Flexible optimized coil designing method using space curves

Caoxiang Zhu¹, Stuart R. Hudson², Yuntao Song³, and Yuanxi Wan¹

¹ University of Science and Technology of China, 96 JinZhai Road, Hefei, Anhui 230026, P. R. China
² Princeton Plasma Physics Laboratory, Princeton University, P.O. Box 451, New Jersey 08543, USA
³ Institute of Plasma Physics, Chinese Academy of Sciences, Hefei, Anhui 230031, P. R. China

Abstract

Given a current distribution, the Biot-Savart law determines the magnetic field; however, it is the inverse of this problem that must be addressed for designing coils to confine optimized stellarator plasmas:given the required magnetic field, what is the current density? In other words, what is the geometry of the external coils? The inverse problem is ill-posed and complicates a fundamental problem of stellarators, the importance of which should not be underestimated. The cancellation of the National Compact Stellarator Experiment (NCSX) was in part due to the complexity of building the coils. Conventional approaches for designing coils presuppose a coil "winding" surface. We explore whether this assumption unnecessarily constrains the coil optimization process.

We represent the coils as arbitrary, closed, one-dimensional curves embedded in three-dimensional space. The Fundamental Theorem of Curves states that such curves are uniquely described (up to rigid displacements) by the curvature and torsion. The geometry of a finite set of discrete coils is allowed to vary freely to minimize the normal magnetic field on the reference plasma boundary, with an additional constraint on the enclosed toroidal flux, with the only engineering constraint being a penalty on the total length of the coils.

A new code, named Flexible Optimized Coils Using Space curves (FOCUS), has been developed. For illustration, and simplicity, we use a Fourier representation to describe the coil geometry, and both the physics and engineering "targets" are differentiated with respect to the Fourier harmonics to enable fast optimization algorithms. We test FOCUS using a simple, rotating elliptical reference plasma boundary, and with an exotic knotatron configuration. With FOCUS, we can reconstruct the W7-X vacuum field (to a reasonable accuracy) with as few as 30 modular coils, compared to the original 50.

1