De-Quan Zhu
Email: 20218131028@m.scnu.edu.cn

Introduction

I join Prof. Jin-Hua Zhao's group in my second year of undergraduate study. The group focuses on the emergence of macroscopic phenomena in complex systems and combinatorial optimization problems in graph theory. Initially, I undertake a semester of research training, primarily focusing on the percolation phenomenon in complex networks, while also studying a classic combinatorial optimization problem in graph theory—the Minimum Vertex Cover problem. Furthermore, I participate in two research projects: the impact of randomly reinforced nodes on the robustness of interdependent networks and an extended study of the Buffon-Laplace Needle Problem. In both projects, my work involves theoretical derivations, numerical computations, and programming simulations.

Research Training on the Percolation Phenomenon in Complex Networks

The percolation phenomena we study refer to the evolution of certain macroscopic structures in a network under random perturbations, such as node or edge removals. Percolation theory provides a mathematical framework to quantify this evolution. During this research training, I reproduce the construction of random graph models, including Erdős–Rényi random graphs, Random Regular graphs, as well as scale-free network models, such as Barabási–Albert, Static, and Configuration Models. I also perform simulation of percolation phenomena on these network models, focusing on the Giant Component (GC), K-core, and core. I follow analytical theory of above percolation models on sparse random graphs which is basically built on self-consistent equations, and apply numerical methods to solve these equations. Finally, I confirm these theoretical results through simulation and reproduce those related result on previous literature.

Research Project on Enhancing Network Robustness

The goal of enhancing network robustness is to ensure that critical infrastructures, such as power grids and communication networks, can continue to function even when subjected to failures or attacks. However, networks often exhibit interdependency between layers. For example, communication networks rely on power networks for electricity, while power networks depend on communication networks for equipment dispatch. This project aims to study the impact of randomly introducing reinforced nodes (immune to all attacks) on network robustness of interdependent networks. This problem setting can be easily formulated into a percolation model. On the simulation side of this project, we implement simulation on interdependent networks with randomly reinforced nodes under K-core pruning process to quantify pertinent structural indicators, such K-core and the GC of K-core. We also derive coupled self-consistent equations to calculate the relative sizes of the network's K-core and its GC. On the theoretical side, we develop an analytical framework for the model on large sparse graphs to calculate fraction of K-core and its GC. In this project, along with co-authors, I finish simulation and analytical calculations, and further test our theory on some random graph models to validate the correctness of our theory. The manuscript is under preparation.

Research Training on the Combinatorial Optimization Problem in Graph Theory

The Minimum Vertex Cover (MVC) problem involves finding the smallest set of vertices in a given undirected graph such that each edge has at least one endpoint in this set. Since this problem is NP-hard, we use an approximate algorithm — the message-passing algorithm. This algorithm has its origins in statistical mechanics of disordered systems, which are distributed in form and powerful for tackling problems with local constraints. It iteratively adjusts the states of nodes through information exchange, allowing them to collaboratively satisfy constraints. I implemented this algorithm and optimized it. Additionally, through analysis, we know that covering a certain type of node is optimal. Therefore, we can use percolation theory to calculate the proportion of these nodes. For those nodes that remain uncovered, we can approximately estimate the size of their MVC using the unstable fixed point. I reproduced the construction of this theoretical framework.

Research Project on Geometric Probability Problems

The Buffon-Laplace Needle Problem (BLNP) considers randomly dropping a thin needle onto a plane with two sets of vertically parallel lines, and studies its statistical properties of intersections between the needle and the lines. As a classic problem in geometric probability, it provides the mathematical foundation for some physical studies, such as the efficiency of filters. However, the original problem only considers short needles in two dimensions, which limits its potential applications. In our work, we extend needles to spherocylinders and two-dimensional setting to three-dimensional ones, thus make it possible to accommodate the needle problem in a more physical setting, such as in filtration processes of rod-like particles with finite volume. We analytically solve the above extended needle problems, and derive intersection probabilities for arbitrary parameter ranges. We finally validate our theoretical predictions with Monte Carlo simulations. In this project, along with co-authors, I complete all the analytical derivations, calculations, and simulations. The manuscript is currently under review in *Physica A: Statistical Mechanics and its Applications*.

Future Work

In my academic experiences, I am always fascinated with the the macroscopic phenomena that emerge in complex systems. My past research mainly revolves understanding the macroscopic behaviors of groups and solving computationally hard problems within complex networks. My approach follows an analytical and algorithmic perspectives, especially with a statistical mechanical background. In my graduate studies, I plan to follow and further extend this line of research by considering complex systems such as neural networks and the brain, and investigating their more intricate macroscopic behaviors. Here are three topics I'd like to explore in the future:

1. Network Science and machine learning methods

Percolation phenomena on networks are relatively simple dynamical processes which permit an analytical treatment for their macroscopic behaviors. Yet there are ubiquitous dynamics on network systems which are basically high-dimensional with multiple states and complex dynamical rules. An incorporation of machine learning methods, especially a low-dimensional representation of network structure and dynamics, could be a promising way to tackle these problems.

2. Computational Neuroscience

I also have a strong interest in brain science. To pursue this interest, I participated in the first Neural Computational Modeling Training Course hosted by Professor Si Wu's research group at Peking University, where I learned the modeling and numerical simulation of neuron and synapse models. This experience sparked my research interest in complex systems like the brain. I am also curious about how it achieves higher-level functions such as information processing, memory storage, and decision-making through the dynamics of neuronal populations.

3. Machine Learning/Deep Learning

I am particularly intrigued by the perplexing phenomena observed in neural networks, including training instability, difficulties in convergence, poor model performance, and weak generalization abilities. I aim to apply analytical theories and statistical physics methods to address these problems. This involves characterizing the global characteristics of the loss landscape, describing phase transition phenomena from "learnable" to "unlearnable," and understanding the fundamental mechanisms underlying generalization.