Principles of Communications

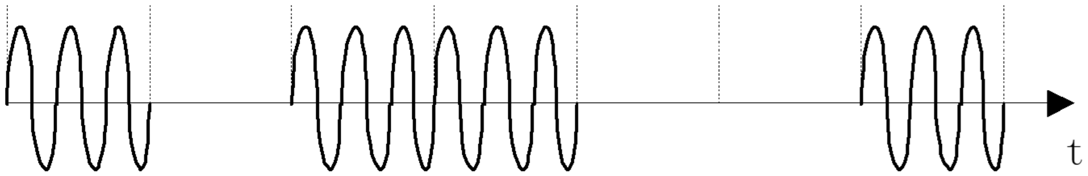
Chapter 6 — Elementary Digital Modulation System

Zhen Chen

Review

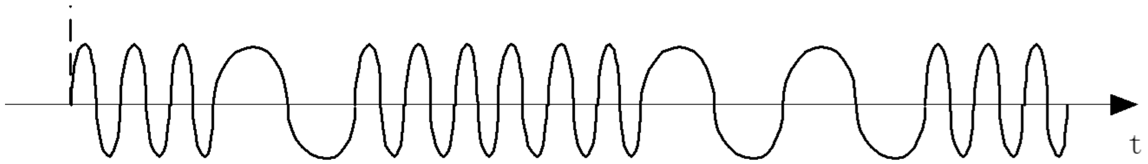
Binary Modulation

Amplitude



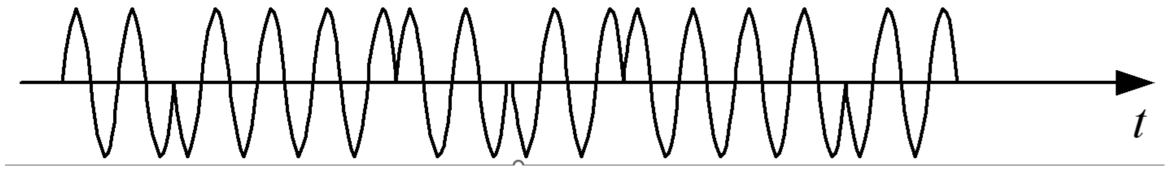
2ASK 2 amplitude states to represent 1 and 0.

Frequency



2FSK 2 frequency states to represent 1 and 0.

Phase



2PSK 2 phase states to represent 1 and 0.

Phase ambiguity → 2DPSK Phase difference to represent 1 and 0.

Generation

Demodulation

Waveform

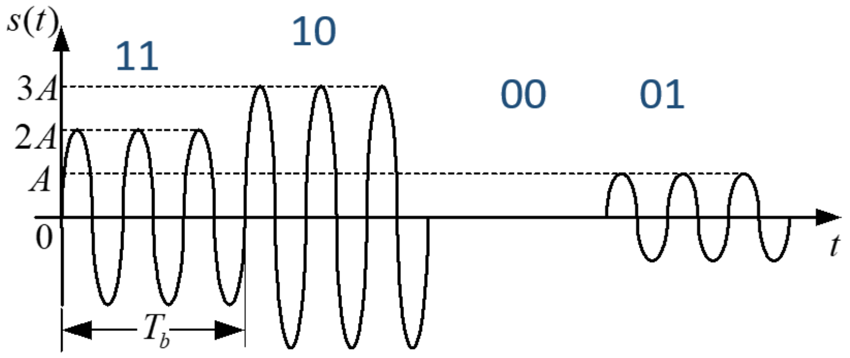
Power spectral density

Review

Multi-ary Modulation

Increase transmission rate of information

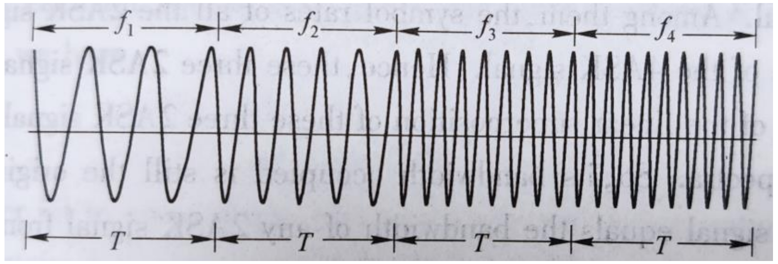
Amplitude



MASK

M amplitude states to represent M symbols.

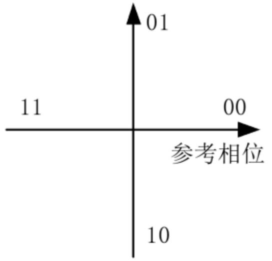
Frequency

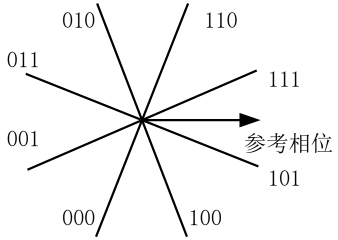


MFSK

M frequency states to represent M symbols.

Phase





MPSK

M phase states to represent 1 and 0.

Outline

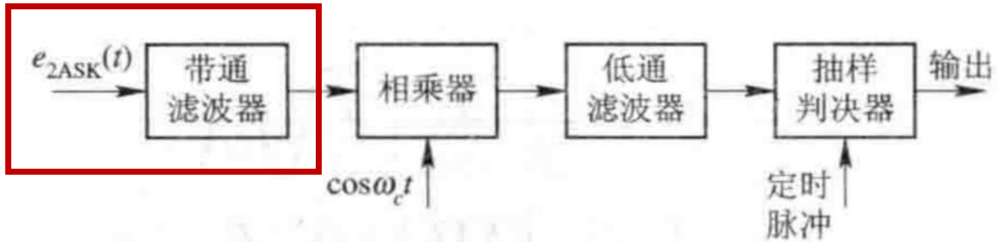
Main Content

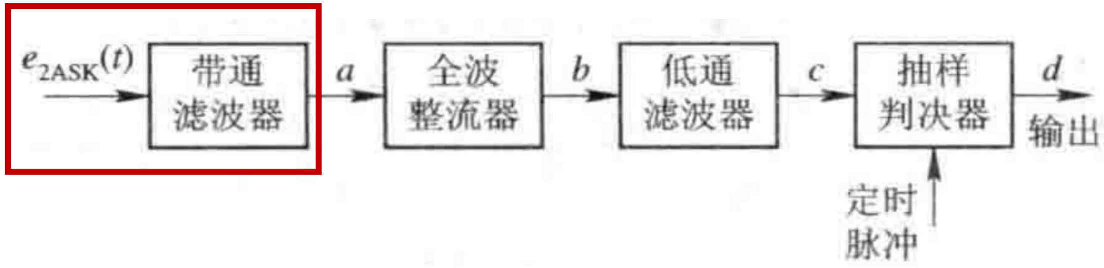
Anti-noise performance

- **2ASK (Coherent & Non-coherent)**
- **2FSK (Coherent & Non-coherent)**
- **2PSK (Coherent)**
- **2DPSK (Coherent & Non-coherent)**

2ASK Demodulation Performance

Coherent & Non-coherent Demodulation





In both methods, signal passes through a BPF:

Assume the signal after the BPF: $y(t) = s(t) + n(t)$

2ASK signal after the BPF

Narrowband Gaussian noise

$$s(t) = \begin{cases} A\cos\omega_0 t, & \text{transmit } 1 \\ 0 \quad , & \text{transmit } 0 \end{cases}$$

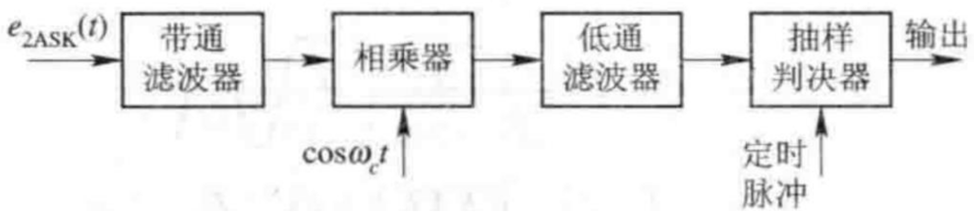
$$n(t) = n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

$$y(t) = \begin{cases} A\cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t, & \text{transmit } 1 \\ n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t, & \text{transmit } 0 \end{cases}$$

$$y(t) = \begin{cases} [A + n_c(t)]\cos\omega_0 t - n_s(t)\sin\omega_0 t \\ n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t \end{cases}$$

2ASK Coherent Demodulation Performance

Coherent Demodulation



After the BPF

$$y(t)=\begin{cases}[A+n_c(t)]\cos\omega_0 t-n_s(t)\sin\omega_0 t\\ n_c(t)\cos\omega_0 t-n_s(t)\sin\omega_0 t\end{cases}$$

After multiplier:

$$y'(t)=\begin{cases}[A+n_c(t)]\cos\omega_0 t\cos\omega_0 t-n_s(t)\sin\omega_0 t\cos\omega_0 t\\ n_c(t)\cos\omega_0 t\cos\omega_0 t-n_s(t)\sin\omega_0 t\cos\omega_0 t\end{cases}$$

$$\Rightarrow$$

$$y'(t)=\begin{cases}\dfrac{[A+n_c(t)]}{2}(\cos 2\omega_0 t+\cos 0)-\dfrac{n_s(t)}{2}(\sin 2\omega_0 t+\sin 0)\\ \dfrac{n_c(t)}{2}(\cos 2\omega_0 t+\cos 0)-\dfrac{n_s(t)}{2}(\sin 2\omega_0 t+\sin 0)\end{cases}$$

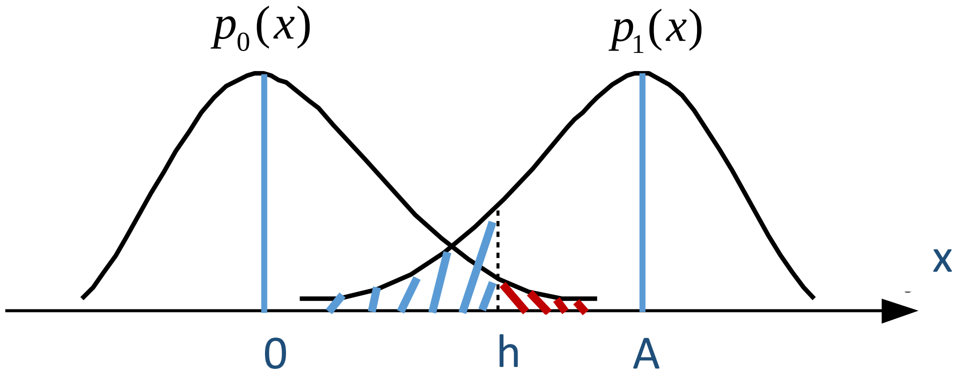
After LPF: Ignore ½ for simplicity

$$x(t)=\begin{cases}A+n_c(t), & \text{transmit } 1\\ n_c(t), & \text{transmit } 0\end{cases}$$

$\Rightarrow$ Use for sampling and decision

2ASK Coherent Demodulation Performance

$$x(t)=\begin{cases}A+n_c(t), & \text{transmit } 1\\ n_c(t), & \text{transmit } 0\end{cases}$$



$$erf[x]=\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$$

$$erfc[x]=1-erf[x]$$

PDF when transmitted 1:

$$p_1(x)=\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left\{-\frac{(x-A)^2}{2\sigma_n^2}\right\}$$

PDF when transmitted 0:

$$p_0(x)=\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left\{-\frac{x^2}{2\sigma_n^2}\right\}$$

When given h, the error probability that judge as 0 when transmit 1:

$$P_{e1}=\int_{-\infty}^{h}p_1(x)dx=1-\frac{1}{2}\{1-erf[(h-A)/\sqrt{2\sigma_n^2}]\}$$

When given h, the error probability that judge as 1 when transmit 0:

$$P_{e0}=\int_{h}^{\infty}p_0(x)dx=\frac{1}{2}\{1-erf[h/\sqrt{2\sigma_n^2}]\}$$

2ASK Coherent Demodulation Performance

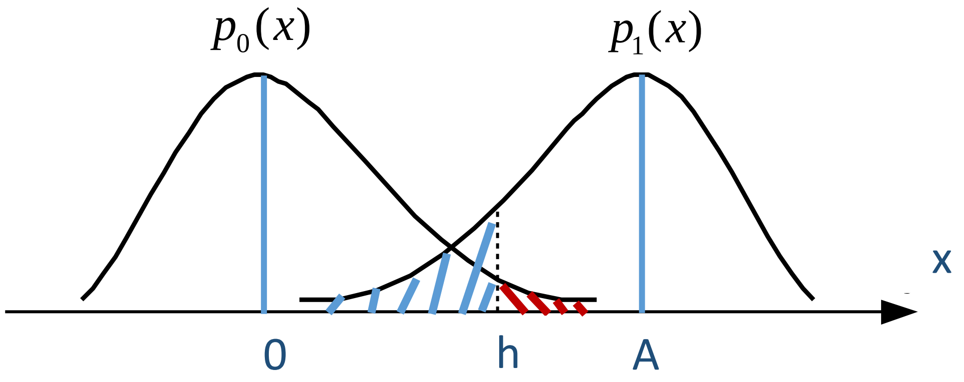
Overall error probability:

$$p_e = p(H_0)P_{e0} + p(H_1)P_{e1}$$

When 0、1 has the same transmitting probability:

$$p_e = \frac{1}{2}p(H_0) + \frac{1}{2}p(H_1)$$

$$= \frac{1}{4}\{1 + erf[(h-A)/\sqrt{2\sigma_n^2}]\} + \frac{1}{4}\{1 - erf[h/\sqrt{2\sigma_n^2}]\}$$



According to optimal threshold condition obtained in Chapter 5: $\dfrac{f(h/H_1)}{f(h/H_0)} = \dfrac{P(H_0)}{P(H_1)}$

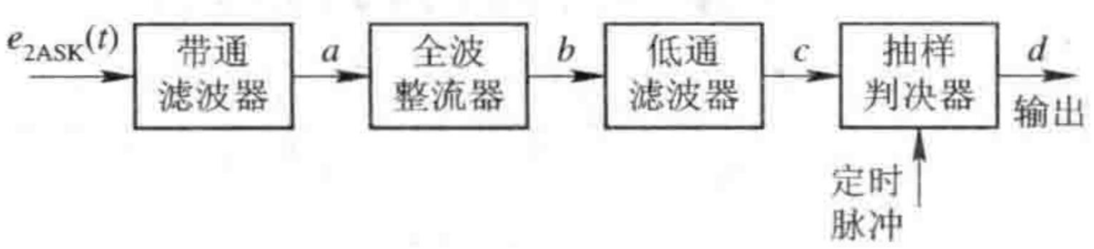
The optimal threshold should be $h^* = \dfrac{A}{2}$

Substitute the optimal threshold to Pe: $p_e = \dfrac{1}{2}erfc[\sqrt{r/4}]$ where $r = \dfrac{A^2}{2\sigma_n^2}$ is SNR

When $r \gg 1$ Pe can be $p_e = \dfrac{1}{\sqrt{\pi r}}e^{-r/4}$

2ASK Non-coherent Demodulation Performance

Non-Coherent Demodulation



After the BPF (High frequency)

$$y(t)=\begin{cases}[A+n_c(t)]\cos\omega_0 t-n_s(t)\sin\omega_0 t\\ n_c(t)\cos\omega_0 t-n_s(t)\sin\omega_0 t\end{cases}$$

Equivalently written in amplitude-phase form:

$$y(t)=\begin{cases}\sqrt{[A+n_c(t)]^2+n_s^2(t)}\cos[\omega_c t+\phi_1(t)], & \text{transmit } 1\\ \sqrt{n_c^2(t)+n_s^2(t)}\cos[\omega_c t+\phi_0(t)], & \text{transmit } 0\end{cases}$$

Non-coherent demodulation: Obtain the envelope of the signal to identify transmitted signal voltage is 0 V or A V.

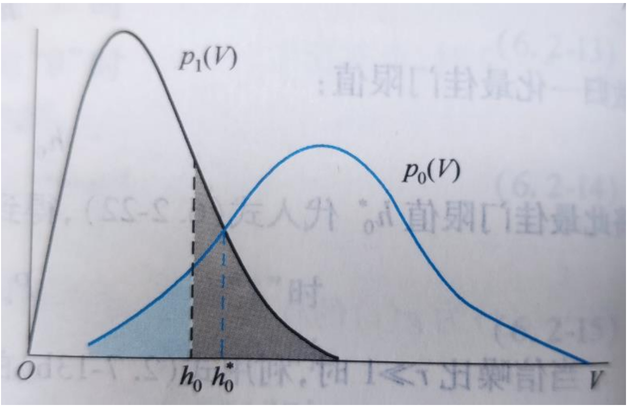
After rectifier and the LPF, the envelope of the signal will be obtained, such that:

$$V(t)=\begin{cases}\sqrt{[A+n_c(t)]^2+n_s^2(t)}, & \text{transmit } 1\\ \sqrt{n_c^2(t)+n_s^2(t)}, & \text{transmit } 0\end{cases}$$

Use for sampling and decision

2ASK Non-coherent Demodulation Performance

$$V(t)=\begin{cases}\sqrt{[A+n_c(t)]^2+n_s^2(t)}, & \text{transmit } 1\\ \sqrt{n_c^2(t)+n_s^2(t)}, & \text{transmit } 0\end{cases}$$



$\mathrm{I}_0(x)$ Modified Bessel function of order 0 of the first kind

$$Q(a,b)=\int_b^{\infty} t\mathrm{I}_0(at)\mathrm{e}^{-(t^2+a^2)/2}dt$$

Rayleigh distribution

PDF when transmitted 1:

$$p_1(V)=\frac{V}{\sigma_n^2}\mathrm{I}_0\left(\frac{AV}{\sigma_n^2}\right)\mathrm{e}^{-\left(V^2+A^2\right)/2\sigma_n^2}$$

PDF when transmitted 0:

$$p_0(V)=\frac{V}{\sigma_n^2}\mathrm{e}^{-V^2/2\sigma_n^2}$$

When given h, the error probability that judge as 0 when transmit 1:

$$P_{e1}=P(V\le h)=\int_0^h\frac{V}{\sigma_n^2}\mathrm{I}_0\left(\frac{AV}{\sigma_n^2}\right)\mathrm{e}^{-\left(V^2+A^2\right)/2\sigma_n^2}dV=1-Q(\sqrt{2r},h_0)$$

When given h, the error probability that judge as 1 when transmit 0:

$$P_{e0}=P(V>h)=\int_h^{\infty}\frac{V}{\sigma_n^2}\mathrm{e}^{-V^2/2\sigma_n^2}dV=e^{-h_0^2/2}$$

2ASK Non-coherent Demodulation Performance

Overall error probability:

$$p_e = p(1)P_{e1} + p(0)P_{e0} = p(1)[1 - Q(\sqrt{2r}, h_0)] + p(0)e^{-h_0^2/2}$$

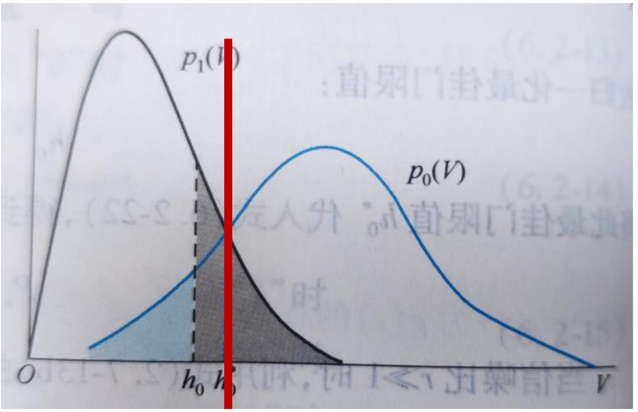
When 0、1 has the same transmitting probability: $p_e = \frac{1}{2}[1 - Q(\sqrt{2r}, h_0)] + \frac{1}{2}e^{-h_0^2/2}$

According to optimal threshold condition obtained in Chapter 5: $\frac{f(V/H_1)}{f(V/H_0)} = \frac{P(H_0)}{P(H_1)}$

The optimal threshold is the intersection of the two PDF functions

$$p_1(V^*) = p_0(V^*) \qquad \frac{V^*}{\sigma_n^2} \mathrm{I}_0\left(\frac{AV^*}{\sigma_n^2}\right) \mathrm{e}^{-\left(V^{*2}+A^2\right)/2\sigma_n^2} = \frac{V^*}{\sigma_n^2} \mathrm{e}^{-V^{*2}/2\sigma_n^2}$$

$$\Rightarrow \quad \frac{A^2}{2\sigma_n^2} = \ln I_0\left(\frac{AV^*}{\sigma_n^2}\right)$$



V^*

2ASK Non-coherent Demodulation Performance

The optimal threshold satisfies this equation: $r = \frac{A^2}{2\sigma_n^2} = \ln I_0(\frac{AV^*}{\sigma_n^2})$

Under 2 conditions

$r \gg 1$ $\qquad r = \frac{A^2}{2\sigma_n^2} = \frac{AV^*}{\sigma_n^2}$ $\qquad h^* = V^* = \frac{A}{2}$

$r \ll 1$ $\qquad \frac{A^2}{2\sigma_n^2} = \frac{1}{4}(\frac{AV^*}{\sigma_n^2})^2$ $\qquad h^* = V^* = \sqrt{2\sigma_n^2}$

Normalized threshold range:

$$\sqrt{2} \le h_0^* = \frac{V^*}{\sigma_n^2} \le \sqrt{r/2}$$

In practice, it is usually assumed that the SNR is larger than 1, so the optimal threshold is $V^* = \frac{A}{2}$

Substitute the optimal threshold to Pe: $p_e = \frac{1}{4} erfc[\sqrt{r/2}] + \frac{1}{2} e^{-r/4}$

Since $\lim_{x \to \infty} erfc(x) = 0$ When $r \to \infty$ The Pe has the lower bound $\inf p_e = \frac{1}{2} e^{-r/4}$

2ASK Non-coherent Demodulation Performance

Example: In a 2ASK transmission system, the symbol rate is R_B=4.8MBaud, and the received signal amplitude is A=1mV. The Gaussian noise power density $n_0=2\times10^{-15}$W/Hz. Please find: (1) The symbol error rate when using envelope detection (2) The symbol error rate when using coherent demodulation.

Solution The BPF filter need the same bandwidth as 2ASK signal: $B \approx \frac{2}{T} = 2R_B = 9.6\times10^6$

The noise power after the BPF: $\sigma_n^2 = n_0 B = 1.92\times10^{-8}$

The output SNR: $r = \frac{A^2}{2\sigma_n^2} \approx 26 \gg 1$

The SER of envelop detector: $p_e = \frac{1}{2}e^{-r/4} = \frac{1}{2}e^{-26/4} = 7.5\times10^{-4}$

The SER of coherent detector: $p_e = \frac{1}{\sqrt{\pi r}}e^{-r/4} = \frac{1}{\sqrt{3.14\times26}}e^{-26/4} = 1.66\times10^{-4}$

Outline

Main

Content

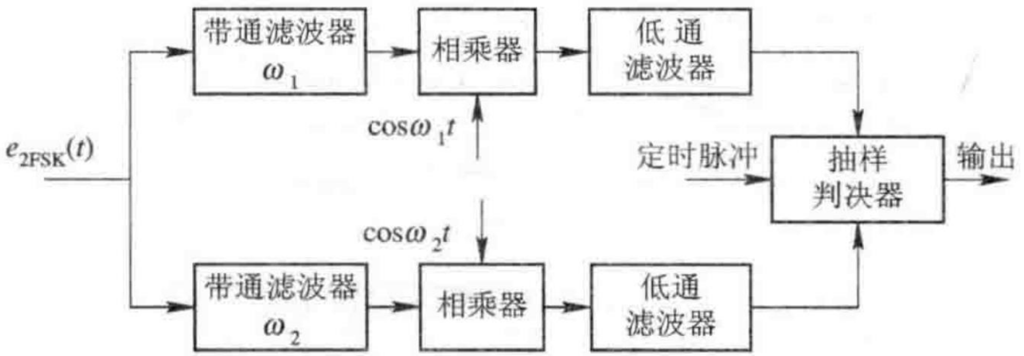
Anti-noise performance

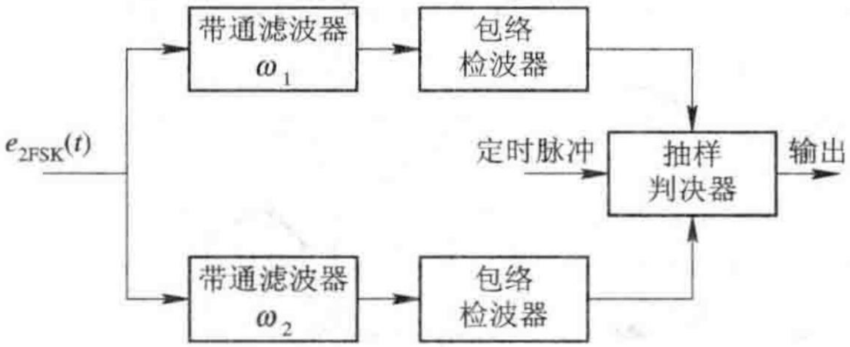
- **2ASK (Coherent & Non-coherent)**
- **2FSK (Coherent & Non-coherent)**
- **2PSK (Coherent)**
- **2DPSK (Coherent & Non-coherent)**

Comparison

2FSK Demodulation Performance

Coherent & Non-coherent Demodulation





In both methods, signal passes through a BPF in 2 paths:

$$y(t)=\begin{cases} A\cos\omega_1 t+n(t), & \text{transmit } 1 \\ A\cos\omega_0 t+n(t), & \text{transmit } 0 \end{cases}$$

$n(t)$ is Gaussian noise

The output of the two BPF in different transmitted symbol:

When 1 is transmitted

$$y_1(t)=[A+n_{1c}(t)]\cos\omega_1 t-n_{1s}(t)\sin\omega_1 t$$
$$y_0(t)=n_{0c}(t)\cos\omega_0 t-n_{0s}(t)\sin\omega_0 t$$

When 0 is transmitted

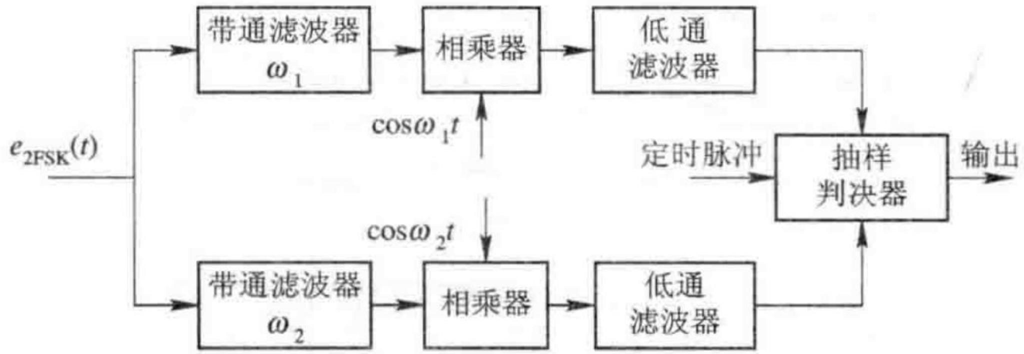
$$y_1(t)=n_{1c}(t)\cos\omega_1 t-n_{1s}(t)\sin\omega_1 t$$
$$y_0(t)=[A+n_{0c}(t)]\cos\omega_0 t-n_{0s}(t)\sin\omega_0 t$$

2FSK Coherent Demodulation Performance

Coherent Demodulation

Take the case of transmitting 1 as example:

$$y_1(t) = [A + n_{1c}(t)]\cos\omega_1 t - n_{1s}(t)\sin\omega_1 t$$
$$y_0(t) = n_{0c}(t)\cos\omega_0 t - n_{0s}(t)\sin\omega_0 t$$



After multiplier and the LPF: $V_1(t) = A + n_{1c}(t)$

(same derivation as previous) $V_0(t) = n_{0c}(t)$

Assume same noise distribution $n_{0c}(t), n_{1c}(t) \sim N(0, \sigma_n^2)$

The distribution of the two output: $V_1(t) \sim N(A, \sigma_n^2)$

$V_0(t) \sim N(0, \sigma_n^2)$

The error occurs when V_0 is greater than V_1

The error probability:

$$P_{e1} = P(V_1 \le V_0) = P[(A + n_{1c}(t)) < n_{0c}(t)] = P(A + n_{1c}(t) - n_{0c}(t) < 0)$$

2FSK Coherent Demodulation Performance

The error probability:

$$P_{e1} = P(V_1 \le V_0) = P[(A + n_{1c}(t)) < n_{0c}(t)]$$
$$= P(A + n_{1c}(t) - n_{0c}(t) < 0)$$

let

$$z = A + n_{1c}(t) - n_{0c}(t)$$
$$z \sim N(A, 2\sigma_n^2)$$

The two noise item has no correlations

Then error probability can be further derived as:

$$P_{e1} = P(z < 0) = \int_{-\infty}^{0} f(z)dz = \frac{1}{\sqrt{2\pi}\sigma_z} \int_{-\infty}^{0} \exp\left\{ -\frac{(z-A)^2}{2\sigma_z^2} \right\} dz = \frac{1}{2} erfc[\sqrt{r/2}]$$

where $r = \frac{A^2}{2\sigma_n^2}$ is SNR

When transmit 0, the error probability derivation is the same: $P_{e0} = \frac{1}{2} erfc[\sqrt{r/2}]$

The overall error probability when 0,1 has the same probability:

$$p_e = p(1) P_{e1} + p(0) P_{e0} = \frac{1}{2} P_{e1} + \frac{1}{2} P_{e0} = \frac{1}{2} erfc[\sqrt{r/2}]$$

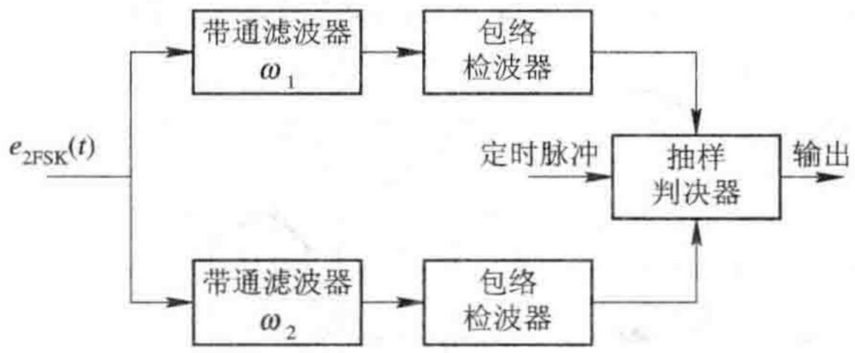
When SNR is large $r \gg 1$ the overall error probability can be approximated as $p_e = \frac{1}{\sqrt{2\pi r}} e^{-r/2}$

2FSK Non-coherent Demodulation Performance

Non-coherent Demodulation

Take the case of transmitting 1 as example:

After the BPF and the envelop detector:



Upper path: $V_1(t)=\sqrt{[A+n_{c1}(t)]^2+n_{s1}^2(t)}$

Lower path: $V_0(t)=\sqrt{n_{c0}^2(t)+n_{s0}^2(t)}$

V1 and V0 follows Rayleigh distribution:

$$P_{e1}=P(V_1<V_0)=\int_0^{\infty}\frac{V}{\sigma_n^2}\mathrm{I}_0\left(\frac{AV_1}{\sigma_n^2}\right)e^{-\left(2V_1^2+A^2\right)/2\sigma_n^2}dV_1$$

$$t=\frac{\sqrt{2}V_1}{\sigma_n} \qquad z=\frac{A}{\sqrt{2}\sigma_n}$$

$$P_{e1}=\frac{1}{2}\int_0^{\infty}t\mathrm{I}_0\left(zt\right)e^{-\left(2t^2+z^2\right)/2}dt$$

It can be further simplified as

$$P_{e1}=\frac{1}{2}e^{-r/2} \quad (r=z^2=\frac{A^2}{2\sigma_n})$$

When transmitting 0, it can be derived similarly:

$$P_{e0}=\frac{1}{2}e^{-r/2}$$

When 0,1 has the same probability, the overall probability:

$$P_e=\frac{1}{2}e^{-r/2}$$

2FSK Non-coherent Demodulation Performance

Example: In a 2FSK transmission system, the transmission bandwidth is 2400Hz. The two frequencies are f_0=980Hz, f_1=1580Hz. The symbol rate is R_B=300Baud. The input SNR of the receiver is 6dB. Please find: (1) The bandwidth of 2FSK (2) The symbol error rate when using envelope detector. (3) The symbol error rate when using coherent demodulation.

Solution

The signal bandwidth: $\Delta f \approx |f_2 - f_1| + 2f_s = |f_2 - f_1| + 2R_B = 1200Hz$

Input SNR: $r_{in} = 6dB = 10^{6/10} \approx 4$

The filter bandwidth: $B = 2R_B = 600Hz$

The bandwidth ratio of input and output: $2400/600 = 4$

Output SNR: $r_{out} = 4 \times 4 = 16$

The Pe of the non-coherent demodulation: $P_e = \frac{1}{2}e^{-\frac{r}{2}} = \frac{1}{2}e^{-8} = 1.7 \times 10^{-4}$

The Pe of the coherent demodulation: $p_e = \frac{1}{2}erfc\left[\sqrt{\frac{r}{2}}\right] = 3.5 \times 10^{-5}$

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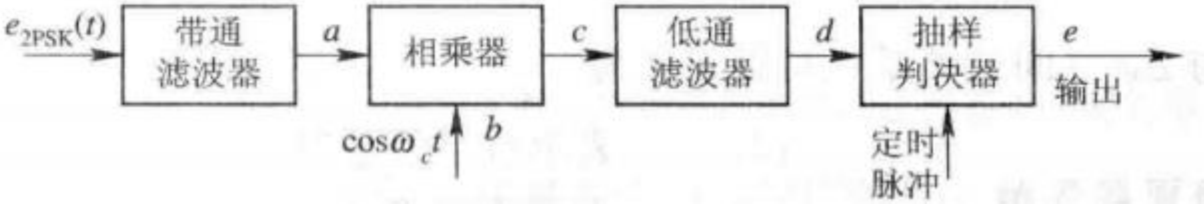
Comparison

2PSK Coherent Demodulation Performance

Coherent Demodulation



After the BPF

$$y(t)=\begin{cases}[A+n_c(t)]\cos\omega_0 t-n_s(t)\sin\omega_0 t\\ [-A+n_c(t)]\cos\omega_0 t-n_s(t)\sin\omega_0 t\end{cases}$$

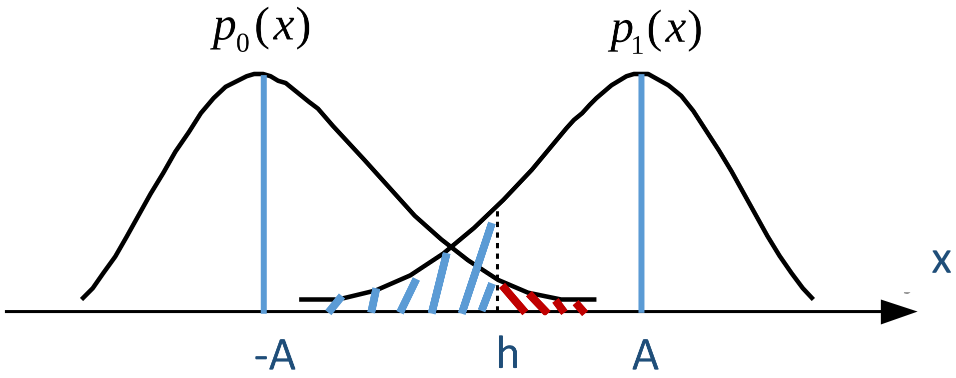
After multiplier:

$$y'(t)=\begin{cases}[A+n_c(t)]\cos\omega_0 t\cos\omega_0 t-n_s(t)\sin\omega_0 t\cos\omega_0 t\\ [-A+n_c(t)]\cos\omega_0 t\cos\omega_0 t-n_s(t)\sin\omega_0 t\cos\omega_0 t\end{cases}$$

After LPF:

$$V(t)=\begin{cases}A+n_c(t), & \text{transmit } 0\\ -A+n_c(t), & \text{transmit } 1\end{cases}$$

$$\begin{cases}V_1(t)\sim N(A,\sigma_n^2) & \text{transmit } 0\\ V_0(t)\sim N(-A,\sigma_n^2) & \text{transmit } 1\end{cases}$$



2PSK Coherent Demodulation Performance

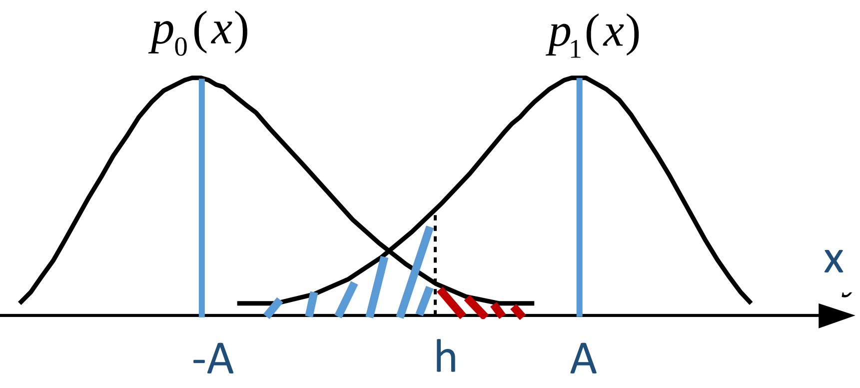
$$\begin{cases} V_1(t) \sim N(A, \sigma_n^2) & \text{transmit } 0 \\ V_0(t) \sim N(-A, \sigma_n^2) & \text{transmit } 1 \end{cases}$$

Shown in the last chapter $\Rightarrow$ The optimal decision threshold:

$$V^* = 0$$

$$\begin{cases} P_{e1} = P(V > 0) = \int_h^{\infty} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{-\frac{(x+A)^2}{2\sigma_n^2}\right\} dx \Rightarrow P_{e1} = \frac{1}{2} erfc[\sqrt{r}] \\ P_{e0} = P(V < 0) = \int_{-\infty}^{h} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{-\frac{(x-A)^2}{2\sigma_n^2}\right\} dx \Rightarrow P_{e0} = \frac{1}{2} erfc[\sqrt{r}] \end{cases}$$

$$(r = \frac{A^2}{2\sigma_n^2})$$



When 0,1 has the same probability, the overall probability:

$$p_e = p(1)P_{e1} + p(0)P_{e0} = \frac{1}{2}P_{e1} + \frac{1}{2}P_{e0} = \frac{1}{2} erfc[\sqrt{r}]$$

When SNR is large $r \gg 1$

the overall error probability can be approximated as

$$p_e \approx \frac{1}{2\sqrt{\pi r}} e^{-r}$$

Outline

Main Content

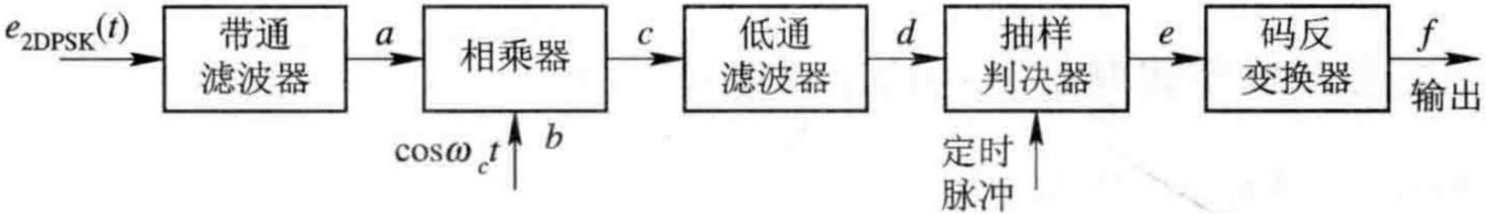
Anti-noise performance

- **2ASK (Coherent & Non-coherent)**
- **2FSK (Coherent & Non-coherent)**
- **2PSK (Coherent)**
- **2DPSK (Coherent & Non-coherent)**

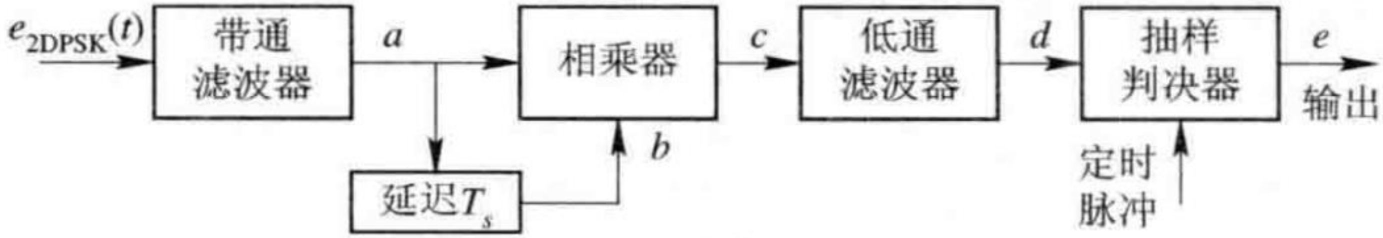
Comparison

2DPSK Demodulation Performance

Coherent Demodulation



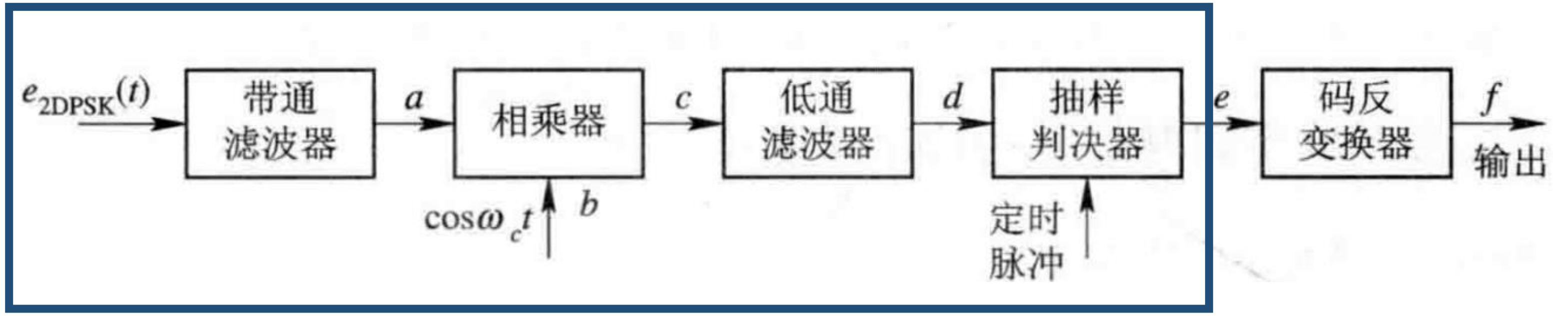
Non-Coherent Demodulation



- Use the phase demodulation on 2DPSK to recover relative code, then transform it into absolute code with code inverse converter

- Compare the phase difference of adjacent symbols to recover the transmitted signal.

2DPSK Coherent Demodulation Performance



Question: When P_e of 2PSK demodulation is known, what is the relationship between P_e' and P_e?

Transmitted Absolute code			0	0	1	0	1	1	0	1	1	1
Transmitted Relative code		0	0	0	1	1	0	1	1	0	1	0
No error	Relative code		0	0	1	1	0	1	1	0	1	0
	Absolute code			0	1	0	1	1	0	1	1	1
1 error	Relative code		0	0	1	0_X	0	1	1	0	1	0
	Absolute code			0	1	1_X	0_X	1	0	1	1	1
2 errors	Relative code		0	0	1	0_X	1_X	1	1	0	1	0
	Absolute code			0	1	1_X	1	0_X	0	1	1	1
5 errors	Relative code		0	0	1	0_X	1_X	0_X	0_X	1_X	1	0
	Absolute code			0	1	1_X	1	1	0	1	0_X	1

N consecutive wrong codes in relative code, there are only 2 wrong codes in the output absolute code signal sequence

2DPSK Coherent Demodulation Performance

Pe’: SER of the absolute code sequence

Pn: probability of n consecutive wrong codes

Pe: SER of the relative code sequence (the error probability is equal and statistically independent)

$$P_e' = 2P_1 + 2P_2 + \cdots + 2P_n + \cdots$$

$$P_n = (1-P_e)P_e^{\,n}(1-P_e) = (1-P_e)^2 P_e^{\,n}$$

Probability of n codes are wrong while the codes at both ends are correct

$$P_e' = 2(1-P_e)^2(P_e + P_e^{\,2} + \cdots + P_e^{\,n} + \cdots) = 2(1-P_e)^2 P_e(1 + P_e + P_e^{\,2} + \cdots + P_e^{\,n} + \cdots)$$

$$1 + P_e + P_e^2 + \cdots = \frac{1}{1-P_e}$$

$$P_e' = 2(1-P_e)P_e \xrightarrow{P_e = \frac{1}{2}erfc[\sqrt{r}]} P_e' = \frac{1}{2}\left[1-\left(erf\sqrt{r}\right)^2\right]$$

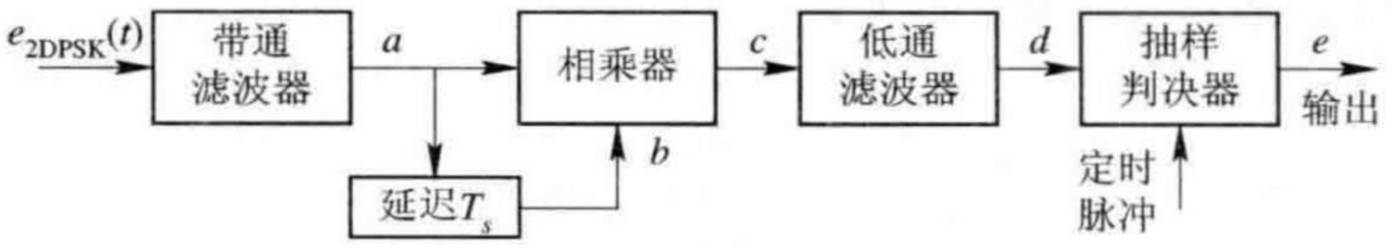
P_e<<1 $\qquad P_e' = 2P_e$

Code inverse converter increase the SER

In practice, 2DPSK is still used instead of 2PSK

2DPSK Non-coherent Demodulation Performance

Non-coherent demodulation: 2DPSK signal differential coherent demodulation & phase comparison method



Compare the two symbols before and after the interval T.

Assuming that the consecutive two relative code is 00:

the BPF output and the delayer output are

$$y_1(t) = A\cos\omega_c t + \boxed{n_1(t)} = [A + n_{1c}(t)]\cos\omega_c t - n_{1s}(t)\sin\omega_c t$$

$$y_2(t) = A\cos\omega_c t + \boxed{n_2(t)} = [A + n_{2c}(t)]\cos\omega_c t - n_{2s}(t)\sin\omega_c t$$

Noise of different moment (independent)

After multiplier and LPF:

$$x(t) = \frac{1}{2}\{[A + n_{1c}(t)][A + n_{2c}(t)] + n_{1s}(t)n_{2s}(t)\}$$

After sampling

$$x = \frac{1}{2}\{[A + n_{1c}][A + n_{2c}] + n_{1s}n_{2s}\}$$

2DPSK Non-coherent Demodulation Performance

The rule of decision:

1、If x>0, decide on “0”——Correct

2、If x<0, decide on “1”——Wrong

Wrong decision Probability:

$$P(1/0)=P\{x<0\}=P\left\{\frac{1}{2}\left[(A+n_{1c})(A+n_{2c})+n_{1s}n_{2s}\right]<0\right\}$$

Use the equation:

$$x_1x_2+y_1y_2=\frac{1}{4}\left\{\left[(x_1+x_2)^2+(y_1+y_2)^2\right]-\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\}$$

$$x_1=a+n_{1c} \qquad x_2=a+n_{2c}$$
$$y_1=n_{1s} \qquad y_2=n_{1s}$$

$$P(1/0)=P\left\{\left[(2A+n_{1c}+n_{2c})^2+(n_{1s}+n_{2s})^2-(n_{1c}-n_{2c})^2-(n_{1s}-n_{2s})^2\right]<0\right\}$$

Let

$$R_1=\sqrt{(2a+n_{1c}+n_{2c})^2+(n_{1s}+n_{2s})^2}$$
$$R_2=\sqrt{(n_{1c}-n_{2c})^2+(n_{1s}-n_{2s})^2}$$

$$p(1/0)=P\{R_1<R_2\}$$

2DPSK Non-coherent Demodulation Performance

$$R_1=\sqrt{(2a+n_{1c}+n_{2c})^2+(n_{1s}+n_{2s})^2}$$
$$R_2=\sqrt{(n_{1c}-n_{2c})^2+(n_{1s}-n_{2s})^2}$$

$$n_{1c},n_{2c},n_{1s},n_{2s}\sim N(0,\sigma_n^2)$$

$$n_{1c}+n_{2c}\sim N(0,2\sigma_n^2)$$
$$n_{1c}-n_{2c}\sim N(0,2\sigma_n^2)$$
$$n_{1s}+n_{2s}\sim N(0,2\sigma_n^2)$$
$$n_{1s}-n_{2s}\sim N(0,2\sigma_n^2)$$

$$p(1/0)=P\{R_1<R_2\}$$

R1 and R2 follows Generalized Rayleigh distribution

$$f(R_1)=\frac{R_1}{2\sigma_n^2}I_0\left(\frac{aR_1}{\sigma_n^2}\right)\mathrm{e}^{-(R_1^2+4a^2)/4\sigma_n^2}$$
$$f(R_2)=\frac{R_2}{2\sigma_n^2}\mathrm{e}^{-R_2^2/4\sigma_n^2}$$

Substitute the PDF to P(1|0)

$$P(1/0)=P\{R_1<R_2\}=\int_0^\infty f(R_1)\left[\int_{R_2=R_1}^\infty f(R_2)\mathrm{d}R_2\right]\mathrm{d}R_1=\int_0^\infty\frac{R_1}{2\sigma_n^2}I_0\left(\frac{aR_1}{\sigma_n^2}\right)\mathrm{e}^{-2(R_1^2+4a^2)/4\sigma_n^2}\mathrm{d}R_1=\frac{1}{2}\mathrm{e}^{-r}$$

Similarly, P(0|1) can be derived as:

$$p(0/1)=p(1/0)=\frac{1}{2}e^{-r}$$

If 1,0 have same probability, the error probability becomes

$$p_e=p(1)P_{e1}+p(0)P_{e0}=\frac{1}{2}P_{e1}+\frac{1}{2}P_{e0}=\frac{1}{2}e^{-r}$$

2DPSK Non-coherent Demodulation Performance

Example: In a 2DPSK transmission system, the symbol rate is R_B=1MBaud. The Gaussian noise power density $n_0=2\times10^{-10}$W/Hz. We want the symbol error rate to be smaller than 10^{-4}. Please find: (1) The required signal power in the receiver input end when using non-coherent demodulation (2) The required signal power in the receiver input end when using coherent demodulation .

Solution The BPF filter need the same bandwidth as DPSK signal: $B = 2R_B = 2\times10^6 Hz$

The noise power after the BPF: $\sigma_n^2 = n_0 B = 4\times10^{-4} W$

(1) The SER : $p_e = \frac{1}{2}e^{-r} \leq 10^{-4} \Rightarrow r = \frac{A^2}{2\sigma_n^2} \geq 8.52$

The required signal power: $r = \frac{A^2}{2} \geq 8.52\sigma_n^2 = 3.4\times10^{-3} W$

(2) The SER : $p_e' = 1 - erfc(\sqrt{r}) \leq 10^{-4} \Rightarrow r = \frac{A^2}{2\sigma_n^2} \geq 7.56$

The required signal power: $\frac{A^2}{2} \geq 7.56\times\sigma_n^2 = 3.02\times10^{-3} W$

Outline

Main Content

Anti-noise performance

- **2ASK (Coherent & Non-coherent)**
- **2FSK (Coherent & Non-coherent)**
- **2PSK (Coherent)**
- **2DPSK (Coherent & Non-coherent)**

Comparison

Comparison

(1) SER:

Modulation	Error probability	
	Coherent	Non-coherent
2ASK	$\frac{1}{2}erfc\left(\sqrt{\frac{r}{4}}\right)$	$\frac{1}{2}e^{-\frac{r}{4}}$（large SNR）
2FSK	$\frac{1}{2}erfc\left(\sqrt{\frac{r}{2}}\right)$	$\frac{1}{2}e^{-\frac{r}{2}}$
2PSK	$\frac{1}{2}erfc\left(\sqrt{r}\right)$	
2DPSK	$\frac{1}{2}[1-erfc\left(\sqrt{r}\right)^{2}]$	$\frac{1}{2}e^{-r}$

1、All of the SER are related to SNR: coherent related to efrc(sqrt(r/k)), non-coherent related to exp(-r/k)

2、For the same digital modulation, SER of the coherent method is lower than that of the non-coherent method

3、In general, 2PSK has the best anti-noise performance, 2FSK has the second best performance, while 2ASK has the worst performance.

Comparison

When fixed SER, the required SNR has relationship among different methods:

$$r_{2\text{ASK}} = 2r_{2\text{FSK}} = 4r_{2\text{PSK}}$$

$$\left(r_{2\text{ASK}}\right)_{\text{dB}} = 3\text{dB} + \left(r_{2\text{FSK}}\right)_{\text{dB}} = 6\text{dB} + \left(r_{2\text{PSK}}\right)_{\text{dB}}$$

When fixed SER, the required SNR

Example

方　式	信噪比 r	
	倍	分贝
2ASK	36.4	15.6
2FSK	18.2	12.6
2PSK	9.1	9.6

$p_e = 10^{-5}$

When fixed SNR, the resultant SER

Example

方　式	误码率 P_e	
	相干解调	非相干解调
2ASK	1.26×10^{-2}	4.1×10^{-2}
2FSK	7.9×10^{-4}	3.37×10^{-3}
2PSK/2DPSK	3.9×10^{-6}	2.27×10^{-5}

$r = 10$

If the SNR is constant, the SER of the 2PSK system is lower than that of the 2FSK system, and the SER of the 2FSK system is lower than that of the 2ASK system.

Comparison

(2) Bandwidth: If the transmitted symbol duration is T

the bandwidth of the 2ASK system and the 2PSK (2DPSK) system is approximated as:

$$B_{2ASK} = B_{2PSK} = \frac{2}{T}$$

The bandwidth of the 2FSK system is approximated as

$$B_{2FSK} = |f_2 - f_1| + \frac{2}{T}$$

From frequency band utilization, the frequency band utilization of 2FSK system is the lowest.

(3) Sensitivity to changes of channels: In practice, the channel parameters change with time

2ASK optimal threshold: A/2

When channel change, A will change, optimal threshold change.

2ASK sensitive to channel changes

2FSK has no decision threshold

Make decision based on upper and lower paths output

2FSK not sensitive

2PSK optimal decision threshold is 0

the decision threshold not change with channel characteristics

2PSK not sensitive

Comparison

Many factors are considered in selection of modulation and demodulation methods

Constant parameter channel transmission:

If higher anti-noise performance :

- 2PSK and 2DPSK should be selected
- 2ASK is the least desirable

If higher frequency band utilization:

- 2PSK and 2DPSK should be selected
- 2FSK is the least desirable

Random parametric channel transmission: 2FSK has better adaptability

Consider hardware complexity: non-coherent demodulation is preferred.

Most used:

- High speed transmission: Coherent 2DPSK
- Low speed transmission: Non-coherent 2FSK

Thank you！

Exercise

Answer briefly

(1) Compare the system performance of 2ASK/2FSK/2PSK/2DPSK in terms of different factors.

(2) Why coherent receiving has lower SER compared to non-coherent receiving?

(3) Describe what the distributions are for 2ASK/2FSK/2PSK/2DPSK coherent/non-coherent demodulations before deciding?(Qualitative analysis)

Exercise

Ex1: In a 2FSK transmission system, the two frequencies are 10MHz and 10.4MHz. The symbol rate is R_B=3MBaud. The amplitude of the input signal is A=40μV. The Gaussian noise power density $n_0=6\times10^{-18}$W/Hz. Please find： (1) The symbol error rate when using envelope detector. (2) The symbol error rate when using coherent demodulation.

Exercise

Ex2: In a OOK transmission system, we use coherent receiving. Know that the probability of transmitting 1 is P, while that of transmitting 0 is 1-P, the amplitude of the signal is a, the narrowband Gaussian noise variance is σ_n^2. Please find： (1) P=0.5, SNR r = 10, what is the optimal threshold h* and the SER Pe. (2) P<0.5, try to analyze the optimal threshold is larger or smaller than h*.

Exercise

Ex3: Know that the symbol rate is R_B=1000Baud, and the power spectrum density of Gaussian noise is $n_0/2=10^{-10}$W/Hz. If the demodulator SER is required to be Pe<=10^{-5}, try to find the required signal power for coherent demodulation OOK, non-coherent demodulation FSK, and differential coherent demodulation DPSK and coherent demodulation of PSK.

MATLAB

(1) Read the bit error rate of BPSK code and write the explanation notes. Compare the theoretical results and the simulation results.

(2) Write code of FSK、DPSK anti-noise performance based on the reference code and draw the bit error rate simulation line. Compare the simulation with the theoretical results