Q1.

0.3. a) Base case: $F_6 = 8 \geq 2^{6/2} = 8$.
Inductive Step: for $n \geq 6$, $F_{n+1} = F_n + F_{n-1} \geq 2^{n/2} + 2^{(n-1)/2} = 2^{(n-1)/2}(2^{1/2}+1) \geq 2^{(n-1)/2} 2 \geq 2^{(n+1)/2}$.

b-c) The argument above holds as long as we have, in the inductive step, $2^{c(n-1)}(2^c+1) \geq 2^{c(n+1)}$, i.e. as long as $2^c \leq \frac{1+\sqrt{5}}{2}$.

For part c), note that we need to find the largest value of c such that
$2^{cn} + 2^{c(n-1)} >= 2^{c(n+1)}$, or
$2^{cn} (1 + 1/2^c) >= 2^{cn} 2^c$, or
$1 + 1/2^c >= 2^c$
Let $2^c = a$
So, we need to solve the equation 1+1/a >= a, or a^2 - a – 1 >= 0
This implies a = (1 + sqrt(5)/2) or a = (1-sqrt(5))/2. The largest value is a = (1 + sqrt(5)/2).
Or c = log (a) = 0.694

Q2.
T(0) = 1
T(1) = 1 + T(1) = 2
T(2) = 1 + T(1) +_T(0) = 1 + 2 + 1 = 4
T(3) = 1 + T(2) + T(1) + T(0) = 1 + 4 + 2 + 1 = 8
Therefore it appears that T(n) = 1 + T(n-1) + T(n-2) + … + T(0) = 2^n
Prove by induction. Certainly true for n=0, n=1, n=2, etc
Assume true for all values upto n, show true for T(n+1)

We have T(n+1) = 1 + T(n) + T(n-1) + … + T(1) + T(0)
= 1 + 2^n + 2^(n-1) + … + 2 + 1
= 1 + (2^(n+1)-1)/(2-1)
= 1 + 2^(n+1) – 1 = 2^(n+1)

Thus, T(n) = 2^n

Q3.

a. $n(n+1) = \Theta(2000n^2)$
b. $100\ n^2 = O(0.01\ n^3)$
c. $\log_2 n = \Theta(\ln n)$
d. $\log_2^2 n = O(\log_2 n^2)$
e. $2^{n-1} = \Theta(2^n)$
f. $(n-1)! = O(n!)$

Q4. The functions in increasing order are:
$\ln^2 n$
$5 \log_2(n+100)^{10}$
$(n)^{1/3}$

$0.001n^4 + 3n^3 + 1$
3^n
2^{2n}
$(n-2)!$