CSE 360: Workshop 2

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https://github.com/zhubiii/CSE360

1. - The camera takes a 14x8 picture. $(u_0 = 7, v_0 = 4)$ represent the optical center.
   - We simply use the equations that convert plane coordinates to pixel coordinates $u = u_0 + \frac{k_u f X_c}{Z_c}$ and $v = v_0 + \frac{k_v f Y_c}{Z_c}$ and $(u = 12, v = 6)$
     - $12 = 7 + \frac{(21)(1)X_c}{15} \implies X_c = \frac{75}{21} = 3.57$
     - $6 = 4 + \frac{(11)(1)Y_c}{15} \implies Y_c = \frac{30}{11} = 2.73$
   - We know that a rotation matrix can transform a point from one frame to another
   - ${}^qR_c p_c = p_q \implies \begin{bmatrix} cos(\pi/2) & 0 & sin(\pi/2) \\ 0 & 1 & 0 \\ -sin(\pi/2) & 0 & cos(\pi/2) \end{bmatrix} \begin{bmatrix} cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.57 \\ 2.73 \\ 15 \end{bmatrix} = \begin{bmatrix} 15 \\ -3.57 \\ -2.73 \end{bmatrix}$

2. We want to find d_g and we know that the inverse of a Rotation matrix gives us the rotation matrix with the frames flipped with respect to each other
   - ${}^qR_g^T = {}^gR_q \implies {}^gR_q d_q = d_g$
   - $\begin{bmatrix} cos(-\pi/6) & -sin(-\pi/6) & 0 \\ sin(-\pi/6) & cos(-\pi/6) & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 15 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{15\sqrt{3}+1}{2} \\ \frac{-\sqrt{3}+15}{2} \\ -3 \end{bmatrix}$

3. H-Transformation matrices have the same frame flipping inverse properties as rotation matrices ${}^wT_q^T = {}^q T_w \implies {}^wT_q^T {}^wT_g = {}^q T_g$

4. - Car 1
     - For car 1, the rotation matrix and the position are both dependent on time
     - The rotation matrix, it will always rotate about the z-axis. The position is a 180-degree rotation CCW which is $\pi/2$ according to right-hand rule Therefore, the rotation at time $t = 0$ is Rot$(z, \pi/2)$
     - we know also need to get the rotation as a function of time. $v_1 * t$ describes it movement along the circumference. This can be referred to as the arc length. From This we know how much theta has changed due to the arc length equation $\frac{ds}{r} = d\theta$
     - Therefore, the rotation matrix w.r.t. the $\{0\}$ frame is Rot$(z, \pi/2 - \frac{(v_1 * t)}{2})$
     - To get the change in position we need to first define the position relative to the $\{0\}$ frame at time $t = 0$. This is simply $(1 + 2cos(\pi/2), 1 + 2sin(\pi/2), 3)$
     - To find the position as a function of time we utilize the same arc length formula as we did for the rotation matrix which gives us the (x,y) position of a circle radius 2 centered at (0,0). To offset we add 1 to x and y. Therefore, the position over time is $(1 + 2cos(\pi/2 - \frac{v_1 * t}{2}), 1 + 2sin(\pi/2 - \frac{v_1 * t}{2}), 3)$
     - Finally, to obtain the transformation matrix we combine our rotation and postion

- $${}^0T_1 = \begin{bmatrix} cos(\pi/2 - \frac{v_1 * t}{2}) & -sin(\pi/2 - \frac{v_1 * t}{2}) & 0 & 1 + 2cos(\pi/2 - \frac{v_1 * t}{2}) \\ sin(\pi/2 - \frac{v_1 * t}{2}) & cos(\pi/2 - \frac{v_1 * t}{2}) & 0 & 1 + 2sin(\pi/2 - \frac{v_1 * t}{2}) \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Car 2
  - For car 2, the rotation is constant about the z-axis clockwise which is negative according to right-hand rule, so the rotation matrix is described as Rot$(z, -\pi/4)$
  - The starting position of car 2 at time $t = 0$ is $(2cos(3\pi/4), 2sin(3\pi/4), 0)$ w.r.t. the center of the table. Multiplied by two due to the radius and 0 because its level with the tabletop
  - To get it with respect to frame $\{0\}$ we simply subtract 1 from x and y such that the starting position of car 2 is $(2cos(3\pi/4) - 1, 2sin(3\pi/4) - 1, 3)$. Note that the z is 3 now because that is the height of the table
  - the radius changes, however, so we need to put the radius term in terms of velocity of time. $((2 - v_2 * t)cos(3\pi/4) - 1, (2 - v_2 * t)sin(3\pi/4) - 1, 3)$
  - Finally, to obtain the transform matrix we combine our rotation matrix and position vector
  - $${}^0T_2 = \begin{bmatrix} cos(-\pi/4) & -sin(-\pi/4) & 0 & (2 - v_2 * t)cos(3\pi/4) - 1 \\ sin(-\pi/4) & cos(-\pi/4) & 0 & (2 - v_2 * t)sin(3\pi/4) - 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
- to find the homogeneous matrix ${}^1T_2$ as a function of time we take the inverse of ${}^0T_1$ and multiply it by ${}^0T_2$ such that ${}^1T_0{}^0T_2 = {}^1T_2$ The inverse of a transform matrix is the transpose of the rotation matrix and then multiplying the position vector by the negative transpose of the rotation matrix $-R^Tp$
- $$-R^Tp = \begin{bmatrix} cos(\frac{v_1 t}{2}) + sin(\frac{v_1 t}{2}) + 2 \\ -cos(\frac{v_1 t}{2}) + sin(\frac{v_1 t}{2}) \\ 3 \end{bmatrix}$$
- ${}^1T_2 =$ Its just the matrix product as described above.