (which are divided into five classes, e.g. heavy inbound, mostly outbound) as the ratio feature. We collect these features of ASes from PeeringDB [32].
- *Peering policy features*: The peering policy features of an AS, including the general policy feature, multiple locations requirement feature, traffic ratio requirement feature and contract requirement feature, will determine which ASes it will peer with. For example, the general policies of ASes are usually divided into *Open* (peer if possible), *Selective* (peer under several additional constraints), or *Restrictive* (peer only by necessity). These features are useful in estimating unknown hop counts between ASes and can be obtained from PeeringDB.
- *Geographic scope feature*: Whether geographic scopes of ASes overlap may affect the establishment of BGP connections between ASes. Therefore, we introduce a geographic scope feature to record the scopes of ASes self-reported to PeeringDB, including global, regional, or more specific regions, e.g. Africa and North America.
- *AS type feature*: This feature takes into account the impact of AS business types on inter-domain topology and routing. We consider there are three AS types: Content ASes (which provide content hosting and distribution systems), Enterprise ASes (which belong to various universities and companies at the network edge that are mostly users of Internet access, transit or content), and Transit/Access ASes (which provide transit or access service). The type feature of each individual AS can be obtained from the CAIDA UCSD AS Classification Dataset [33].

Although PeeringDB is known to contain some errors and be incomplete and the AS Classification Dataset is not so accurate, our SGMF with a neural network can still learn much useful information from the noisy datasets. As experiments in §V-B show, these AS side-information are effective in improving the performance of estimating unknown hop counts and predicting unseen AS links.

B. Framework Design

Based on the above side-information, we develop an AS side-information assisted GMF method (SGMF) by integrating them into GMF to learn more expressive AS latent vectors.

Figure 2 shows the framework of our SGMF method. The input layer consists of row (column) AS identity n_i (n_j) encoded by N-dimension one-hot vector r_i (c_j) and each above-mentoned feature of AS n_i (n_j) encoded by one-hot vector f_i^k (f_j^k) $\in \mathbb{R}^{1\times L_k}$, where k means the k^{th} feature and L_k means the total number of categories contained in the feature. For example, if AS n_i is located in the clique tier, the corresponding AS tier feature vector $f_i^1 = [1, 0, 0, 0]$. Above the input layer is the embedding layer, which stores embeddings to project each sparse one-hot vector into a dense vector. Then the embedded row (column) AS identity vector and corresponding embedded feature vectors are concatenated to construct a more informative row (column) AS latent vector x_i (y_j). Later, we continue to use two neural layers of the GMF model to take the new row AS and column AS latent vectors as inputs and generate an estimated hop count between AS n_i and AS n_j. The mapping function of the first layer is:

$$\phi_1(x_i, y_j) = x_i \circ y_j \tag{2}$$

where $\circ$ denotes the element-wise product of vectors. Then the vector ϕ_1 is taken as input to the output layer to obtain an estimated hop count:

$$\mathbf{H}'_{i,j} = a_{out}((x_i \circ y_j)h^T) \tag{3}$$

where a_{out} denotes the activation function, and h^T denotes the weights of the output layer, which allows varying importance of latent dimensions. Our SGMF method uses the Rectifier function (which is suitable for sparse data and makes the model less likely to be overfitting [12]) as a_{out} and learn h from training data, which has greater learning capability than the inner product used in popular matrix factorization methods.

Compared with machine learning algorithms or even deep neural networks, the above two-layer neural network is more suitable for our problem. First, neural networks usually have a stronger learning ability and can work better than traditional machine learning algorithms in the case of learning from noisy data. Second, considering that the relationship between AS latent vectors and hop counts is not so much complex, we choose to use the neural network with two layers instead of deep neural networks to avoid overfitting.

C. Leaning SGMF

After constructing the framework of SGMF, we need to learn model parameters (e.g. embeddings and h) from known AS hop count entries. To ensure that the estimated hop counts of SGMF fit all known hop count entries closely, we define the loss function as follows:

$$L = \frac{1}{n} \sum_{(i,j)\in\Omega} (\mathbf{H}^{'}_{i,j} - \mathbf{H}_{i,j})^2 \tag{4}$$

Where Ω denotes the set of locations of known AS hop count entries in the matrix and n denotes the number of known entries. Given the loss function, we need an optimization method to learn model parameters to optimize this function. We randomly initialize model parameters with a Gaussian distribution with a mean of 0 and standard deviation of 0.01. To learn optimal model parameters, we perform training by using the mini-batch Adam method to minimize the above mean square error loss between estimated hop counts and known hop counts in the training set. The Adam optimization method is an extension to the stochastic gradient descent algorithm, which outperforms other optimization methods [34].

Then the trained SGMF model can be used to predict hop count and whether a link exists between any pair of ASes by taking their identity and feature vectors as inputs. The smaller the hop count is, the more likely there exists a link between the AS pair.

V. EXPERIMENTAL ANALYSIS

In this section, we conduct experiments to evaluate how the SGMF method performs in the two problems completing AS hop count matrix and predicting unseen AS links.

A. Experimental Settings

Datasets. We extract a set of observed AS paths from the BGP routing table snapshot on March 29, 2021 that are observed by 1,134 VPs in RouteViews [35] and RIPE RIS [36]. The AS path dataset consists of N ($N = 72034$) ASes in total, so a $N \times N$ AS hop count matrix $\mathbf{H}$ is needed to hold these observed AS paths. The matrix $\mathbf{H}$ is severely incomplete, including only 1.46×10^7 known hop count entries and 5.17×10^9 unknown entries. To complete $\mathbf{H}$ and predict unseen AS links in the unknown part, our SGMF algorithm also needs another important AS side-information dataset. Therefore, we calculate AS tier features based on BGP routing table data, collect AS traffic, peering policy and geographic scope features from PeeringDB on April 9, 2021, and collect AS type features from CAIDA on July 1, 2020.

Evaluation Metrics. We use the Root Mean Squared Error (RMSE) to evaluate the overall performance in completing AS hop count matrix. As for the overall performance in predicting unseen AS links, it can be measured by the area under the receiver operating characteristic curve (AUC). It represents the ability of a classifier in distinguishing between links (positive class) and non-links (negative class), which equals the probability that the classifier correctly ranks a randomly chosen positive-negative pair. Then we use the true positive rate (TPR) and false positive rate (FPR) to measure the performance of predicted links in guiding topology measurements. TPR is the fraction of links within the unknown part can be correctly predicted, which is a measure of the benefits introduced by conducting guided measurements. FPR is the fraction of non-links within the unknown part are mistaken as links, which is a measure of the resources that will be wasted in conducting unnecessary measurements. The higher the TPR and the lower the FPR, the more efficient the predicted links in guiding measurements.

Evaluation Methodology. Considering the unknown AS hop counts and unseen AS links are mainly caused by the scarcity of VPs, we randomly divide the 1,134 VPs in Route-Views and RIPE RIS into $\mathbb{T}$ (for training) and $\mathbb{V}$ (for testing), which respectively contain 85% and 15% of all the VPs. How the trained model using observed AS paths from $\mathbb{T}$ performs on unique observations from $\mathbb{V}$ can approximate the performance of the model on Internet-scale unseen observations. This evaluation method has been widely used in route or link prediction literature [1], [37], [38].

More specifically, we use the AS paths observed from $\mathbb{T}$ to construct an AS hop count matrix $\mathbf{H}^t$ as a training set. To test the performance of the trained SGMF model in completing the AS hop count matrix, we use the AS paths observed from $\mathbb{V}$ to construct an AS hop count matrix $\mathbf{H}^v$ and add unique AS hop counts, which are known in $\mathbf{H}^v$ but unknown in $\mathbf{H}^t$, into a hop count test set. To test the performance of the trained SGMF model in predicting unseen links, we need both labeled positive samples (links) and negative samples (non-links). However, it is difficult to obtain non-links because there is no evidence to tell whether there is no link between an AS pair or there exists a link that has not been observed. To deal with the problem, we assume if a large number of AS paths observe that the shortest hop count between an AS pair is greater than one, the AS pair tends to have a non-link. Therefore, we check all unique AS connectivity information in $\mathbf{H}^v$ and add AS links ($\mathbf{H}^v$=1 and $\mathbf{H}^t$ is unknown) as positive samples and non-links ($\mathbf{H}^v$ >1 and $\mathbf{H}^t$ is unknown) as negative samples into a link test set.

We create 20 random splits of the BGP measurement data into training and test sets for cross-validation and the average results over the splits are final evaluation results.

Hyperparameters Settings. We set the dimensions of embedded feature vectors to fixed values equal to the dimensions of corresponding input feature vectors. To determine other hyperparameters (e.g. learning rate, batch size, the dimension of the embedded AS identity vector) of SGMF, we conduct

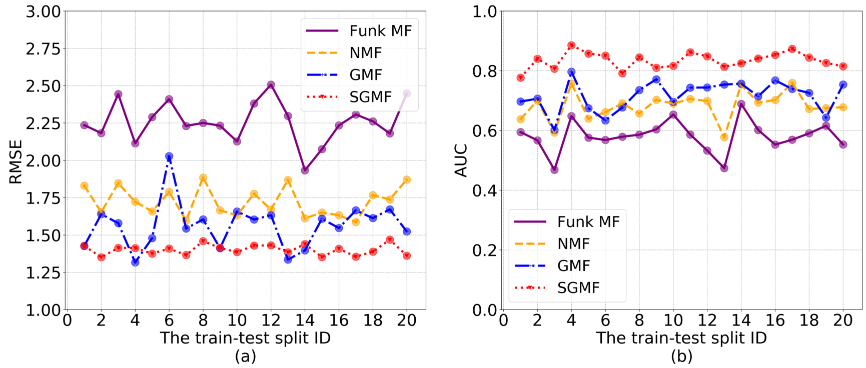

Fig. 3. (a) RMSE of Funk MF, NMF, GMF and SGMF on different train-test splits; (b) AUC of Funk MF, NMF, GMF, SGMF on different train-test splits.

the process of hyperparameter tuning. Specifically, we vary the learning rate in [0.0001, 0.0005, 0.001, 0.005], vary the batch size in [64, 128, 256] and vary the dimension of the embedded AS identity vector in [50, 100, 200]. Under a randomly created train-test split, we train multiple SGMF models with different combinations of the hyperparameters and evaluate their AUC performance on the link test set. We find the AUC performance is optimal when the learning rate is set to 0.0001, the batch size is set to 128 and the dimension of the embedded AS identity vector is set to 50.

B. Comparison with Other Completion Methods

We compare SGMF with other MF-based completion methods and the GMF method without side-information to illustrate the effectiveness of our method. We choose the Funk matrix factorization method (Funk MF)[1] and nonnegative matrix factorization method (NMF) [39] as baseline MF-based completion methods because they are widely used in solving matrix completion problems [20]. Funk MF uses a stochastic gradient descent (SGD) to map rows and columns to corresponding latent feature vectors so that their inner products fit known matrix entries closely. Similarly, NMF uses a stochastic gradient descent with a specific choice of step size to ensure the non-negativity of latent feature vectors. GMF replaces the inner product with a neural network but does not consider any side-information. We have carefully tuned hyperparameters of these methods to optimize resulting AUC values on the link test set which was used for tuning hyperparameters of SGMF.

Figure 3 plots the RMSE and AUC performance of Funk MF, NMF, GMF and SGMF on 20 random train-test splits. We can see that our SGMF method can achieve an average RMSE of 1.40 hops, which is lower than an average RMSE of 2.26 for Funk MF, 1.72 for NMF, and 1.56 for GMF. The SGMF method works better in estimating unknown AS hop counts, which improves the performance by 10.3% even compared with GMF. Besides, the higher the AUC, the better the method is at distinguishing between links and non-links. We observe that the average AUC of our SGMF method (0.834) has an improvement of 43.8% than that of Funk MF (0.580), 22.3% than that of NMF (0.682), and 16.5% than that of GMF (0.716). The improvements indicate SGMF has a better class separation capacity in the AS link prediction problem.

C. Impact of the Number and Distribution of Known Measurement VPs

In addition, the performance of our SGMF algorithm may be influenced by the known AS hop count matrix used for training. To investigate the robustness of the performance of SGMF with respect to known AS hop matrices that are observed from different numbers or distributions of known VPs, we conduct sensitivity analysis in this subsection.

Specifically, for each train-test split, we randomly select 10%, 20%, 30%, 50%, 70% of the known VPs in $\mathbb{T}$ (divided for training) and use their observed AS hop information as training sets. Except for the inputs of AS hop counts, all other initial model parameters and hyperparameter settings are fixed to previously mentioned values. Figure 4(a) shows the performance evaluation results on two random train-test splits. It can be seen that as the number of VPs reserved for training increases from 10% to 50%, the RMSE and AUC performance of trained SGMF models have improved significantly. However, as more randomly selected VPs are added for training, the performance remains stable. The results indicate that randomly deploying more VPs in the future will bring a lot of redundant AS hop count information, which does not help improve the performance of our SGMF model in hop count estimation and link prediction.

To further study the impact of the geographical distribution of known VPs on the algorithm performance, we first need to accurately geolocate known VPs. To this end, we extract city-level locations of VPs from the RouteViews website and use the paid IP2location DB9 database to map RIPE RIS VPs. Then, for each random train-test split, we reserve known VPs in $\mathbb{T}$ that are located in each region Africa (AF), Asia-Pacific (AP), Europe (EU), Latin America (LA), and North America (NA), respectively, and construct corresponding observed AS hop count matrices as training sets.

[1]Proposed in the context of the Netflix Prize by Simon Funk http://sifter.org/simon/journal/20061211.html

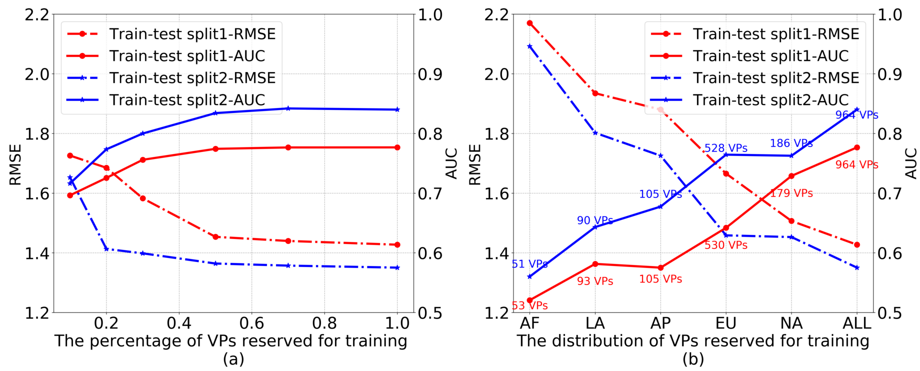

Fig. 4. (a) The RMSE and AUC performance of SGMF with respect to different percentages of VPs reserved for training; (b) The RMSE and AUC performance of SGMF with respect to different distributions of VPs reserved for training.

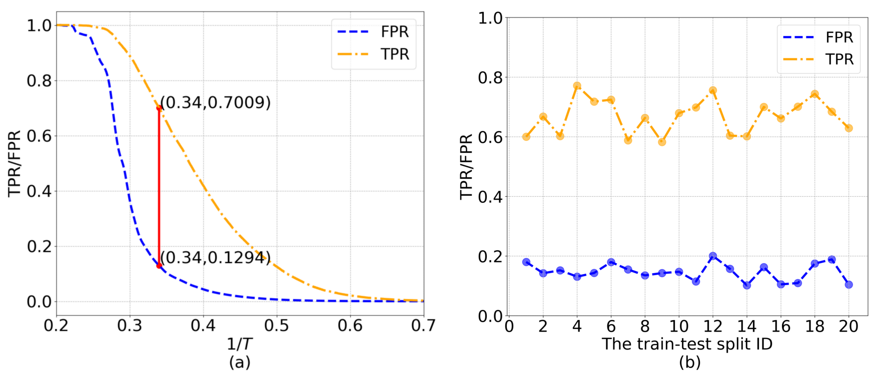

Fig. 5. (a) The FPR and TPR of the trained SGMF model under different T; (b) The distribution of FPR and TPR of trained SGMF models under other different train-test splits with T=2.94.

Figure 4(b) shows the performance of SGMF with respect to different training sets observed from biased distributions of VPs. We can see that the imbalance of known VP locations will influence the RMSE and AUC performance of our SGMF model. Taking the first train-test split as an example, the SGMF model trained on the AS hop count matrix observed from 530 VPs in EU can only achieve an AUC of 0.6420 (an RMSE of 1.6656), which reduces the performance of 17.34% (16.70%) than that of reserving all the known VPs in $\mathbb{T}$. In comparison, as shown in Figure 4(a), an equal number of known VPs (530, more than 50% of the VPs in $\mathbb{T}$) that are distributed in diverse geographic locations can help achieve much better performance. The results indicate that deploying unbiased distribution of VPs in the future will bring much informative AS hop count information, thereby can further improve the performance of SGMF.

D. Unseen Link Prediction Results

Using SGMF, we now have a completed AS hop count matrix. By comparing predicted hop counts with a given threshold T, we will obtain link prediction results. The performance of the predicted links in guiding topology measurements will be evaluated in this subsection.

Figure 5(a) shows the FPR and TPR of the trained SGMF model under different T on a random train-test split. The choice of T gives a trade-off between the proportion of unseen links missed and the proportion of non-links incorrectly predicted as existing. To some extent, we would like to miss a certain proportion of unseen links in exchange for greatly reducing the proportion of non-links that are mistakenly classified as links, thereby saving a large amount of resources that will be wasted in deploying unnecessary VPs and conducting unnecessary measurements. As a result, we choose T to be 2.94 to achieve a TPR of 70.09% and a low FPR of 12.94%. In other words, in the link test set, our SGMF method can filter out 87.06% (1-FPR) non-links, which significantly saves resources of deploying unnecessary VPs and conducting unnecessary topology measurements in trying to discover unseen links. Meanwhile, it can bring significant topology improvements of observing 70.09% unseen links.

To evaluate the generalization ability of the model, we conduct cross-validation and show the distribution of FPR and TPR of trained SGMF models under other 20 random train-test splits with T=2.94 in Figure 5(b). There is an average TPR of 66.87% and an average FPR of 14.65%, which is close to the performance on the above link test set. Later, the trained SGMF model is applied to shed light on the unknown part of the entire AS-level topology (see §V-A), predicting there exist 9.28×10^8 AS links and 4.25×10^9 non-links.

E. Validation Using Different Datasets

Since the Internet is a large-scale distributed system without centralized authoritative control, it is almost impossible to obtain the ground truth of a complete AS-level Internet topology for validation. The dearth of ground truth is notorious in many Internet-scale inference researches [1], [40]–[42] (e.g. AS business relationship inference, Internet path inference and IP2geo inference) and has not been solved well. The above cross-validation leaves partial BGP snapshots out as ground truth and obtains performance evaluation results, which might be questioned that it may overestimate the performance because the training and test sets come from the same dataset. To make the evaluation results more reliable, we leverage multi-source AS topology datasets, such as traceroute-based topology measurements conducted on the Archipelago (Ark) infrastructure and topology collection through Looking Glass (LG) servers, for further validation.

The specific ways to collect AS topology from two data sources CAIDA Ark and LG will be introduced in the following.

- CAIDA Ark: CAIDA deploys about 118 geographically distributed Ark monitors and periodically conducts coordinated traceroute measurements to all the routed /24 networks in the IPv4 address space. In this paper, we use Ark's IPv4 Routed /24 AS Links Dataset [43] on May 17, 2020. The dataset contains direct AS links which have two adjacent hops in traceroute paths and indirect AS links which are separated by one or more unmapped or non-responding traceroute hops. We discard indirect AS links and obtain 74K AS-level links.
- Looking Glass: Many network operators deploy LG VPs and allow users to run some popular measurement commands (e.g. traceroute and BGP) on these VPs. We notice AS paths returned by the *show BGP neighbor ip advertised (or received)* command can help construct the AS-level Internet topology. Therefore, we develop a tool [44] to automate the command querying process to LG sites published on PeeringDB [32], Traceroute.org [45], BGP4.as [46], and BGPLookingglass.com [47]. The tool successfully collects AS-level routing paths from 14 LG VPs and obtains 100K AS links.

After removing the collected AS links that have been observed by the BGP collector projects RouteViews and RIPE RIS, there remain 27K new AS links from the Ark source and 15K new AS links from the LG source. These new AS links can be used to further validate the performance of the predicted AS links in guiding topology measurements.

Figure 6 shows the number of observed AS links, new AS links and successfully predicted AS links from each data source. On the one hand, new links from the two data sources actually verify the existence of a large number of predicted AS links (17K and 11K respectively). Although this is only a lower bound obtained through small-scale measurements, it can indicate the potential high benefits of observing unseen links in the set of our predicted links. On the other hand,

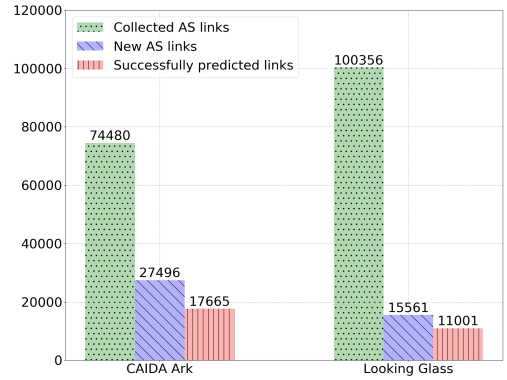

Fig. 6. The number of observed AS links, new AS links and successfully predicted AS links from each dataset.

we can see that a large proportion of new links from the two data sources have been correctly predicted. There are 64.25% new AS links from Ark and 70.70% new AS links from LG in the set of the predicted AS links. The validation results are consistent with the TPR performance (66.87%) obtained by the above cross-validation, which means we have the potential to observe at least 64% of all unseen links on the Internet in the set of our predicted links. In short, whether in terms of quantity or proportion, the validation results show that significant topological completeness improvements will be achieved if we conduct large-scale measurements under the guidance of the predicted AS links.

VI. Conclusion and Future Work

In this paper, we develop an AS hop count matrix completion method SGMF to predict Internet-scale unseen links, thereby guiding measurements towards significant topology completeness improvements. Our SGMF method[2] utilizes the powerful learning capability of neural networks to mine information-rich observed AS paths and integrates well-designed AS side-information to further enhance prediction performance. It can be seen from the experimental results that SGMF improves AS hop count estimation performance and link prediction performance respectively by at least 10.3% and 16.5% compared with other matrix completion methods. Moreover, the cross-validation evaluation and the validation using multi-source datasets both show that our AS link prediction results have the potential to bring great benefits in improving topology completeness. Future work would focus on conducting large-scale measurements under the guidance of our predicted AS links to discover unseen AS links efficiently.

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[2]The source code of our SGMF method is publicly accessible at https://github.com/zhuangshuying18/Unseen_Link_Prediction

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