**Problem 1**

**Problem 2**

**Problem 3**

We prove that $K(x, y) = (x^\intercal y + c)^d$ is a kernel by the construction rules.

- $K_1(x, y) = x^\intercal y$ is a kernel by Rule 5, where $B = I$.
- $K_2(x, y) = c$ is a kernel by Rule 4, where $\phi(z) = \sqrt{c}, \phi : \mathbb{R}^n \to \mathbb{R}^m, m = 1$ and $K_3(x, y) = x^\intercal y$
- $K_3(x, y) = K_1(x, y) + K_2(x, y)$ is a kernel by Rule 1
- $K_4(x, y) = K_3(x, y)^d$ is a kernel by recursive application of Rule 3

Thus $K(x, y) = (x^\intercal y + c)^d$ is a valid kernel.

**Problem 4**

**Problem 5**

**Problem 6**

**Problem 7**