Lecture 6 Projectors

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1 Prerequisite

2 Solutions

2.1 Exercise 6.1

Proof. Since $\boldsymbol{P}$ is an orthogonal projector, then we have $\boldsymbol{P}^2 = \boldsymbol{P}$ and $\boldsymbol{P}^\star = \boldsymbol{P}$, hence

$$(\boldsymbol{I} - 2\boldsymbol{P})^\star(\boldsymbol{I} - 2\boldsymbol{P}) = \boldsymbol{I} - 2\boldsymbol{P}^\star - 2\boldsymbol{P} + 4\boldsymbol{P}^\star\boldsymbol{P} = 0, \tag{1}$$

which means that $\boldsymbol{I} - 2\boldsymbol{P}$ is unitary. A geometric interpretation is given by **??**. □

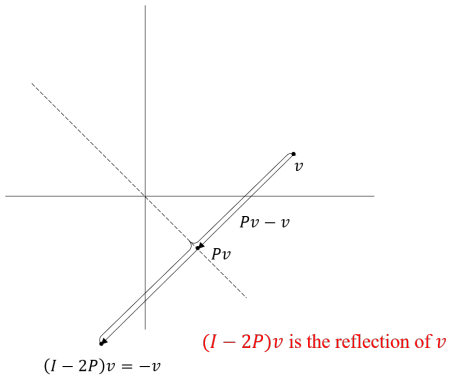

Figure 1: Geometric interpretation of $\boldsymbol{I} - 2\boldsymbol{P}$.