4. $u_x = n\cos nx \sinh ny$

$u_{xx} = -n^2 \sin nx \sinh ny$

$u_y = \sin nx \cdot n\cos$

$\sinh ny = n^2 \sinh ny.$

$\Rightarrow u_{xx} + u_{yy}$

$= -n^2 \sin(nx)\sinh(ny)$

$\quad + n^2 \sin nx \sinh(ny)$

$= 0.$

5.(2). $u_t - u_{xx} + xu = 0.$ 二阶线性非齐次.

(4). 二阶, 非线性, 齐次.

(6). $u_x(1+u_x^2)^{-\frac{1}{2}} + u_y(1+u_y^2)^{-\frac{1}{2}} = 0.$

一阶 非线性齐次

(8) $u_t + u_{xxxx} + \sqrt{1+u} = 0.$

四阶非线性齐次.

6.(1). $u_x + 2xy^2 u_y = 0.$

$\dfrac{\partial u}{\partial x} + 2xy^2 \dfrac{\partial u}{\partial y} = 0 \Rightarrow \dfrac{dy}{dx} = 2xy^2.$

$\Rightarrow y = \dfrac{1}{c - x^2} \Rightarrow x^2 + \dfrac{1}{y} = C \Rightarrow u(x,y) = f(x^2 + \dfrac{1}{y}).$

(2). $(1+x^2)u_x + u_y = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1+x^2}. \Rightarrow y = \arctan x + C.$

$\Rightarrow y - \arctan x = C. \Rightarrow u(x,y) = f(y - \arctan x)$

(3). $au_x + bu_y + cu = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{b}{a} \Rightarrow y - \dfrac{b}{a}x = C. \quad u = Ce^{-\frac{c}{a}x}.$

$\Rightarrow u(x,y) = f(y - \dfrac{b}{a})x \cdot e^{-\frac{c}{a}x}.$

7. $yu_{xx} + u_{yy} = 0.$

二阶，线性齐次方程．