Handbook on basic analysis

Andriy Zhugayevych

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§1. Constants

$\pi \approx 3.1416$, $\mathrm{e} \approx 2.7183$, $\ln 10 \approx 2.3026$, $\lg 2 \approx 0.30103$.
Euler's constant $\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^{n} k^{-1} - \ln n\right) \approx 0.5772$.

§2. Sums and products

2.1. Series expansion

Taylor's formula:

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k + R_n(x),$$

where

$$R_n(x) = \frac{1}{n!}\int_0^x f^{(n+1)}(y)(x-y)^n \,\mathrm{d}y = \frac{f^{(n+1)}(\xi)}{(n+1)!} x^{n+1},\ 0 < \xi < x.$$

Multidimensional Taylor's formula:

$$f(x_1,\dots,x_d) = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\sum_{i=1}^{d} x_i \partial_i\right)^n f = \sum_{n_1,\dots,n_d \geqslant 0} f^{(n_1,\dots,n_d)} \frac{x_1^{n_1}\dots x_d^{n_d}}{n_1!\dots n_d!}.$$