Problem 1

$$\begin{aligned}
\mathbb{E}[x] &= \int x \cdot \sum_k \pi_k \mathcal{N}(x|\mu_k, \Sigma_k)\,\mathrm{d}x \\
&= \sum_k \int \pi_k \mathcal{N}(x|\mu_k, \Sigma_k)\,\mathrm{d}x \\
&= \sum_k \pi_k \int \mathcal{N}(x|\mu_k, \Sigma_k)\,\mathrm{d}x \\
&= \sum_k \pi_k \mu_k
\end{aligned}$$

Problem 2

Problem 3