

# Handbook on basic analysis

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October 9, 2011

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## §1. Constants

$\pi \approx 3.1416$ ,  $e \approx 2.7183$ ,  $\ln 10 \approx 2.3026$ ,  $\lg 2 \approx 0.30103$ .

Euler's constant  $\gamma = \lim_{n \rightarrow \infty} (\sum_{k=1}^n k^{-1} - \ln n) \approx 0.5772$ .

## §2. Sums and products

### 2.1. Series expansion

Taylor's formula:

$$f(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k + R_n(x),$$

where

$$R_n(x) = \frac{1}{n!} \int_0^x f^{(n+1)}(y) (x-y)^n dy = \frac{f^{(n+1)}(\xi)}{(n+1)!} x^{n+1}, \quad 0 < \xi < x.$$

Multidimensional Taylor's formula:

$$f(x_1, \dots, x_d) = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \sum_{i=1}^d x_i \partial_i \right)^n f = \sum_{n_1, \dots, n_d \geq 0} f^{(n_1, \dots, n_d)} \frac{x_1^{n_1} \dots x_d^{n_d}}{n_1! \dots n_d!}.$$