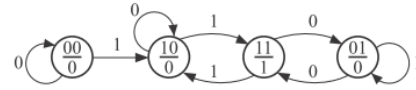


13.3 (a)  $A^+ = AK'_A + A'J_A = A(B' + X) + A'(BX' + B'X)$   
 $B^+ = B'J'_B + BK'_B = AB'X + B(A' + X')$   
 $Z = AB$

A B		X	
		0	1
00		0	1
01		1	0
11		0	1
10		1	1

Present State AB	Next State $A^+B^+$		Z
	X=0	X=1	
00	00	10	0
01	11	01	0
11	01	10	1
10	10	11	0

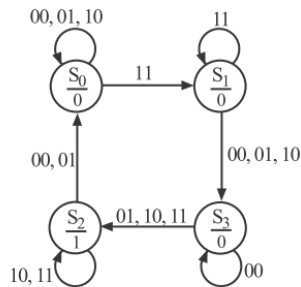


13.3 (b)  $X = 0 \ 1 \ 1 \ 0 \ 0$   
 $AB = 00 \ 00 \ 10 \ 11 \ 01 \ 11$   
 $Z = (0) \ 0 \ 0 \ 1 \ 0 \ 1$

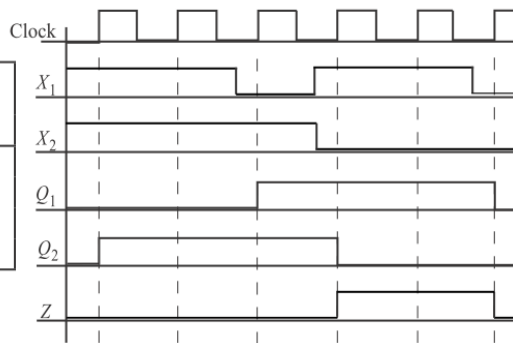
13.3 (c) See FLD p. 758 for solution.

13.9 (a)  $Q_1^+ = D_1 = (X_1' + X_2' + Q_1)(Q_1 + Q_2)(X_1' + Q_2)$   
 $Q_2^+ = D_2 = (X_1'X_2' + Q_1)(X_1X_2 + Q_2)$   
 $Z = Q_1Q_2'$

State	Present State $Q_1Q_2$	Next State $X_1X_2$				Z
		00	01	11	10	
$S_0$	00	00	01	01	00	0
$S_1$	01	11	11	01	11	0
$S_3$	11	11	10	10	10	0
$S_2$	10	10	10	00	00	1

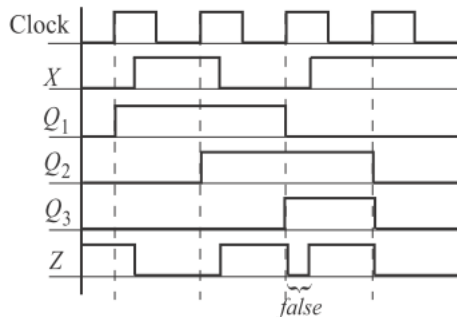


13.9 (b)



13.9 (c)  $Z = (0)000110$

13.13



Correct output:  $Z = 1011$

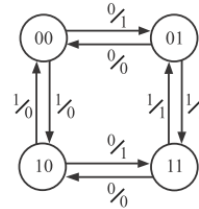
## 13 Solutions

13.21

Clock Cycle	Information Gathered
1	$Q_1Q_2 = 00, X = 0 \Rightarrow Z = 1, Q_1^+Q_2^+ = 01$
2	$Q_1Q_2 = 01, X = 0 \Rightarrow Z = 0; X = 1 \Rightarrow Z = 1, Q_1^+Q_2^+ = 11$
3	$Q_1Q_2 = 11, X = 1 \Rightarrow Z = 1; X = 0 \Rightarrow Z = 0, Q_1^+Q_2^+ = 10$
4	$Q_1Q_2 = 10, X = 0 \Rightarrow Z = 1; X = 1 \Rightarrow Z = 0, Q_1^+Q_2^+ = 00$
5	$Q_1Q_2 = 00, X = 1 \Rightarrow Z = 0, Q_1^+Q_2^+ = 10$
6	$Q_1Q_2 = 10, X = 1 \Rightarrow (Z = 0); X = 0 \Rightarrow (Z = 1), Q_1^+Q_2^+ = 11$
7	$Q_1Q_2 = 11, X = 0 \Rightarrow (Z = 0); X = 1 \Rightarrow (Z = 1), Q_1^+Q_2^+ = 01$
8	$Q_1Q_2 = 01, X = 1 \Rightarrow (Z = 1); X = 0 \Rightarrow (Z = 0), Q_1^+Q_2^+ = 00$
9	$Q_1Q_2 = 00, X = 0 \Rightarrow (Z = 1)$

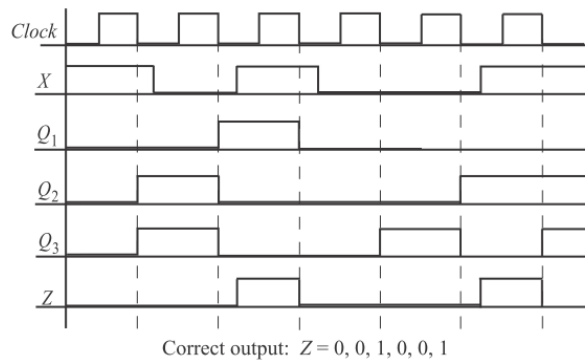
Note: Information inside parentheses was already obtained in a previous clock cycle.

Present State	Next State		Z	
$Q_1Q_2$	$X=0$	$X=1$	$X=0$	$X=1$
00	01	10	1	0
01	00	11	0	1
10	11	00	1	0
11	10	01	0	1



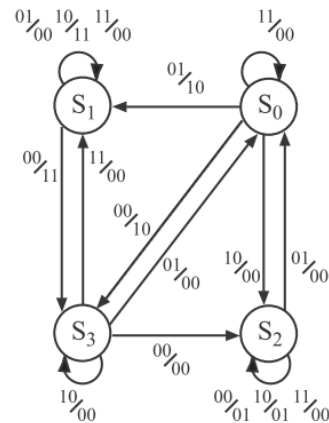
13.27 Transition table using a straight binary state assignment:

State	Present State	Next State		Z	
	$Q_1Q_2Q_3$	$X=0$	$X=1$	$X=0$	$X=1$
$S_0$	000	001	011	0	0
$S_1$	001	010	011	0	0
$S_2$	010	001	011	0	1
$S_3$	011	100	000	0	0
$S_4$	100	011	000	0	1



13.30 (cont.)

State	Present State	$A^+B^+$				$Z_1Z_2$			
		$X_1X_2 =$				$X_1X_2 =$			
	$AB$	00	01	10	11	00	01	10	11
$S_0$	00	11	01	10	00	10	10	00	00
$S_1$	01	11	01	01	01	11	00	11	00
$S_2$	10	10	00	10	10	01	00	01	00
$S_3$	11	10	00	11	01	00	00	00	00



- 14.6** This should be solved in the same way as Example 3 on FLD p. 471–472. Assign a state to each possible input (00, 01, 11, 10) with an output of 0, and another state to each input with an output of 1. This gives eight states. See FLD p. 762 for the state table.

State	Z = 0	State	Z = 1
$S_0$	Last input was 00	$S_4$	Last input was 00
$S_1$	Last input was 01	$S_5$	Last input was 01
$S_2$	Last input was 11	$S_6$	Last input was 11
$S_3$	Last input was 10	$S_7$	Last input was 10

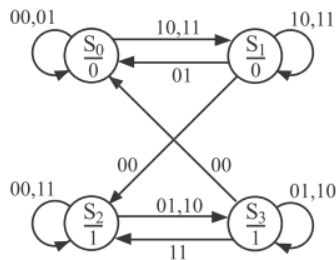
Each input takes you to the state defined by that input (e.g. an input of 01 takes you to either  $S_1$  or  $S_5$ ). The only thing in question is whether the output is 0 or 1. Determine the output by checking whether the last two inputs correspond to the three input sequences.

*Alternate Solution:* Notice that when  $Z = 0$ , “causes the output to become 0” is the same as remaining constant, and “causes the output to become 1” is the same as toggling the output. The situation is similar when  $Z = 1$ . So we can use only four states, as follows:

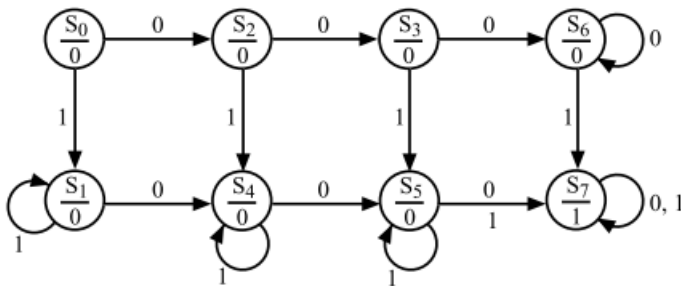
State	Meaning
$S_0$	$Z = 0$ and last input was either 00 or 01
$S_1$	$Z = 0$ and last input was either 10 or 11
$S_2$	$Z = 1$ and last input was either 00 or 11
$S_3$	$Z = 1$ and last input was either 01 or 10

State	Next State	Z
	$X_1 X_2 = 00 \ 01 \ 10 \ 11$	
$S_0$	$S_0 \ S_0 \ S_1 \ S_1$	0
$S_1$	$S_2 \ S_0 \ S_1 \ S_1$	0
$S_2$	$S_2 \ S_3 \ S_3 \ S_2$	1
$S_3$	$S_0 \ S_3 \ S_3 \ S_2$	1

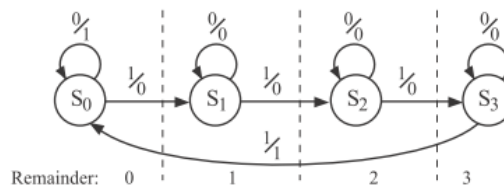
Note: The state table with 8 states reduces to this 4-state table using methods in Unit 15.



- 14.21** Plot 0's horizontally. Plot 1's vertically. Receiving a 0 takes us one state to the right. Receiving a 1 takes us one state down. The output is a 1 only in the “three 0's or more, one 1 or more” state:

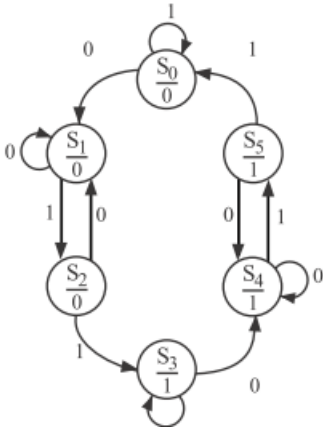


- 14.24 (a)** We need four states to describe the 1's received, as there are four possible remainders when dividing by four. An input of 1 takes us to the next state in cyclic fashion. An input of zero leaves us in the same state.



**4.26** There are two identical parts: one with an output of 0 and one with an output of 1.

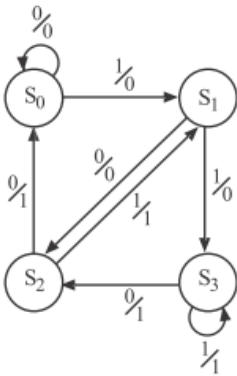
State	Meaning
$S_1, S_4$	Previous input was 0
$S_2, S_5$	Previous inputs were 01
$S_3, S_0$	Previous input was 1 / Reset ( $S_0$ )



**14.34** To delay by two clock periods, we need to remember the previous two inputs. So we have four states, one for each combination of two inputs:

State	Next State		Z	
	$X=0$	$X=1$	$X=0$	$X=1$
$S_0$	$S_0$	$S_1$	0	0
$S_1$	$S_2$	$S_3$	0	0
$S_2$	$S_0$	$S_1$	1	1
$S_3$	$S_2$	$S_3$	1	1

State	Meaning
$S_0$	Previous two inputs were 00
$S_1$	Previous two inputs were 01
$S_2$	Previous two inputs were 10
$S_3$	Previous two inputs were 11



*Note:* Just go to the state that represents the last two inputs.

**14.45**

Present State	Next State				$Z_1 Z_2$			
	00	01	10	11	00	01	10	11
$S_0$	$S_1$	$S_0$	$S_2$	$S_2$	01	10	01	01
$S_1$	$S_1$	$S_2$	$S_0$	$S_0$	00	11	00	00
$S_2$	$S_2$	$S_1$	$S_0$	$S_0$	00	00	00	00

15.3

$S_0$	$S_5 - a$ $S_1 - b$	<del><math>S_5 - a</math> <math>S_1 - b</math></del>	<del></del>
$S_1$	<del><math>S_5 - a</math> <math>S_6 - b</math></del>	$S_5 - a$ $S_6 - c$	<del></del>
$S_2$	<del><math>S_2 - a</math> <math>S_6 - b</math></del>	<del><math>S_2 - a</math> <math>S_6 - c</math></del>	<del></del>
$S_3$	<del></del>	<del></del>	$S_0 - a$ $S_1 - b$
$S_4$	<del><math>S_4 - a</math> <math>S_3 - b</math></del>	<del><math>S_4 - a</math> <math>S_3 - c</math></del>	<del></del>
$S_5$	$S_0 - a$ $S_1 - b$	<del><math>S_0 - a</math> <math>S_1 - b</math></del>	<del></del>
$S_6$	<del></del>	<del></del>	$S_5 - a$ $S_1 - b$
	$a$	$b$	$c$

$S_0 \equiv a$   
 $S_1 \equiv b$   
 $S_3 \equiv c$   
 $S_5 \equiv a$   
 $S_6 \equiv c$   
 $S_2$  and  $S_4$  have no  
 equivalent states.

15.3 (a)  $a \equiv S_0, S_5$   
 $b \equiv S_1$   
 $c \equiv S_3, S_6$

Since  $S_2$  and  $S_4$  do not have corresponding states, the circuits are *not* equivalent.

15.3 (b) Starting from  $S_0$ , it is not possible to reach  $S_2$  or  $S_4$ .  
 So then the circuits would perform the same.

15.9

		$Q_1$	$Q_2$	$Q_3$
Assign	$S_0$	1	0	0
	$S_1$	0	1	0

$$D_1 = X'Q_1 + XY'Q_3$$

$$D_2 = XQ_1 + YQ_3 + X'Q_2$$

$$D_3 = XQ_2 + X'Y'Q_3$$

$$P = XQ_1 + Y'Q_3 + XQ_2$$

$$S = X'Q_1 + XY'Q_3$$

15.11 (a)

<i>b</i>	<del><i>d-e</i></del> <del><i>f-g</i></del>					
<i>c</i>						
<i>d</i>	<del><i>b-e</i></del> <del><i>f-g</i></del>	✓				
<i>e</i>	<del><i>e-g</i></del> <del><i>f-g</i></del>	<del><i>d-g</i></del>		<del><i>b-g</i></del>		
<i>f</i>			<del><i>b-e</i></del> <del><i>c-d</i></del>			
<i>g</i>			✓			<del><i>b-e</i></del> <del><i>e-d</i></del>
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>

$b \equiv d$   
 $c \equiv g$

State	Next State		Output	
	$X=0$	$X=1$	$X=0$	$X=1$
<i>a</i>	<i>e</i>	<i>c</i>	0	1
<i>b</i>	<i>b</i>	<i>f</i>	0	1
<i>c</i>	<i>e</i>	<i>c</i>	1	0
<i>e</i>	<i>c</i>	<i>f</i>	0	1
<i>f</i>	<i>b</i>	<i>b</i>	1	0

15.11 (b) Input: 000

Output starting in state *a*:

001 (state *a* → state *e* → state *g* → state *e*)

Output starting in state *b*:

000 (state *b* → state *d* → state *b* → state *d*)

**15.20 (a)** Invert all three columns of assignment (iv), and then swap the first and last columns. Then (iii) and (iv) are the same, therefore, Assignment (iii)  $\equiv$  Assignment (iv).

**15.20 (c)** Many state assignments are not equivalent to (i) through (v), for example:

101	or	011
000		101
011		000
100		100
010		010
110		110

**15.20 (b)** Equivalent assignments to each column having 000 as the starting state. Invert any column with 1 in the first row.

	$(ii) - (c'_2)$	$iii - c'_1$	$iv - c'_1c'_2$	$v - c'_3$
$S_0$	000	000	000	000
$S_1$	101	001	100	110
$S_2$	011	100	001	100
$S_3$	100	101	101	010
$S_4$	010	011	110	001
$S_5$	110	010	010	011



15.24 (a) Equations for one-hot state assignment:

$$D_A = X(A + B + D + E), D_B = X'(A + D),$$

$$D_C = X'B, D_D = XC, D_E = X'(C + E), z = D$$

15.24 (b) Guidelines:

1.  $(A, D) \times 2 (C, E) (A, B, D, E)$
2.  $(A, B) \times 2 (A, C) (D, E) (A, E)$

The following assignment satisfies all but (A, E),  
(A, C) and (B, D):

		Q <sub>1</sub>	
		0	1
Q <sub>2</sub> Q <sub>3</sub>	00	A	-
	01	B	-
	11	E	C
	10	D	-

Q <sub>1</sub> Q <sub>2</sub> Q <sub>3</sub>	Q <sub>1</sub> <sup>+</sup> Q <sub>2</sub> <sup>+</sup> Q <sub>3</sub> <sup>+</sup>		z
	X=0	1	
000	001 000		0
010	111 000		0
011	011 000		0
010	001 000		1
110	---	---	-
111	011 010		0
101	---	---	-
100	---	---	-

$$D_1 = X'Q_2'Q_3, D_2 = X'Q_3 + Q_1, D_3 = X',$$

$$z = Q_2Q_3'$$

**15.35** By inspecting incoming arrows, we get:

$$Q_0^+ = D_0 = X'YQ_0 + Y'Q_1 + X'YQ_2$$

$$Q_1^+ = D_1 = XY'Q_0 + XYQ_1 + Y'Q_2$$

$$Q_2^+ = D_2 = XYQ_0 + X'Y'Q_0 + X'YQ_1 + XYQ_2$$

$$Z = X'YQ_1 + XYQ_2 + X'YQ_2 = X'YQ_1 + YQ_2$$

15.40  $S_7 \sim S_9$  and  $S_8 \sim S_{10}$  so the table reduces to

Present State	Next State		Output (Z)	
	$X=0$	$X=1$	$X=0$	$X=1$
$S_0$	$S_1$	$S_2$	0	0
$S_1$	$S_3$	$S_4$	0	0
$S_2$	$S_5$	$S_6$	0	0
$S_3$	$S_7$	$S_8$	0	0
$S_4$	$S_8$	$S_7$	0	0
$S_5$	$S_8$	$S_7$	0	0
$S_6$	$S_7$	$S_8$	0	0
$S_7$	$S_0$	$S_0$	1	0
$S_8$	$S_0$	$S_0$	0	1

Now  $S_3 \sim S_6$  and  $S_4 \sim S_5$  so the table reduces to

Present State	Next State		Output (Z)	
	$X=0$	$X=1$	$X=0$	$X=1$
$S_0$	$S_1$	$S_2$	0	0
$S_1$	$S_3$	$S_4$	0	0
$S_2$	$S_4$	$S_3$	0	0
$S_3$	$S_7$	$S_8$	0	0
$S_4$	$S_8$	$S_7$	0	0
$S_7$	$S_0$	$S_0$	1	0
$S_8$	$S_0$	$S_0$	0	1

The implication chart verifies the answer.

