Handbook on matrices

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§1. Conventions

Capitals A are matrices, normal symbols a are vectors, and Greek symbols α are scalars. Upper index enumerates vectors and matrices while lower indexes enumerates their elements. For a matrix component A_{ij} i means row and j means column. Throughout this paper we denote n to be the dimension of matrices and vectors. We denote identity matrix simply by 1 so that $\lambda - A$ means $(\lambda - A)_{ij} = \lambda\delta_{ij} - A_{ij}$. Transposed matrix is denoted by $A^\top$: $(A^\top)_{ij} = A_{ji}$. Hermitian conjugated matrix is denoted by A^+: $(A^+)_{ij} = \overline{A_{ji}}$.

There are two kinds of linear transformations. A linear map given by a nondegenerate matrix T transforms $x \to Tx$ so that $(Ax) \to T(Ax)$ and thus $A \to TAT^{-1}$. Two matrices are similar (or equivalent) if there exists a transformation mapping one matrix into another. The second kind of linear transformations is a basis change (basis is an outer object for the matrix theory). Let a new basis is given by $e'_i = \sum_j e_j B_{ji}$, where B is a nondegenerate matrix. A vector defined as $x = \sum_i x_i e_i$ transforms as follows: $x' = B^{-1}x$. A matrix defined by the scalar product $(x, Ay) = \sum_{ij} \overline{x}_i A_{ij} y_j$ transforms as $A' = B^+AB$. In particular the basis overlap matrix $S_{ij} = (e_i, e_j)$ transforms as $S' = B^+SB$. Both kinds of transformations are equivalent iff the transformation matrix is unitary.

We call matrix A reducible if there is such a partition of its index set $\{1, \ldots, n\} = \Xi \cup \Gamma$ $(\Xi \cap \Gamma = \emptyset)$ that $\forall\xi \in \Xi\ \forall\gamma \in \Gamma\ A_{\xi\gamma} = 0$. If in addition $A_{\gamma\xi} = 0$ than the matrix A is decomposable. Thus the natural chain is {decomposable}⊃{indecomposable reducible}⊃{irreducible}.

As default real symmetric matrices are symbolized by S, hermitian by H, real orthogonal by O, unitary by U, projective by P.