

Problem 1

$$\begin{aligned}\mathbb{E}[x] &= \int_{-\infty}^{\infty} x \cdot \sum_k \pi_k \mathcal{N}(x|\mu_k, \Sigma_k) \, dx \\ &= \sum_k \int_{-\infty}^{\infty} \pi_k \mathcal{N}(x|\mu_k, \Sigma_k) \, dx \\ &= \sum_k \pi_k \int_{-\infty}^{\infty} \mathcal{N}(x|\mu_k, \Sigma_k) \, dx \\ &= \sum_k \pi_k \mu_k = \mu\end{aligned}\tag{1.1}$$

$$\begin{aligned}\text{Cov}[x] &= \mathbb{E}[xx^\top] - \mathbb{E}[x]\mathbb{E}[x]^\top \\ &= \mathbb{E}[xx^\top] - \mu\mu^\top && \text{(by 1.1)} \\ &= \mu\mu^\top + \Sigma_k - \mu\mu^\top && \text{(by Bishop 2.62)} \\ &= \Sigma_k\end{aligned}$$

Problem 2**Problem 3**