Some mathematical formulas *

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Definite integrals:

$$\int_{0}^{\pi/2} \sin^{\mu} x \cos^{\nu} x \, dx = \frac{\Gamma\left(\frac{\mu+1}{2}\right) \Gamma\left(\frac{\nu+1}{2}\right)}{2\Gamma\left(\frac{\mu+\nu}{2}+1\right)}, \quad \int_{0}^{1} t^{p-1} (1-t)^{q-1} \, dt = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)},$$

$$\int_{-\infty}^{\infty} e^{-ax^{2}+bx} \, dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^{2}}{4a}},$$

$$\int_{0}^{\infty} x^{\mu-1} e^{-ax} \, dx = \frac{\Gamma(\mu)}{a^{\mu}}, \quad \int_{0}^{\infty} x^{\mu-1} e^{-ax^{2}} \, dx = \frac{\Gamma(\mu/2)}{2a^{\mu/2}}.$$

Special functions:

$$\begin{split} &\Gamma(z+1) = z\Gamma(z), \quad \Gamma(n) = (n-1)!, \quad \Gamma(1/2) = \sqrt{\pi}. \\ &H_{n+1} = 2\xi H_n - 2nH_{n-1}. \\ &\rho^2 Z'' + \rho Z' + (\lambda^2 \rho^2 - \nu^2) Z = 0 \implies Z = c_1 J_{\nu}(\lambda \rho) + c_2 N_{\nu}(\lambda \rho). \end{split}$$

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