

vector analysis

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1 triple product

三重积的值是括号里面的两个矢量的线性组合，中间的矢量的系数是正的，例子：

$$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$$

2 球坐标

$$\begin{cases} x = r \sin \theta \cos \varphi \\ y = r \sin \theta \sin \varphi \\ z = r \cos \theta \end{cases}$$

$$f = \sqrt{\left(\frac{\partial x}{\partial r}\right)^2 + \left(\frac{\partial y}{\partial r}\right)^2 + \left(\frac{\partial z}{\partial r}\right)^2} = 1 \quad g = r \quad h = r \sin \theta$$

$$\nabla y = \frac{1}{f} \frac{\partial y}{\partial r} + \frac{1}{g} \frac{\partial y}{\partial \theta} + \frac{1}{h} \frac{\partial y}{\partial \varphi}$$

$$\nabla \cdot \vec{y} = \frac{1}{fgh} \left(\frac{\partial}{\partial r} (gh \frac{\partial y}{\partial r}) + \frac{\partial}{\partial \theta} (fh \frac{\partial y}{\partial \theta}) + \frac{\partial}{\partial \varphi} (fg \frac{\partial y}{\partial \varphi}) \right)$$

$$\nabla^2 y = \frac{1}{fgh} \left(\frac{\partial}{\partial r} \left(\frac{gh}{f} \frac{\partial y}{\partial r} \right) + \frac{\partial}{\partial \theta} \left(\frac{fh}{g} \frac{\partial y}{\partial \theta} \right) + \frac{\partial}{\partial \varphi} \left(\frac{fg}{h} \frac{\partial y}{\partial \varphi} \right) \right)$$