

Handbook on matrices

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1	Conventions	1
2	Basic matrix theory	2
2.1	Types of matrices	2
2.2	Characteristic matrix $\lambda - A$	2
2.3	Spectrum	2
3	Canonical forms of matrices	3
3.1	Diagonal form	3
3.2	Jordan normal form	4
3.3	Schur form	5
3.4	Rectangular matrices, singular values and pseudoinverse	5
4	Matrix functions	5
4.1	Using Jordan normal form	5
4.2	Interpolating polynomial	6
4.3	Special relations	7
5	Miscellaneous	7
5.1	Linear equations	7
5.2	Bilinear forms	7
5.3	Matrices with positive elements	7
6	Special relations for matrices	7
6.1	Determinant	7
6.2	Special matrices	8

§1. Conventions

Capitals A are matrices, normal symbols a are vectors, and Greek symbols α are scalars. Upper index enumerates vectors and matrices while lower indexes enumerates their elements. For a matrix component A_{ij} i means row and j means column. Throughout this paper we denote n to be the dimension of matrices and vectors. We denote identity matrix simply by 1 so that $\lambda - A$ means $(\lambda - A)_{ij} = \lambda \delta_{ij} - A_{ij}$. Transposed matrix is denoted by A^\top : $(A^\top)_{ij} = A_{ji}$. Hermitian conjugated matrix is denoted by A^+ : $(A^+)_{ij} = \overline{A_{ji}}$.

There are two kinds of linear transformations. A linear map given by a nondegenerate matrix T transforms $x \rightarrow Tx$ so that $(Ax) \rightarrow T(Ax)$ and thus $A \rightarrow TAT^{-1}$. Two matrices are similar (or equivalent) if there exists a transformation mapping one matrix into another. The second kind of linear transformations is a basis change (basis is an outer object for the matrix theory). Let a new basis is given by $e'_i = \sum_j e_j B_{ji}$, where B is a nondegenerate matrix. A vector defined as $x = \sum_i x_i e_i$ transforms as follows: $x' = B^{-1}x$. A matrix defined by the scalar product $(x, Ay) = \sum_{ij} \bar{x}_i A_{ij} y_j$ transforms as $A' = B^+ A B$. In particular the basis overlap matrix $S_{ij} = (e_i, e_j)$ transforms as $S' = B^+ S B$. Both kinds of transformations are equivalent iff the transformation matrix is unitary.

We call matrix A reducible if there is such a partition of its index set $\{1, \dots, n\} = \Xi \cup \Gamma$ ($\Xi \cap \Gamma = \emptyset$) that $\forall \xi \in \Xi \forall \gamma \in \Gamma A_{\xi\gamma} = 0$. If in addition $A_{\gamma\xi} = 0$ then the matrix A is decomposable. Thus the natural chain is $\{\text{decomposable}\} \supset \{\text{irreducible}\}$.

As default real symmetric matrices are symbolized by S , hermitian by H , real orthogonal by O , unitary by U , projective by P .