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Hessian Matrix Used for Stellarator Coil Design and Error Fields Prediction

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Coil design and error fields control are important for accurately reproducing the 3D magnetic fields for stellarators. The new coil design code, FOCUS [1], has been implemented to remove the dependence of the winding surface, by representing coils with 3D curves. The chi-squared cost function χ^2 consists of multiple physics and engineering constraints. With analytically calculated gradient, FOCUS uses the steepest descent method for minimizing χ^2. But it's relatively slow near the minimum.

The square matrix of second order derivatives, called the Hessian matrix in mathematics, contains rich information that can be used for Hessian-based minimization algorithms and more interestingly for measuring the magnetic field response to coil deformations. We differentiate χ^2 with respect to coil parameters (currents and Fourier coefficients) to construct accurate Hessian matrix. A modification of Powell hybrid method [2] and a truncated Newton method [3] are applied here to find the minimum of χ^2. Numerical examples of designing modular coils for W7-X [4] show that Hessian-based optimizations can accelerate the convergence.

Another application for the Hessian matrix is to analyze the error fields. We demonstrate that a quadratic approximation to χ^2 is valid in the neighborhood of the minimum. For different perturbations with fixed magnitudes, the strongest response in the magnetic field happens in the direction of eigenvector with the largest eigenvalue of the Hessian matrix. Thus, it requires the most precision during manufacture and assembling. And those deformations that are not sensitive could be relaxed to reduce the cost with little sacrifice in accuracy. A proof-of-principle illustration on the CNT stellarator [5] has been carried out. The results coincide with previous work [6].

References

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