Principles of Communications

Chapter 5 — Presentation and Transmission of Baseband Signal

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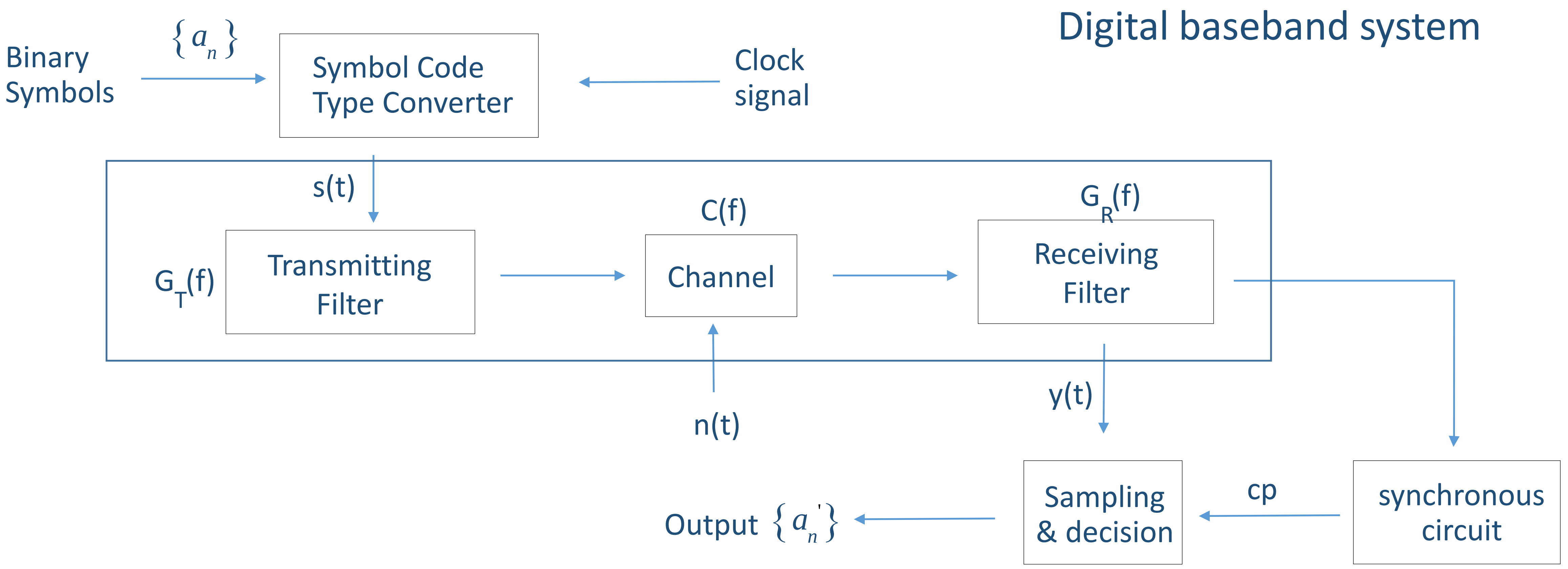
Review


Digital baseband system
Binary Symbols
$\{a_n\}$
Symbol Code Type Converter
Clock signal
s(t)
$G_T(f)$
Transmitting Filter
C(f)
Channel
$G_R(f)$
Receiving Filter
n(t)
y(t)
Output $\{a_n'\}$
Sampling & decision
cp
synchronous circuit

Review

Commonly used symbol types

Basic symbol type
- Unipolar NRZ/RZ
- Bipolar NRZ/RZ
- Differential code

More useful symbol type
- AMI code
- HDB3 code
- Biphase code

Characteristic indicators:

1、With/without D.C. component

2、With/without timing information

3、Error detection capability

…

Review

General form of the power density of baseband signal

$$P_b(f) = f_b P(1-P)\left|\left[G_1(f) - G_2(f)\right]\right|^2 + \sum_{m=-\infty}^{\infty}\left|f_b\left[PG_1(mf_b) + (1-P)G_2(mf_b)\right]\right|^2 \delta(f - mf_b)$$

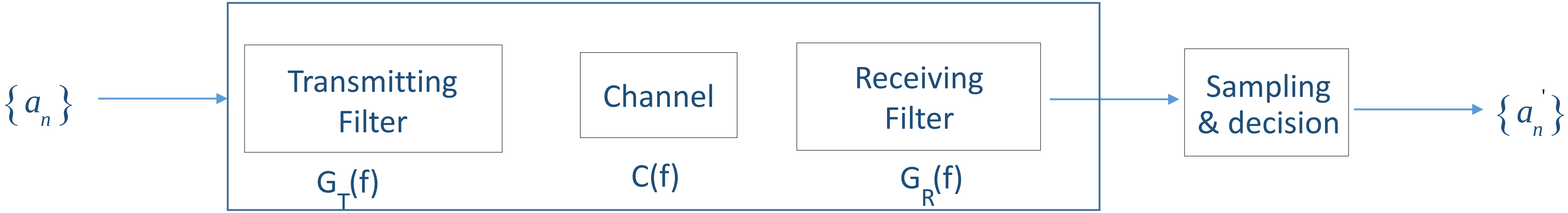
From the power density, we can obtain the information about:

- 1、D.C. component
- 2、timing information
- 3、bandwidth

…

Baseband transmission system

How to quantitively express the baseband signal transmission



Input baseband signal can be expressed as

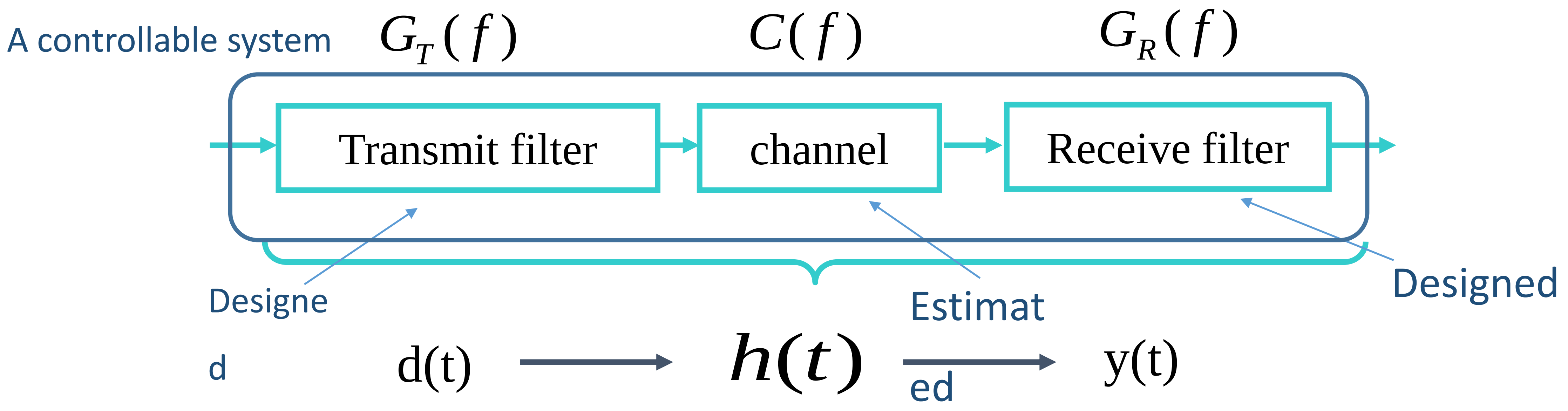
$$d(t) = \sum_{n=-\infty}^{\infty} a_n \boxed{\delta(t - nT_b)}$$

n(t)

Symbol duration Tb

Baseband symbol (can be 0,1 or -1,+1)

Baseband transmission system



Generalized channel (frequency domain): $H(f) = G_T(f) \cdot C(f) \cdot G_R(f)$

Generalized channel (time domain): $h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df$

Baseband transmission system

The signal at the receiver end:

$$y(t) = h(t) * d(t) + n_R(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT_b) + n_R(t)$$

The sampling & decision block will sample the signal and decide what the transmitted signals are

$$y(kT_b + t_0) = \sum_{n=-\infty}^{\infty} a_n h[(k-n)T_b + t_0] + n_R(t)$$

k-th symbol sampling

The time delay of the channel

Baseband transmission system

The sampling value at the receiver can be further expanded as

$$y(kT_b+t_0)=\sum_{n=-\infty}^{\infty}a_n h[(k-n)T_b+t_0]+n_R(t)$$

$$=a_k h(t_0)+\sum_{\substack{n=-\infty\\ n\neq k}}^{\infty}a_n h[(k-n)T_b+t_0]+n_R(t)$$

Noise interference

Originating from the desired symbol

The interference from other symbols other than k-th symbol

Baseband transmission system

To make the decision of what symbol has transmitter transmitted:

Find a decision threshold: V_d

If $y(kT_b + t_0) > V_d$ The estimated ak is decided as "1"

If $y(kT_b + t_0) < V_d$ The estimated ak is decided as "0"

However, due to the intersymbol interference and noise, the estimated ak may not be the same as the true ak

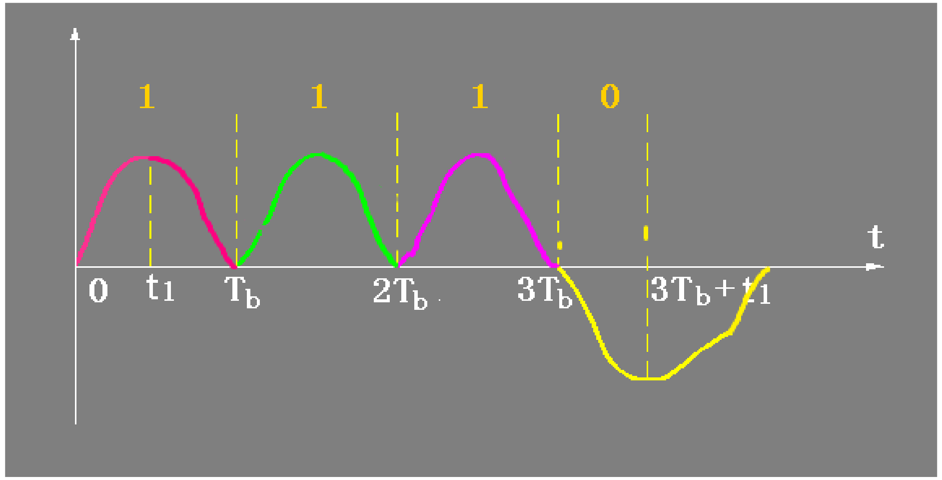
Intersymbol interference

Focus on intersymbol interference:

Ideally, if the transmit filter and receiving filter is well designed, then

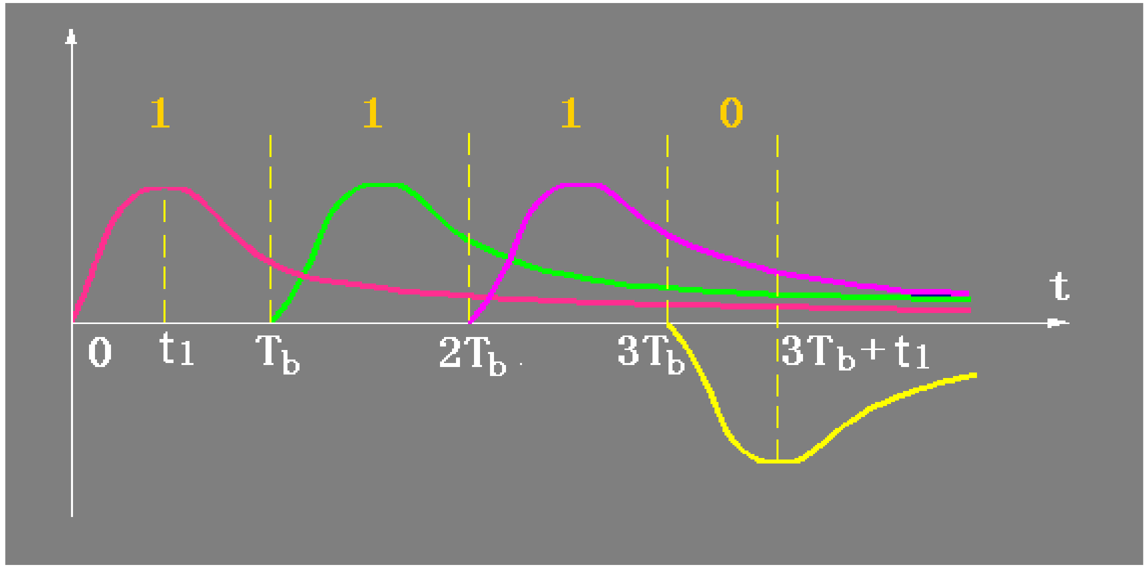
$$h[(k-n)T_b] = \begin{cases} h[0] & k = n \\ 0 & k \neq n \end{cases}$$

The symbol will only be contained in a symbol duration such that:



Intersymbol interference

In practice, the transmit filter and receiving filter cannot be made with very high accuracy, then the receiver will receive the symbols with long tails



When the sum of the interference is too large, then the desired symbol will be covered by the interference.

Intersymbol interference

How to cancel the intersymbol interference?

$$\sum_{\substack{n=-\infty \\ n\neq k}}^{\infty} a_n h[(k-n)T_b + t_0] = 0$$

Two intuitive ideas

1、Make the equation 0 by canceling each other out;

(Almost impossible due to the randomness of a_n

2、Let the interference be 0 at the sampling time:

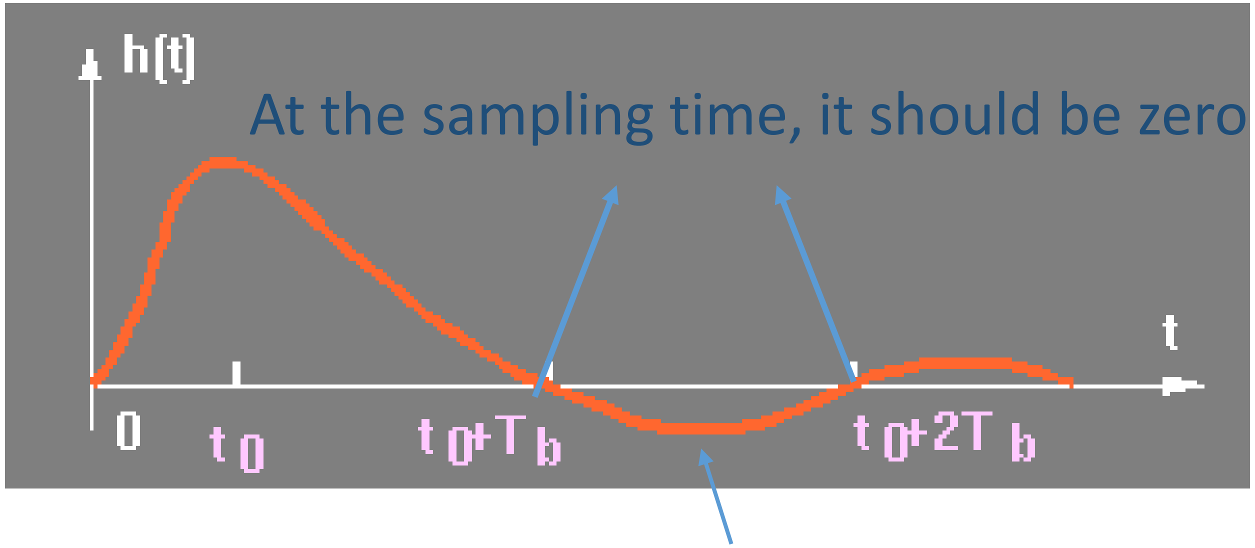
(allow the interference not be 0 when it's not the sampling time)

$$h[(k-n)T_b + t_0] = 0$$

Intersymbol interference

How to cancel the intersymbol interference?

Let the interference be 0 at the sampling time:



Allow a tail when it is not the sampling time

Intersymbol interference

The condition of no intersymbol interference:

$$h(kT_b) \equiv h_k = \begin{cases} \text{constant} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

1、At the time of 0, it should not be 0 so as to sample the desired symbol

2、At the time of k*Tb, it should be 0 then it will not influence other symbol decision

Intersymbol interference

The condition of no intersymbol interference (frequency domain):

Fourier transformation relationship:

$$h(t)=\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}d\omega$$

We consider each sampling time:

$$h(kT_b)=\int_{-\infty}^{\infty} H(\omega)e^{j\omega kT_b}d\omega$$

Split into the sum of multiple tiny intervals

$$h(kT_b)=\frac{1}{2\pi}\sum_{i=-\infty}^{\infty}\int_{-(2i-1)\pi/T_b}^{(2i+1)\pi/T_b} H(\omega)e^{j\omega kT_b}d\omega$$

Intersymbol interference

Employ variable substitution (w'=w-2m pi /Tb)

$$h(kT_b)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\int_{-\pi/T_b}^{\pi/T_b}H(\omega'+2\pi m/T_b)e^{j\omega' kT_b}d\omega'=\frac{1}{2\pi}\int_{-\pi/T_b}^{\pi/T_b}\sum_{m=-\infty}^{\infty}H(\omega'+2\pi m/T_b)e^{j\omega' kT_b}d\omega'$$

Notice

$$\begin{cases} F(\omega)=\sum_{n=-\infty}^{\infty}f_n e^{-jn\omega T_b} \\ f_n=\frac{T_b}{2\pi}\int_{-\frac{\pi}{T_s}}^{\frac{\pi}{T_s}}F(\omega)e^{jn\omega T_b}d\omega \end{cases} \Rightarrow \frac{1}{T_b}\sum_{m=-\infty}^{\infty}H(\omega+2\pi m/T_b)=\sum_{k=-\infty}^{\infty}h(kT_b)e^{-j\omega kT_b}$$

$$\Rightarrow \frac{1}{T_b}\sum_{m=-\infty}^{\infty}H(\omega+2\pi m/T_b)=\text{constant},\quad |\omega|<\frac{\pi}{T_b}$$

Intersymbol interference

The condition of no intersymbol interference (frequency domain):

$$\frac{1}{T_b}\sum_{m=-\infty}^{\infty} H(\omega + 2\pi m/T_b) = \text{constant}, \quad |\omega| < \frac{\pi}{T_b}$$

Also called Nyquist first criterion

It provides a method to test whether a system will guarantee no intersymbol interference

Any baseband transmit system that satisfies this requirement, the intersymbol interference will be cancelled out.

Intersymbol interference

The physics meaning of the frequency condition:

$$\frac{1}{T_b}\sum_{m=-\infty}^{\infty} H(\omega + 2\pi m/T_b) = \text{constant}, \quad |\omega| < \frac{\pi}{T_b}$$

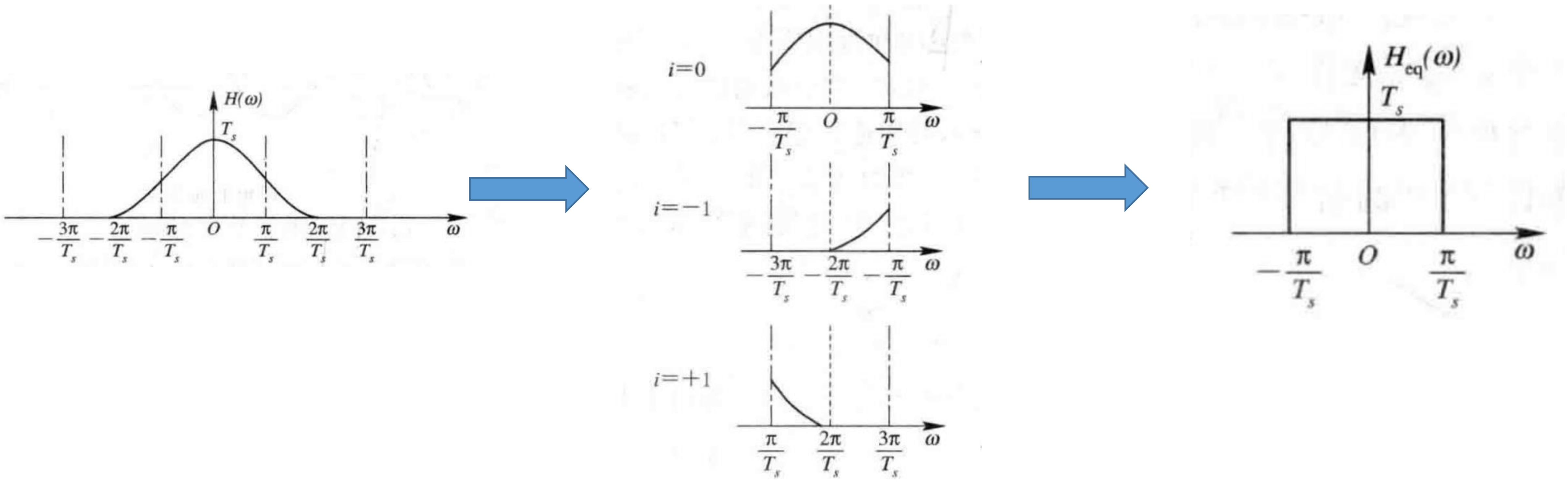
1、Cut H (w) with a period of 2π/Tb and superimpose it. If the superimposed result is constant, there will be no intersymbol interference, otherwise there will be intersymbol interference.

2、The above formula have no any other conditions as long as the superimposed results are constant.

Intersymbol interference

The physics meaning of the frequency condition:

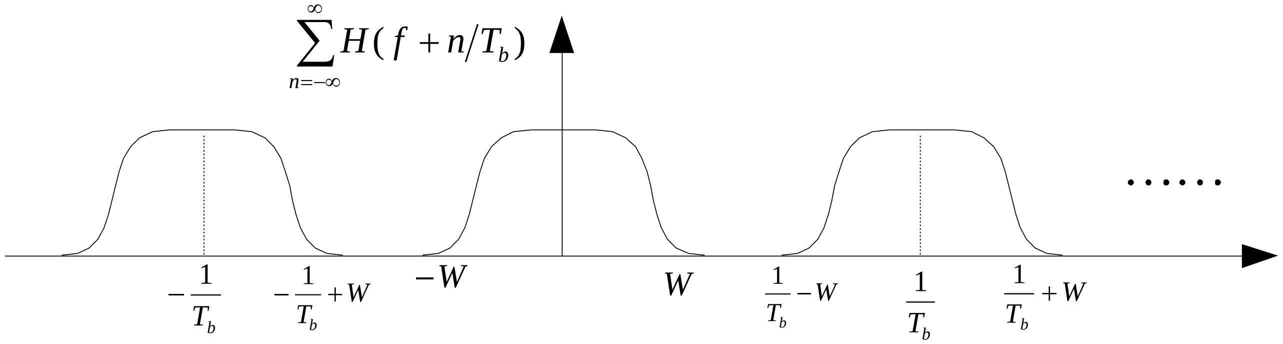
$$\frac{1}{T_b}\sum_{m=-\infty}^{\infty} H(\omega + 2\pi m/T_b) = \text{constant}, \quad |\omega| < \frac{\pi}{T_b}$$



Intersymbol interference

The physics meaning of the frequency condition:

(1) fb>2W（the symbol rate is 2 times larger than the system bandwidth）

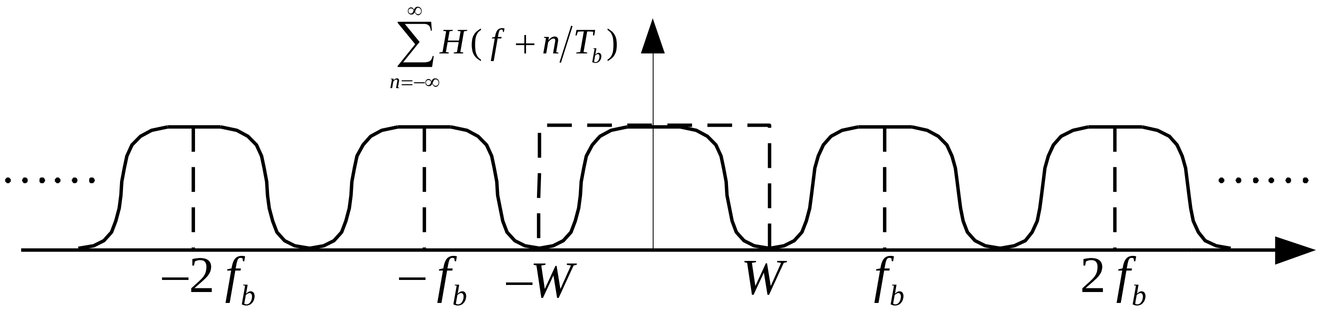


Conclusion: When the symbol rate is greater than twice the bandwidth of the baseband transmission system, intersymbol interference cannot be cancelled out

Intersymbol interference

The physics meaning of the frequency condition:

(2) fb =2W （the symbol rate is equal to twice of the system bandwidth）



Conclusion： The only possible transmission system without intersymbol interference

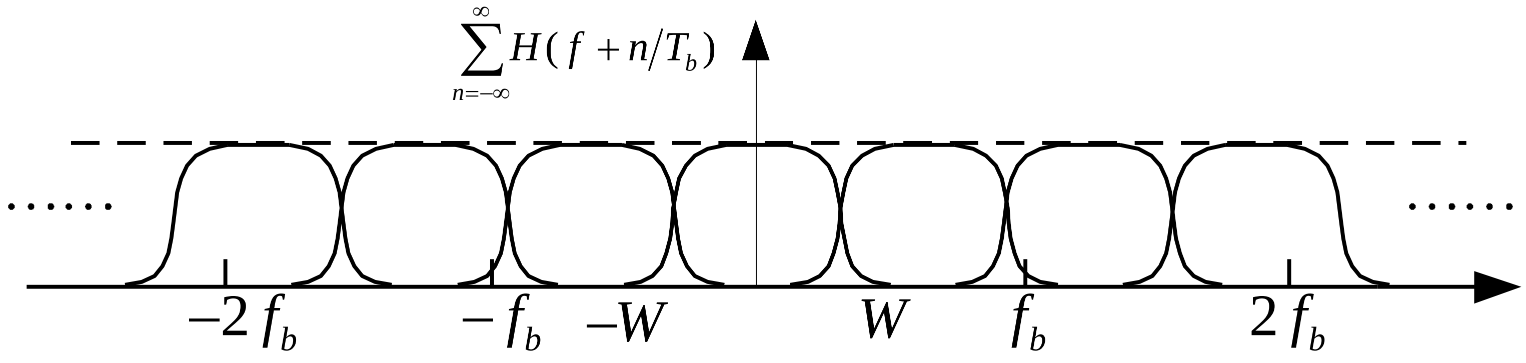
$$H(f)=\begin{cases}\text{constant} & |f|<W \\ 0 & otherwise\end{cases}$$

$$h(t)=\frac{\sin(\pi t/T_b)}{\pi t/T_b}=\sin c\left(\frac{\pi t}{T_b}\right)$$

Intersymbol interference

The physics meaning of the frequency condition:

(3) fb<2W (the symbol rate less than twice of the system bandwidth)



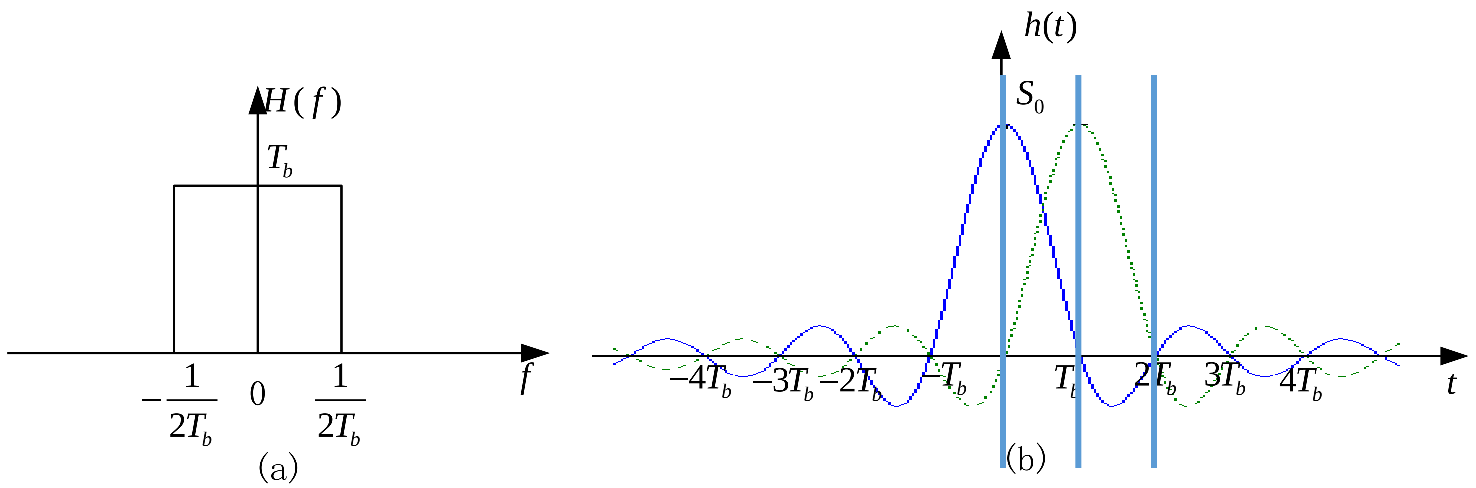
Conclusion: Multiple H (f) superimpose, so that the condition will be satisfied

$$\frac{1}{T_b}\sum_{m=-\infty}^{\infty} H(\omega+2\pi m/T_b)=\text{constant}$$

Intersymbol interference

The systems without intersymbol interference:

1、Ideal low pass system $H_{eq}(f) = H(f) = \begin{cases} T_b, & |f| \le \dfrac{f_b}{2} \\ 0, & |f| > \dfrac{f_b}{2} \end{cases}$



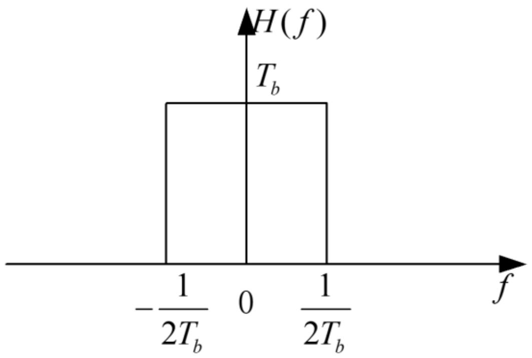
Intersymbol interference

Transmission limits

If no intersymbol interference, the symbol rate of the transmitter should be fb <=2W

The maximum transmitter symbol rate is fb =2W (ideal low pass system)

(Nyquist rate)



The symbol rate is Rb=1/Tb

Bandwidth W = 1/2Tb

(Nyquist bandwidth)

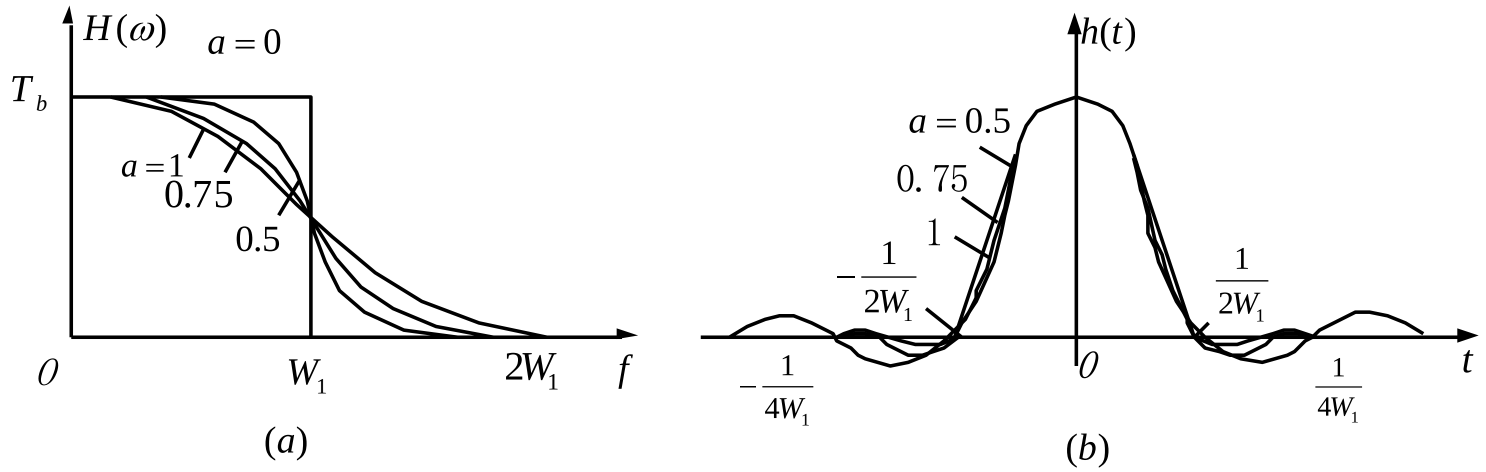
The maximum frequency utilization under the condition of no intersymbol interference:

$$\eta = \frac{R_b}{W} = 2$$

Intersymbol interference

The problems of the ideal low pass system

- The physical realization of ideal rectangular properties is difficult；
- h(t) has a long tail, and decay slowly. When there is timing skew, there will be serious intersysmbol interference；
- Solutions: Correcting the ideal rectangular cut-off margin to a smooth roll-off shape

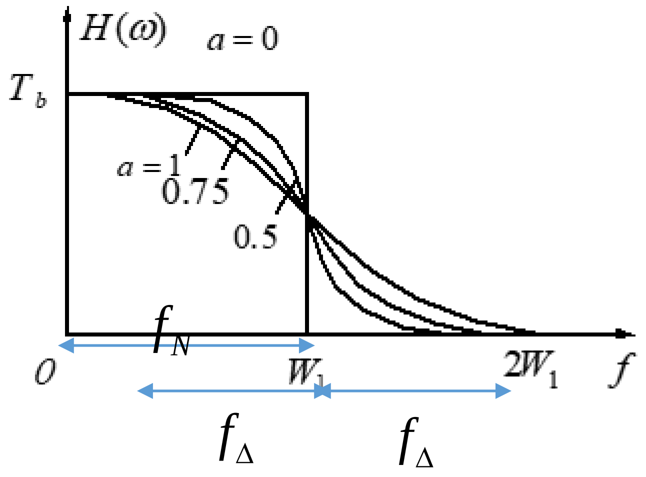


Intersymbol interference

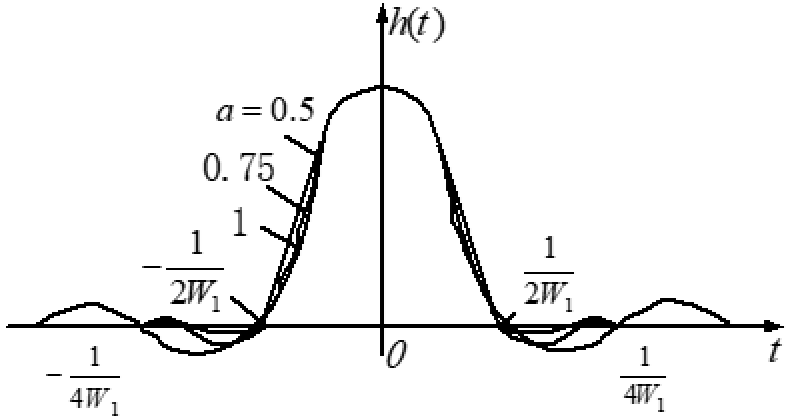
Roll-off characteristic

$$\alpha = f_\Delta / f_N$$

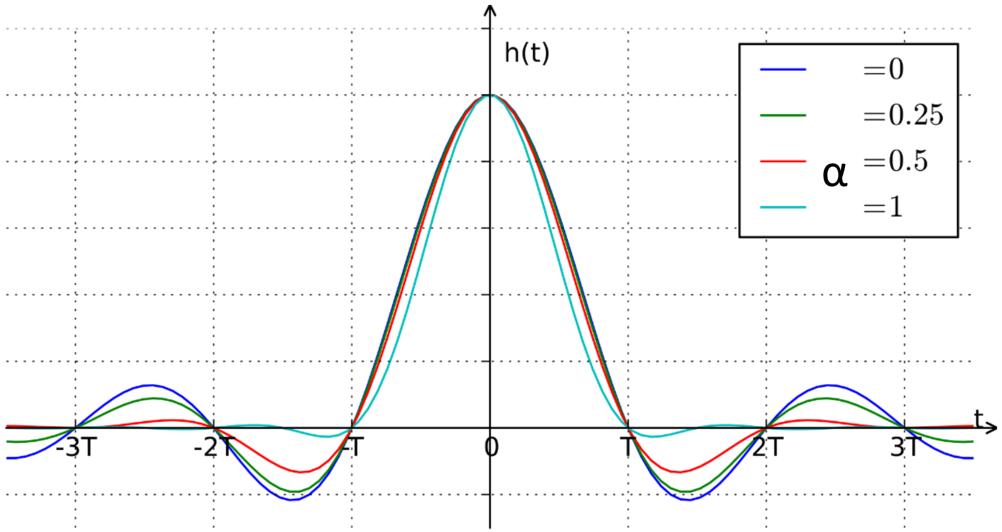
$$H(f)=\begin{cases} T_b & 0\le|f|\le\dfrac{1-\alpha}{2T_b} \\ \dfrac{T_b}{2}\left\{1+\cos\left[\dfrac{\pi T_b}{\alpha}\left(|f|-\dfrac{1-\alpha}{2T_b}\right)\right]\right\} & \dfrac{1-\alpha}{2T_b}\le|f|\le\dfrac{1+\alpha}{2T_b} \\ 0 & |f|\ge\dfrac{1+\alpha}{2T_b}\end{cases}$$



$$h(t)=\frac{\sin(\pi t/T_b)}{\pi t/T_b}\bullet\frac{\cos\pi\alpha t/T_b}{1-4\alpha^2t^2/T_b{}^2}=\sin c(\pi t/T_b)\frac{\cos\pi\alpha t/T_b}{1-4\alpha^2t^2/T_b{}^2}$$



Intersymbol interference



Roll-off system with different parameters

Larger α, the tail decay faster

The synchronous accuracy can not be very strict

However, the frequency utilization decreases if α is larger:

The bandwidth of the roll off system: $W = f_{\Delta} + f_N$

Frequency utilization : $\eta = \dfrac{R_b}{W} = \dfrac{2 f_N}{(1+\alpha) f_N} = \dfrac{2}{(1+\alpha)}$

Intersymbol interference

Specially, when α=1

$$H(f)=\begin{cases}\frac{T_b}{2}(1+\cos\pi fT_b) & |f|\le\frac{1}{T_b}\\ 0 & |f|>\frac{1}{T_b}\end{cases}$$

$$h(t)=\frac{\sin(\pi t/T_b)}{\pi t/T_b}\bullet\frac{\cos\pi t/T_b}{1-4t^2/T_b{}^2}$$

The tail decay fast (Inversely proportional to t^3)

→ Good for low synchronous accuracy and reduced interference

Frequency utilization is half of the limit:

→ $$\eta=\frac{2}{(1+\alpha)}=\frac{2}{(1+1)}=1$$

Partial Response Waveforms

A raised problem:

- The ideal low-pass transmission characteristic band utilization can reach the theoretical maximum of 2B/Hz, but it cannot be realized, and h(t) has a large tail and slow convergence, so the timing requirements are very strict；
- The cosine roll-off characteristic overcomes the above shortcomings, the required frequency band is widened, and the frequency band utilization rate of 2 baud/hertz is lowered
- Is there a transmission characteristics that can be achieved in practice with a frequency band utilization rate of 2B/Hz, large attenuation of the "tail" and fast convergence?

Partial Response Waveforms

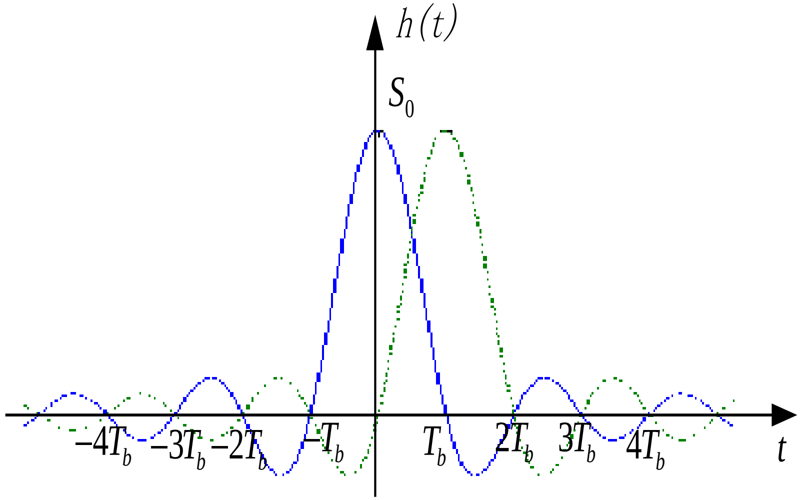
Solutions： Nyquist second criterion

- Controlling the intersymbol interference at the sampling time of some symbols, while no interference at the sampling time of other symbols, then the band utilization can be increased to the theoretical maximum and the timing accuracy can be reduced.
- This waveform is usually referred to as a partial response waveform .
- A baseband transmission system that uses a partial response waveform for transmission is called a partial response system .

Partial Response Waveforms

Partial Response Waveforms of the First Class

- The “tail" of the two sinx/x waveforms with one symbol interval apart is just the opposite of positive and negative
- Using such a combination of waveforms can certainly form a waveform that decays quickly

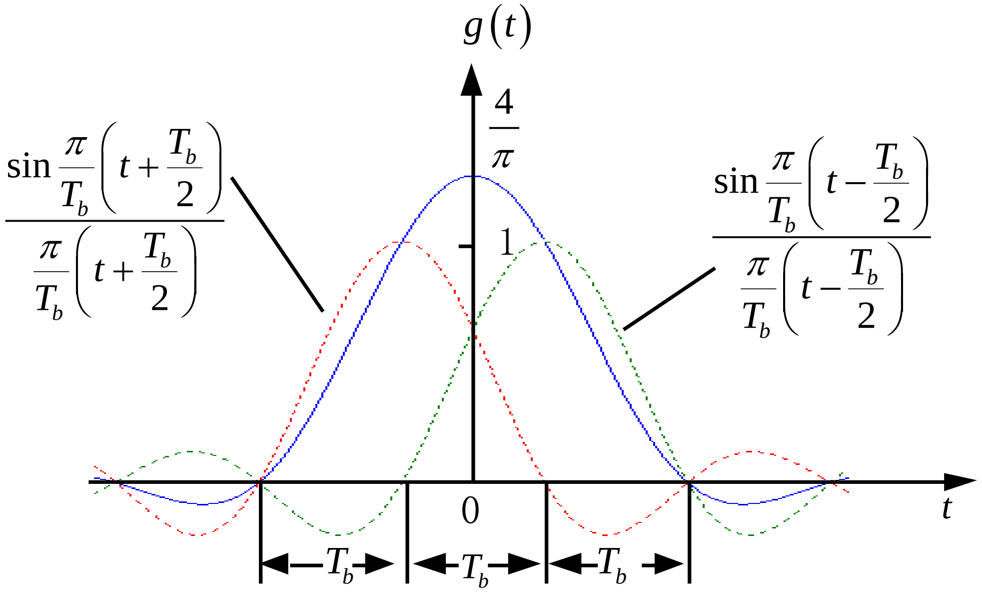


(b)

Partial Response Waveforms

Partial Response Waveforms of the First Class

- Add two sinx/x intervals of one symbol width Tb



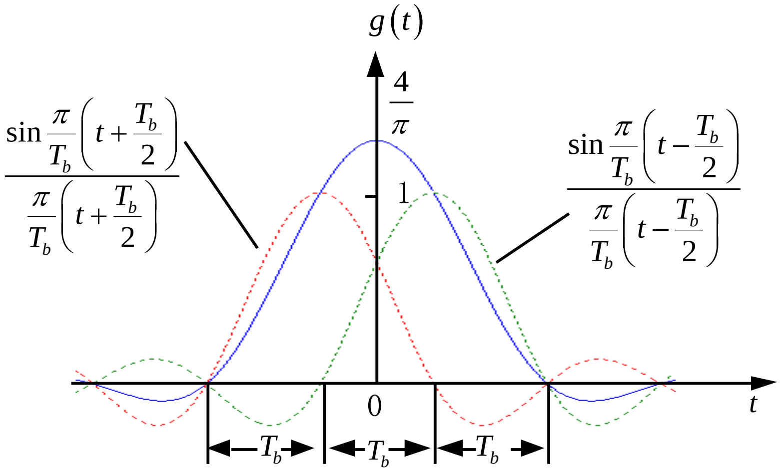
(a)

$$g(t)=\frac{\sin\left[\frac{\pi}{T_b}\left(t+\frac{T_b}{2}\right)\right]}{\frac{\pi}{T_b}\left(t+\frac{T_b}{2}\right)}+\frac{\sin\left[\frac{\pi}{T_b}\left(t-\frac{T_b}{2}\right)\right]}{\frac{\pi}{T_b}\left(t-\frac{T_b}{2}\right)}$$

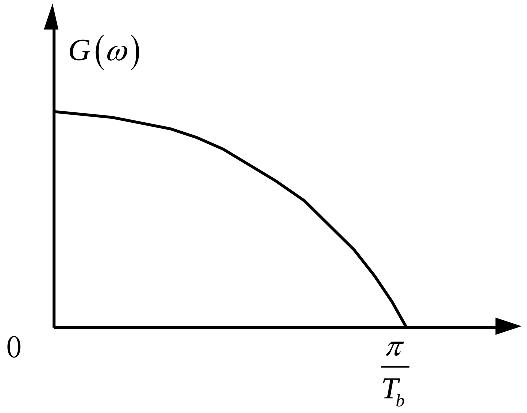
Partial Response Waveforms

Partial Response Waveforms of the First Class

- Frequency spectrum



(a)



(b)

$$G(\omega)=\begin{cases} 2T_b\cos\dfrac{\omega T_b}{2}, & |\omega|\le\dfrac{\pi}{T_b} \\ 0, & |\omega|>\dfrac{\pi}{T_b} \end{cases}$$

- The frequency band utilization rate is η=RB/B=2 baud/Hz, reaching the theoretical limit value of the baseband system when transmitting binary sequences

Partial Response Waveforms

Partial Response Waveforms of the First Class

- The characteristic of g(t)

$$g(t)=\frac{\sin\left[\frac{\pi}{T_b}\left(t+\frac{T_b}{2}\right)\right]}{\frac{\pi}{T_b}\left(t+\frac{T_b}{2}\right)}+\frac{\sin\left[\frac{\pi}{T_b}\left(t-\frac{T_b}{2}\right)\right]}{\frac{\pi}{T_b}\left(t-\frac{T_b}{2}\right)} \Longrightarrow g(t)=\frac{4}{\pi}\left(\frac{\cos\frac{\pi t}{T_b}}{1-\frac{4t^2}{T_b^2}}\right)$$

$$g(0)=4/\pi\,,\ g\left(\pm\frac{T_s}{2}\right)=1,\ g\left(\frac{kT_s}{2}\right)=0,\ k=\pm 3\,,\pm 5\,,\cdots$$

Except g(t)=1 at the adjacent sampling time t=±Tb/2, g(t) has equally spaced zeros at other sampling time points

Partial Response Waveforms

General form of a partial response

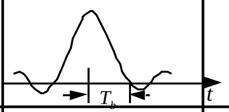
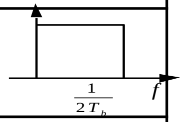
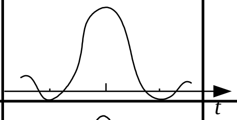
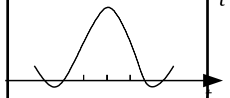
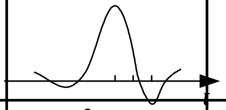
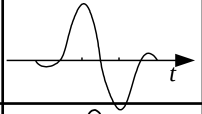
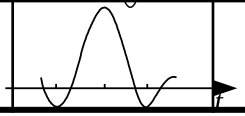
- The general form of a partial response waveform is the sum of N consecutive sinx/x waveforms spaced Tb

$$g(t) = R_1 \frac{\sin \frac{\pi}{T_b} t}{\frac{\pi}{T_b} t} + R_2 \frac{\sin(\frac{\pi}{T_s}(t - T_b))}{\frac{\pi}{T_b}(t - T_b)} + \cdots + R_N \frac{\sin \frac{\pi}{T_b}[t - (N-1)T_b]}{\frac{\pi}{T_b}[t - (N-1)T_b]}$$

- R1, R2,…, RN are weighting coefficients, and their values are positive, negative integers or zero. For example, when R1=1, R2=1, and other coefficients Ri=0, it is the type I partial response waveform mentioned above 。

Partial Response Waveforms

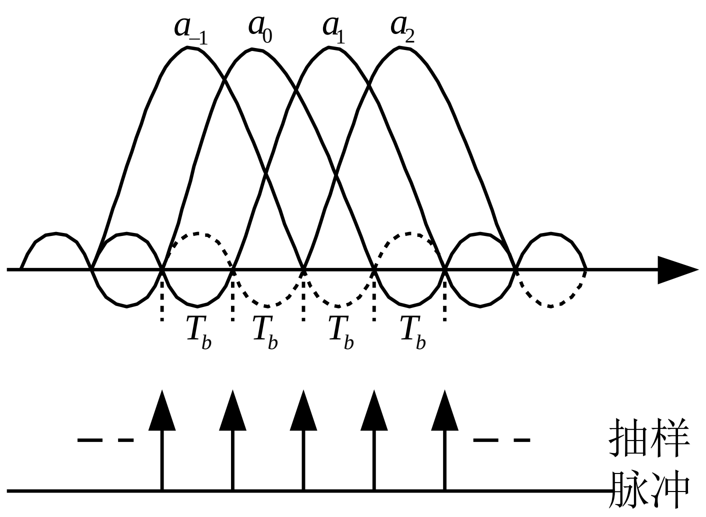
Table: partial response signal

Type	R1	R2	R3	R4	R5	g(t)	|G(ω)|
0	1						
I	1	1					
II	1	2	1				
III	2	1	-1				
IV	1	0	-1				
V	-1	0	2	0	-1		

Partial Response Waveforms

Can g (t) be used as the transmitted waveform?

- If g(t) is used as the transmission waveform, and the symbol interval is Tb, there will be interference；
- Interference occurrence time: sampling time
- Where Interference occurs: the same amplitude samples of the previous symbol
- Conclusion: Intersymbol interference is controllable, and symbols can still be transmitted at a transmission rate of 1/Tb



Thank you！

Exercise

Answer briefly

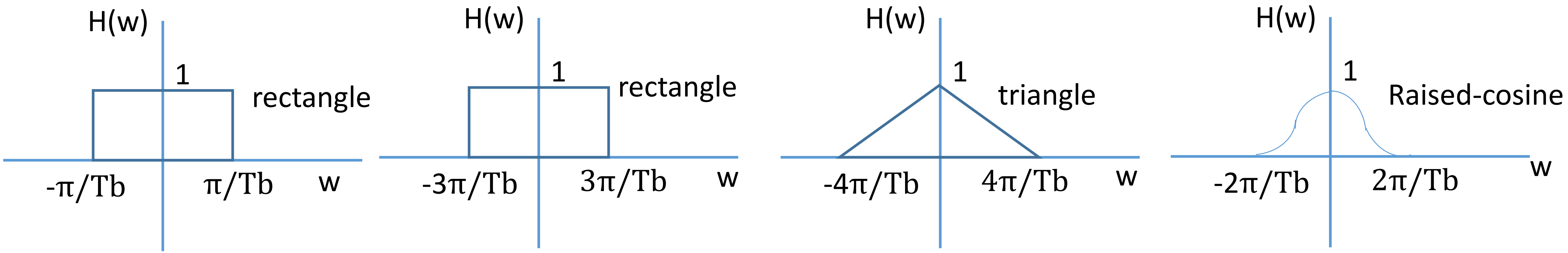
(1) When the baseband transmission system characteristics are ideal low-pass with a bandwidth of 2400Hz, what are the highest symbol rate and frequency band utilization rate without intersymbol interference under the ideal low pass, 50% cosine roll-off and 100% cosine roll-off?

(2) What is intersymbol interference and how does it affect the communication?

(3) What is the partial Response Waveforms?

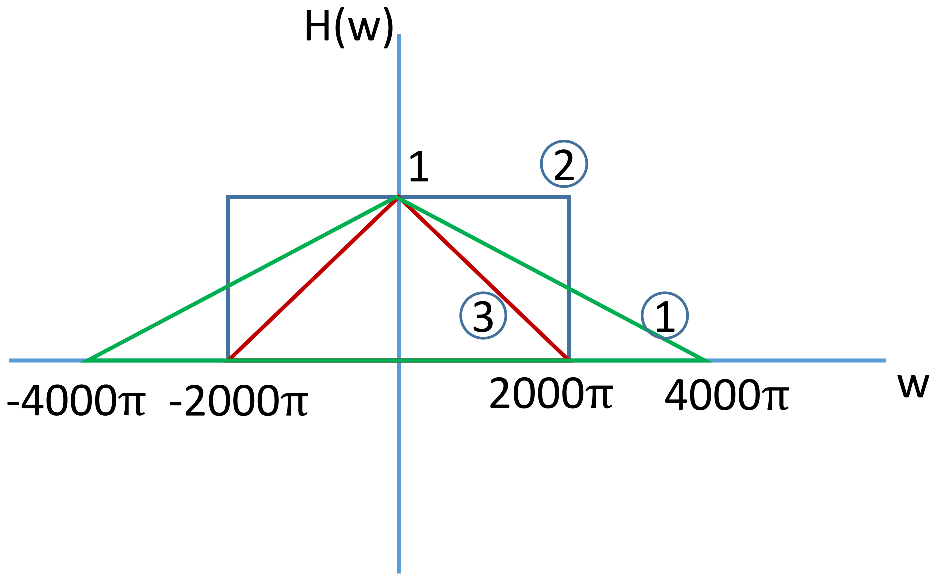
Exercise

Ex1: Please determine whether the following system satisfies the no intersymbol interference conditions when the symbol rate is 2/Tb.



Exercise

Ex2: To support the transmission of the digital baseband signal whose symbol rate is Rb=1000B, which of the following system has the better transmission characteristics and give the reasons. (Hint: Consider the frequency band utilization, tail decay, practical realization)

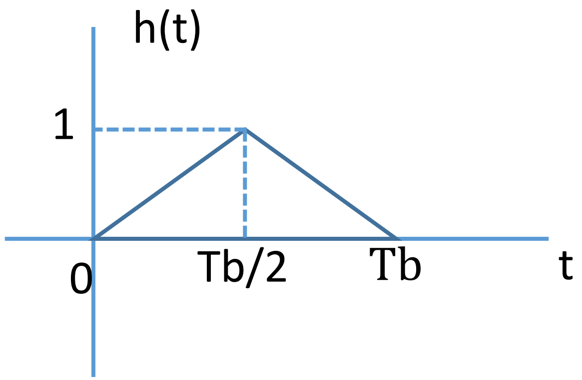


Exercise

Ex3: A baseband transmission system is shown as follows, please determine

(1) The frequency spectrum of the transmission system

(2) If the channel C(w)=1, and the transmitting filter and the receiving filter has the same transmission function (i.e. $G_T(w)=G_R(w)$), find the expression of the $G_T(w)$ or $G_R(w)$



MATLAB

(1) Plot the roll-off system with alpha=0,0.5,1 respectively in both time and frequency domain (can use the function “rcosdesign”) under the condition of Nyquist rate being 10kBaud and sampling rate of 1e5kHz

(2) In a roll-off system with alpha=0.5, assume that the sampling rate is 1e5kHz, please (1) generate a binary sequence with length of 100 (2) upsample the sequence to make symbols (zero-padding) (3) let the Nyquist rate be 5kBaud and 20kBaud (4) generate the corresponding roll-off systems (5) signal and systems convolution (6) plot both the original signal and the received signal