Principles of Communications

Chapter 3 — Analog Modulation System

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Introduction

Baseband Signal

Source
- text
- music
- figure

transform $\Longrightarrow$ low frequency electrical signal $\Downarrow$ Baseband Signal

- Low pass spectrum
- Have a high ratio bandwidth $B_R = \frac{f_H}{f_L}$.

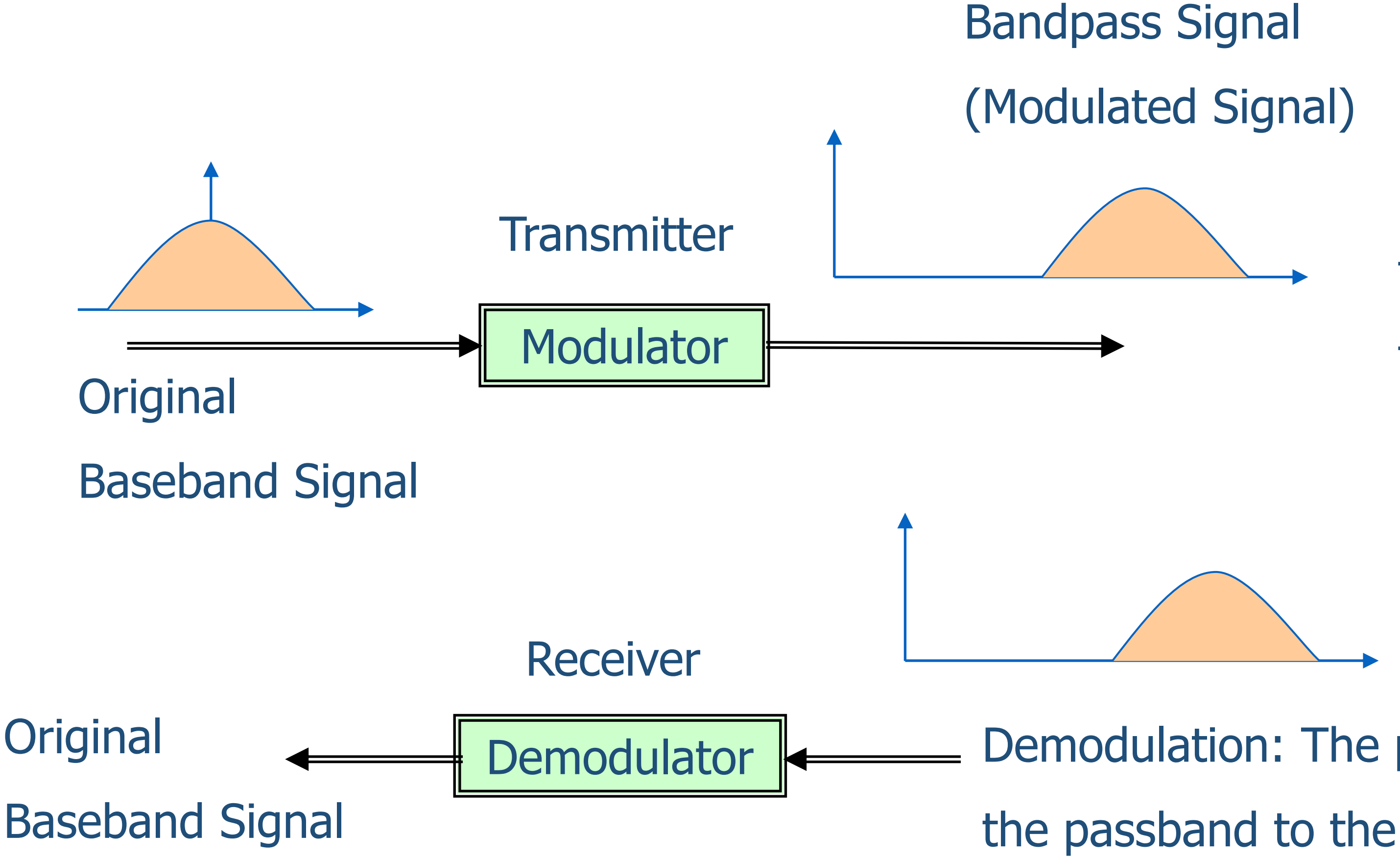
Ex: the frequency range of the telephone signal is 300-3000Hz

Baseband Signal
- Can: transmit through wired channels such as overhead wires and cables
- Can't: transmit over wireless channels

But: On a wired channel, only one signal can be transmitted on a pair of lines, which is very uneconomical

Introduction

Bandpass Signal & Modulation



Modulation: A process of moving the baseband signal spectrum into the passband of a given channel

Demodulation: The process of restoring the spectrum in the passband to the baseband signal

Introduction

Why we need to modulate the signal in communication?

1 Moves the baseband signal spectrum to a certain frequency range to meet the requirements of the channel (Due to different pathloss in different frequencies)

2 Easy to radiate (Higher frequency, the radiation antenna is smaller)

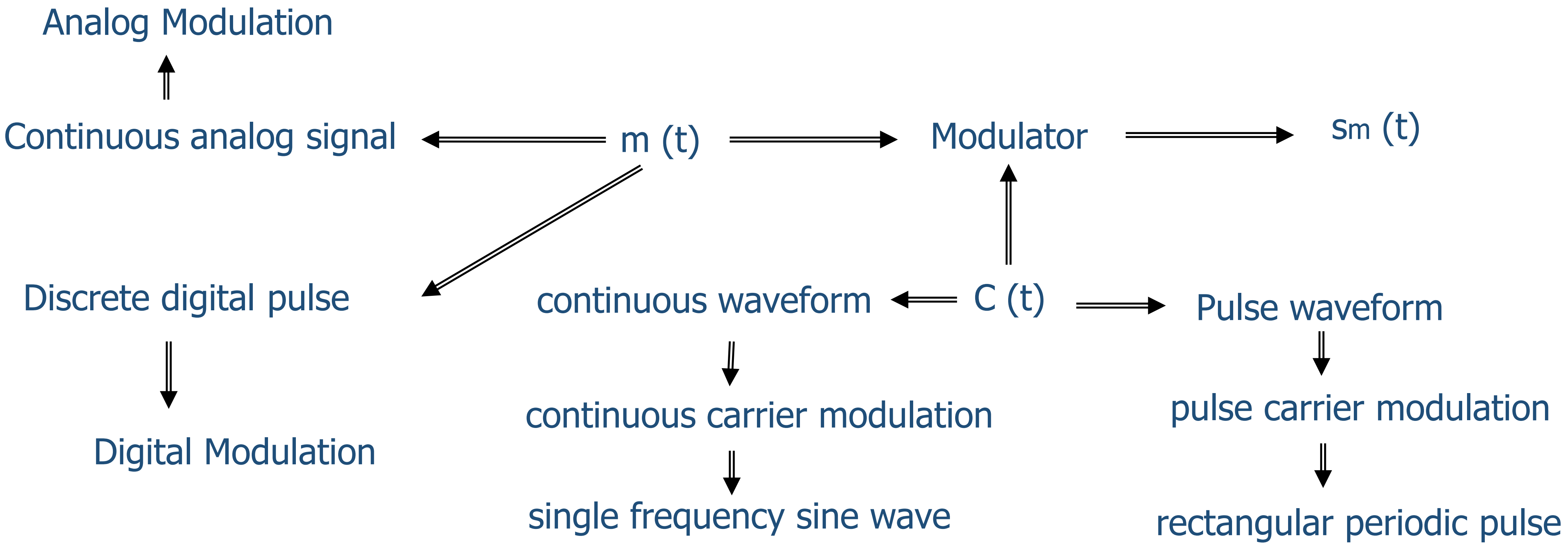
3 Easy to Propagate (Higher frequency has more power and can travel farther)

4 Frequency-division multiplexing (To avoid interference)

5 Reduce the impact of noise and interference, improve the system anti-interference ability

Introduction

Modulation Classification (Analog Modulation & Digital Modulation)

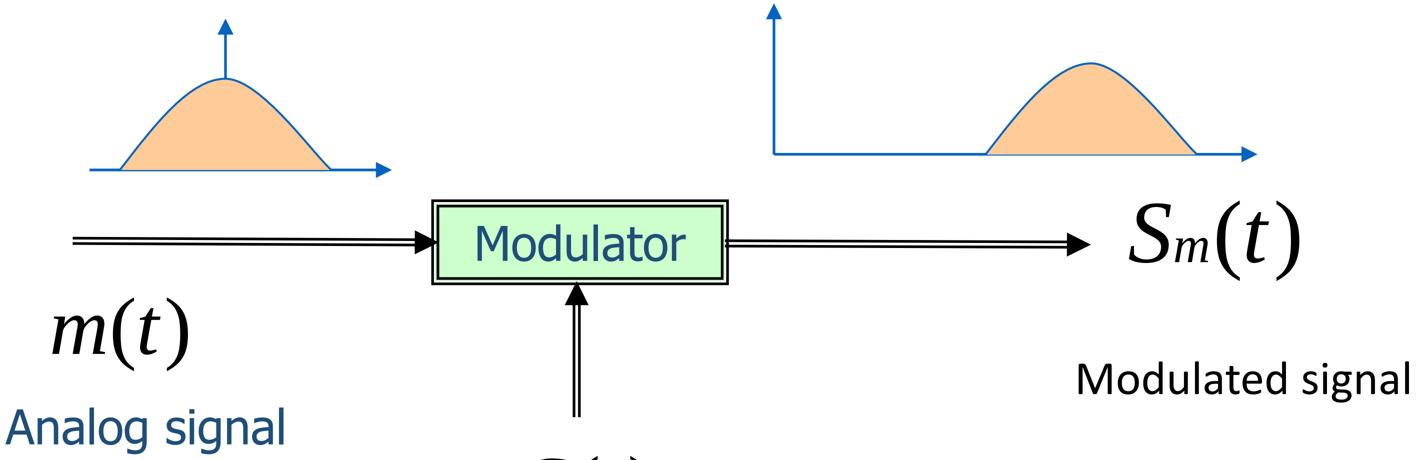


The application of analog modulation is less frequently used currently due to the digital communication.

But it is the most basic modulation mode.

Introduction

Analog Modulation



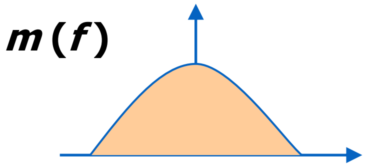
$C(t)$ Carrier: a deterministic periodic waveform

Usually, $c(t) = A\cos(\omega_0 t + \varphi_0) = A\cos(2\pi f_0 t + \varphi_0)$

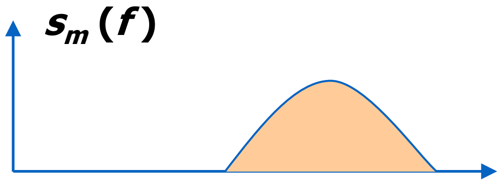
High frequency

Introduction

Analog Modulation Classification

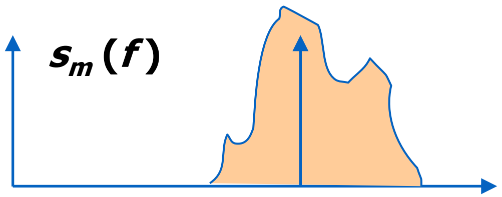


Linear Modulation



Displacement of modulating signal along frequency axis: AM、DSB 、 SSB 、 VSB

Non-linear Modulation



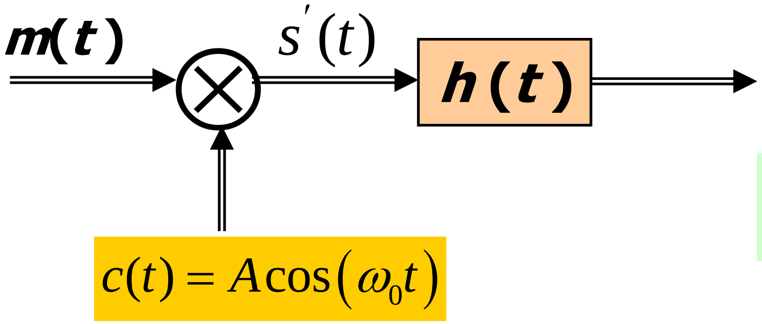
The spectrum of modulated signal is different from the original spectrum, not just the displacement in frequency axis: FM、PM

Linear Modulation

Basic concept

Let the carrier be $c(t) = A\cos\omega_0 t = A\cos 2\pi f_0 t$

Let the modulating signal be $m(t)$



$$s_m(t) = Am(t)\cos(\omega_c t + \varphi_0)$$

$$S_m(f) = \frac{A}{2}[M(f + f_c) + M(f - f_c)]$$

m(t) and c(t) are multiplied:

$$s'(t) = m(t)A\cos\omega_0 t$$

Usually, the multiplied signal passes through a bandpass filter h(t) and gets $s_m(t)$

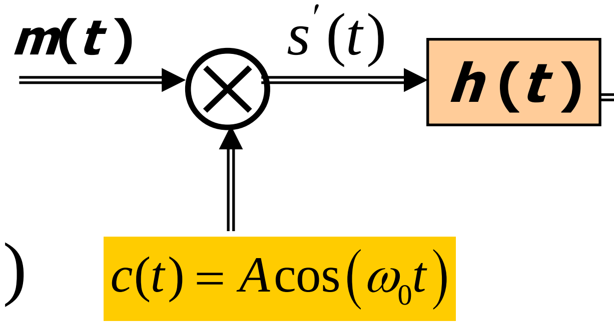
In frequency domain $$m(t) \Leftrightarrow M(f)$$

$$m(t)A\cos\omega_0 t \Leftrightarrow S'(f)$$

where $$S'(f) = \frac{A}{2}\left[M(f - f_0) + M(f + f_0)\right]$$

Linear Modulation

Basic concept



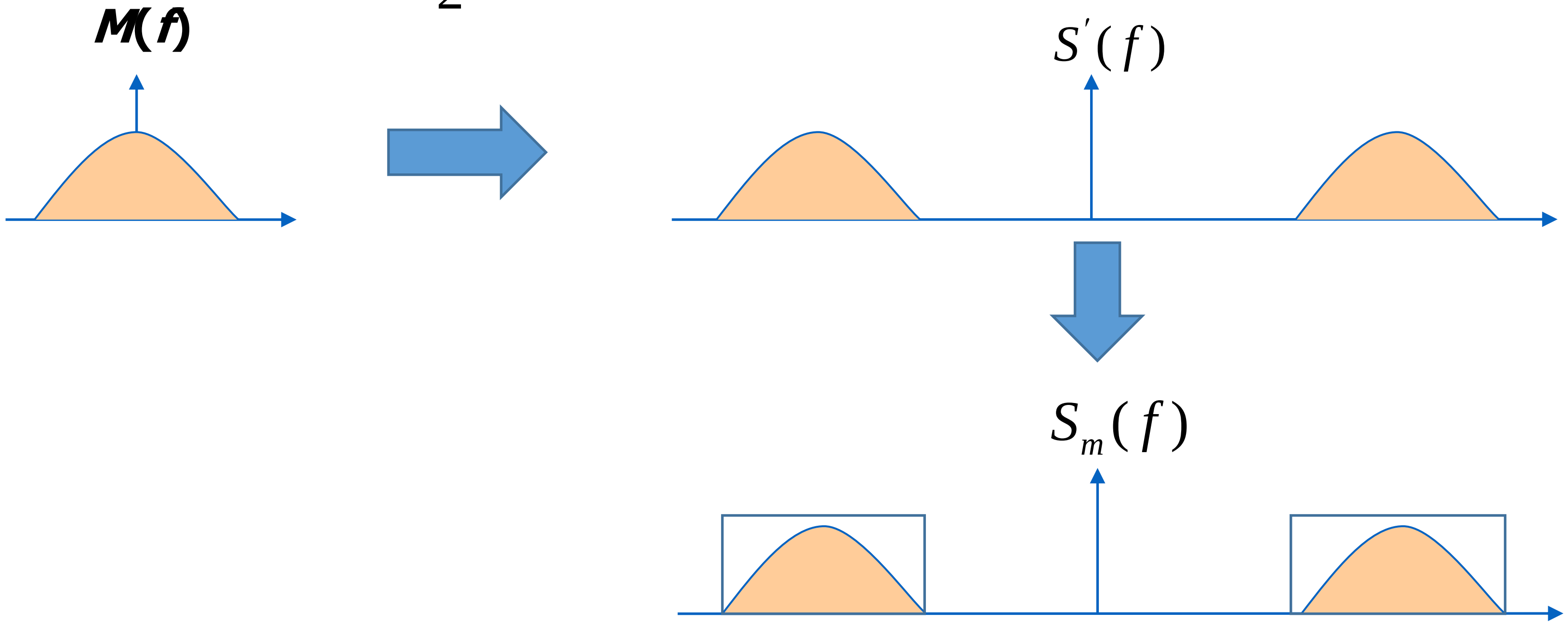
$$s_m(t) = Am(t)\cos\left(\omega_c t + \varphi_0\right)$$

$$S_m(f) = \frac{A}{2}[M(f+f_c) + M(f-f_c)]$$

In frequency domain $\quad m(t) \Leftrightarrow M(f)$

$$m(t)A\cos\omega_0 t \Leftrightarrow S'(f)$$

where $\quad S'(f) = \frac{A}{2}\left[M(f-f_0) + M(f+f_0)\right]$



Linear Modulation

Amplitude Modulation

Assume modulating signal m(t) contains D. C. component and A.C component

$$m(t) = 1 + m'(t)$$

D. C. component   A.C component $|m'(t)| \le 1$

The modulated signal: $s'(t) = [1 + m'(t)] A\cos\omega_0 t$

Why there should be D.C component?

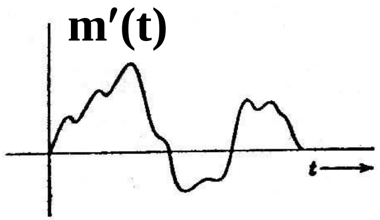
The envelope of s'(t) isn't less than 0, can be detected with simple low pass filter

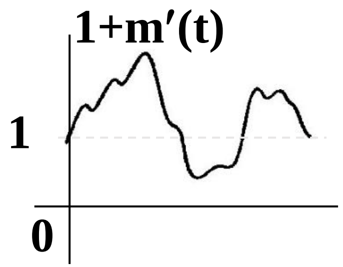
For modulating signal without D. C. component ,in order to obtain amplitude modulation,other simpler modulator circuit is often used , and the method of adding D.C component is not used.

Linear Modulation

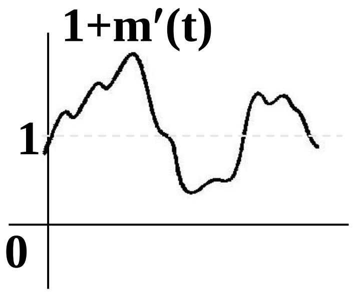
Amplitude Modulation

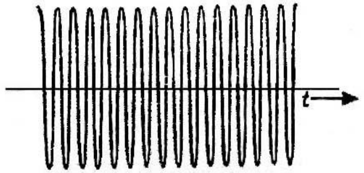
Time domain

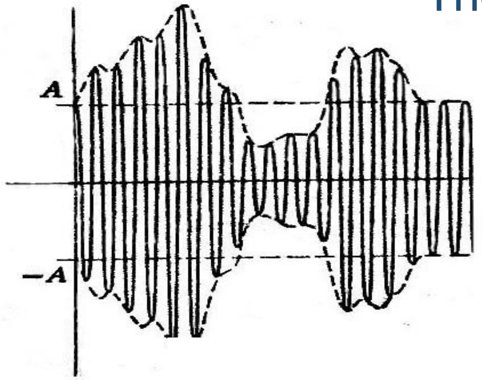




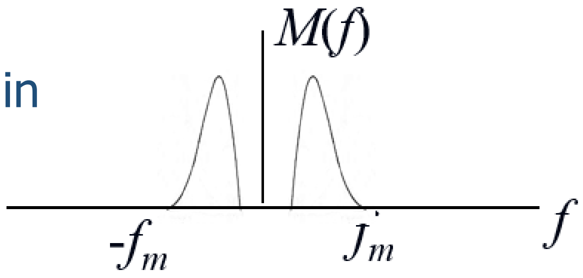
The envelope is positive

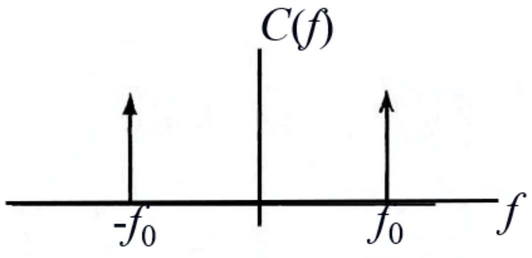


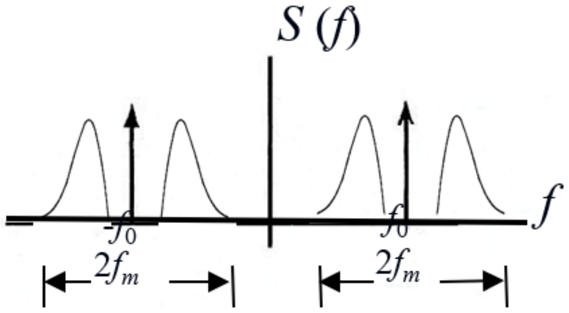




Frequency domain







Linear Modulation

Amplitude Modulation

How to detect the envelope?

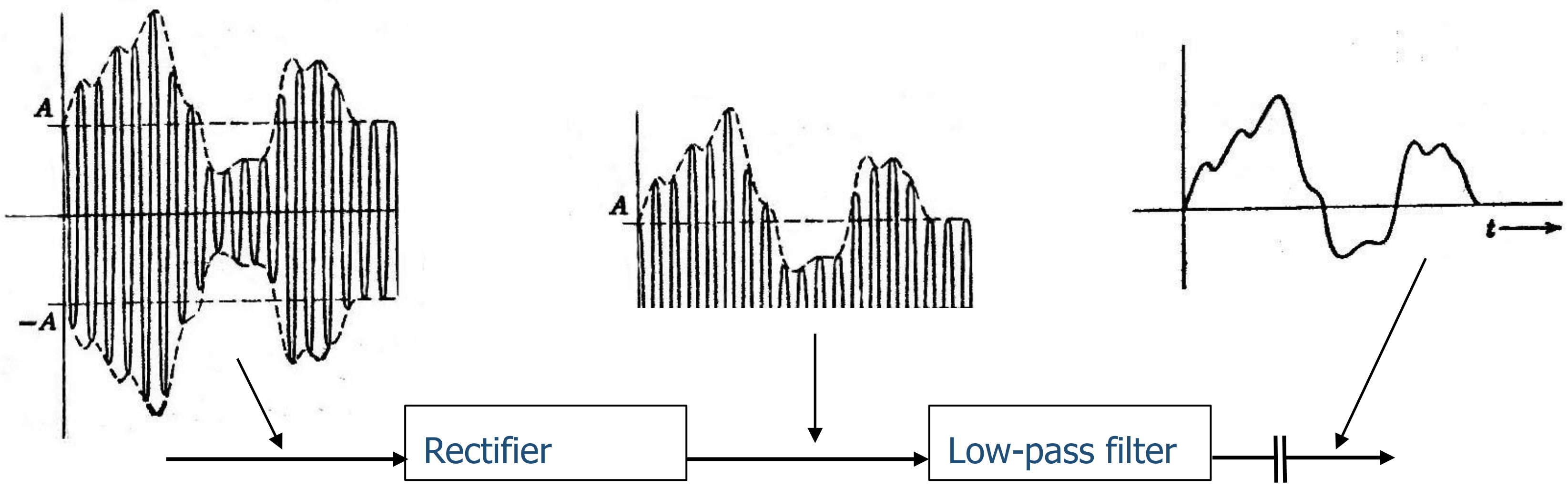


Figure 3.2.4 Envelope detector

The shape of the envelope of the modulated signal is the same as the shape of the modulating signal

Linear Modulation

Amplitude Modulation

What's the advantages of the AM?

Simple: Envelope detection method can be used for demodulation without local synchronization carrier signal

What's the problem of the AM?

1 The envelope detector could fail when the D.C component is not appropriate.

2 The power utilization of AM signal is relatively low

Linear Modulation

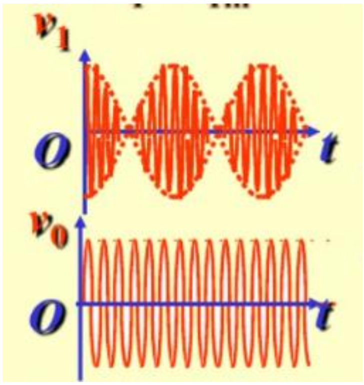
Amplitude Modulation

Problem 1: The envelope detector could fail when the D.C component is not appropriate.

→ The envelope detector can only detect the positive envelope. When $|m'(t)| \geq 1$ the recovered signal is distorted.

Possible solution: To ensure distortion-free demodulation, synchronous demodulation can be used

mixing the signal with a replica of the unmodulated carrier



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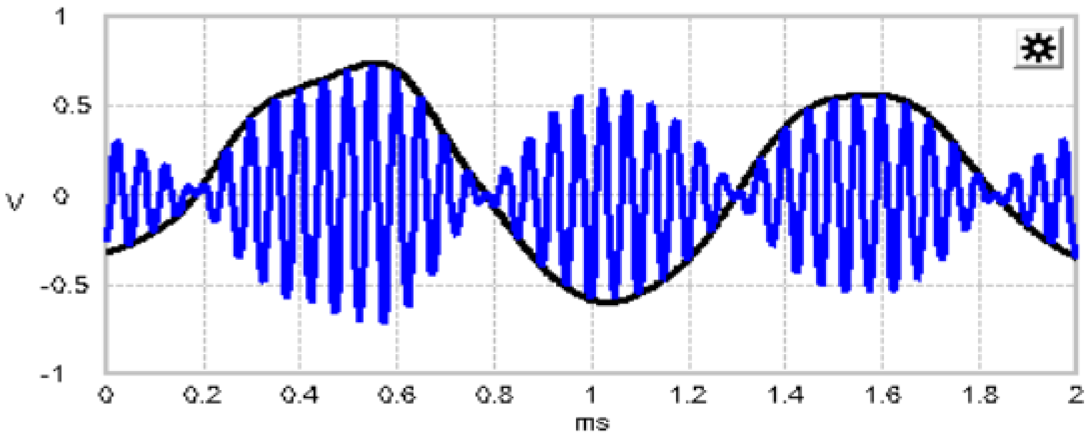
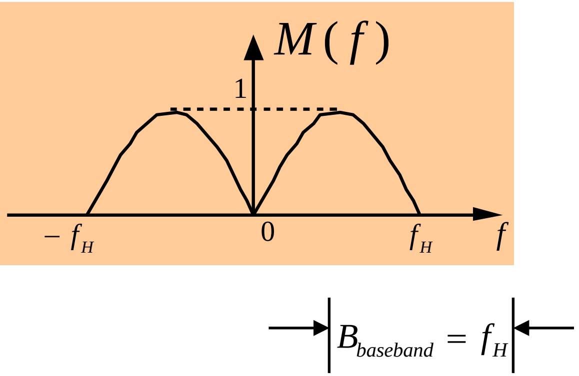
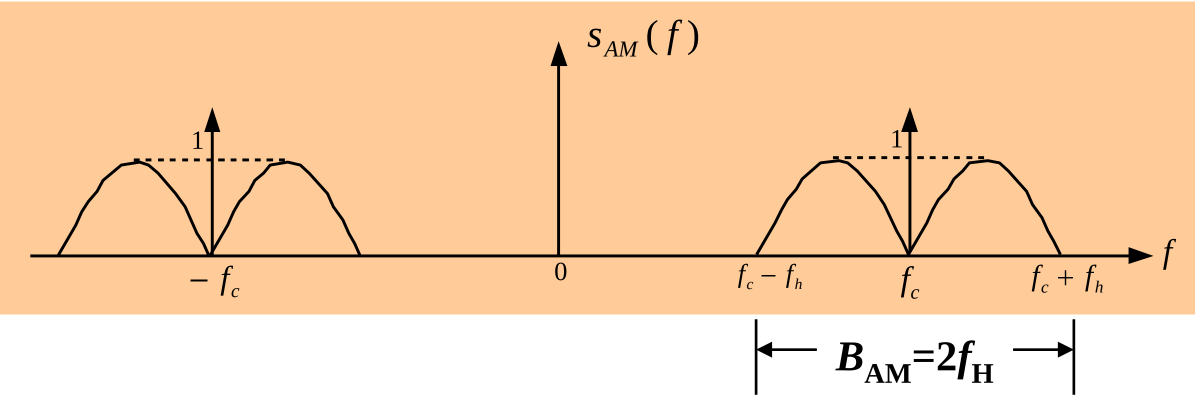


Linear Modulation

Amplitude Modulation

Problem 2: The power utilization of AM signal is relatively low .





1. $S_{AM}(f)$ is composed of carrier frequency components

2. The spectral structure of the upper sideband is the same as that of the original modulated signal, and the lower sideband is the mirror image of the upper sideband

3. The AM signal is a double-sideband signal with a carrier, and its bandwidth is twice the bandwidth of the baseband signal f_H

Linear Modulation

Amplitude Modulation

Problem 2: The power utilization of AM signal is relatively low .

Assume the input voltage of the envelope detector is

$$y(t) = \left\{\left[1 + m'(t)\right]A + n_c(t)\right\}\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

Input noise of the detector

$$n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

The envelope of $y(t)$

$$V_y(t) = \sqrt{\left\{\left[1 + m'(t)\right]A + n_c(t)\right\}^2 + n_s^2(t)}$$

Be approximate:

$$V_y(t) \approx \left[1 + m'(t)\right]A + n_c(t)$$

After detection, the output voltage:

$$v(t) = m'(t)A + n_c(t)$$

Before detection, the SNR:

$$r_i = E\left\{\frac{1}{2}\left[1 + m'(t)\right]^2 A^2 / n^2(t)\right\}$$

After detection, the SNR:

$$r_o = E\left[m'^2(t)A^2 / n_c^2(t)\right]$$

Linear Modulation

Amplitude Modulation

Problem 2: The power utilization of AM signal is relatively low .

Before detection, the SNR: $r_{\mathrm{i}} = E\left\{ \frac{1}{2}\left[1 + m'(t)\right]^2 A^2 / n^2(t) \right\}$

After detection, the SNR: $r_{\mathrm{o}} = E\left[m'^2(t) A^2 / n_{\mathrm{c}}^2(t) \right]$

The SNR ratio: $\frac{r_0}{r_{\mathrm{i}}} = E\left\{ \frac{m'^2(t) A^2 / n_{\mathrm{c}}^2(t)}{\frac{1}{2}\left[1 + m'(t)\right]^2 A^2 / n^2(t)} \right\} = E\left[\frac{2m'^2(t)}{\left[1 + m'(t)\right]^2} \right]$

Assume

$E\left[n_{\mathrm{c}}^2(t)\right] = E\left[n^2(t)\right]$

Obviously ,ro/ri is less than 1 for m'(t) <1. SNR has decreased after detection

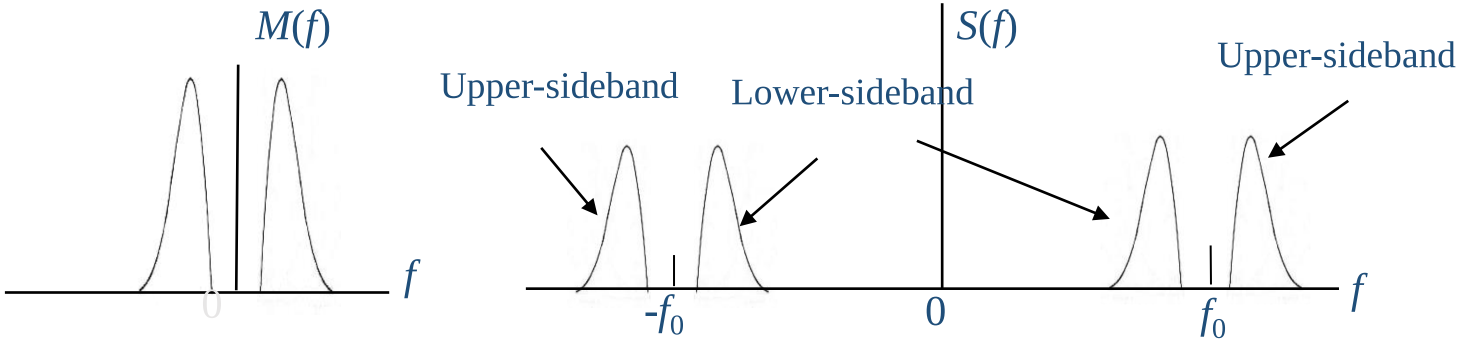
Because most part of power in signal before detection is occupied by the carrier, which carries no information.

Linear Modulation

Double sideband Modulation (double-sideband suppressed carrier AM)

When the modulation signal m(t) has no D.C. component, DSB signal is obtained

Frequency spectrum: the two sidebands contain identical information



(a) Frequency spectral density of modulating signal

(b) Frequency spectral density of modulated signal

Figure 3.2.5 Spectrum of double-sideband modulation signal

Linear Modulation

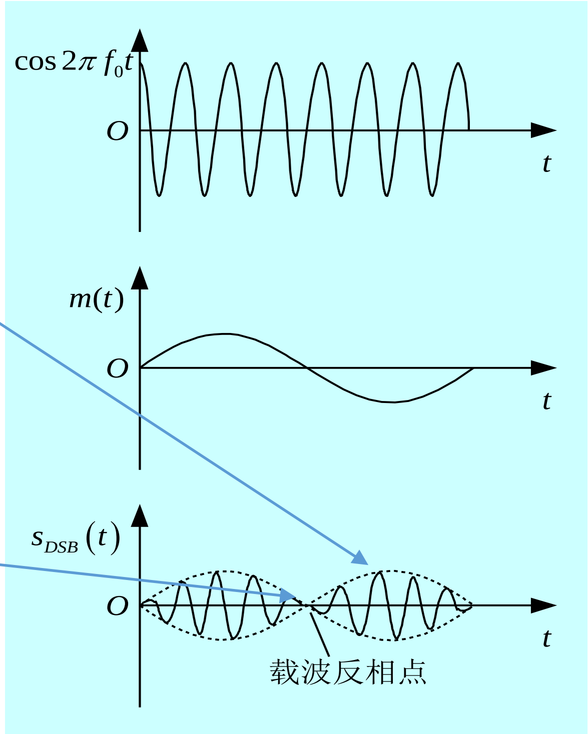
Double sideband Modulation (double-sideband suppressed carrier AM)

Since no D.C component, the carrier of the DSB signal is not transmitted and the power of the carrier is saved

However, the detection becomes difficult compared to AM.

The envelope of the DSB signal is not consistent with the that of the modulating signal

At the zero-crossing point of the modulation signal m(t), the high-frequency carrier phase has a 180° mutation



Linear Modulation

Double sideband Modulation (double-sideband suppressed carrier AM)

How to detect the DSB signal?

- Receiver circuit is more complex.
- Need to have a carrier in the receiver for demodulation
- the frequency and phase of the carrier in the receiver should be the same as that in the transmitter. (synchronous demodulation)

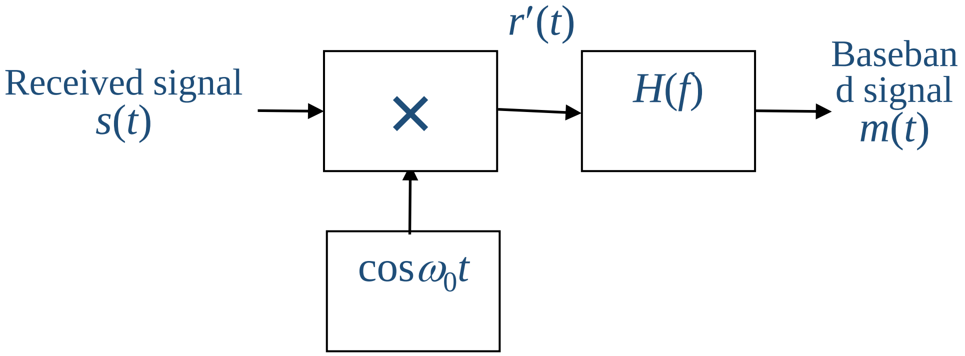


Figure 3.2.6 Block diagram of DSB signal demodulator

Linear Modulation

Double-sideband Modulation (DSB)

Let received DSB signal be $m'(t)\cos(\omega_0 t)$.

local carrier at the receiver $\cos\left[\left(\omega_0+\Delta\omega\right)t+\varphi\right]$

Frequency and phase asynchronous

The product of the received signal and the local carrier

$$r'(t)=m'(t)\cos\omega_0 t\cos\left[\left(\omega_0+\Delta\omega\right)t+\varphi\right]=\frac{1}{2}m'(t)\left\{\cos(\Delta\omega t+\varphi)+\boxed{\cos\left[\left(2\omega_0+\Delta\omega\right)t+\varphi\right]}\right\}$$

The demodulated output signal $\frac{1}{2}m'(t)\cos(\Delta\omega t+\varphi)$.

filter out by the low-pass filter

If the local carrier has no frequency and phase error $m'(t)/2$ no distortion

Linear Modulation

Double-sideband Modulation (DSB)

The benefits of the DSB signal:

DSB signal saves carrier power and improves power utilization

What can be improved?

The frequency bandwidth is still twice the bandwidth of the modulating signal

The upper and lower sidebands are completely symmetrical

Question: Can only one of the sidebands be transmitted to save bandwidth?

Linear Modulation

Single-sideband Modulation (SSB)

Motivated by the problem of DSB: → SSB

- Since the two sidebands contain the same information for DSB signal, from the perspective of information transmission, it is enough to transmit one sideband.
- This communication idea that only transmits one sideband is called single-sideband communication.
- How to achieve this: filter

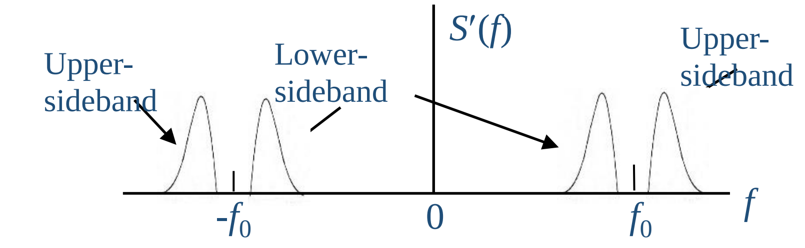
Linear Modulation

Single-sideband Modulation (SSB)

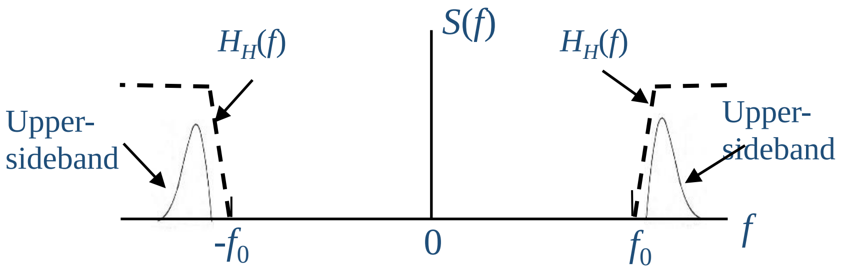
The basic idea of SSB : filter out one upper sideband and one lower sideband

How to obtain in transmitter?

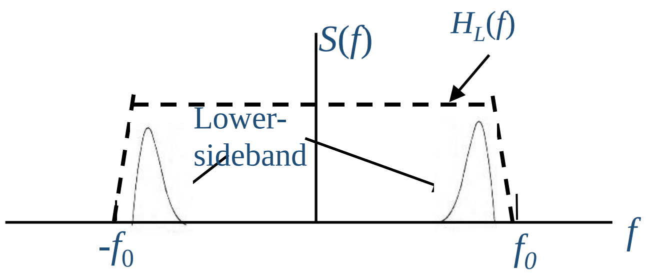
- When apply the high-pass filter, upper sideband signal is obtained.
- When apply the low-pass filter, lower sideband signal is obtained.



(a) Signal frequency spectrum before filtering



(b) Upper-sideband filter characteristic and signal frequency spectrum



(c) Lower-sideband filter characteristic and signal frequency spectrum

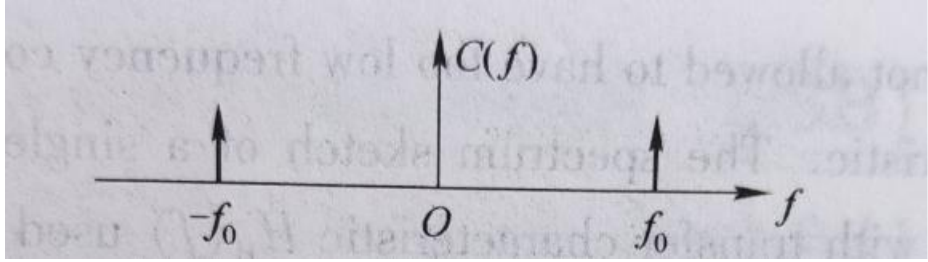
Linear Modulation

Single-sideband Modulation (SSB)

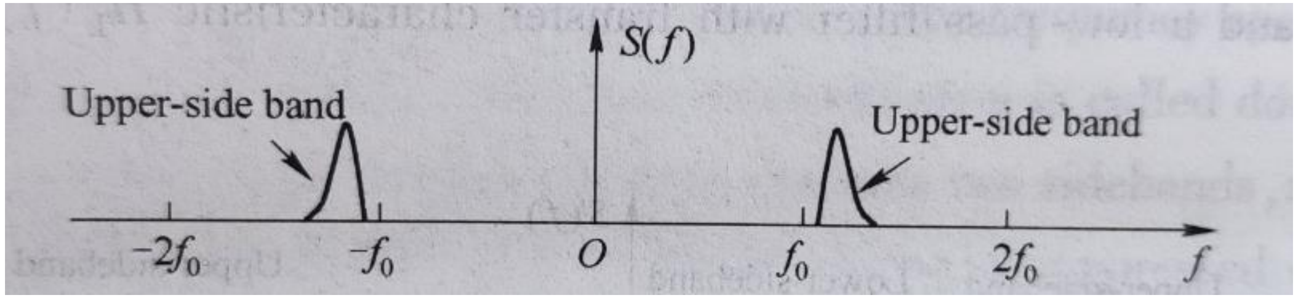
How to restore the signal from SSB?

1 Multiply a carrier

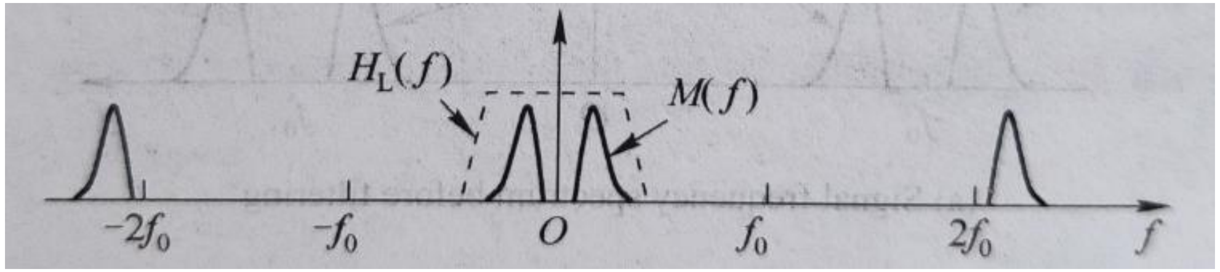
2 Low pass filter



Spectrum of carrier



Spectrum of the received SSB



Multiply the carrier in time domain

Linear Modulation

Single-sideband Modulation (SSB)

- The SSB not only saves the transmission power, but also occupies half of the bandwidth of DSB and AM. Therefore, it is currently an important modulation method in communication.
- The demodulation of SSB signal cannot adopt simple envelope detection, because SSB signal is also a modulated signal that suppresses the carrier, and its envelope cannot directly reflect the change of the modulated signal, so synchronous demodulation is required.

Linear Modulation

Vestigial-sideband Modulation (VSB)

The problem of SSB: Difficult to achieve in practice.

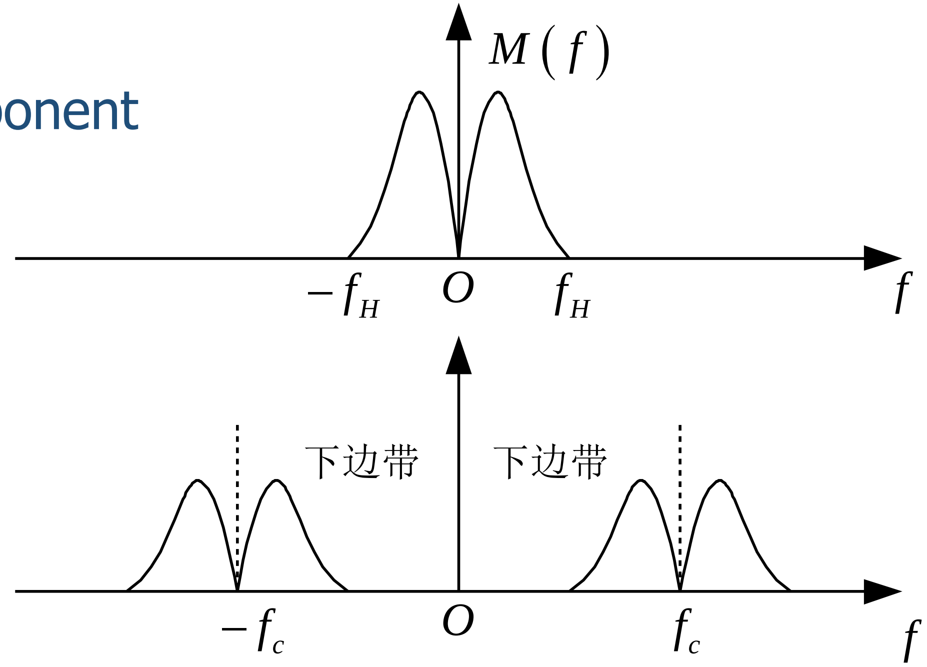
If the modulating signal has very low frequency component

↓

The filter requires to be sharp cut-off to separate the upper and lower sidebands (Not practical)

↓

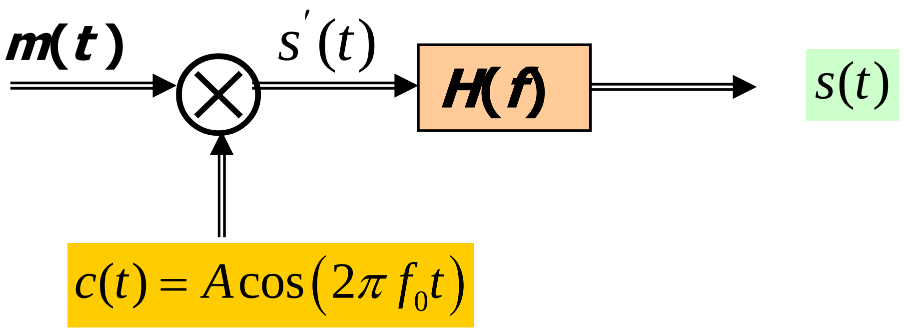
To counterbalance: Not completely suppress a sideband → VSB



Linear Modulation

Vestigial-sideband Modulation (VSB)

How to design the VSB (signal and the filter)?



In the transmitter:

The output signal after the modulating signal multiplied by

the carrier: $$S'(f) = \frac{A}{2}\left[M(f - f_0) + M(f + f_0)\right]$$

Assume the filter of VSB is H(f), the modulated signal is:

$$S(f) = \frac{A}{2}\left[M(f - f_0) + M(f + f_0)\right]H(f)$$

Linear Modulation

Vestigial-sideband Modulation (VSB)

How to design the VSB (signal and the filter)?

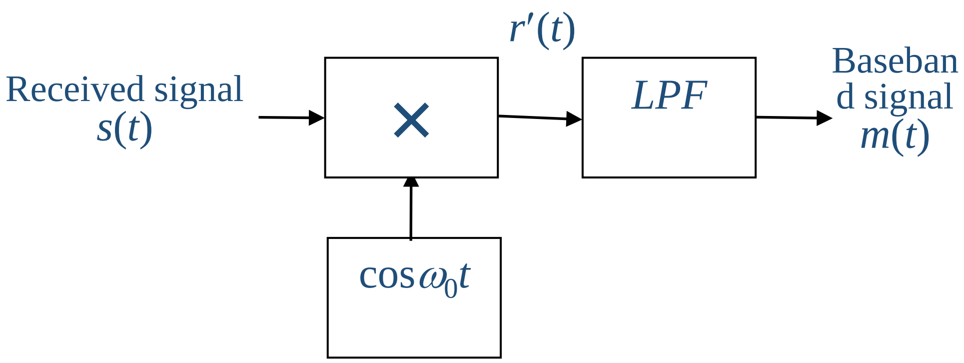


Figure 3.2.6 Block diagram of DSB signal demodulator

In the receiver:

Output of r′(t): $\frac{1}{2}\left[S(f+f_0)+S(f-f_0)\right]$

$$=\frac{A}{4}\left\{\left[M(f+2f_0)+M(f)\right]H(f+f_0)+\left[M(f-2f_0)+M(f)\right]H(f-f_0)\right\}$$

can be filtered out by LPF

After LPF: $\frac{A}{4}M(f)\left[H(f+f_0)+H(f-f_0)\right]$

For distortion-less transmission: $H(f+f_0)+H(f-f_0)=C$

Linear Modulation

Vestigial-sideband Modulation (VSB)

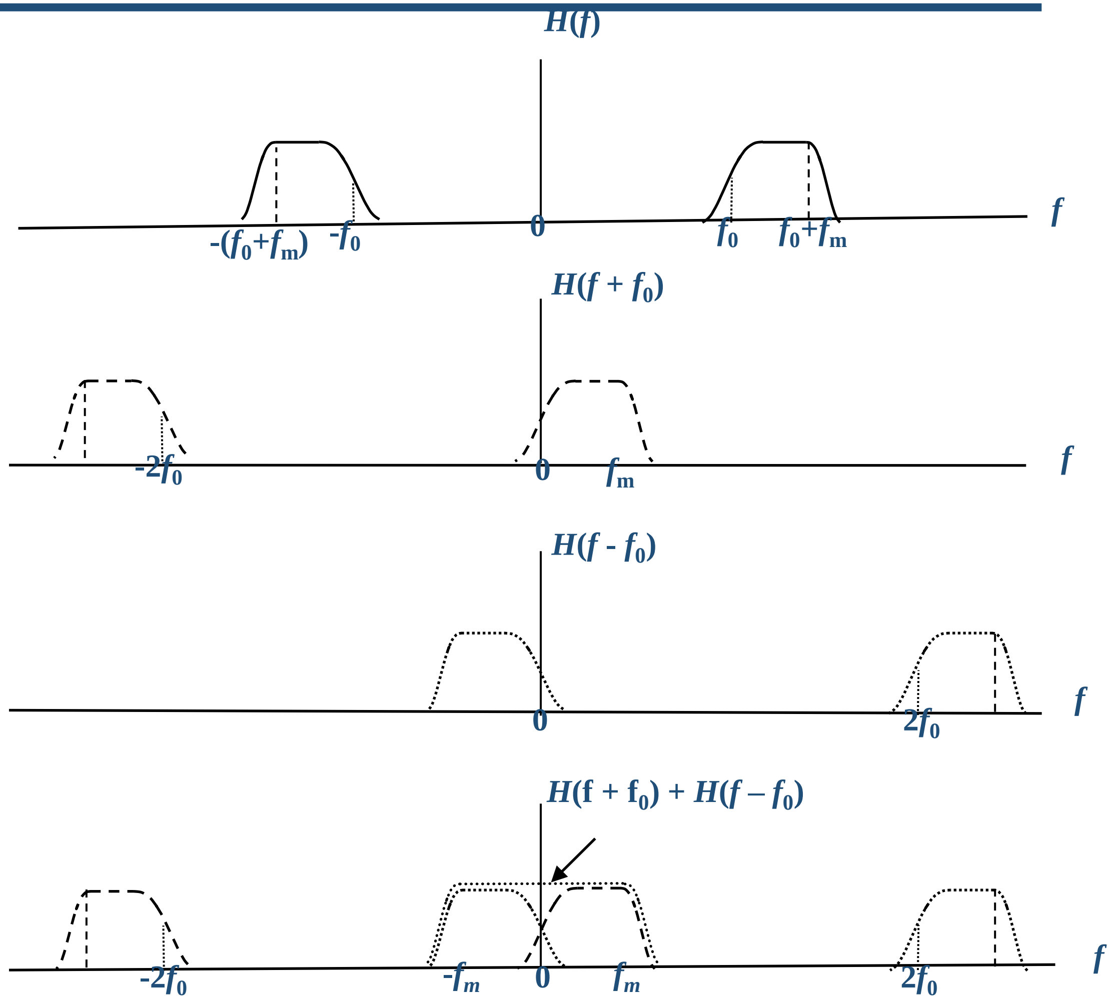
How to design the VSB (signal and the filter)?

The VSB filter needs to satisfy (show in the figure):

$$H(f+f_0)+H(f-f_0)=C$$

The cut-off characteristic of the filter is complementary symmetry

A part of the carrier frequency and a small part of another sideband frequency spectrum are reserved



Main disadvantage: occupy slightly wider bandwidth than SSB

Nonlinear Modulation

Basic concept

Nonlinear modulation (Angle modulation):

modulating signal is carried on the phase of the carrier.

The carrier: has single frequency component and constant amplitude. (sin & cos)

↓

After the modulation: frequency and phase of the carrier vary with the modulating signal

(The information is inserted in the frequency or phase of the carrier)

Nonlinear Modulation

Basic concept

A carrier: $c(t) = A\cos\varphi(t) = A\cos\left(\omega_0 t + \varphi_0\right)$

Constant

After modulation:

Instantaneous phase

$\varphi = \omega_0 t + \varphi_0$

$\omega_0 = \mathrm{d}\varphi(t)/\mathrm{d}t$

Instantaneous frequency $\omega_i(t) = \mathrm{d}\varphi(t)/\mathrm{d}t$

Instantaneous phase $\varphi(t) = \int \omega_i(t)\mathrm{d}t + \varphi_0$

A function of time

(Vary with the information)

Nonlinear Modulation

Basic concept

If phase varies with m(t), it is phase modulation(PM): $\varphi(t)=\omega_0 t+\varphi_0+k_\mathrm{p} m(t)$

The instantaneous frequency: $\omega_i(t)=\omega_0+k_\mathrm{p}\dfrac{\mathrm{d}}{\mathrm{d}t}m(t)$

The modulated signal: $s_\mathrm{p}(t)=A\cos\left[\omega_0 t+\varphi_0+k_\mathrm{p} m(t)\right]$

If frequency varies with m(t), it is frequency modulation(FM): $\omega_i(t)=\omega_0+k_f m(t)$

The instantaneous phase: $\varphi(t)=\int\omega_i(t)\mathrm{d}t+\varphi_0=\omega_0 t+k_f\int m(t)\mathrm{d}t+\varphi_0$

The modulated signal: $s_f(t)=A\cos\left[\omega_0 t+\varphi_0+k_f\int m(t)dt\right]$

Nonlinear Modulation

Basic concept

No difference between FM and PM substantially

- If m(t) is integrated first, the phase of carrier is modulated, it is FM
- If m(t) is differentiated first, the frequency of carrier is modulated, it is PM

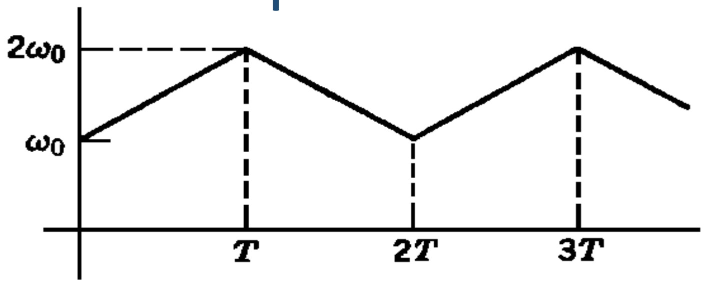
For both FM and PM, the amplitude of the signal is constant.

↓

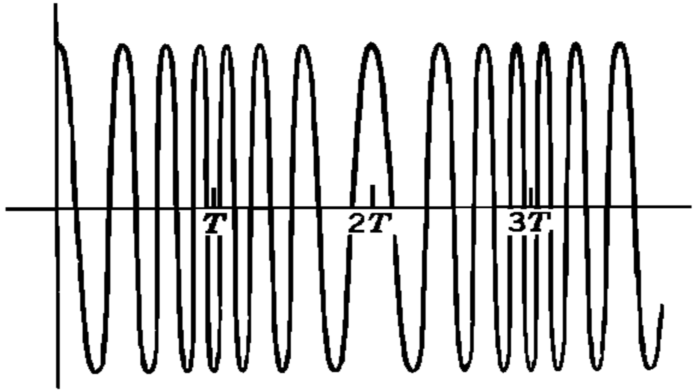
Impossible to distinguish FM and PM by modulated signal waveform.

Nonlinear Modulation

Basic concept



Relation between instantaneous frequency and time



Waveform of modulated signal

From the figures,

- If m(t) varies along a straight line with t, it is FM
- If m(t) varies with t^2 , it is PM

Nonlinear Modulation

Frequency spectrum and Bandwidth of modulated signal

The bandwidth of angle modulated signal may be much larger than that of baseband signal

Assume the modulating signal is $m(t) = \cos \omega_m t$

The instantaneous angular frequency after FM:

$$\omega_i(t) = \omega_0 + k_f m(t) = \omega_0 + k_f \cos \omega_m t$$

The largest frequency deviation of the carrier angular frequency:

$$\Delta\omega = k_f \quad (\mathrm{rad/s})$$

$$m_f = \Delta f / f_m = \Delta\omega / \omega_m = k_f / \omega_m$$

Frequency modulation index

The modulated signal is (assume initial phase is 0):

$$s_f(t) = A\cos\left[\omega_0 t + k_f \int \cos\omega_m t dt\right] = A\cos\left[\omega_0 t + \left(\Delta\omega / \omega_m\right)\sin \omega_m t\right]$$

Nonlinear Modulation

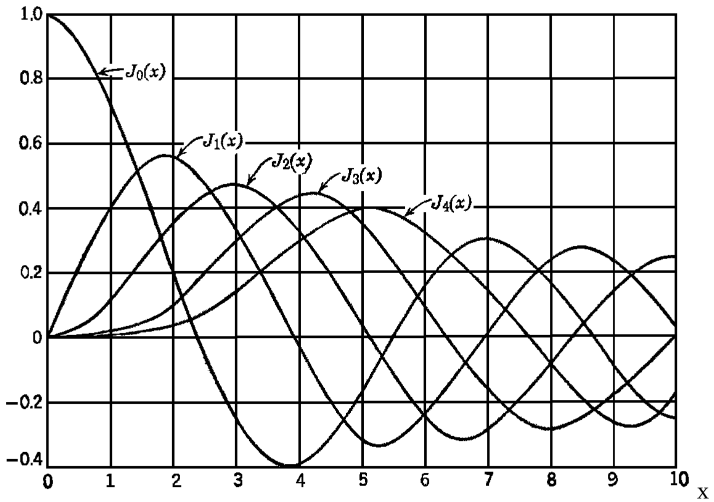
Frequency spectrum and Bandwidth of modulated signal

$$s_f(t) = A\cos\left[\omega_0 t + k_f \int \cos\omega_m t dt\right] = A\cos\left[\omega_0 t + (\Delta\omega / \omega_m)\sin\omega_m t\right]$$ (a cos contain a sin)

It can be proven that Sf(t) can be expanded:

$$\begin{aligned} s_f(t) = A\{ & J_0(m_f)\cos\omega_0 t + J_1(m_f)\left[\cos(\omega_0+\omega_m)t - \cos(\omega_0-\omega_m)t\right] \\ & + J_2(m_f)\left[\cos(\omega_0+2\omega_m)t + \cos(\omega_0-2\omega_m)t\right] \\ & + J_3(m_f)\left[\cos(\omega_0+3\omega_m)t - \cos(\omega_0-3\omega_m)t\right] + \cdots \} \end{aligned}$$

$J_n(m_f)$ is the Bessel function of the first kind of order n.



Nonlinear Modulation

Frequency spectrum and Bandwidth of modulated signal

Based on the properties of the Bessel function:

$$\left.\begin{array}{ll} \mathrm{J}_n\left(m_f\right)=\mathrm{J}_{-n}\left(m_f\right) & \text{when } n \text{ is even} \\ \mathrm{J}_n\left(m_f\right)=-\mathrm{J}_{-n}\left(m_f\right) & \text{when } n \text{ is odd} \end{array}\right\}$$

Sf(t) can be rewritten as:

$$s_f(t)=A\sum_{n=-\infty}^{\infty} J_n\left(m_f\right)\cos\left(\omega_0+n\omega_m\right)t$$

There are side frequencies in pair with angular frequency on 2 sides of carrier frequency

$$\left(\omega_0\pm\omega_m\right),\left(\omega_0\pm 2\omega_m\right),\left(\omega_0\pm 3\omega_m\right),\cdots$$

Seems the bandwidth is infinite

Nonlinear Modulation

Frequency spectrum and Bandwidth of modulated signal

Most of the power concentrates within a limited bandwidth

When mf <<1, the bandwidth basically equals $2\omega_M$ – called narrow band FM

The bandwidth: $B \approx 2\omega_m$

When mf >1, it is known as broad band FM:

The bandwidth:
$$B \approx 2(\Delta\omega + \omega_m)$$
$$B \approx 2(\Delta f + f_m)$$

Nonlinear Modulation

Reception of Angular Modulated Signal

The amplitude of the signal after transmission changes due to fading.

There is no loss of information for angular modulated signal for amplitude change.

↓

Reason: The amplitude doesn't contain information and the fading has greater influence on amplitude than phase and frequency.

↓

Strong anti-jamming ability

To eliminate the influence of fading and noise:

- Use an amplitude limiter (remove the amplitude variation)
- Demodulate by frequency/phase discriminator

Thank you！

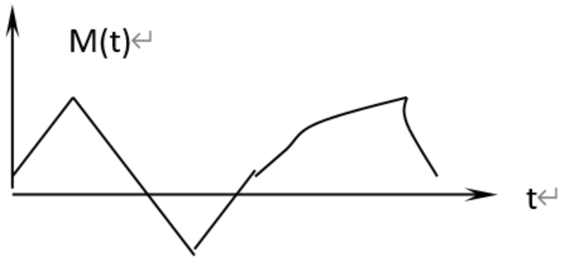
Exercise

SAQ

Assume the original signal waveform is shown

(1) try to draw the waveform of AM and DSB modulation

(2) Draw the result of envelope detection of AM and DSB modulation



Answer briefly:

Why we need modulation?

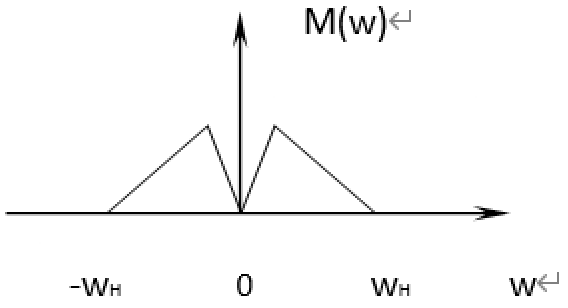
What's the difference between linear modulation and non-linear modulation?

What is the advantage of SSB compared to AM/DSB?

what is the requirement on the filter characteristic of VSB?

Exercise

A modulation block diagram is shown below, and the frequency spectrum of known m(t) is shown on the right. The carrier frequency w1<<w2, w1>wH, and the cut-off frequency of the ideal low-pass filter is w1, try to find the output signal s(t), and explain what kind of modulation signal s(t) is. (You can illustrate the process with drawing)



Hint:

$$\cos(2\pi\xi_0 x) \leftrightarrow \frac{1}{2}(\delta(\xi-\xi_0)+\delta(\xi+\xi_0))$$

$$\sin(2\pi\xi_0 x) \leftrightarrow \frac{1}{2j}(\delta(\xi-\xi_0)-\delta(\xi+\xi_0))$$

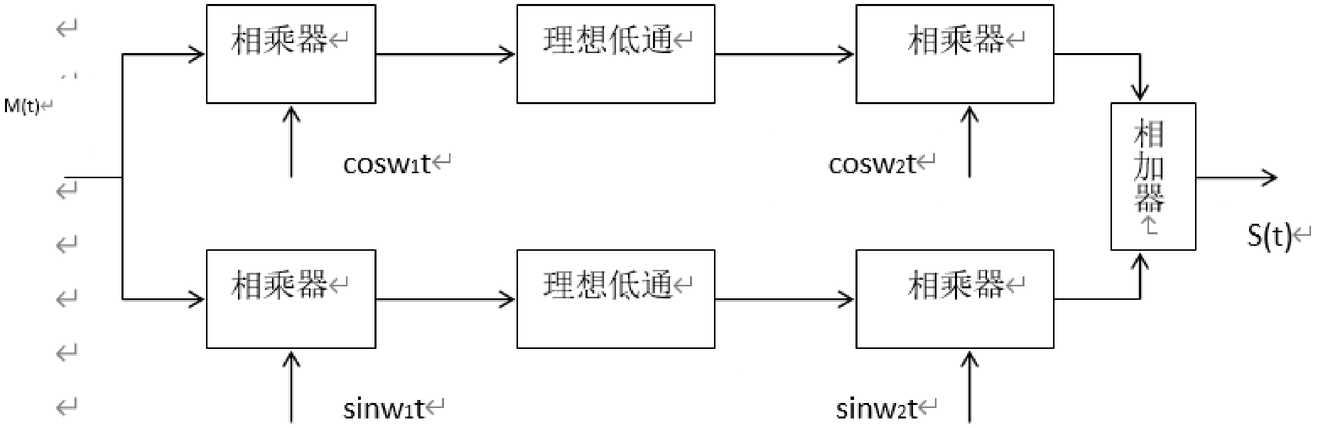


图 3-14 调制信号方框图

MATLAB

Assume that the baseband signal is $m(t) = \cos(2\pi \times 10 \times t)$

1 Generate the AM and DSB modulated signal (DC component for AM be 2, carrier frequency be 1kHz)

2 Recover the baseband signal from AM and DSB, with envelope detector and synchronous demodulation, respectively.

3 Generate the PM modulated signal

(give the plots in both time and frequency domain)

Hint: there is an “envelope” function in MATLAB to get the envelope