Approximating the Held-Karp Bound for Metric TSP in Nearly Linear Work and Polylog Depth

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THE HEBREW UNIVERSITY OF JERUSALEM

ETH zürich

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- **APX**-hard [Lampis '14]
- $3/2$ approximation [Christofides '76] [Serdyukov '78]
- $3/2 - 10^{-36}$ approximation [Karlin, Klein, Oveis Gharan '22]

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- Metric TSP on $(G, c) \equiv$ TSP on the metric completion $(\hat{G}, \hat{c})$

$$\hat{G} = \text{Complete graph on } V$$
$$\hat{c}_{uv} = \text{Shortest path length between } u \text{ and } v \text{ in } G$$

Subtour Elimination LP

[Dantzig, Fulkerson, Johnson '54]

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\min\ & \hat{c}^\top x \\
\text{s. t.}\ & \sum_{v} x_{uv} = 2 && \forall\, u \in V \\
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Conjecture: The LP integrality gap is at most $4/3$ [Goemans '95].

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- LP relaxation of the 2-edge-connected spanning multisubgraph problem:

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  - **Multiplicative weight update (MWU)**

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Main Result [KWY '25]

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Framework: Width-independent epoch-based MWU.

[Garg, Könemann '07] [Fleischer '00] [Luby, Nisan '93] [Young '01]

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- Terminate when $\|w\|_\infty$ is big.

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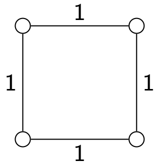

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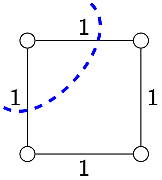

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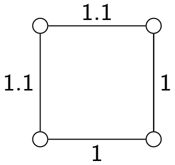

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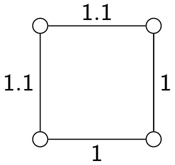

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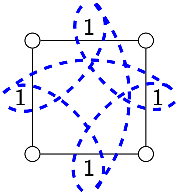

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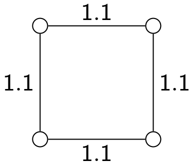

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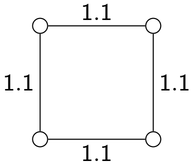

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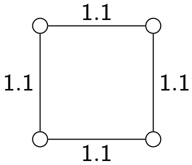

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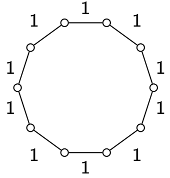
$\Longrightarrow$ $\tilde{O}(1/\varepsilon^4)$ iterations because $|\mathcal{C}^*| = O(n^2)$ for cuts.

Core-Sequence

- Parallel MWU can incur $\Omega(n^2)$ work.



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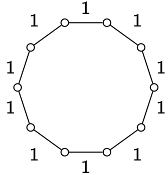

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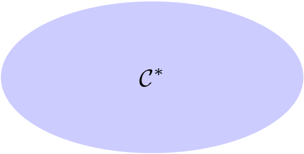

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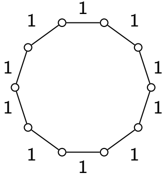

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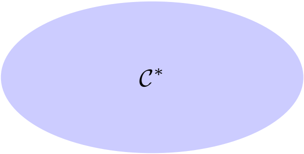

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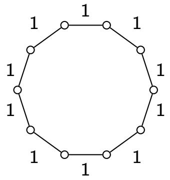

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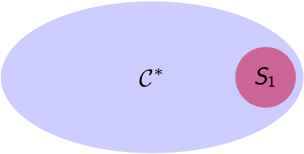

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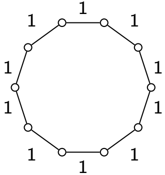

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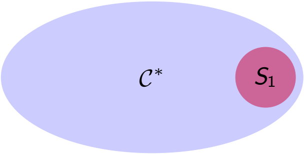

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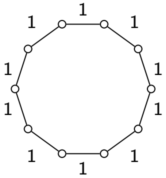

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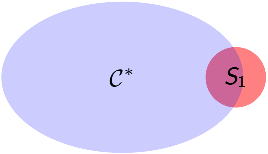

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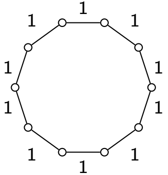

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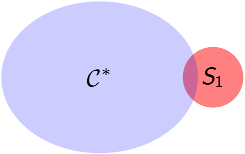

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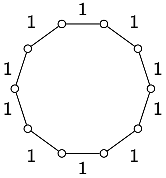

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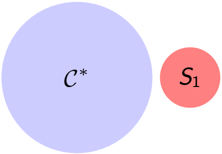

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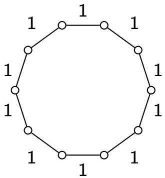

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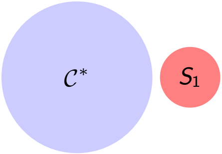

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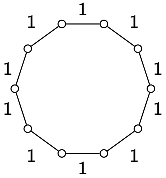

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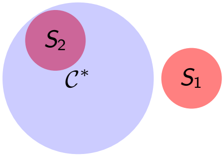

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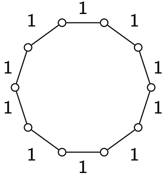

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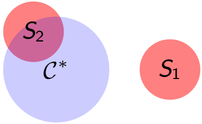

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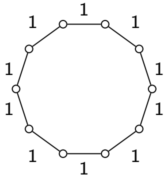

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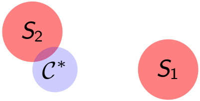

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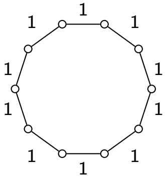

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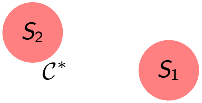

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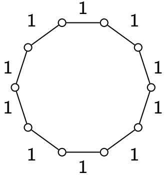

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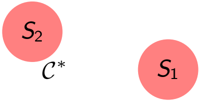

Definition

The sequence $\mathcal{S} = (S_1, \ldots, S_\ell)$ of representative sets is called a core-sequence of the epoch.

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There is a parallel FPTAS for the Held–Karp bound that runs in $\tilde{O}(m/\varepsilon^4)$ work and $\tilde{O}(1/\varepsilon^4)$ depth.

Finding the Core-Sequence

Tree Packing: Compute $O(\log n)$ spanning trees $\mathcal{T}$ such that w.h.p., every cut in $\mathcal{C}^*$ intersects ≤ 2 edges of some $T \in \mathcal{T}$. [Karger '00]

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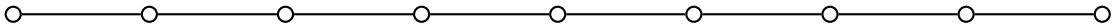

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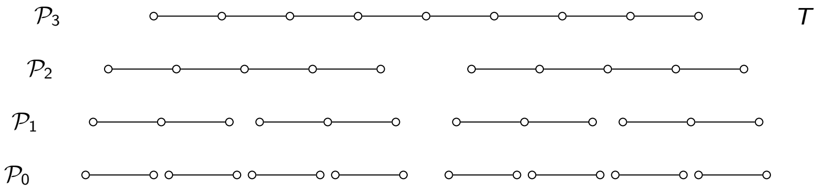

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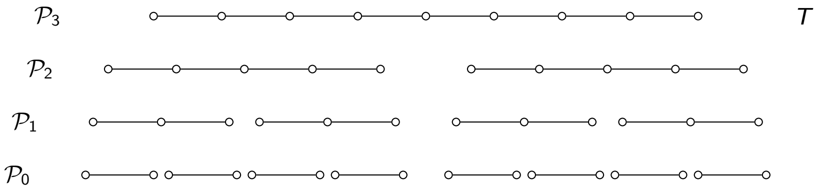

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$$S_i := \{\text{Cuts in } \mathcal{C}^* \text{ that intersect} \leq 2 \text{ edges of } T \text{ on some } P \in \mathcal{P}_i\},$$

then $|S_i| = O(n)$ for all i.

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Thank You!