Principles of Communications

Chapter 5 — Presentation and Transmission of Baseband Signal



Analog signal ADC — 0/1 sequence (Digital baseband signal)

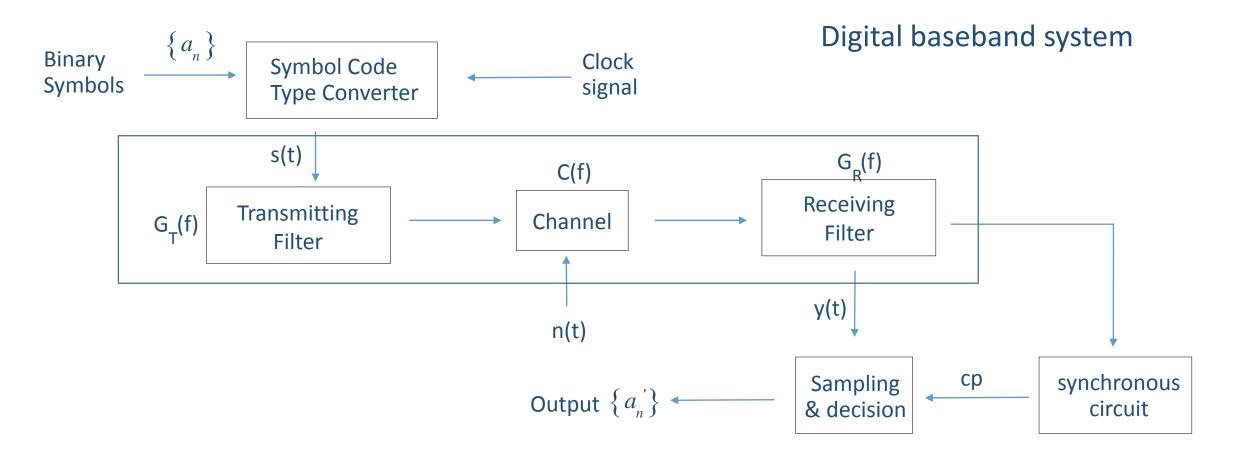
- \bullet Digital baseband signal: Unmodulated digital signal, which occupies a spectrum of low frequencies $_{\circ}$
- Digital Baseband Transmission System: A system that directly transmits
 digital baseband signals without carrier modulation, often used when the
 transmission distance is not too far
- Digital Bandpass Transmission System: Transmission systems including modulation and demodulation processes



Why we need to investigate the digital baseband transmission system?

- 1. Widely used in short-range digital communication systems
- 2. The baseband transmission method also has a rapid development trend
- 3. Many fundamental issues in baseband transmission are also considered in bandpass digital transmission.
- 4. Any bandpass transmission system using linear modulation can be equivalent to a baseband transmission system for investigation.



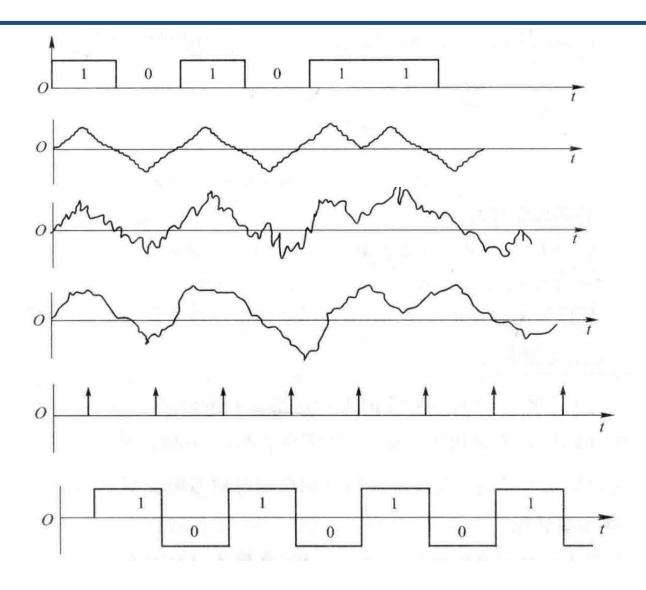




- **Symbol Code Type Converter**: Transform the original baseband signal to the code type suitable for channel transmission
- Transmitting Filter: The output by the converter are based on rectangle pulse. The filter transforms it into a smooth waveform, which is beneficial to compressing the bandwidth
- Channel: A medium that allows baseband signals to pass through, and usually does not meet the conditions of undistorted transmission. In addition, the channel will introduce noise, which generally is considered as AWGN
- Receiving Filter: Filter out-of-band noise, equalize the channel characteristics, and make the output baseband waveform conducive to sampling & decision
- Sampling & decision: Under the condition of unsatisfactory transmission characteristics and noise, the output waveform is sampled and judged at the specified time to restore or regenerate the baseband signal.



The illustrations of waveform at each point of the baseband digital communication system





Binary signals (0/1) How to represent the digits by voltage in the circuit

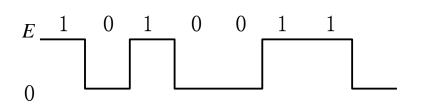
For better illustration, we use rectangular pulse for the introductions of several basic

waveforms:

- 1. Unipolar nonreturn-to-zero(NRZ) waveform
- 2 Bipolar nonreturn-to-zero waveform
- 3. Unipolar return-to-zero(RZ) waveform
- 4 Bipolar return-to-zero waveform
- 5 Differential waveform
- 6. Multi-level waveform



Unipolar NRZ waveform



- 1. Use voltage 0 to represent the digit 0
- 2. Use voltage E to represent the digit 1
- 3. Signal pulse width equals the symbol width

Characteristics:

- 1、Has D.C. component;
- 2. Hard to stabilize the decision threshold;
- 3. Hard to obtain timing information;

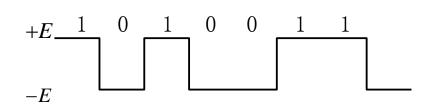
Used compatibly:

Short distance transmission between

points connected by wires



Bipolar NRZ waveform

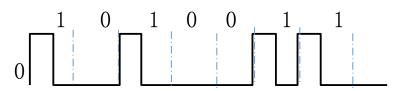


- 1. Use voltage -E to represent the digit 0
- 2. Use voltage E to represent the digit 1
- 3. Signal pulse width equals the symbol width

Characteristics:

- 1. No D.C. component when 0,1 has equal probability, otherwise contain D.C.
- 2. Decision threshold is 0;
- 3. No timing information;

Unipolar RZ waveform



- 1. Use voltage 0 to represent the digit 0
- 2. Use voltage E to represent the digit 1
- 3. Signal pulse width less than the symbol width

Characteristics:

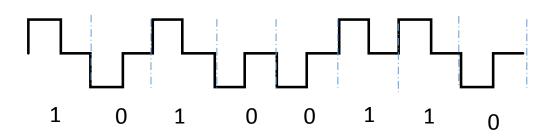
- 1、Has D.C. component;
- 2. The voltage will return to zero before next symbol comes;
- 3. Can obtain timing information directly;

Often met:

Signal processing circuit



Bipolar RZ waveform

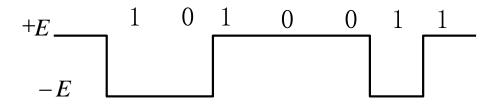


- 1. Use voltage -E to represent the digit 0
- 2. Use voltage E to represent the digit 1
- 3. Signal pulse width less than the symbol width

Characteristics:

- 1. No D.C. component when 0,1 has equal probability, otherwise contain D.C.
- 2. The voltage will return to zero before next symbol comes;
- 3. No timing information but can obtain timing information with rectifier

Differential waveform

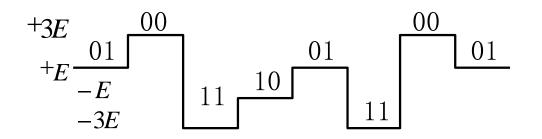


The sequence is not represented by the level of the symbol itself, but represented by the change of adjacent symbols. When 1 appears, voltage jumps, when 0 appears, voltage doesn't change.

Characteristics:

- 1. The code with a differential waveform can eliminate the carrier phase ambiguity problem in the phase modulation system (Discuss in the next chapter)
- 2. The previous code is called an absolute code while this is called a relative code

Multi-level waveform



There are many voltage levels representing the binary sequence. One pulse of this waveform can represent multiple binary symbols, and the information rate can be increased when the symbol rate is constant, so it is widely used in high-speed digital transmission systems;

The previous waveforms has limited uses in practice



They don't quite adapt to the transmission in the channel

Some requirements on the symbol code types to adapt to transmission in channel:

- 1. No D.C. component and very small low frequency components
- 2. Containing timing information of symbols
- 3. High transmission efficiency
- 4. Certain error-detecting ability
- 5. The above performance should be independent of the statistic characteristic of the information source.



AMI code

Full term: Alternative Mark Inverse code

Coding rule: 1 in the message symbols are converted to +1 and -1 alternatively, and 0 in the message symbols keep 0

Example: Message symbol:

1 0 0 1 1 0 0 0 0 0 0 0 1 1 0 0 1 1...

AMI code:

+1 0 0 -1 +1 0 0 0 0 0 0 0 -1 +1 0 0 -1 +1...



AMI code

+1 represents voltage +V; -1 represents voltage -V and 0 is voltage 0

Advantages of AMI code:

- 1 no D.C component because +1 and -1 appear alternatively
- 2. AMI can be converted to NR unipolar symbol after a rectifier to obtain timing information
- 3 Easy to find error: if +1 or -1 appears continuously, that means an error occurs.

Main disadvantages of AMI code:

When a long string of appears, there will be no way to get timing information in receiver since the voltage doesn't change at all.



HDB3 code

To overcome the disadvantage of no time information when continuous 0 in AMI

Full term: 3rd order High Density Bipolar code

Coding rule:

- 1 check continuous 0s in AMI code
 - (1) when no more than 4 (including 4) continuous 0s, no changes (same as AMI)
 - (2) when more than 4 (including 4) continuous 0s, introduce violation symbol.
- 2 use AMI code rules



HDB3 code

How to involve the violation symbol?

- (1) The replacing rule: The 4 continuous 0s will be replaced by 000V or B00V. When there is odd number of 1 in between the two adjacent V, 0000 will be replace by 000V, while there is even number of 1 in between, 0000 will be replaced by B00V.
- (2) Determine the polarity of B and V: V has the same polarity as that of the first previous non-zero code. The polarities of 1s and Bs satisfy the polarity alternatively inverse rule.



Example:

Message symbol

100001 00001 10000 1 1

AMI code

-1 0 0 0 0 +1 0 0 0 0 -1 +1 0 0 0 0 -1 +1

Step 1 (check continuous 0s and replace by 000V)

1 0 0 0 V 1 0 0 0 V 1 10 0 0 V 1 1

Step 2 (check the number of 1 between the adjacent V to replace 000V by B00V)

1 0 0 0 V 1 0 0 0 V 1 1 B 0 0 V 1 1

Step3 (Determine the polarity)

-1 0 0 0 -V +1 0 0 0 +V -1 +1 -B 0 0 -V +1 -1

Final code (B and V are 1)

 $-1 \ 0 \ 0 \ 0 \ -1 \ +1 \ 0 \ 0 \ 0 \ +1 \ -1 \ +1 \ -1 \ 0 \ 0 \ -1 \ +1 \ -1$



Decoding rule:

- 1. Find the two adjacent 1 with the same polarity, and the latter one is V
- 2 the 3 codes in front of V changes to 0
- 3 ±1 changes back to 1

HDB3

$$-1 \ 0 \ 0 \ 0 \ -1 \ +1 \ 0 \ 0 \ 0 \ +1 \ -1 \ +1 \ -1 \ 0 \ 0 \ -1 \ +1 \ -1$$

Step1

Step2

$$-1$$
 0 0 0 0 +1 0 0 0 0 -1 +1 0 0 0 0 +1 -1

Decoded

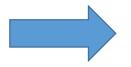
100001 00001 10000 1 1



Biphase code

Also called: Manchester code

Coding rule: Each binary symbol is converted into a period of square wave with different phases

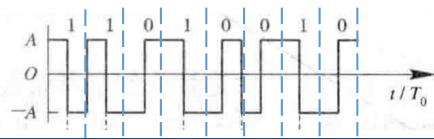


0 can be represented by 01, corresponding to phase π

1 can be represented by 10, corresponding to phase 0

Message symbol: 1 1 0 1 0 0 1 0

Biphase code: 10 10 01 10 01 10 01





Biphase code

Characteristics:

- 1. The biphase code has only two levels with opposite polarities. The biphase code has a level jump during a symbol period, so it contains timing information
- 2. The positive and negative polarity of this code are half and half, so there is no DC component, and the encoding process is simple
- 3. The bandwidth is double of the original code
- 4. Bi-phase code is suitable for short-distance transmission. Local data networks often use this code as a transmission code type, and the information rate achieve 10Mb/s



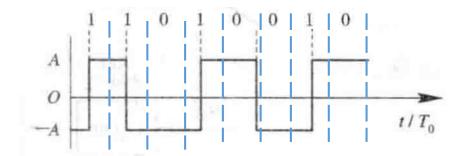
Miller code

Also called: delay modulation code

It is a transformation of biphase code

Coding rule:

- 1. 1 is represented by the voltage jump at the middle point of the symbol duration, i.e. 01/10
- 2. 0 is represented by no changes during the symbol duration, i.e. 11/00
- 3 when 0 appears continuously, there is a voltage jump between 0s, i.e. 00 and 11



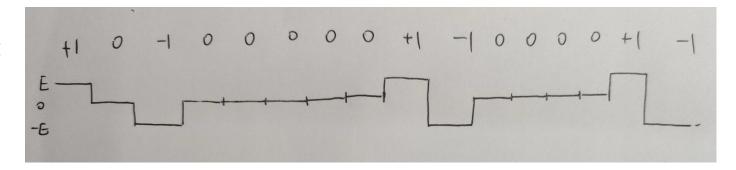


Example: Knowing that the information code is 1010000011000011, please determine the corresponding AMI、HDB3、Biphase code and differential waveform. Please draw the waveform as well.

Solutions:

Message symbols: 1 0 1 0 0 0 0 0 1 1 0 0 0 0 1 1

AMI: +10-100000+1-10000+1-1





Example: Knowing that the information code is 1010000011000011, please determine the corresponding AMI、HDB3、Biphase code and differential waveform. Please draw the waveform as well.

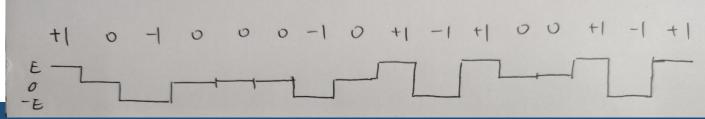
Solutions: Message symbols: 1 0 1 0 0 0 0 0 1 1 0 0 0 0 1 1

HDB3: 101000V011000V11

1 0 1 0 0 0 V 0 1 1 B 0 0 V 1 1

+10 -1 000-V 0+1-1+B0 0+V-1+1

+10 -1 000-1 0+1-1+10 0+1-1+1

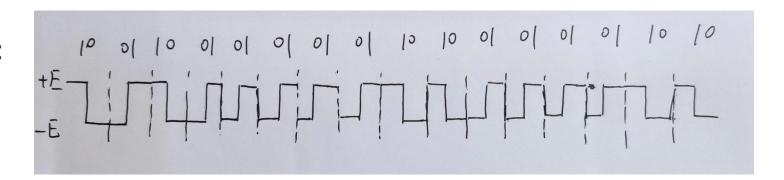




Example: Knowing that the information code is 1010000011000011, please determine the corresponding AMI、HDB3、Biphase code and differential waveform. Please draw the waveform as well.

Solutions:

Message symbols: 1 0 1 0 0 0 0 0 1 1 0 0 0 0 1 1



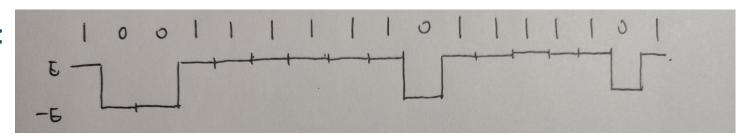


Example: Knowing that the information code is 1010000011000011, please determine the corresponding AMI、HDB3、Biphase code and differential waveform. Please draw the waveform as well.

Solutions:

Message symbols: 1 0 1 0 0 0 0 0 1 1 0 0 0 0 1 1

Differential code: 1 0 0 1 1 1 1 1 1 0 1 1 1 1 0 1





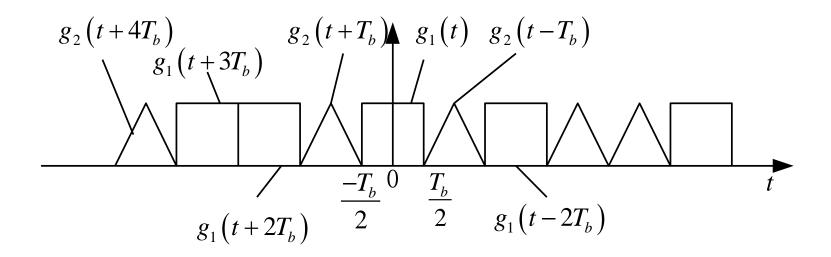
It is discussed qualitatively whether the waveform contains D.C. component or timing information



How to determine quantitatively whether the waveform contains D.C. component or timing information



It is possible to use the frequency characteristic of the baseband signal to determine whether the waveform is suitable to be transmitted in the channel



g1(t) - "0", appearance probability is P

g2(t) - "1", appearance probability is 1-P fb - symbol rate

Tb – symbol duration



The derivation of the power density of the baseband signal

The baseband signal can be expressed as:

$$s(t) = \sum_{n=-\infty}^{\infty} s_n(t) \qquad \text{where} \quad s_n(t) = \begin{cases} g_1(t - nT_b), P \\ g_2(t - nT_b), 1 - P \end{cases}$$

It can also be expressed as the sum of 2 part:

Stationary wave
$$v(t) = \sum_{n=-\infty}^{\infty} \left[Pg_1 \left(t - nT_b \right) + (1 - P)g_2 \left(t - nT_b \right) \right] = \sum_{n=-\infty}^{\infty} v_n(t)$$

$$s(t) = u(t) + v(t)$$
 Alternating wave
$$u(t) = \sum_{n=-\infty}^{\infty} u_n(t) = \sum_{n=-\infty}^{\infty} a_n \left[g_1 \left(t - nT_b \right) - g_2 \left(t - nT_b \right) \right]$$
 where
$$a_n = \begin{cases} 1 - P, \text{ appear with probability } P \\ -P, \text{ appear with probability } (1 - P) \end{cases}$$



Stationary wave can be written as (statistical mean)

$$v(t) = \sum_{n=-\infty}^{\infty} \left[Pg_1(t - nT_b) + (1 - P)g_2(t - nT_b) \right]$$

It is a periodic signal with period Tb, so it can be expanded as a Fourier series:

$$v(t) = \sum_{m = -\infty}^{\infty} C_m e^{j2\pi m f_b t} \qquad f_b = \frac{1}{T_b} \qquad G_1(m f_b) = \int_{-\infty}^{\infty} g_1(t) e^{-j2\pi m f_b t} dt$$
 where
$$C_m = \frac{1}{T_b} \int_{-T_s/2}^{T_s/2} v(t) e^{-j2\pi m f_b t} dt = f_b \left[PG_1\left(m f_b\right) + (1 - P)G_2\left(m f_b\right) \right] \qquad G_2\left(m f_b\right) = \int_{-\infty}^{\infty} g_2(t) e^{-j2\pi m f_b t} dt$$

Therefor, the power density of the stationary wave is:

$$P_{v}(f) = \sum_{m=-\infty}^{\infty} \left| C_{m} \right|^{2} \delta \left(f - m f_{b} \right) = \sum_{m=-\infty}^{\infty} \left| f_{b} \left[P G_{1} \left(m f_{b} \right) + (1 - P) G_{2} \left(m f_{b} \right) \right] \right|^{2} \delta \left(f - m f_{b} \right)$$

Alternating wave can be written as a truncated signal:

$$u_{\rm T}(t) = \sum_{n=-N}^{N} u_n(t) = \sum_{n=-N}^{N} a_n \left[g_1(t - nT_b) - g_2(t - nT_b) \right]$$

Obtain its frequency spectral density:

$$U_{T}(f) = \int_{-\infty}^{\infty} u_{T}(t) e^{-j2\pi f t} dt = \sum_{n=-N}^{N} a_{n} e^{-j2\pi f n T_{b}} \left[G_{1}(f) - G_{2}(f) \right]$$

$$|U_{T}(f)|^{2} = \sum_{m=-N}^{N} \sum_{n=-N}^{N} a_{m} a_{n} e^{i2\pi f (n-m)T_{b}} \left[G_{1}(f) - G_{2}(f) \right] \left[G_{1}(f) - G_{2}(f) \right]^{*}$$

$$E\left[a_{m}^{2} \right] = P(1-P)$$

$$E\left[a_{m} a_{n} \right] = 0$$

$$E[|U_{\mathrm{T}}(f)|^{2}] = \sum_{n=-N}^{N} E[a_{n}^{2}]|G_{1}(f) - G_{2}(f)|^{2} = (2N+1)P(1-P)|G_{1}(f) - G_{2}(f)|^{2}$$

Therefore, the power density of the alternating wave is:

$$P_{u}(f) = \lim_{T \to \infty} \frac{\mathbf{E} \left[|\mathbf{U}_{T}(f)|^{2} \right]}{T} = \lim_{N \to \infty} \frac{(2N+1)P(1-P) \left| G_{1}(f) - G_{2}(f) \right|^{2}}{(2N+1)T_{b}} = f_{b}P(1-P) \left| G_{1}(f) - G_{2}(f) \right|^{2}$$



General expression of the frequency spectrum of the baseband digital signal

The frequency spectrum of g1 and g2

$$P_b\left(f\right) = P_u(f) + P_v(f) = f_b P\left(1 - P\right) \left[\left[G_1\left(f\right) - G_2\left(f\right) \right] \right]^2 \\ + \sum_{m = -\infty}^{\infty} \left| f_b \left[PG_1\left(mf_b\right) + \left(1 - P\right)G_2\left(mf_b\right) \right] \right|^2 \delta\left(f - mf_b\right)$$
 Discrete spectrum (stationary wave) – determine

Continuous spectrum (alternative wave) whether there is D.C. component or timing information

determine the signal bandwidth

The conclusions drawn from the expression:

- The spectrum Ps(f) contains continuous component and discrete component. The D.C.
 component and timing information can be determined by the discrete spectrum.
- The continuous spectrum always exists. The waveform of g1(t) and g2(t) can't be the same, hence, G1(f) ≠ G2(f). The shape of the spectrum depends on the spectrum of g1(t) and g2(t) and their appearance probabilities P.
- Whether the discrete spectrum exists, it depends on the waveform g1(t) and g2(t) and their probabilities P. In general, it usually exists. However, for bipolar signal (g1(t) = g2(t) = g(t)), whose has equal probabilities (P=1/2), there is no discrete component δ (f mfs).



Example 1: Please determine the spectrum of unipolar NRZ and RZ rectangular pulse

Solution:

For unipolar waveform, assume g1(t) = 0, g2(t) = g(t) and substitute into the general spectrum expression:

$$P_{S}(f) = f_{S}P(1-P)|G_{1}(f) - G_{2}(f)|^{2} + \sum_{m=-\infty}^{\infty} |f_{S}[PG_{1}(mf_{S}) + (1-P)G_{2}(mf_{S})]|^{2} \delta(f - mf_{S})$$

Then obtain:

$$P_{S}(f) = f_{S}P(1-P)|G(f)|^{2} + \sum_{m=-\infty}^{\infty} |f_{S}(1-P)G(mf_{S})|^{2} \delta(f-mf_{S})$$

Assume the equal probability, i.e. P=1/2:

$$P_{S}(f) = \frac{1}{4} f_{S} |G(f)|^{2} + \frac{1}{4} f_{S}^{2} \sum_{m=-\infty}^{\infty} |G(mf_{S})|^{2} \delta(f - mf_{S})$$



Example 1: Please determine the spectrum of unipolar NRZ and RZ rectangular pulse

Solution:
$$P_S(f) = \frac{1}{4} f_S |G(f)|^2 + \frac{1}{4} f_S^2 \sum_{m=-\infty}^{\infty} |G(mf_S)|^2 \delta(f - mf_S)$$

For NRZ:

$$g(t) = \begin{cases} 1, & |t| \le \frac{T_S}{2} \\ 0, & otherwise \end{cases}$$



$$g(t) = \begin{cases} 1, & |t| \le \frac{T_S}{2} \\ 0, & otherwise \end{cases}$$
 FT
$$G(f) = T_S \left(\frac{\sin \pi f T_S}{\pi f T_S}\right) = T_S Sa(\pi f T_S)$$

When f=m fs: if m=0, then G(0)=Ts $Sa(0) \neq 0$

if $m \neq 0$, then G(m fs)=0

Contain D.C. component

No timing information

The spectrum:
$$P_S(f) = \frac{1}{4} f_S T_S^2 \left(\frac{\sin \pi f T_S}{\pi f T_S} \right) + \frac{1}{4} \delta(f) = \frac{T_S}{4} Sa^2 (\pi f T_S) + \frac{1}{4} \delta(f)$$

Example1: Please determine the spectrum of unipolar NRZ and RZ rectangular pulse

Solution:
$$P_S(f) = \frac{1}{4} f_S |G(f)|^2 + \frac{1}{4} f_S^2 \sum_{m=-\infty}^{\infty} |G(mf_S)|^2 \delta(f - mf_S)$$

For RZ (signal duration is half of the symbol duration):

$$g(t) = \begin{cases} 1, & 0 \le t \le \frac{T_S}{2} \\ 0, & otherwise \end{cases}$$
 FT
$$G(f) = \frac{T_S}{2} Sa(\frac{\pi f T_S}{2})$$

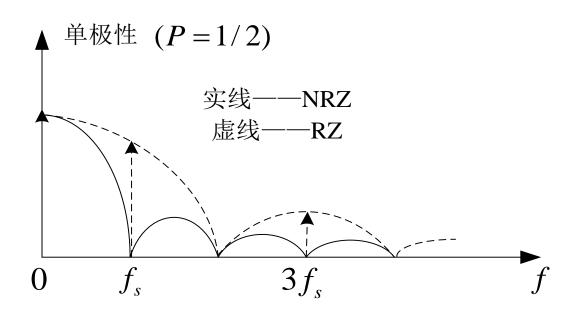
When f=m fs: if m=0, then $G(0) = Ts Sa(0)/2 \neq 0$ Contain D.C. component

if m is odd num,
$$G(mf_S) = \frac{T_S}{2} Sa(\frac{m\pi}{2}) \neq 0$$
 Contain timing information (m=1) if m is even num, $G(mf_S) = \frac{T_S}{2} Sa(\frac{m\pi}{2}) = 0$

The spectrum:
$$P_S(f) = \frac{T_S}{16} Sa^2 (\frac{\pi f T_S}{2}) + \frac{1}{16} \sum_{m=-\infty}^{\infty} Sa^2 (\frac{m\pi}{2}) \delta(f - mf_S)$$



The spectrum density can be plotted like this:





Example 2: Please determine the spectrum of bipolar NRZ and RZ rectangular pulse

Solution:

For unipolar waveform, assume g1(t) = -g2(t) = g(t) and substitute into the general spectrum expression:

$$P_{S}(f) = f_{S}P(1-P)|G_{1}(f) - G_{2}(f)|^{2} + \sum_{m=-\infty}^{\infty} |f_{S}[PG_{1}(mf_{S}) + (1-P)G_{2}(mf_{S})]|^{2} \delta(f - mf_{S})$$

Then obtain:

$$P_{S}(f) = 4f_{S}P(1-P)|G(f)|^{2} + \sum_{m=-\infty}^{\infty} |f_{S}(2P-1)G(mf_{S})|^{2} \delta(f-mf_{S})$$

Assume the equal probability, i.e. P=1/2:

$$P_{S}(f) = f_{S} |G(f)|^{2}$$



Example 2: Please determine the spectrum of bipolar NRZ and RZ rectangular pulse

Solution:

$$P_{S}(f) = f_{S} |G(f)|^{2}$$

For NRZ:
$$g(t) = \begin{cases} 1, & |t| \le \frac{T_S}{2} \\ 0, & otherwise \end{cases}$$
 FT
$$G(f) = T_S \left(\frac{\sin \pi f T_S}{\pi f T_S}\right) = T_S Sa(\pi f T_S)$$

$$G(f) = T_{S} \left(\frac{\sin \pi f T_{S}}{\pi f T_{S}} \right) = T_{S} Sa(\pi f T_{S})$$

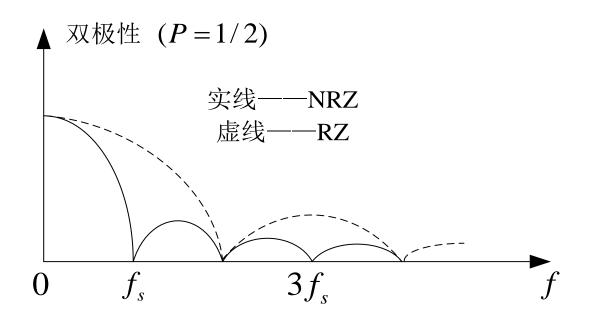
The spectrum:
$$P_S(f) = T_S Sa^2(\pi f T_S)$$

For RZ:
$$g(t) = \begin{cases} 1, & 0 \le t \le \frac{T_S}{2} \\ 0, & otherwise \end{cases}$$
 FT
$$G(f) = \frac{T_S}{2} Sa(\frac{\pi f T_S}{2})$$

FT
$$G(f) = \frac{T_S}{2} Sa(\frac{\pi f T_S}{2})$$

The spectrum:
$$P_S(f) = \frac{T_S}{4} Sa^2 (\frac{\pi}{2} fT_S)$$

The spectrum density can be plotted like this:





It can be seen from the 2 examples:

- The bandwidth of the baseband signal depends on the spectrum G1(f) and G2(f) . The smaller duty ratio (ratio of signal duration and symbol duration) is, the wider spectrum bandwidth is occupied. Calculated from the first zero point, the bandwidth of NRZ(τ = Ts) is $1/\tau$ = fs , the bandwidth of RZ(τ = Ts / 2)is $1/\tau$ = 2fs , where fs = 1/Ts is the frequency of the timing signal, which is the same as the symbol rate.
- 2. Whether a unipolar baseband signal has a discrete line spectrum depends on the duty ratio of the rectangular pulse. There is no timing component in the unipolar NRZ signal; the unipolar RZ signal contains the timing component, it can be extracted directly. Bipolar signals of equal probabilities have no discrete spectrum(no DC components and timing components)



Thank you!

Exercise

Ex1: There is a binary sequence 110010001, please draw the waveform of unipolar NRZ/RZ, bipolar NRZ/RZ and the differential code.

Ex2: There is a binary sequence 1000,0000,0011, please find the AMI code, HDB3 code and the Manchester code.



Exercise

Ex3: Assume that in a random binary sequence, "0" and "1" are represented by g(t) and - g(t) , respectively. g(t) is a rectangular pulse whose height is 1 and the width of it is $\tau = \frac{1}{3}T_s$. The probability of "1" is 1/4, while the probability of "0" is 1/4

- (1) Find the spectrum density of this baseband signal
- (2) Can the frequency component of $f_s = \frac{1}{T_s}$ be extracted directly? If yes, calculate the component.
- (3) Find the symbol rate and the bandwidth of this baseband signal if Ts=0.0001s



MATLAB

Generate a random binary sequence

- (1) plot the unipolar RZ waveform, the bipolar NRZ waveform of the sequence
- (2) Write a function to generate the differential code and HDB3 code.
- (3) Assume that we use triangular pulse to represent 1 while a zero level to represent 0, where 1 and 0 appear with equal probabilities, find the theoretic power density and try to plot the curve.

