ECSE 4965/6965

Introduction to Deep Learning

Assignment #1,

Due date: 4pm, Feb. 8

1. Given three RVs, X,Y,Z, and their joint probability p(X,Y,Z)=p(X)p(Y)p(Z|X,Y), prove that X and Y are marginally independent of each other but become conditionally dependent given Z

2. Suppose you roll a die with six possible values i=1 ,2,..,6. Let the θ_i be the probability of the die landing on the number i, and θ_1=0.25, θ_2=0.3, θ_3=0.1, θ_4=0.1, θ_5=0.15, and θ_6=0.1 Assume the die was rolled 100 times, compute the probability that among the 100 rolls, 20 times, it comes up 1, 15 times it comes up 2, 10 times it comes up 3, 5 times it comes up 4, 20 times it comes up 5, and 30 times it comes up 6.

3. Given three binary RVs, X $\in$\{0,1\} , Y$\in$\{0,1\}, and Z$\in$\{0,1\}, as well as their joint probabilities p(X,Y) as follows:

x	y	z	p(x,y,z)
0	0	0	1/12
0	0	1	1/6
0	1	0	1/6
0	1	1	1/12
1	0	0	1/6
1	0	1	1/12
1	1	0	1/12
1	1	1	1/6

Prove X$\perp$Y, Z$\perp$Y, and X$\perp$Z

4. Let Σ be the covariance matrix for RV $X^{N\text{x}1}$, perform eigen and SVD decomposition of Σ , and compare them.

5. Given $f(\mathbf{X}) = \log\{(A\mathbf{X}+b)^t(C\mathbf{X}+d)+\lambda\mathbf{X}^t\mathbf{X}\}$, where **X** is a N x 1 vector, **A** and **C** are MxN matrix, **b** and **d** are a M x 1 vector, and λ is a scalar, **A, C, b,** and **d** are not function of **X**. Derive the equation for computing $\frac{\partial f(\mathbf{X})}{\partial \mathbf{X}}$