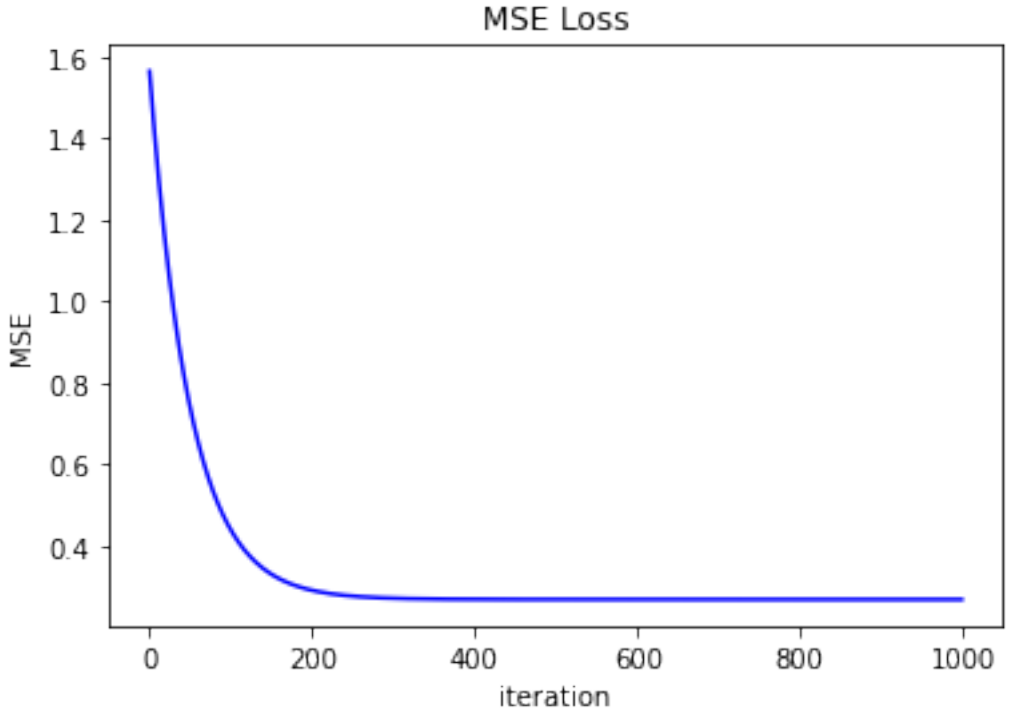
0819823 陳子祈 Image recognition HW1

Part 1

1. MSE loss



2. Final testing data MSE = 0.068705834617267

3. weight = 0.8179331278291383
intercept = 0.7845389075277833

4.
(1) **Gradient Descent**: Send all the data into the network at one time. Although lot of data makes the gradient easy to find the general direction, but when the amount of data is large, each round of training will take a lot of time
(2) **Mini-Batch Gradient Descent**: Each time you input m data to train the model, the gradient of this method is easier to find the roughly correct direction than the gradient of SGD, and because the data size of each round of training is m, it is easier for the computer to process this vectorized data
(3) **Stochastic Gradient Descent**: Input one data at a time to train the model. Although each round of training in this method is fast, it is easy for the gradient to find the correct direction, and the chaotic travel direction may not necessarily help it find the lowest point faster.

Part2

1. (10%) Suppose that we have three colored boxes R (red), B (blue), and G (green). Box R contains 3 apples, 4 oranges, and 3 guavas, box B contains 2 apples, 0 orange, and 2 guavas, and box G contains 12 apples, 4 oranges, and 4 guavas. If a box is chosen at random with probabilities p(R)=0.2, p(B)=0.4, p(G)=0.4, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting guava? If we observe that the selected fruit is in fact an apple, what is the probability that it came from the blue box?

Box	apples	oranges	guavas	choosen
R	3	4	3	0.2
B	2	0	2	0.4
G	12	4	4	0.4

Let the event of selecting guavas be g

, the event of choosing box R, box G, box B be R, G, B

① $P(g \cap R) + P(g \cap B) + P(g \cap G)$

$= P(g|R)P(R) + P(g|B)P(B) + P(g|G)P(G)$

$= \frac{3}{10} \times 0.2 + \frac{2}{4} \times 0.4 + \frac{4}{20} \times 0.4$

$= 0.06 + 0.2 + 0.08 = 0.34$

② Let the event of selecting apples be a

$$P(B|a) = \frac{P(a \cap B)}{P(a \cap R) + P(a \cap B) + P(a \cap G)}$$

$$= \frac{\frac{2}{4} \times 0.4}{\frac{3}{10} \times 0.2 + \frac{2}{4} \times 0.4 + \frac{12}{20} \times 0.4} = \frac{0.2}{0.06 + 0.2 + 0.24}$$

$$= \frac{0.2}{0.5} = 0.4$$

※

2. (15%) Consider two nonnegative numbers a and b, and show that, if $a \leq b$, then $a \leq (ab)^{1/2}$. Use this result to show that, if the decision regions of a two-class classification problem are chosen to minimize the probability of misclassification, this probability will satisfy

$$p(mistake) \leq \int \{p(x, C_1)\, p(x, C_2)\}^{1/2}\, dx\,.$$

(Hint: Please refer to the textbook 1.5. Decision Theory)

Let R_1 be the distribution area of class C_1,
and R_2 be the area of class C_2

$$p(\text{mistake}) = \int_{R_1} p(x, C_2)\, dx + \int_{R_2} p(x, C_1)\, dx \quad \cdots ①$$

In the error made in R_1 we always have $p(C_1|x) \geq p(C_2|x)$.

$$\Rightarrow p(C_2|x) \leq [p(C_1|x)\, p(C_2|x)]^{1/2}$$

$$\Rightarrow \int_{R_1} p(x, C_2)\, dx = \int_{R_1} p(C_2|x)\, p(x)\, dx$$

$$\leq \int_{R_1} [p(C_1|x)\, p(C_2|x)]^{1/2} p(x)\, dx = \int_{R_1} [p(x|C_1)\, p(x|C_2)]^{1/2}\, dx \quad \cdots ②$$

and similar situations apply for errors in R_2, $p(C_2|x) \geq p(C_1|x)$ when in R_2

$$\Rightarrow p(C_1|x) \leq [p(C_1|x)\, p(C_2|x)]^{1/2}$$

$$\Rightarrow \int_{R_2} p(x, C_1)\, dx = \int_{R_2} p(C_1|x)\, p(x)\, dx$$

$$\leq \int_{R_2} [p(C_1|x)\, p(C_2|x)]^{1/2} p(x)\, dx = \int_{R_2} [p(x|C_1)\, p(x|C_2)]^{1/2}\, dx \quad \cdots ③$$

Substitute ②, ③ back to ①:

$$p(\text{mistake}) = \int_{R_1} p(x, C_2)\, dx + \int_{R_2} p(x, C_1)\, dx$$

$$\leq \int_{R_1} [p(x, C_1)\, p(x, C_2)]^{1/2} dx + \int_{R_2} [p(x, C_1)\, p(x, C_2)]^{1/2} dx$$

$$= \int [p(x, C_1)\, p(x, C_2)]^{1/2}\, dx$$

※

3. (15%) Consider two variables x and y with joint distribution $p(x, y)$. Prove the following two results

$$E[x] = E_y\left[E_x[x|y]\right]$$

$$var[x] = E_y\left[var_x[x|y]\right] + var_y\left[E_x[x|y]\right].$$

Here $E_x[x|y]$ denotes the expectation of x under the conditional distribution $p(x|y)$, with a similar notation for the conditional variance.

① If x, y are continuous variables

$$E_y[E_x[x|y]]$$
$$= \int_{-\infty}^{\infty} E_x[X|Y=y] f_Y(y)\, dy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{X|Y}(x|y)\, dx\, f_Y(y)\, dy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{X|Y}(x|y) f_Y(y)\, dx\, dy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{X,Y}(x,y)\, dy\, dx$$
$$= \int_{-\infty}^{\infty} x \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy\, dx$$
$$= \int_{-\infty}^{\infty} x f_X(x)\, dx = E[x]$$

If x, y are discrete variables

$$E_y[E_x[x|y]]$$
$$= \sum_{y\in Y} E_x[x|Y=y]\, p(y)$$
$$= \sum_{y\in Y}\left(\sum_{x\in X} x p(x|y)\right) p(y)$$
$$= \sum_{y\in Y}\sum_{x\in X} x p(x|y) p(y)$$
$$= \sum_{x\in X}\sum_{y\in Y} x p(x,y)$$
$$= \sum_{x\in X} x \sum_{y\in Y} p(x,y)$$
$$= \sum_{x\in X} x p(x) = E[x]$$

② $E_y[var_x[x|y]]$
$$= E_y\left[E_x[x^2|y] - (E_x[x|y])^2\right]$$
$$= E_y\left[E_x[x^2|y]\right] - E_y\left[E_x[x|y]^2\right] \quad \cdots ⓐ$$
$$var_y[E_x[x|y]]$$
$$= E_y\left[E_x[x|y]^2\right] - \{E_y[E_x[x|y]]\}^2 \quad \cdots ⓑ$$

ⓐ + ⓑ : $E_y[var_x[x|y]] - var_y[E_x[x|y]]$
$$= E_y[E_x[x^2|y]] - \{E_y[E_x[x|y]]\}^2$$

due to ①: $$= E[x^2] - (E[x])^2 = var[x]$$

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