# Principles of Communications

Chapter 6 — Elementary Digital Modulation System

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# Outline

# Main Content

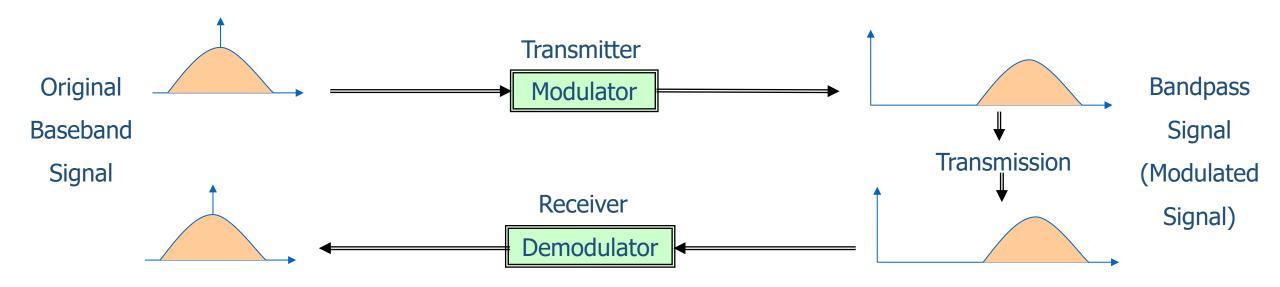
- Introduction
- Binary modulation
  - 2ASK
  - 2FSK
  - 2PSK &2DPSK
- Multi-ary Modulation
  - MASK
  - MFSK
  - MPSK



### Introduction

#### Modulation & Demodulation

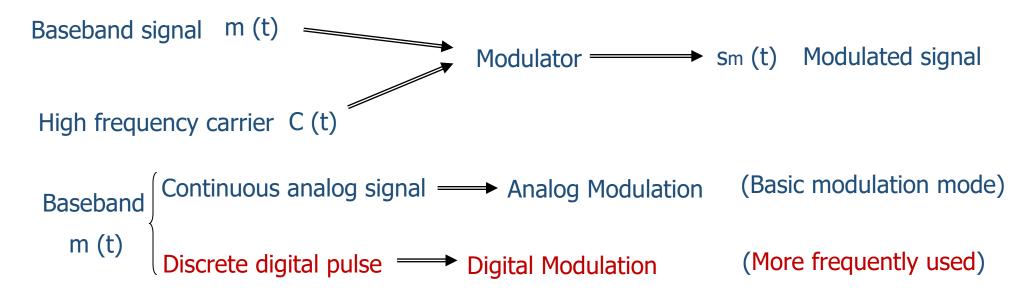
Modulation: A process of moving the baseband signal spectrum into the passband of a given channel



Demodulation: The process of restoring the spectrum in the passband to the baseband signal

### Introduction

#### Modulation & Demodulation



#### Why digital modulation?

- Most channels have bandpass transmission characteristics
- Digital baseband signals have rich low-frequency components (not suitable for direct bandpass transmission)
- · Like analog signals, digital baseband signals can be modulated



### Introduction

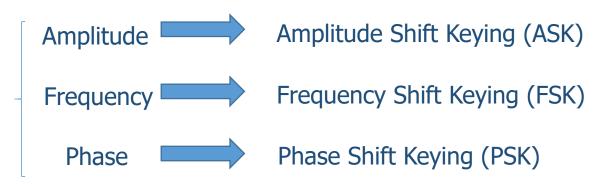
Digital Modulation & Digital bandpass transmission system

Digital modulation: the process of controlling a certain parameter of the carrier with a digital signal.

The purpose of digital transmission system modulation

move the signal spectrum to the optimal frequency band for multiplexing and improve transmission quality

#### Digital modulation methods:



#### Binary modulation

 there are only two possible values for signal parameters

Multi-ary modulation

signal parameters may have M (M>2) values



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- Introduction
- Binary modulation
  - 2ASK
  - 2FSK
  - 2PSK &2DPSK
- Multi-ary Modulation
  - MASK
  - MFSK
  - MPSK



#### Basic principle

ASK uses the amplitude of the carrier to represent digital information

2ASK: 2 amplitude states to represent 1 and 0.

Most simple and common 2ASK: On Off Keying (OOK)

$$e_{OOK}(t) = \begin{cases} A\cos\omega_c t, & \text{transmit 1} \\ 0, & \text{transmit 0} \end{cases}$$

#### A general form of 2ASK:

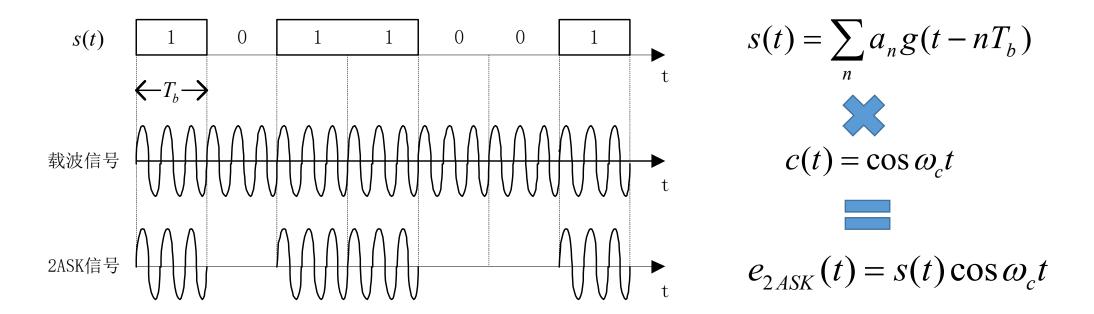
Baseband signal 
$$s(t) = \sum_{n} a_{n} g(t - nT_{b})$$

$$g(t) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, & 其它 \end{cases}$$

2ASK signal 
$$e_{2ASK}(t) = s(t) \cos \omega_c t$$

$$a_n = \begin{cases} 1, & \text{transmit probability of } p \\ 0, & \text{transmit probability of } 1-p \end{cases}$$

2ASK is a digital modulation where the amplitude of the carrier varies with the baseband

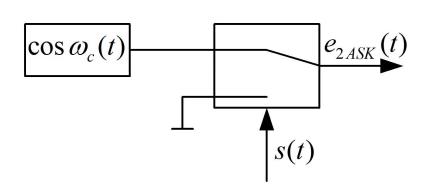


The digit 0 and 1 are represented by the varying amplitude of the carrier

#### Two ways of generation of 2ASK:

The baseband signal and the carrier pass through a multiplier directly

Switching method

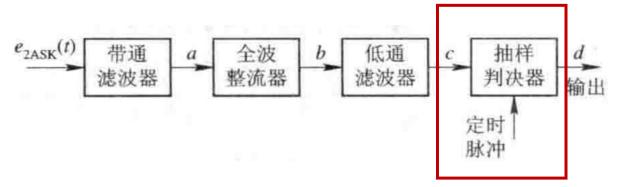


The baseband signal control a switch. When the digit is 1, switch to the carrier, while the digit is 0, switch to the ground.

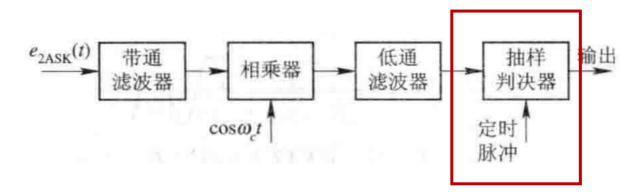


#### 2ASK demodulation:

Noncoherent demodulation (envelope detection)



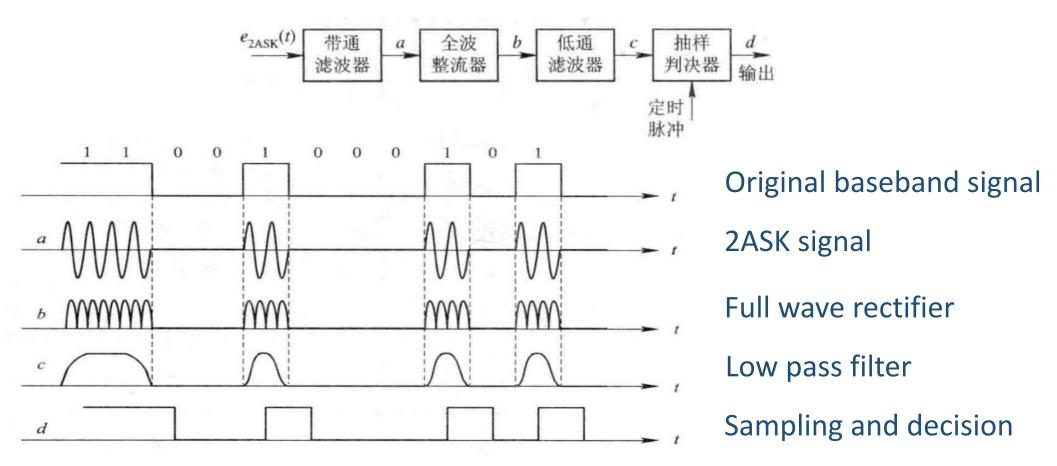
Coherent demodulation (synchronous detection)



- 1. The demodulation process is almost the same as that of the analog communication system.
- 2 The only difference is that in digital transmission system, a sampling & decision modulus is included.

#### 2ASK demodulation:

#### An illustration of noncoherent demodulation





Power spectral density of 2ASK

The 2ASK signal is expressed as 
$$e_{2ASK}(t) = \left[\sum_{n} a_{n}g(t-nT_{b})\right]\cos\omega_{c}t$$

2ASK is linear modulation, so the spectral density is a shift of baseband spectral density

$$P_{2ASK}(f) = \frac{1}{4} [P_s(f + f_c) + P_s(f - f_c)]$$

$$P_s(f) \text{ is spectral density of baseband signal}$$

Assume the baseband signal is unipolar signal with same probability of 0 and 1:

$$P_{s}(f) = f_{b}P(1-P)|G(f)|^{2} + \sum_{m=-\infty}^{\infty} |f_{b}(1-P)G(mf_{b})|^{2} \delta(f-mf_{b})$$

$$= [T_{b}Sa^{2}(\pi fT_{b}) + \delta(f)]/4$$
Obtain from chapter 5

#### The final spectral density of 2ASK is expressed as:

$$P_{2ASK}(f) = \frac{1}{4} [P_s(f + f_c) + P_s(f - f_c)] = \frac{1}{16} f_s [|G(f + f_c)|^2 + |G(f - f_c)|^2] + \frac{1}{16} f_s^2 |G(0)|^2 [\delta(f + f_c) + \delta(f - f_c)]$$

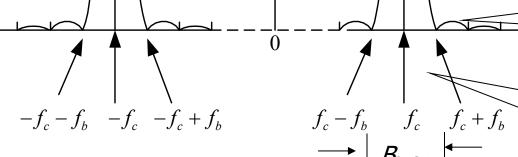
0

$$= \frac{T_b}{16} \left[ \left| \frac{\sin \pi (f + f_c) T_b}{\pi (f + f_c) T_b} \right|^2 + \left| \frac{\sin \pi (f - f_c) T_b}{\pi (f - f_c) T_b} \right|^2 \right] + \frac{1}{16} \left[ \delta (f + f_c) + \delta (f - f_c) \right]$$

Discrete Spectrum: Determined by Carrier Components

The bandwidth of 2ASK

$$B_{2ASK} = 2f_h$$



Continuous spectrum: determined by the baseband signal waveform g(t)

> The peak of the first side lobe is attenuated by 14dB compared with the main peak

The bandwidth of 2ASK is twice of the baseband signal



#### Basic principle

FSK uses the frequency of the carrier to represent digital information

2FSK can be expressed as: 
$$e_{2FSK}(t) = \begin{cases} A\cos(\omega_1 t + \varphi_n), & \text{transmit 1} \\ A\cos(\omega_2 t + \theta_n), & \text{transmit 0} \end{cases}$$

A general form of 2FSK: 
$$e_{2FSK}(t) = \left[\sum_{n} a_n g(t - nT_b)\right] \cos(\omega_1 t + \varphi_n) + \left[\sum_{n} \overline{a_n} g(t - nT_b)\right] \cos(\omega_2 t + \theta_n)$$

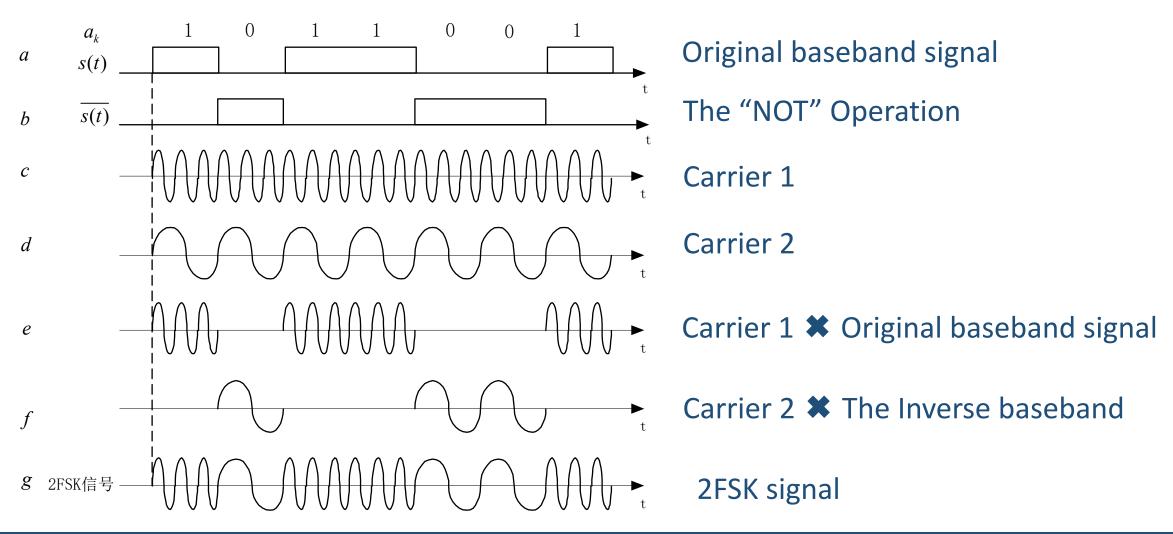
$$g(t) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, &$$
其它  $\end{cases}$   $a_n = \begin{cases} 1, & \text{transmit probability of } p \\ 0, & \text{transmit probability of } 1-p \end{cases}$   $a_n = \begin{cases} 0, & \text{transmit probability of } p \\ 1, & \text{transmit probability of } 1-p \end{cases}$ 

Since  $\phi_n$  and  $\theta_n$  carry no information, usually  $\phi_n$  and  $\theta_n$  can be made zero

$$e_{2FSK}(t) = \left[\sum_{n} a_n g(t - nT_b)\right] \cos \omega_1 t + \left[\sum_{n} \overline{a_n} g(t - nT_b)\right] \cos \omega_2 t$$



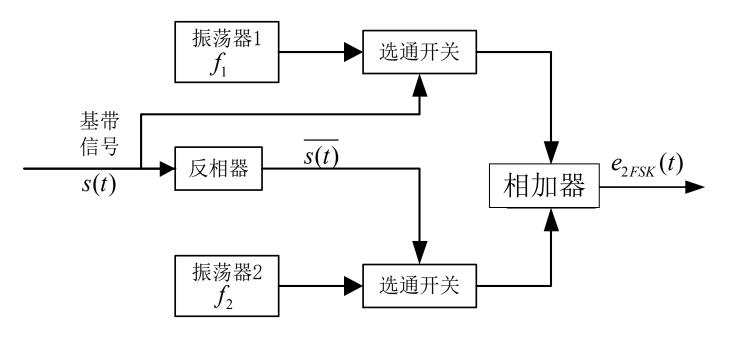
#### An illustration of 2FSK





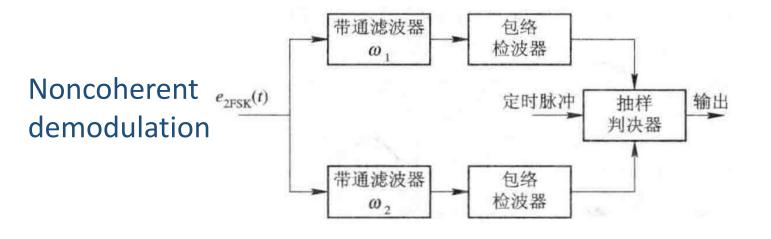
Generation of 2FSK: Frequency modulation & Switching method

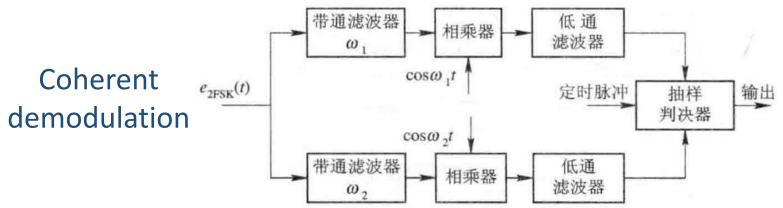
The diagram of switching method



- The baseband signal control 2
   switches. When the digit is 1, the
   carrier f1 switch on , while the
   digit is 0, the carrier f2 switch on.
- During a certain symbol Tb, only one of the two carriers f1 or f2 is output

#### 2FSK demodulation:





The principle of demodulation:

- 1 decompose the 2FSK signal into upper and lower 2ASK signals
- 2 demodulate them separately
- 3 yiudge the output signal by comparing the sampling values of the upper and lower channels.



#### Power spectral density of 2FSK

The 2FSK signal is expressed as 
$$e_{2FSK}(t) = \left[\sum_{n} a_n g(t - nT_b)\right] \cos \omega_1 t + \left[\sum_{n} \overline{a_n} g(t - nT_b)\right] \cos \omega_2 t$$
  
=  $s_1(t) \cos \omega_1 t + s_2(t) \cos \omega_2 t$ 

The spectral density of 2FSK can be regarded as the sum of two 2ASK spectral density

$$P_{2FSK}(f) = \frac{1}{4} \Big[ P_{s_1}(f+f_1) + P_{s_1}(f-f_1) \Big] + \frac{1}{4} \Big[ P_{s_2}(f+f_2) + P_{s_2}(f-f_2) \Big]$$

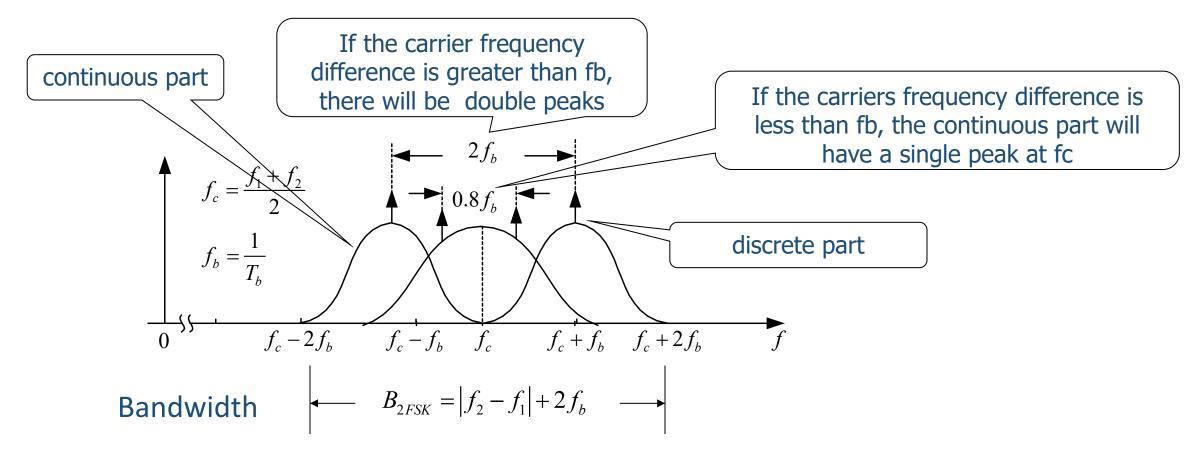
Assume the baseband signal is unipolar signal with same probability of 0 and 1:

$$P_{2FSK}(f) = \frac{T_b}{16} \left[ \left| \frac{\sin \pi (f + f_1) T_b}{\pi (f + f_1) T_b} \right|^2 + \left| \frac{\sin \pi (f - f_1) T_b}{\pi (f - f_1) T_b} \right|^2 \right] + \frac{T_b}{16} \left[ \left| \frac{\sin \pi (f + f_2) T_b}{\pi (f + f_2) T_b} \right|^2 + \left| \frac{\sin \pi (f - f_2) T_b}{\pi (f - f_2) T_b} \right|^2 \right] + \frac{1}{16} \left[ \delta (f + f_1) + \delta (f - f_1) + \delta (f + f_2) + \delta (f - f_2) \right]$$



Illustration of Power spectral density of 2FSK

Sum of two spectral density, contains both and discrete part





#### Basic principle

PSK uses the phase of the carrier to represent digital information

2PSK: 2 carrier phase states to represent 1 and 0.

A general form of 2PSK: 
$$e_{2PSK}(t) = \left[\sum_{n} a_{n}g(t-nT_{s})\right] \cos \omega_{c}t$$

$$g(t) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, & \sharp \stackrel{\sim}{\to} \end{cases}$$

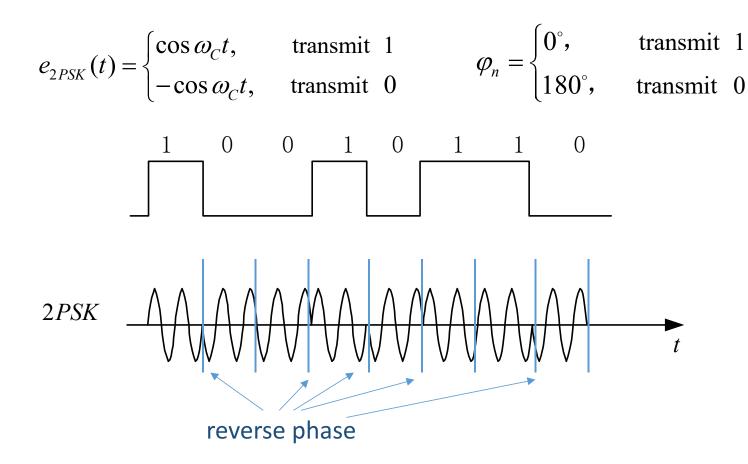
$$g(t) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, & \sharp \ \end{cases} \qquad a_n = \begin{cases} 1, & \text{transmit probability of } p \\ -1, & \text{transmit probability of } 1-p \end{cases}$$

If g(t) is a rectangular pulse with pulse width Tb and height 1

$$e_{2PSK}(t) = \begin{cases} \cos \omega_C t, & \text{transmit } 1\\ -\cos \omega_C t, & \text{transmit } 0 \end{cases}$$

$$\varphi_n = \begin{cases} 0^{\circ}, & \text{transmit } 1\\ 180^{\circ}, & \text{transmit } 0 \end{cases}$$

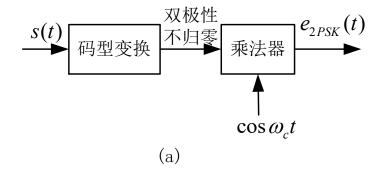
#### An illustration of 2PSK



2PSK, which uses different phases of the carrier to directly represent the digital information, is usually called the absolute phase shifting method.

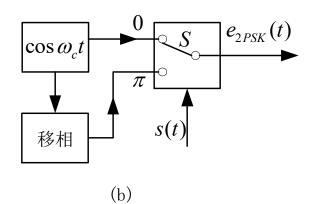
#### Generation of 2PSK:

Multiplication method



- The baseband symbols are first transformed to bipolar signal
- The bipolar NRZ code multiple with the carrier

Switching method

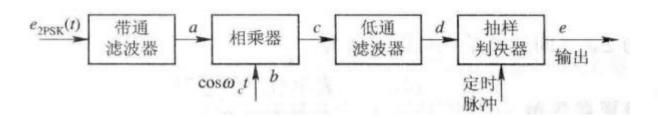


- The baseband signal control a switch.
- When the digit is 1, switch to the carrier.
- While the digit is 0, switch to the carrier with reverse phase.

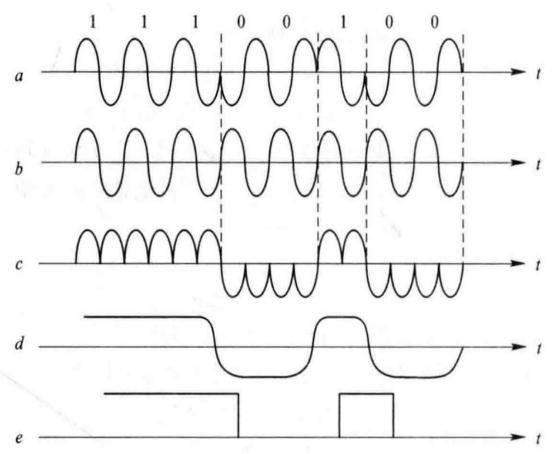


#### 2PSK demodulation:

It can only use coherent demodulation



The process is almost the same as that of analog demodulation, while an extra sampling and decision modulus is needed.





The problem of 2PSK: In practice, it is rarely used

- 1. The phase of carrier is difficult to determined in the receiver end
- The condition of correct demodulation is consistent reference phase
- Phase ambiguity exists in 2PSK carrier extraction (inverse reference phase decision)
- If inverse reference phase happens, the decision becomes all wrong
- 2. The start and stop instants are hard to be identified
  - When continuous "0" or "1", received signal is a long continuous sine waveform
  - The start and stop instants are hard to identify
  - The sampling and decision instant can't be decided.



Basic principle

To overcome the phase ambiguity of 2PSK, 2DPSK is proposed

2DPSK: Utilizes the relative value of the carrier phases of the adjacent symbols to express "0" and "1"

Assume the phase difference of the current symbol and the previous symbol is

$$\Delta \varphi = \begin{cases} 0, & \text{When 0 is transmitted} \\ \pi, & \text{When 1 is transmitted} \end{cases}$$

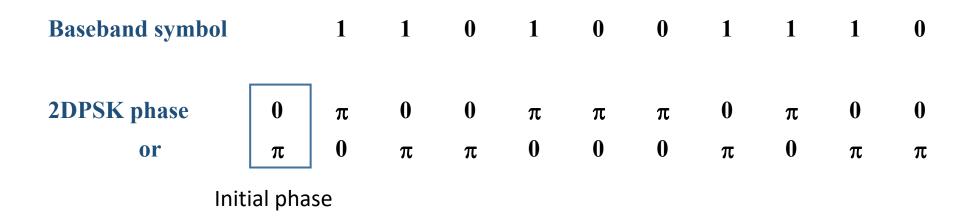
Then, the signal symbol can be expressed as  $e_{DPSK} = \cos(\omega_c t + \varphi + \Delta \varphi)$ 

Angular frequency of carrier Previous symbol phase Phase difference

$$\omega_c = 2\pi f_c$$

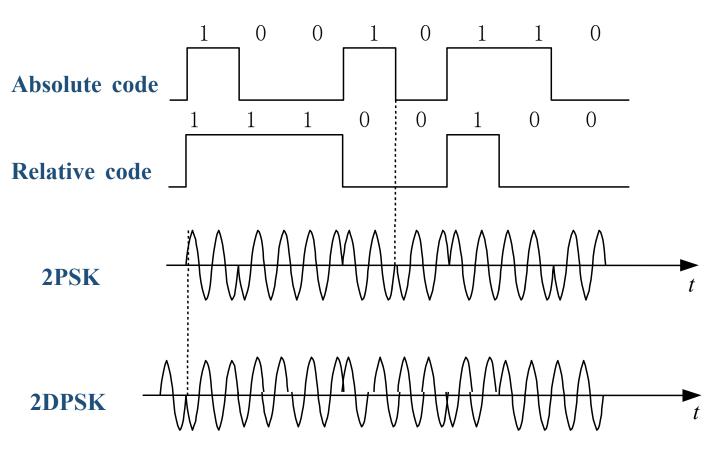
$$\varphi$$

#### Illustration



- After deciding the initial phase (reference phase), use the phase changes to represent the information;
- "1" is expressed with the phase change of the carrier:  $0 > \pi / \pi > 0$ ;
- "0" is expressed with the phase remain of the carrier:  $0 > 0 / \pi > \pi$ ;

#### Comparison



- Simply looking at waveform,2DPSK & 2PSK are the same.
- When demodulating 2DPSK signals, don't care about the phase reference value.
- If the relative phase relationship is kept, the digital can be recovered, which avoids the problem in the 2PSK mode.

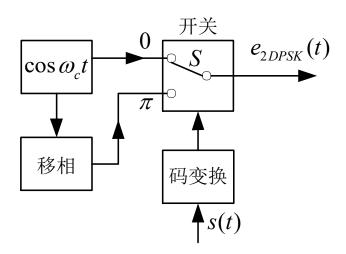
#### Two ways of drawing the waveform of 2DPSK:

- 1. The relative code is first obtained from the original symbols according to the rule of "1 changes while 0 unchanged", and then the 2DPSK waveform is drawn based on the relative code
- 2. Apply the rule of "1 change to 0 unchanged" directly to the modulated carrier phase corresponding to the adjacent symbols: (1) if the symbol is "1", (first phase) and Phase) the first phase of the current symbol and the last phase of previous symbol is opposite (disruption point appears); (2) if the symbol is "0", the phase is the same (no discontinuity point).



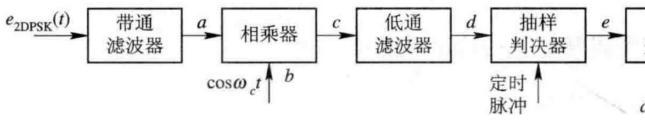
#### 2DPSK modulation:



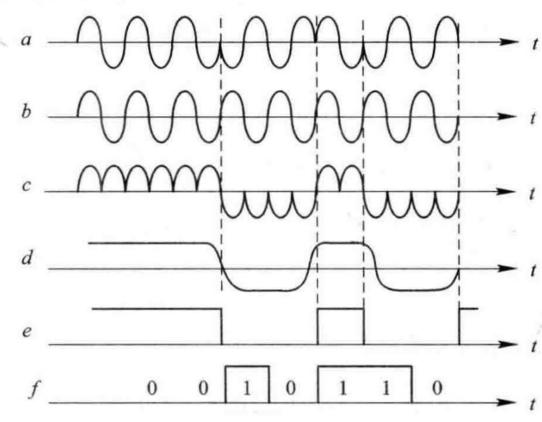


- 1 Differentially encode the binary digital
  baseband signal, convert the absolute code to
  represent the binary information into the
  relative code to represent the binary information
- 2 Perform absolute phase modulation to generate a binary differential phase keying signal

#### 2DPSK demodulation: Coherent demodulation

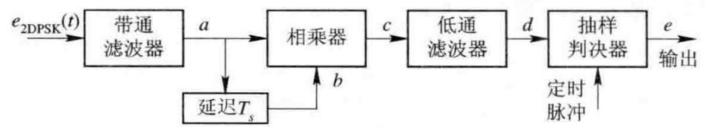


- Use the phase demodulation on 2DPSK to recover relative code, then transform it into absolute code with code inverse converter
- If  $180^{\circ}$  phase ambiguity, demodulated relative code will have inversion  $\pi$ , but after the code inverse converter, the problem of phase ambiguity will not exist

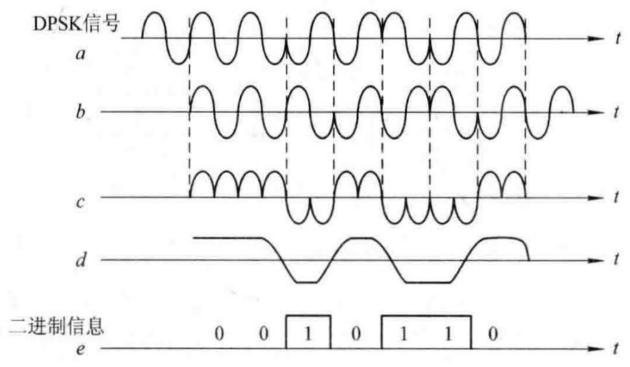




2DPSK demodulation: Phase comparison demodulation (non-coherent demodulation)



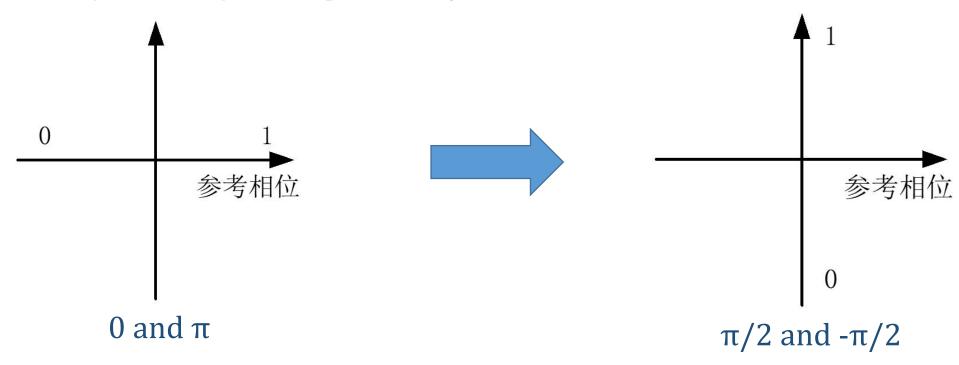
- Compare the phase difference of adjacent symbols to recover the transmitted signal.
- The code inverse conversion is completed while demodulating, so code inverse converter is not needed
- This demodulation method does not require a special coherent carrier.





To solve problem 2: start and stop instants are hard to be identified

Use  $\pi/2$  and  $-\pi/2$  to represent symbol 0 and 1



When continuous 1, the waveform will have no change

There must be changes when the symbol period ends



Power spectral density of 2PSK and 2DPSK

It is impossible to distinguish 2DPSK and 2PSK signal if only observed from receiver



The power spectral density2DPSK and 2PSK signal are quite the same

2PSK is regarded as bipolar signal multiplied by a sinusoidal carrier

$$P_{2PSK}(f) = \frac{1}{4} [P_s(f + f_c) + P_s(f - f_c)]$$
  $P_s(f)$  is spectral density of baseband signal

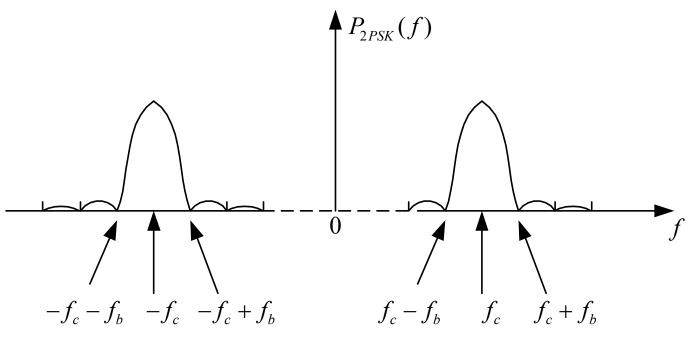
If baseband signal is a rectangular pulse, the power density of the 2PSK signal is

$$P_{2PSK}(f) = \frac{T_b}{4} \left[ \left| \frac{\sin \pi (f + f_c) T_b}{\pi (f + f_c) T_b} \right|^2 + \left| \frac{\sin \pi (f - f_c) T_b}{\pi (f - f_c) T_b} \right|^2 \right]$$
The probability of "1" symbol and the "0" symbol is equal



#### Illustration of Power spectral density of 2PSK

$$P_{2PSK}(f) = \frac{T_b}{4} \left[ \left| \frac{\sin \pi (f + f_c) T_b}{\pi (f + f_c) T_b} \right|^2 + \left| \frac{\sin \pi (f - f_c) T_b}{\pi (f - f_c) T_b} \right|^2 \right]$$



- 1 The shape of 2PSK power density is similar to that of 2ASK.
- 2 The bandwidth is twice of that in baseband.
- 3. In general, the power density consists of continuous part and discrete part. Specially, when "0" and "1" has equal probability, no discrete part.

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# M-ary Digital Keying

Multi-ary digital phase modulation system

When channel frequency band is limited

- Need to increase the transmission rate of information (i.e. R<sub>b</sub>)
- improve the utilization rate of the frequency band
- improve the effectiveness of the digital transmission system

Definition of M-ary Digital Keying:

- In interval  $0 \le t \le T_s$ , there are M types of symbols that may be sent:  $s_i(t)$ , i=1, 2, ..., M.
- $\triangleright$  Each symbol can carry  $\log_2 M$  bit information, so the transmission rate of information can be increased, and the utilization rate of the frequency band can be improved
- Multi-ary digital modulation is proposed to meet the requirements of mobile communication



## **MASK**

#### Multi-ary amplitude shift keying (MASK)

#### **Definition:**

- MASK uses M possible carrier amplitude, and represents a symbol with one of amplitudes.
- MASK, also known as multi-level modulation, is an extension of the 2ASK

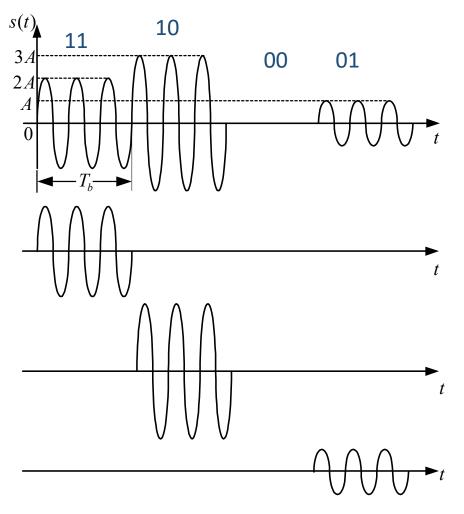
#### MASK is a highly effective transmission modulation (high $R_b$ ):

- Under the same symbol rate, more bit will be transmitted
- MASK has the same bandwidth with 2ASK
- For binary system (bandpass), the highest channel frequency band utilization is 1bit/s/Hz.
- For multi-level systems, the highest channel frequency band utilization will exceed 1bit/s/Hz



# **MASK**

#### Illustration of MASK



MASK can be expressed as the multiplication of

the M-ary baseband signal and the carrier:

$$e_{MASK}(t) = \sum_{n} a_{n}g(t - nT_{s})\cos\omega_{c}t$$
Baseband
waveform
Symbol
interval

$$a_n = \begin{cases} A_1, & \text{Transmit Probability } P_1 \\ A_2, & \text{Transmit Probability } P_2 \\ \vdots & & \sum_{i=1}^M P_i = 1 \end{cases}$$

$$A_M, & \text{Transmit Probability } P_M$$

# **MFSK**

Multi-ary frequency shift keying (MFSK)

Definition: MFSK uses M possible carrier frequency, and represents a symbol with one of frequencies.

Only one of the M frequencies is chosen during Ts 
$$s_i(t) = a \cos \left( 2\pi f_c t + \frac{i\pi}{T_s} t \right)$$
 Ts is symbol interval

Each of the M FSK signals have the same energy and Orthogonal

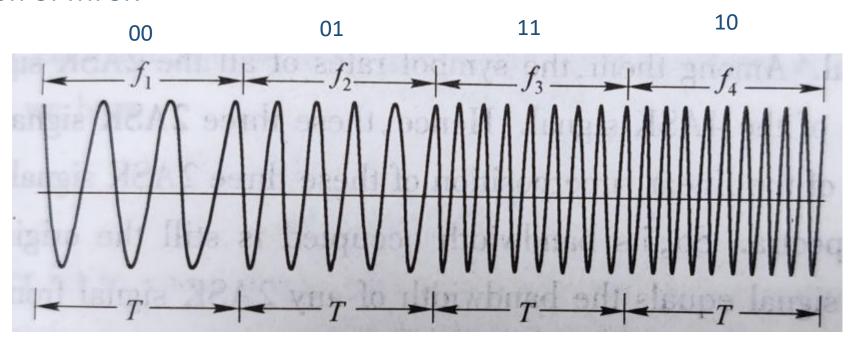
$$\int_{0}^{T_{s}} s_{i}(t) s_{j}(t) = \begin{cases} E_{i} & i = j \\ 0 & i \neq j \end{cases}$$
 The frequency interval should be 1/(2Ts)Hz

MFSK is generally applied in occasions where the modulation rate is not high

Bandwidth of MFSK: 
$$B = |f_M - f_1| + \frac{2}{T_s}$$
 Has a wide frequency band, so its channel band utilization is not high

# **MFSK**

#### Illustration of MFSK



Frequency interval 
$$|f_{i+1} - f_i| = \frac{1}{2T_s}$$
 Satisfy Orthogonal Condition

Multi-ary phase shift keying (MPSK)

Definition: MPSK uses M possible carrier phase, and represents a symbol with one of phase.

It can be expressed as  $e_{MPSK}(t) = \sum_{n} g(t - nT_s) \cos(\omega_c t + \varphi_n)$ 

Signal envelope, usually rectangular wave with amplitude of 1

Symbol interval

The phase corresponding to the n-th symbol (M values in total)

It is usually written in Orthogonal form (IQ):

$$e_{MPSK}(t) = \left[\sum_{n} g(t - nT_s)\cos\varphi_n\right]\cos\omega_c t - \left[\sum_{n} g(t - nT_s)\sin\varphi_n\right]\sin\omega_c t$$

$$= \left[\sum_{n} a_n g(t - nT_s)\right]\cos\omega_c t - \left[\sum_{n} b_n g(t - nT_s)\right]\sin\omega_c t$$

$$= I(t)\cos\omega_c t - Q(t)\sin\omega_c t$$

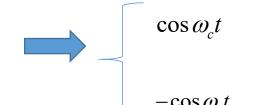
## Multi-ary phase shift keying (MPSK)

$$e_{MPSK}(t) = I(t)\cos\omega_c t - Q(t)\sin\omega_c t$$

 $I(t) = \sum_{n} a_{n}g(t - nT_{s})$   $Q(t) = \sum_{n} b_{n}g(t - nT_{s})$ 

For 4PSK: 
$$\begin{cases} \text{When } a_n = 0, b_n = \pm 1 \\ \text{When } b_n = 0, a_n = \pm 1 \end{cases} \qquad \begin{cases} a_n = \pm 1 \\ b_n = \pm 1 \end{cases}$$

$$\begin{cases} a_n = \pm 1 \\ b_n = \pm 1 \end{cases}$$



 $-\sin \omega_c t$ 

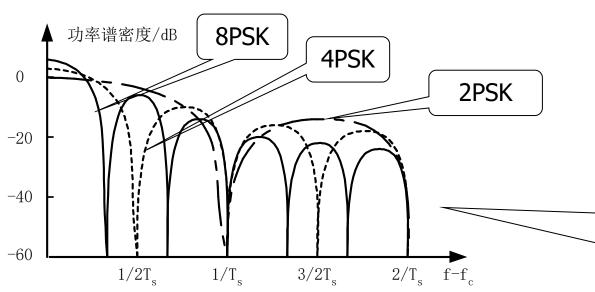
represent

4 states

 $-\cos\omega t$ 

 $\sin \omega_c t$ 

4 symbols

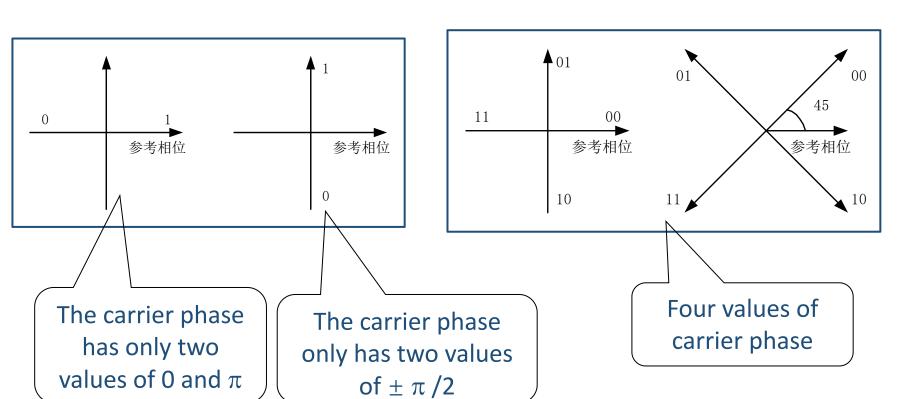


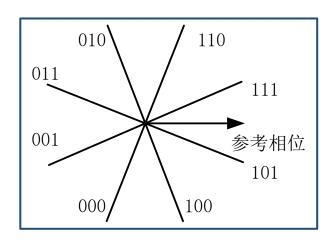
When the information rate is the same, the larger M, the narrower the main lobe, and the higher frequency band utilization

Multi-ary phase shift keying (MPSK)

2PSK signal vector illustration

QPSK (4PSK) signal vector illustration 8PSK signal vector illustration





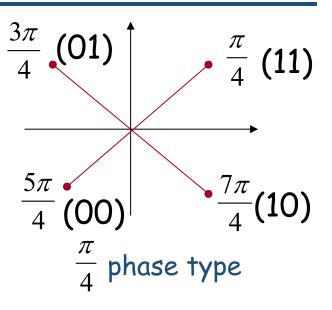


## A typical MPSK: QPSK

4 symbol types: 00, 01, 10, 11

4 carrier phases: Can be  $\frac{\pi}{4} = \frac{3\pi}{4} = \frac{3\pi}{4} = \frac{7\pi}{4}$ 

Can be



## Take the first type as example:

$$11 \rightarrow \frac{\pi}{4} \qquad 00 \rightarrow \frac{5\pi}{4}$$

$$01 \rightarrow \frac{3\pi}{4} \qquad 10 \rightarrow \frac{7\pi}{4}$$
Gary code rule

the baseband signal is 11, the carrier is  $A\cos(2\pi f_c t + \frac{\pi}{4})$  the baseband signal is 01, the carrier is  $A\cos(2\pi f_c t + \frac{3\pi}{4})$  the baseband signal is 00, the carrier is  $A\cos(2\pi f_c t + \frac{5\pi}{4})$  the baseband signal is 10, the carrier is  $A\cos(2\pi f_c t + \frac{7\pi}{4})$ 

# Thank you!

### Answer briefly

- (1) What is absolute phase modulation, what is relative phase modulation? What's the purpose of relative phase modulation?
- (2) What's the characteristics of the power spectral density of 2ASK, 2FSK, 2PSK and 2DPSK? Compare them.
- (3) How to generate and demodulate 2ASK, 2FSK, 2PSK and 2DPSK?
- (4) What are the benefits of Multi-ary digital modulation?



Ex1: Assume that the baseband sequence is 011011100010, the bit rate is 1000bit/s, carrier frequency is 1kHz

- (1) Draw the waveform of 2ASK、2FSK (f2=2kHz) 、2PSK、2DPSK
- (2) If the carrier frequency is 1.5kHz, draw the waveform of 2ASK、2FSK (f2=3kHz) 、 2PSK、2DPSK



Ex2: For a 2ASK system, symbol rate is 1200B, carrier frequency is 6000Hz, the baseband signal is 10110

- (1) Draw the 2ASK modulation diagram
- (2) Draw the power spectral density of 2ASK
- (3) Find the bandwidth of 2ASK



Ex3: symbol rate is 1200B, carrier frequency is 1800Hz, the baseband signal is 011000010

- (1) If phase shift  $\Delta \varphi = 0$  to represent 0 and  $\Delta \varphi = \pi$  to represent 1, draw the waveform of 2DPSK
- (2) If phase shift  $\Delta \varphi = -\pi/2$  to represent 0 and  $\Delta \varphi = \pi/2$  to represent 1, draw the waveform of 2DPSK



## **MATLAB**

- (1) Generate a 0-1 sequence with equal probability, modulate it with 2FSK, plot the signal waveform and the power spectral density
- (2) modulate the sequence with 2PSK, plot the signal waveform and the power spectral density
- (3) modulate the sequence with 2DPSK, plot the signal waveform

