A new stellarator coil design tool using space curves

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Motivations and Summary

- The difficulties in designing stellarator coils has been a critical problem for long time, even partly causing the termination of NCSX [1] and the delay of the W7-X construction [2].
- On the plasma surface ∂S, the total magnetic field, which is the sum of fields generated by coils and plasma currents, has zero normal components. So coil optimization problem is trying to find a set of coil parameters that minimizes the total normal field, and meets some essential engineering constraints.

$$(\boldsymbol{B}_{coils}+\boldsymbol{B}_{plasma})\cdot\boldsymbol{n}=\boldsymbol{0}$$

- All existing codes assume coils lying on a toroidal "**winding surface**":
  - NESCOIL[3] uses Green's function to solve a surface current potential and then discretizes for coils;
  - ONSET[4], COILOPT[5] non-linearly optimize the Fourier coefficients representing coil filaments as planar curves on winding surface;
  - COILOPT++[6] uses cubic spline to shape the coils on the surface
- Before optimizing coils or current potentials, an optimal winding surface is needed. Then it keeps fixed and binds coils on it.
- Do we really need the winding surface? Is the winding surface over-constraining the coils?

❖ We are developing a new coil optimization code named **FOCUS** (Finding Optimized Coils Using Space curves) :

1) throw away the winding surface;
2) use 3-D space curves to represent coil filaments directly;
3) Calculate the 1st and 2nd derivatives of penalty functions over all the free variables analytically;
4) construct quick and robust minimizing methods, like the steepest descent and Newton method.

❖ Illustrations of a simple two-period rotating ellipse stellarator and the W7-X are shown.

Space Curve Representation

- Coils are treated as arbitrary closed space curves
  $\boldsymbol{r}(t)=[x(t),y(t),z(t)]$
- One dimensional Fourier series is an good choice

$$\begin{cases} x= X_{c,0}+\sum_{n=1,N}\left[X_{c,n}\cos(nt)+X_{s,n}\sin(nt)\right] \\ y= Y_{c,0}+\sum_{n=1,N}\left[Y_{c,n}\cos(nt)+Y_{s,n}\sin(nt)\right] \\ z= Z_{c,0}+\sum_{n=1,N}\left[Z_{c,n}\cos(nt)+Z_{s,n}\sin(nt)\right] \end{cases}$$

$t\in[0,2\pi]$ *is a angle-like parameter.*

- **Simple but effective**

  use $N=6$ to reparameterize W7-X coils and produce a highly accurate magnetic field (seen in Fig. 1)

- **Global and differential**

$$\begin{cases} \dfrac{\partial x}{\partial t}=\sum_{n=1,N} -n\,X_{c,n}\sin(nt)+n\,X_{s,n}\cos(nt) \\ \dfrac{\partial x}{\partial X_{c,n}}=\cos(nt) \\ \dfrac{\partial x}{\partial X_{s,n}}=\sin(nt) \end{cases}$$

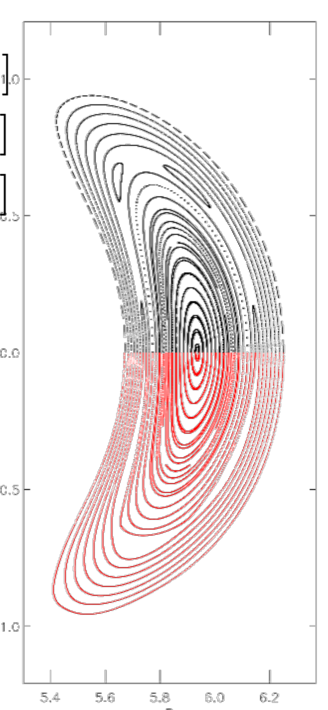

Figure 1. Poincare plot comparison of fitted coils (upper) and original one (below)

Target Function

- A target function covering both physical requirements and engineering constraints needs to be constructed.

$$\chi=\sum_i w_i\left(\frac{O_i-O_i^{target}}{O_i^{target}}\right)^2$$

- Physical requirements are for reconstructing target magnetic field, like the averaged squared error of $\boldsymbol{B}\cdot\boldsymbol{n}$, the maximum error of $\boldsymbol{B}\cdot\boldsymbol{n}$, toroidal flux error, the magnetic well, rotational transform profile, etc.
- Engineering constraints includes the total coil length, coil-plasma separation, coil-coil separation, the maximum coil torsion, magnetic forces, etc.
- Three fundamental object functions with explicitly calculated derivatives have been well constructed.

∗ *note: For conciseness, we assume vacuum field (no plasma currents) and only part of the derivatives are listed.*

Average Bnormal error

$$O_b=\iint_S\frac{1}{2}\left(\frac{\boldsymbol{B}\cdot\boldsymbol{n}}{|\boldsymbol{B}|}\right)^2\mathrm{d}s$$

$$=\frac{1}{2}\int_0^{2\pi}\int_0^{2\pi}\frac{(B_xn_x+B_yn_y+B_zn_z)^2}{{B_x}^2+{B_y}^2+{B_z}^2}\sqrt{g}\,\mathrm{d}\theta\,\mathrm{d}\zeta$$

$$\boldsymbol{B}=\frac{\mu_0}{4\pi}\sum_{i=1,Ncoils}I_i\int_{C_i}\frac{\mathrm{d}\boldsymbol{l}'\times(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$$

$$\frac{\partial B_x}{\partial X^i_{c,n}}=\frac{\mu_0}{4\pi}I_i\int_0^{2\pi}-\frac{3(\Delta y\;\dot{z}'-\Delta z\;\dot{y}')\;\Delta x\;cos(nt)}{|\boldsymbol{r}-\boldsymbol{r}'|^5}\,\mathrm{d}t$$

$$\frac{\partial O_b}{\partial X^i_{c,n}}=\int_0^{2\pi}\int_0^{2\pi}\left(\frac{B_xn_x+B_yn_y+B_zn_z}{{B_x}^2+{B_y}^2+{B_z}^2}\left(\frac{\partial B_x}{\partial X^i_{c,n}}n_x+\frac{\partial B_y}{\partial X^i_{c,n}}n_y+\frac{\partial B_z}{\partial X^i_{c,n}}n_z\right)-\frac{(B_xn_x+B_yn_y+B_zn_z)^2}{({B_x}^2+{B_y}^2+{B_z}^2)^2}\left(\frac{\partial B_x}{\partial X^i_{c,n}}B_x+\frac{\partial B_y}{\partial X^i_{c,n}}B_y+\frac{\partial B_z}{\partial X^i_{c,n}}B_z\right)\right)\sqrt{g}\,\mathrm{d}\theta\,\mathrm{d}\zeta$$

Average toroidal flux error

$$O_\Psi=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\left(\frac{\Psi_i-\Psi_o}{\Psi_o}\right)^2\mathrm{d}\zeta$$

$$\boldsymbol{A}=\frac{\mu_0}{4\pi}\sum_{i=1,Ncoils}I_i\int_{C_i}\frac{\mathrm{d}\boldsymbol{l}'}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$\frac{\partial A_x}{\partial X^i_{c,n}}=\frac{\mu_0}{4\pi}I_i\int_0^{2\pi}\left(-\frac{n\;sin(nt)}{\sqrt{\Delta x^2+\Delta y^2+\Delta z^2}}-\frac{\dot{x}'\Delta x\;cos(nt)}{(\Delta x^2+\Delta y^2+\Delta z^2)^{3/2}}\right)\mathrm{d}t$$

$$\Psi=\iint_{S_{tor}}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}=\int_l\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{l}$$

$$\frac{\partial\Psi}{\partial X^i_{c,n}}=\int_0^{2\pi}\left(\frac{\partial A_x}{\partial X^i_{c,n}}\frac{\partial x}{\partial\theta}+\frac{\partial A_y}{\partial X^i_{c,n}}\frac{\partial y}{\partial\theta}+\frac{\partial A_z}{\partial X^i_{c,n}}\frac{\partial z}{\partial\theta}\right)\mathrm{d}\theta$$

$$\frac{\partial O_\Psi}{\partial X^i_{c,n}}=\frac{1}{2\pi}\int_0^{2\pi}\frac{\Psi_i-\Psi_o}{{\Psi_o}^2}\frac{\partial\Psi_i}{\partial X^i_{c,n}}\mathrm{d}\zeta$$

Total coil length

$$L_i=\int_0^{2\pi}\sqrt{\dot{x}(t)^2+\dot{y}(t)^2+\dot{z}(t)^2}\,\mathrm{d}t$$

$$\frac{\partial L_i}{\partial X^i_{c,n}}=\int_0^{2\pi}\frac{-\dot{x}\,n\,sin(nt)}{\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}}\,\mathrm{d}t$$

$$O_L=\frac{1}{Ncoils}\sum_{i=1,Ncoils}\frac{e^{L_i}}{e^{L_o^i}}$$

$$\frac{\partial O_L}{\partial X^i_{c,n}}=\frac{1}{Ncoils}\frac{e^{L_i}}{e^{L_o^i}}\frac{\partial L_i}{\partial X^i_{c,n}}$$

Optimization

- The target function has been constructed.

$$\chi=w_b\,O_b+w_\Psi\,O_\Psi+w_L\,O_L$$

$$\frac{\partial\chi}{\partial\boldsymbol{x}}=w_b\frac{\partial O_b}{\partial\boldsymbol{x}}+w_\Psi\frac{\partial O_\Psi}{\partial\boldsymbol{x}}+w_L\frac{\partial O_L}{\partial\boldsymbol{x}}$$

$\boldsymbol{x}=\{X^i_{c,n},X^i_{s,n},Y^i_{c,n},Y^i_{c,n},Z^i_{c,n},Z^i_{s,n},I^i\}$ denotes all the free variables

- The Steepest Descent method can then be applied. Defining an artificial "time" τ, the descent direction is given by
- $\dfrac{\partial\boldsymbol{x}}{\partial\tau}=-\nabla\chi \qquad \dfrac{\partial\chi}{\partial\tau}=\dfrac{\partial\chi}{\partial\boldsymbol{x}}\dfrac{\partial\boldsymbol{x}}{\partial\tau}=-\left(\dfrac{\partial\chi}{\partial\boldsymbol{x}}\right)^2$
- Use NAG:D02BJF to integrate a system of first-order ordinary differential equations.

Applications

- Two cases are selected to validate the code:

Two-periods rotating ellipse

- $Nfp=2,\ R=3.0m, a=0.36m, b=0.24m$
- Initiate with 16 planar circular coils (r=0.75m)
- Normalize weights to $w_bO_b=1.0, w_\Psi O_\Psi=0.5, w_LO_L=0.02$
- turn on all the constraints and set $\tau_{max}=100$

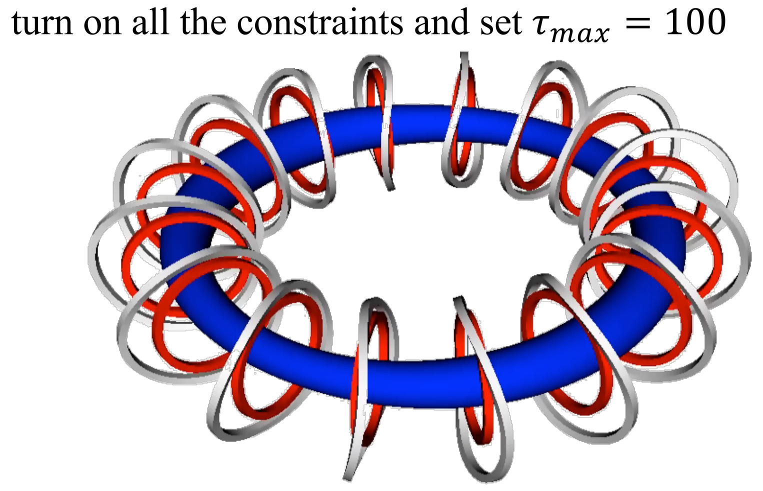

Figure 2. Plasma (blue), initial coils (red) and final coils (silver) of simple rotating ellipse

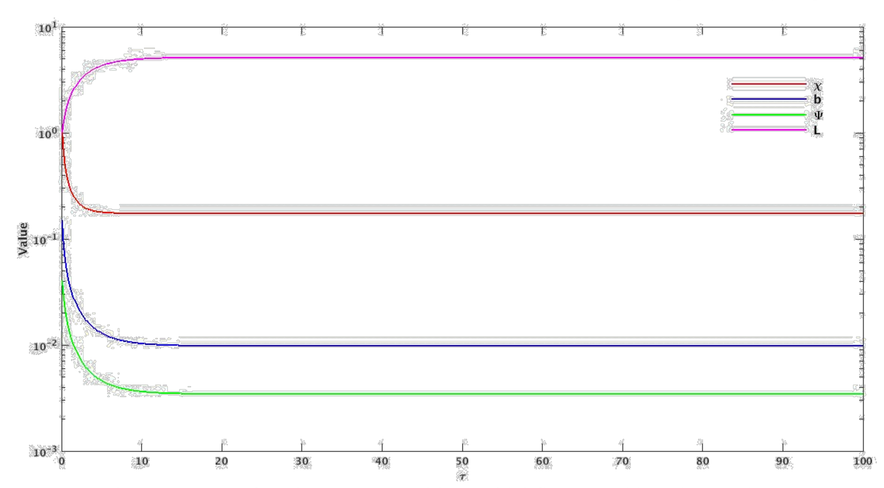

Figure 3. Target function χ, Bnormal b, toroidal flux Ψ and length L evolve over τ

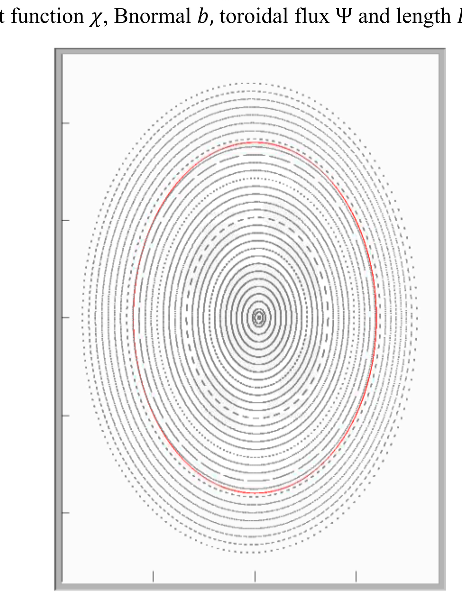

Figure 4. Free boundary reconstructing (red line is the target plasma boundary)

W7-X

- Plasma boundary generated by W7-X as-built coils (OP1.1 limiter configuration)
- Initiate with 50 planar circular coils (r=1.25m)
- Normalize weights to $w_bO_b=1.0, w_\Psi O_\Psi=0.5, w_LO_L=0.05, \tau_{max}=10^4$
- w_bO_b decreased to about 10^{-3}

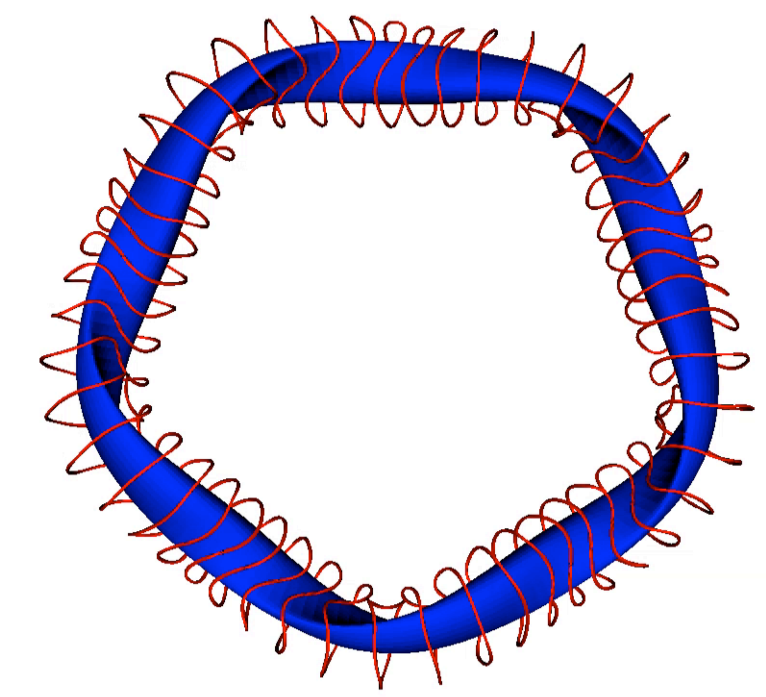

Figure 5. W7-X plasma boundary (blue) and optimized coils (red) produced by FOCUS.

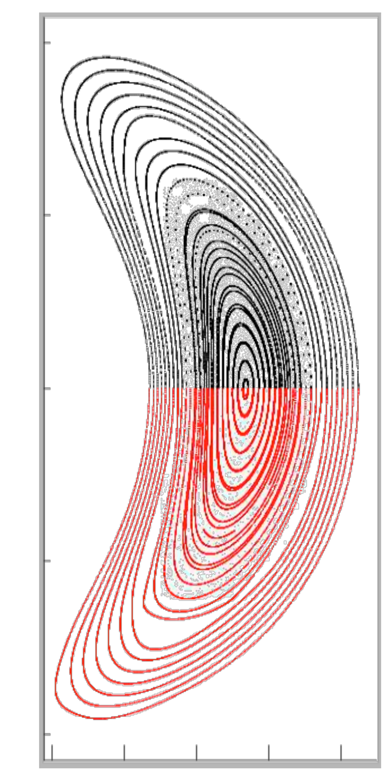

Figure 6. Free boundary reconstructing (upper half fro m new FOCUS produced coils; below half from W7-X built coils)

Conclusions and Outlook

- Letting coils to evolve freely in space and ignoring the winding surface are workable, as long as proper constraints are given.
- Analytically calculated derivatives offer effective methods to minimize the target function.
- Even though only a few constraints are added, the coils still perform quite robustly.

❑ Keep constructing more penalty functions for different uses;
❑ Develop other minimizing algorithms;
❑ Explore possible improvements for current stellarator coil designs.

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