Principles of Communications

Chapter 4 — Digitization of Analog Signal

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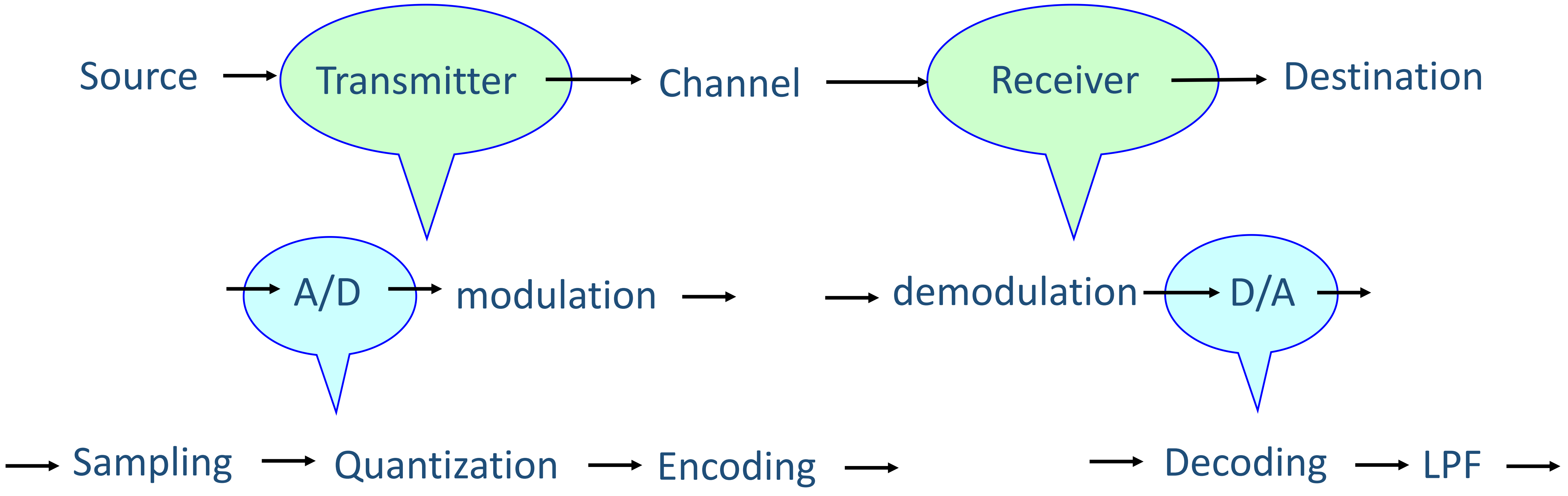
Review

The framework of digital communication system



Review

ADC includes 3 steps, including sampling, quantization and encoding

Sampling：Analog signal with continuous time and continuous value

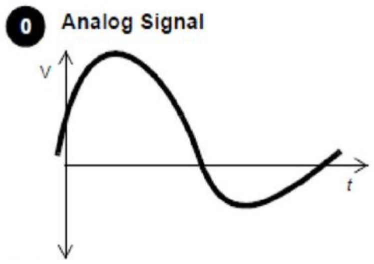
Time-discrete, value-continuous signal

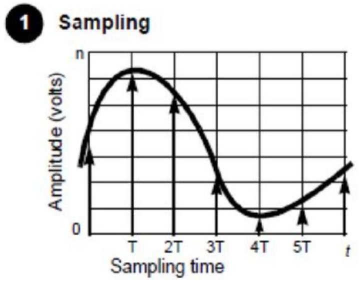
Quantization：Time-discrete, value-continuous signal

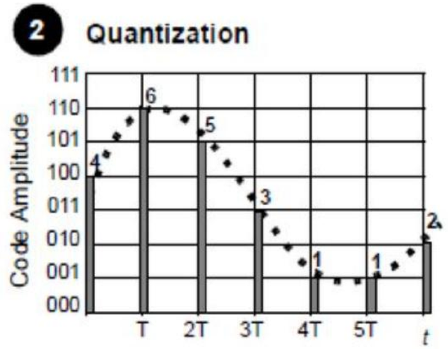
Time-discrete, value-discrete multilevel signal

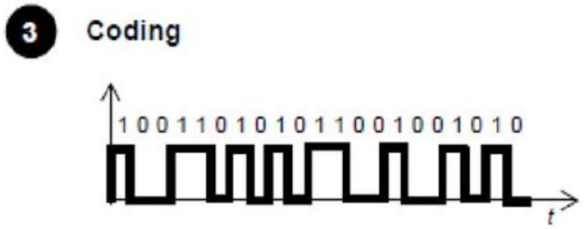
Coding: time-discrete, value-discrete multi-level signal

binary PCM signal



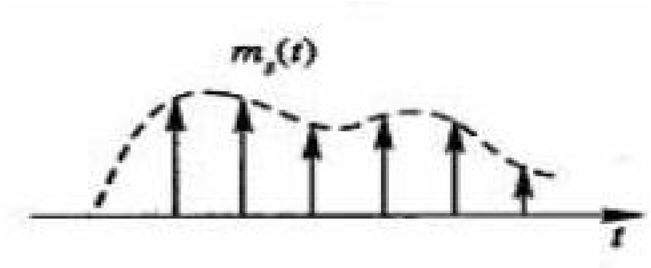






Review

Sampling：Analog signal with continuous time → Analog signal with discrete time



$$m_s(t) = m(t)\delta_T(t)$$

Sampling theorem:

1、Low-pass: if the sampling rate fs satisfies the condition fs>=2fh, then m(t) can be determined completely by these samples

2、Band-pass: for a band-pass signal with high frequency, the minimum sampling rate for this signal can be approximated as 2B.

Review

Quantization：Value-continuous signal → Value-discrete signal

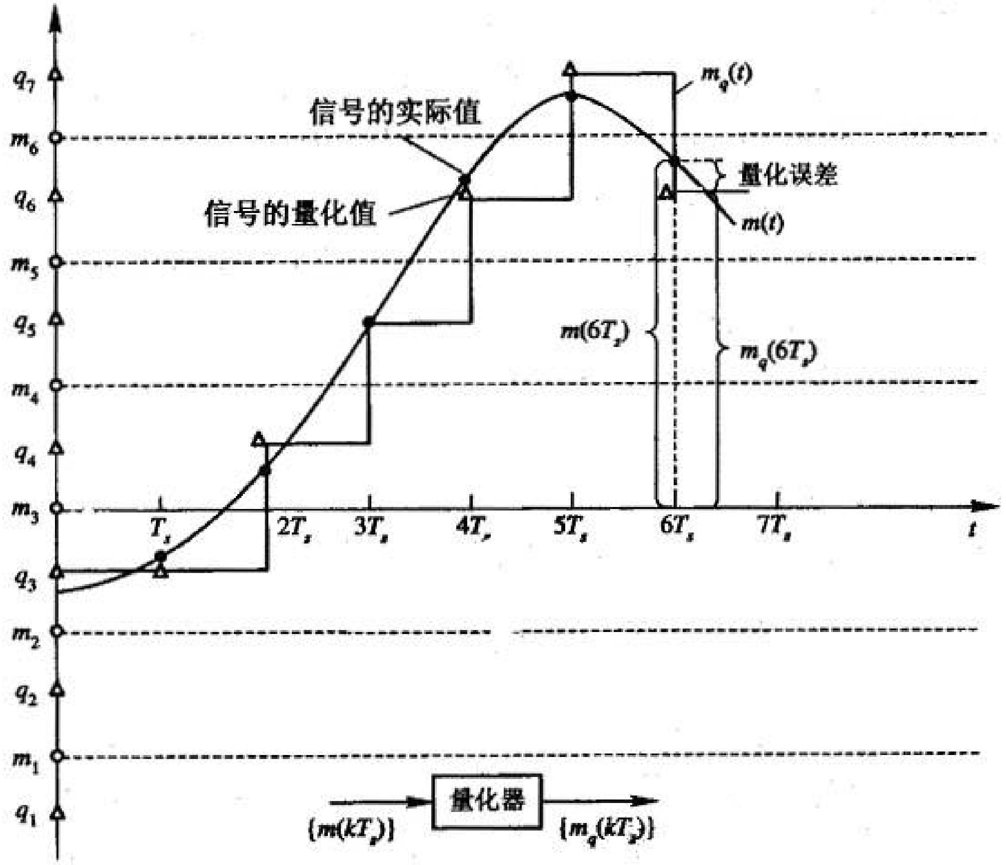
1、Uniform quantization (uniform quantization level and interval)

↓

not good for small signals

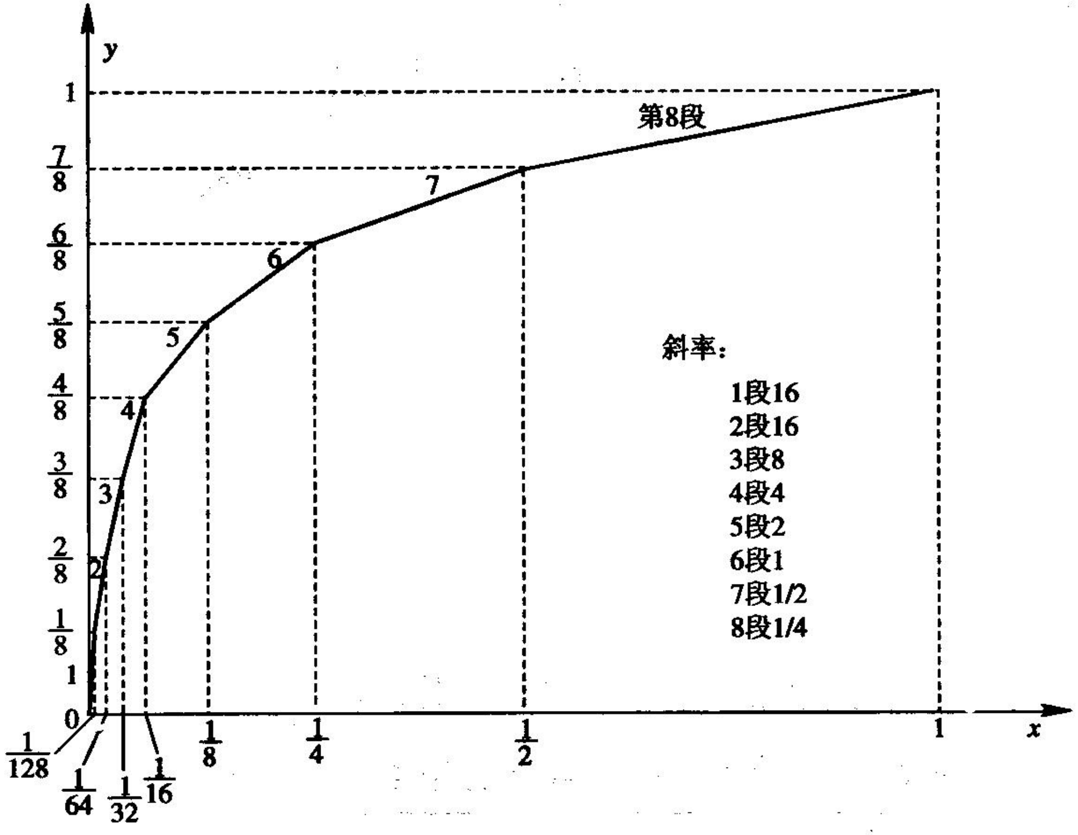
↓

2、Nonuniform quantization (Quantization steps vary)

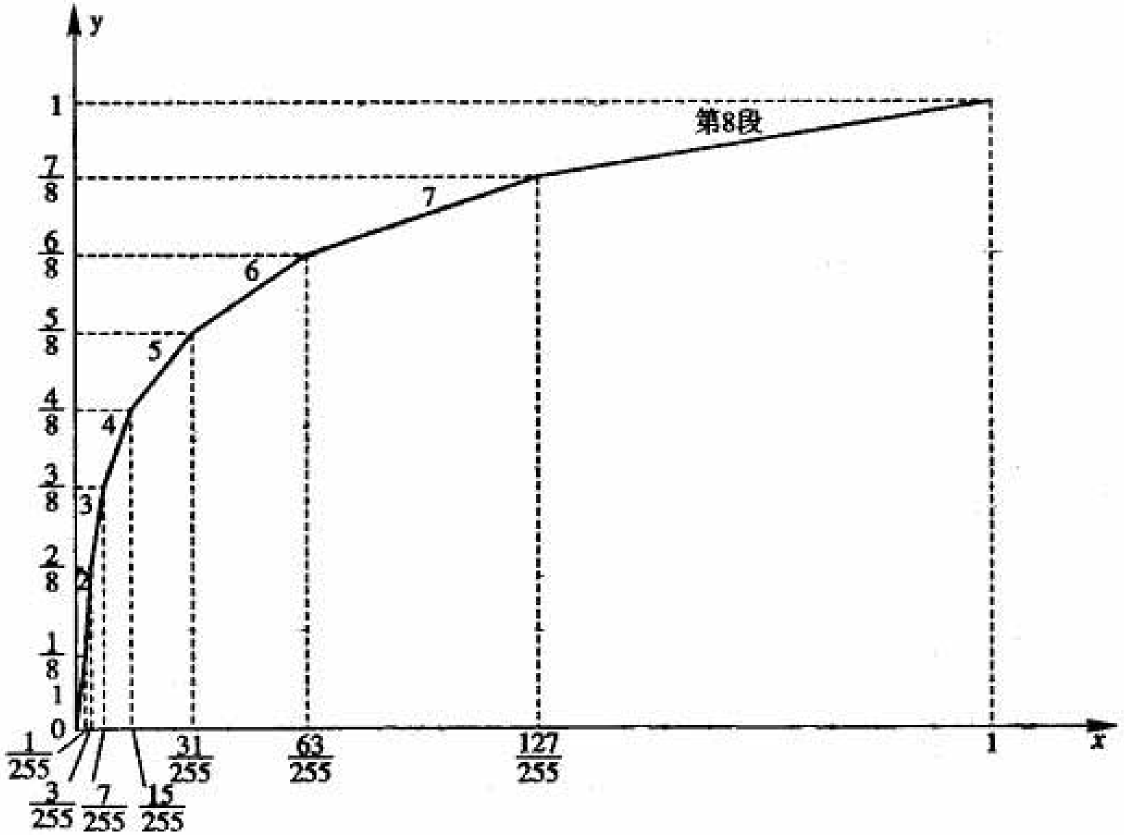


Review

Quantization: Nonuniform Quantization



A-law 13 broken lines



μ-law 15 broken lines

Encoding and Decoding

Basic principles of Pulse Code Modulation (PCM)

Encoding: time-discrete, value-discrete multi-level signal → Binary signal

One quantized value → Codeword (0-1 sequence)

Why we need encoding?

Compared with multi-level signal transmission, binary signal has the properties of anti-interference and easy to generate

Commonly used encoding: PCM

Encoding and Decoding

Codeword

Three often used codes to encode the quantized voltage:

- Natural binary codeword
  - Encoding based on natural binary
- Fold binary codeword
  - The encoding of positive and negative polarity has image relationship
- Gray binary codeword
  - The encoding that one bit changes state from one position to another

Encoding and Decoding

Codeword

Natural binary code

The first bit: represent the polarity

- 0: Negative Polarity
- 1: Positive Polarity

Quantization levels are binary coded in ascending order

(small level has small codeword)

Advantage: Easy to encode and decode, the circuit is simple

Negative Polarity	Positive Polarity
Natural binary codeword	Natural binary codeword
0000 (0)	1000 (8)
0001 (1)	1001 (9)
0010 (2)	1010 (10)
0011 (3)	1011 (11)
0100 (4)	1100 (12)
0101 (5)	1101 (13)
0110 (6)	1110 (14)
0111 (7)	1111 (15)

Encoding and Decoding

Codeword

Fold binary code

The first bit: represent the polarity

- 0: Negative Polarity
- 1: Positive Polarity

After deciding the polarity, quantization levels are binary coded in ascending order regardless of the polarity (the absolute small level has small codeword)

Negative Polarity	Positive Polarity
Fold binary codeword	Fold binary codeword
0000 (7)	1000 (8)
0001 (6)	1001 (9)
0010 (5)	1010 (10)
0011 (4)	1011 (11)
0100 (3)	1100 (12)
0101 (2)	1101 (13)
0110 (1)	1110 (14)
0111 (0)	1111 (15)

Encoding and Decoding

Codeword

The advantage of fold code over natural code:

1、bipolarity voltage can be processed by unipolarity voltage encoding method (the circuit and process are simplified.)

2、Codeword error has less influence on small voltage.

Example:

Codeword: 0000
- Natural: 0^{th} quantization level
- Fold: 7^{th} quantization level

→

Error Codeword: 1000
- Natural: 8^{th} quantization level
- Fold: 8^{th} quantization level

Encoding and Decoding

Codeword

Gray binary code

The first bit: represent the polarity

0: Negative Polarity

1: Positive Polarity

The rest bits: only one bit changes state from one position to another

Advantage: less error in digital communications

Negative Polarity	Positive Polarity
Gray binary codeword	Gray binary codeword
0000 (0)	1100 (8)
0001 (1)	1101 (9)
0011 (2)	1111 (10)
0010 (3)	1110 (11)
0110 (4)	1010 (12)
0111 (5)	1011 (13)
0101 (6)	1001 (14)
0100 (7)	1000 (15)

Encoding and Decoding

Encoding and Decoding in PCM

More bits in a codeword increase signal quantization noise ratio → Conflict ← More bits means more transmission and storage quantities.

↓

Tradeoff

↓

In speech communication, PCM codeword with 8 bits (ensure satisfactory communication quality.)

Encoding and Decoding

Encoding and Decoding in PCM

PCM codeword with 8 bits based on A-law 13 broken lines

Quantization levels number:

$$M = 2^8 = 256 \begin{cases} \text{128 quantization levels in the positive polarity} \\ \text{128 quantization levels in the negative polarity} \end{cases}$$

In the positive (negative) polarity the quantization levels are determined by the 13 broken line

(There are 8 segments in a unipolarity)

In each segment, the segment is evenly divided into 16 quantization levels.

The length of each segment is uneven, the 8 segments of the positive or negative input are divided into

$8 \times 16 = 128$ uneven quantization levels.

Encoding and Decoding

Encoding and Decoding in PCM

Polarity bit	Segment bits	Inner Segment bits
C1	C2 C3 C4	C5 C6 C7 C8

Polarity bit : Determine the signal is positive or negative

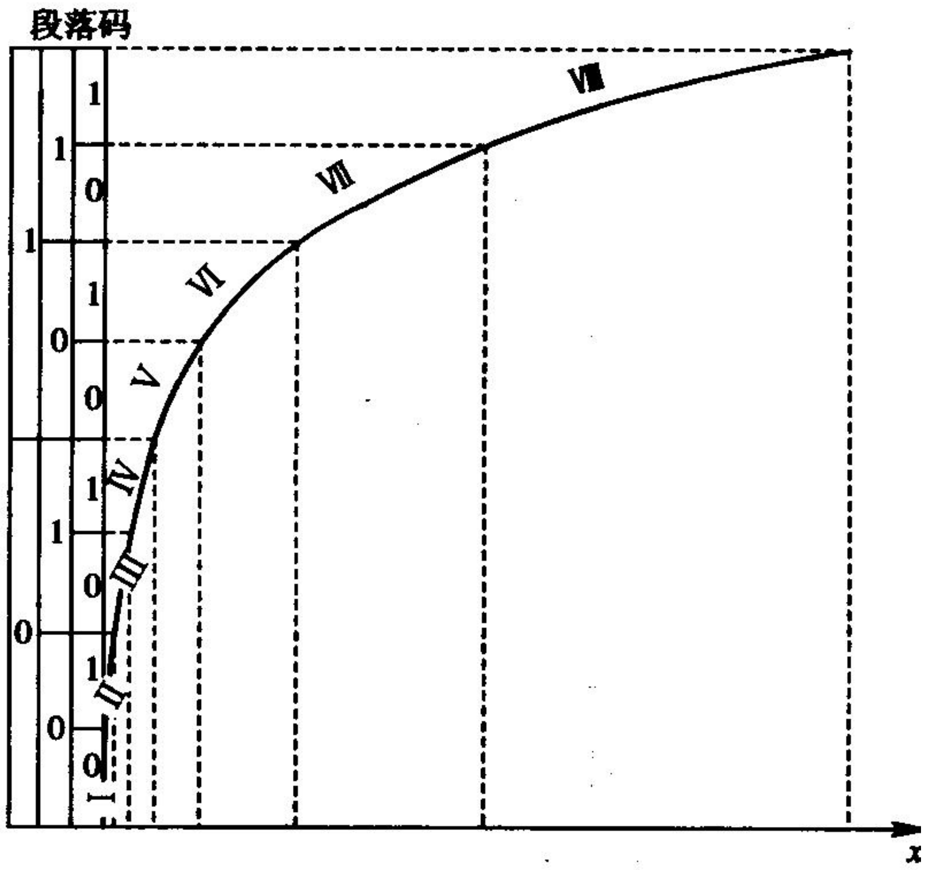
$$C_1 = \begin{cases} 1, & \text{positive} \\ 0, & \text{negative} \end{cases}$$

Segment bits : Determine the signal is in which segment of 13 broken line in unipolarity

Inner-Segment bits : Determine the signal is in which sub-segment in a segment

Encoding and Decoding

Encoding and Decoding in PCM



Index	Segment bits
	$C_2\ C_3\ C_4$
8	1 1 1
7	1 1 0
6	1 0 1
5	1 0 0
4	0 1 1
3	0 1 0
2	0 0 1
1	0 0 0

Encoding and Decoding

Encoding and Decoding in PCM

In each segment: the segment is divided uniformly into 16 quantization levels.

电平序号	段内码 $C_5C_6C_7C_8$	电平序号	段内码 $C_5C_6C_7C_8$
15	1 1 1 1	7	0 1 1 1
14	1 1 1 0	6	0 1 1 0
13	1 1 0 1	5	0 1 0 1
12	1 1 0 0	4	0 1 0 0
11	1 0 1 1	3	0 0 1 1
10	1 0 1 0	2	0 0 1 0
9	1 0 0 1	1	0 0 0 1
8	1 0 0 0	0	0 0 0 0

Encoding and Decoding

Encoding and Decoding in PCM

量化段序号 $i=1\sim8$	电平范围 (Δ)	段落码 C_1	C_2	C_3	段落起始电平 $I_{si}(\Delta)$	量化间隔 $\Delta_i(\Delta)$	段内码对应权值/Δ C_5	C_6	C_7	C_8
8	1024～2048	1	1	1	1024	64	512	256	128	64
7	512～1024	1	1	0	512	32	256	128	64	32
6	256～512	1	0	1	256	16	128	64	32	16
5	128～256	1	0	0	128	8	64	32	16	8
4	64～128	0	1	1	64	4	32	16	8	4
3	32～64	0	1	0	32	2	16	8	4	2
2	16～32	0	0	1	16	1	8	4	2	1
1	0～16	0	0	0	0	1	8	4	2	1

Encoding and Decoding

Encoding and Decoding in PCM

How to encode the sampled signal to a 8-bit sequence?

1. Determine the polarity of the sampled value to determine the polarity bit C1
2. Determine the sampled value is in which segment to determine the segment bit C2 C3 C4
3. Determine the sampled value is in which sub-segment in the segment to determine the inner-segment bit C5 C6 C7 C8

Encoding and Decoding

Example: The signal sampling value is -361Δ, try to encode with 8 bits according to the A-law 13 broken line

Solution: It has negative polarity: $C_1 = 0$

The segment bits: $256\Delta < 361\Delta < 512\Delta$

→ The sampling value is in the 6th segment

→ 6th segment bits: $C_2C_3C_4 = 101$

Encoding and Decoding

Example: The signal sampling value is -316Δ, try to encode with 8 bits according to the A-law 13 broken line

Solution:

Inner-segment bits: In 6th segment, the segment is divided into 16 sub-segment:

For each sub-segment, the interval is $\dfrac{512\Delta - 256\Delta}{16} = 16\Delta$

Quantization value is in the 4th sub-segment: $\dfrac{316\Delta - 256\Delta}{16\Delta} = 3 \cdots 12\Delta$

Inner segment bits: $C_5C_6C_7C_8 = 0011$

The final code: $C_1C_2C_3C_4C_5C_6C_7C_8 = 01010011$

Quantization error: 12Δ

Encoding and Decoding

Encoding and Decoding in PCM

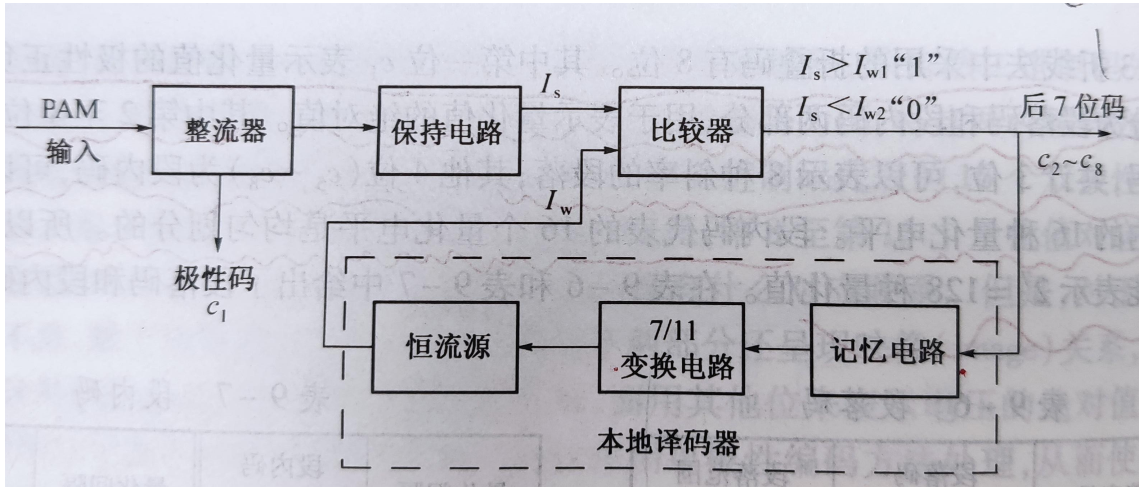
7/11 transformation

successive comparison coding

Transform the 7 bit non-linear

codes into 11 bit linear code

to control the current



Encoding and Decoding

Encoding and Decoding in PCM

7/11 transformation

Obtain the sample and encoded into 7 bit non-linear codes after the PCM

The quantization value can also be represents by 11 bit linear code.

Ex: 316Δ will be quantized as 304Δ, which can be written as a 7 bit non-linear codes

Meanwhile, $304 = 256 + 32 + 16 = 2^8 + 2^5 + 2^4$

which can be written into a 11-bit code, such that 00100110000

Encoding and Decoding

Encoding and Decoding in PCM

7/11 transformation

The weights corresponding to the 11-bit linear codes b11~b1 are 1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1

1. Segment code → where the segment is located → the starting point level of the segment → the corresponding bi = 1；

2. Write the inner-segment code in the back of bi, and the rest of the bits are 0 。

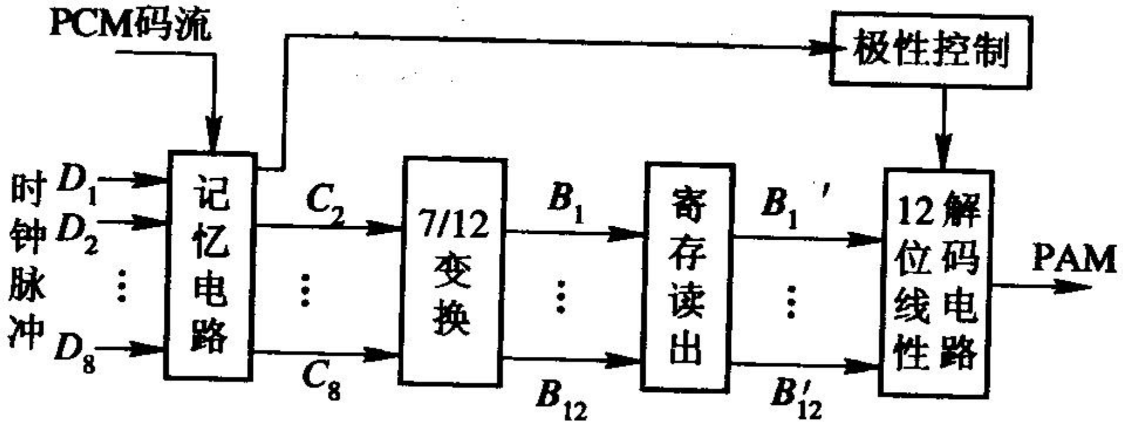
Encoding and Decoding

Encoding and Decoding in PCM

7/12 transformation

In the decoder, user 7/12 transformation to control the current.

By adding half of the quantization interval to the original quantization, the decoding quantization error will be decreased



Encoding and Decoding

Encoding and Decoding in PCM

7/12 transformation

Ex: 316Δ will be quantized as 304Δ, the quantization error is 12Δ

In the decoder, to reduce the quantization error, the quantization value is viewed as 312Δ

Then the quantization error is reduced from 12Δ to 4Δ

In this way, the quantization error will not exceed half of the quantization interval

Encoding and Decoding

Encoding and Decoding in PCM

The corresponding weights of 12-bit linear codes (b11~b0) are 1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1, 0.5

1、First convert C2~C8 of the 7-bit nonlinear code into b11~b1 (same as 7/11 conversion)

2、set the bit behind the inner-segment code to be 1.

Encoding and Decoding

Encoding and Decoding in PCM

表 6－8　A 律 13 折线非线性码与线性码间的关系

段落序号	非线性码(幅度码)								线性码(幅度码)											
	起始电平(Δ)	段落码			段内码的权值				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}^*
		C_2	C_3	C_4	C_5	C_6	C_7	C_8	1024	512	256	128	64	32	16	8	4	2	1	$\Delta V/2$
8	1024	1	1	1	512	256	128	64	1	C_5	C_6	C_7	C_8	1 *	0	0	0	0	0	0
7	512	1	1	0	256	128	64	32	0	1	C_5	C_6	C_7	C_8	1 *	0	0	0	0	0
6	256	1	0	1	128	64	32	16	0	0	1	C_5	C_6	C_7	C_8	1 *	0	0	0	0
5	128	1	0	0	64	32	16	8	0	0	0	1	C_5	C_6	C_7	C_8	1 *	0	0	0
4	64	0	1	1	32	16	8	4	0	0	0	0	1	C_5	C_6	C_7	C_8	1 *	0	0
3	32	0	1	0	16	8	4	2	0	0	0	0	0	1	C_5	C_6	C_7	C_8	1 *	0
2	16	0	0	1	8	4	2	1	0	0	0	0	0	0	1	C_5	C_6	C_7	C_8	1 *
1	0	0	0	0	8	4	2	1	0	0	0	0	0	0	0	C_5	C_6	C_7	C_8	1 *

注：① $C_5 \sim C_8$ 码以及 $B_1 \sim B_{12}$ 码下面的数值为该码的权值。

② B_{12}^* 和 1^* 项为收端解码时 $\Delta V/2$ 补差项，此表用于编码时，没有 B_{12}^* 项，且 1^* 项为零。

Encoding and Decoding

Example: If the nonlinear code is 0111011 (no polarity), please determine the corresponding 11-bit linear code

Solution: The segment bits: 011

→ The 4th segment, starting point of the segment is 64

→ $B_5 = 1$

Inner segment bits: 1011

→ $B_6 B_7 B_8 B_9 = 1011$

Zero padding for the rest: $B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8 B_9 B_{10} B_{11} = 00001101100$

Encoding and Decoding

Example: the signal sample value is 183 and is encoded with PCM using A-law 13 broken line

(1) Write the corresponding 8-bit codeword

(2) Write the transformation result of 7/11 and 7/12

(3) Determine the quantization value of 7/11, 7/12 and their quantization error

Solution: (1) It has positive polarity: $C_1 = 1$

The segment bits: $128 < 183 < 256$ the 5th segment

→ 5th segment bits: $C_2C_3C_4 = 100$

Inner segment : $\frac{183-128}{8} = 6 \cdots 7$ → 7th segment bits: $C_5C_6C_7C_8 = 0110$

8-bit codeword $C_1C_2C_3C_4C_5C_6C_7C_8 = 11000110$

Encoding and Decoding

Example: the signal sample value is 183 and is encoded with PCM using A-law 13 broken line

(1) Write the corresponding 8-bit codeword

(2) Write the transformation result of 7/11 and 7/12

(3) Determine the quantization value of 7/11, 7/12 and their quantization error

Solution: (2) 7/11: $C_2C_3C_4C_5C_6C_7C_8 = 1000110$

the 5th segment $B_4 = 1$

Inner segment bits: $B_5B_6B_7B_8 = 0110$

Zero-padding: $B_1B_2B_3B_4B_5B_6B_7B_8B_9B_{10}B_{11} = 00010110000$

7/12: $B_9 = 1$ $\Rightarrow$ $B_1B_2B_3B_4B_5B_6B_7B_8B_9B_{10}B_{11}B_{12} = 000101101000$

Encoding and Decoding

Example: the signal sample value is 183 and is encoded with PCM using A-law 13 broken line

(1) Write the corresponding 8-bit codeword

(2) Write the transformation result of 7/11 and 7/12

(3) Determine the quantization value of 7/11, 7/12 and their quantization error

Solution: (3) 7/11 quantization value: $128+6\times 8=176$

Quantization error: $183-176=7$

7/12 quantization value: $128+6\times 8+4=180$

Quantization error: $183-180=3$

Encoding and Decoding

Quantization Noise in PCM system

When uniform quantization is considered, the average signal quantization noise power ratio at the output is

$$\frac{S_o}{n_q} = \frac{E\left[m_o^2(kT_s)\right]}{E[n_q^2(t)]} = M^2 = 2^{2k}$$

For a low-pass signal (frequency band is fH), sampling rate no less than 2fH. The symbol rate for PCM is 2*N*fH (bit/s), so the required system bandwidth is B=N*fH (discuss in the next chapter), thus, the signal quantization noise power ratio becomes:

$$\frac{S_o}{n_q} = 2^{2(B/f_H)}$$

Encoding and Decoding

Differential PCM (DPCM)

The source signal is sampled with a certain frequency

↓

a strong correlation between adjacent sampled values

↓

the difference between adjacent sampled values can be encoded instead of the sampled value itself

↓

the number of encoding bits can be significantly reduced when the quantization step remains unchanged

↓

the signal bandwidth is compressed.

Encoding and Decoding

Differential PCM (DPCM)

Let x_n be the sample value of the source at the current moment, $\tilde{x}_n$ be the predicted value of x_n, which is a weighted linear combination of the past sample values, defined as

$$\tilde{x}_n = \sum_{i=1}^{k} a_i x_{n-i}$$

a_i are predictive parameters.

The error between the prediction and the true symbol

$$e_n = x_n - \tilde{x}_n = x_n - \sum_{i=1}^{k} a_i x_{n-i}$$

Find a_i to minimize the error

In the receiver end, $$x_n = e_n + \sum_{i=1}^{k} a_i x_{n-i}$$

Encoding and Decoding

Adaptive DPCM (ADPCM)

ADPCM can adaptively change the quantization level and prediction parameters, where the output signal-to-noise ratio can be greatly improved.

1、Adaptive differential pulse code modulation can achieve 64kb/s PCM digital telephony quality at a bit rate of 32kb/s 。

2、ADPCM has become an international general voice coding method in long-distance transmission.

3、ADPCM is developed on the basis of differential pulse code modulation (DPCM).

Encoding and Decoding

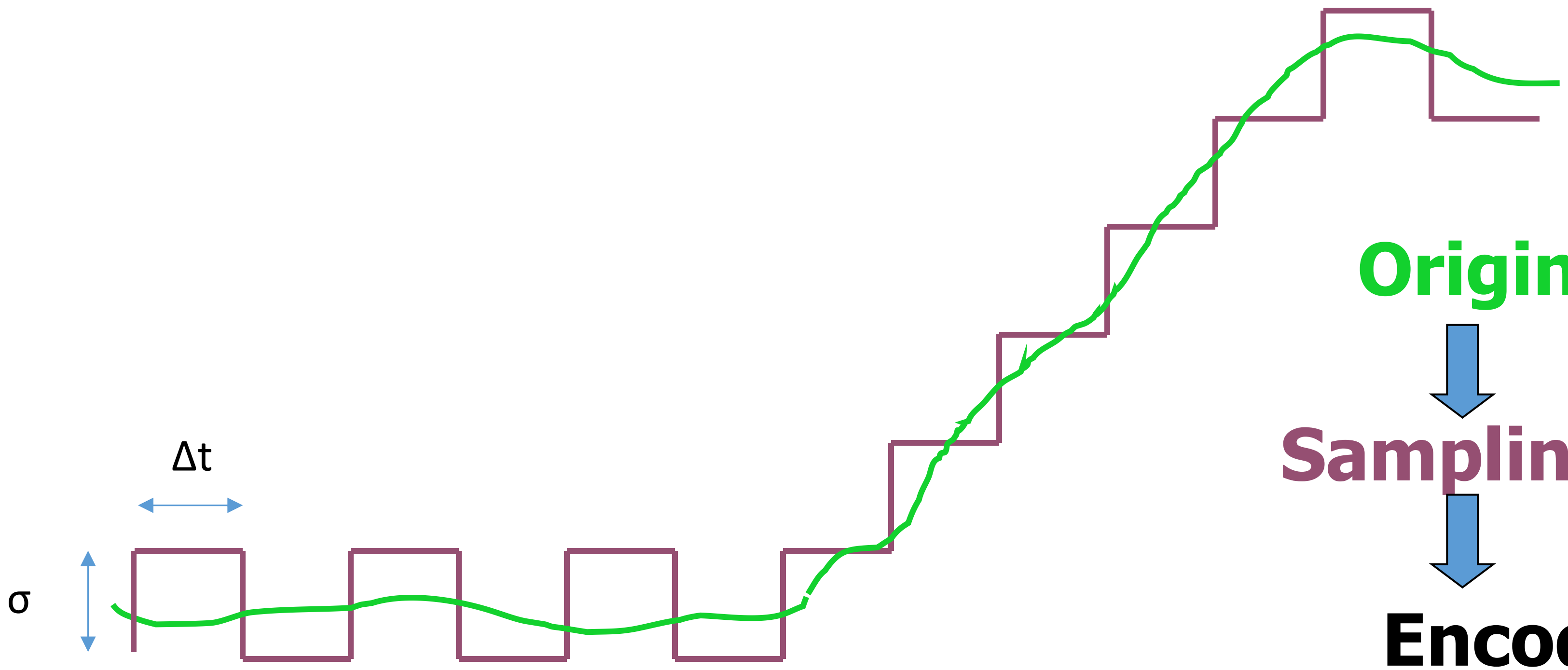
Delta Modulation (ΔM)

ΔM is another analog signal digital transmission method developed on the basis of the PCM method, which can be regarded as a special case of DPCM

Different from the PCM method, ΔM converts the analog signal into a digital signal sequence composed of only one binary code to represent the relative size of adjacent signals, and reflects the change law of analog signals through the relative changes of adjacent sampled values.

Encoding and Decoding

Delta Modulation (ΔM)



Original signal

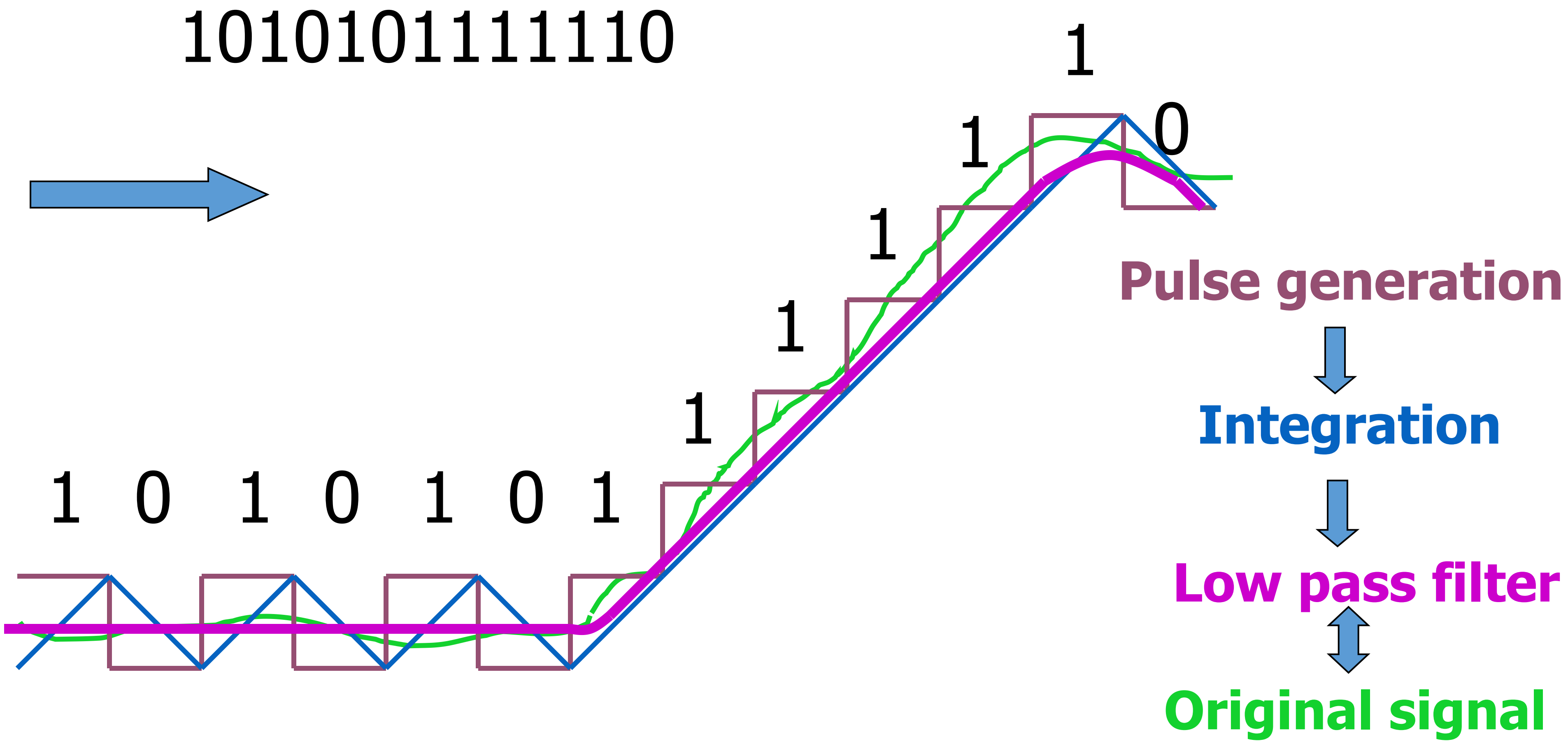
Sampling and quantization

Encoding

1010101111110

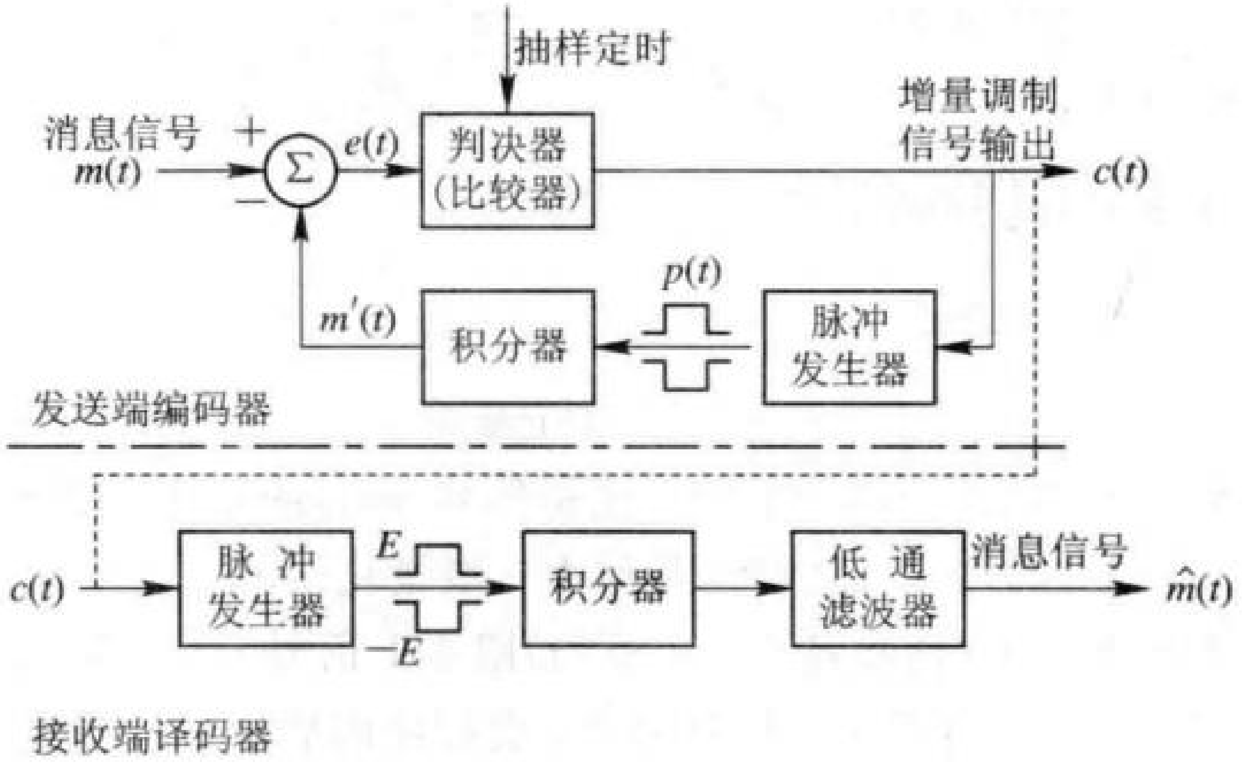
Encoding and Decoding

Delta Modulation (ΔM)



Encoding and Decoding

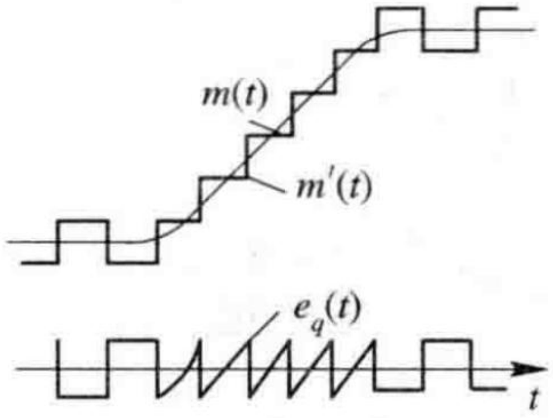
Delta Modulation (ΔM)



Encoding and Decoding

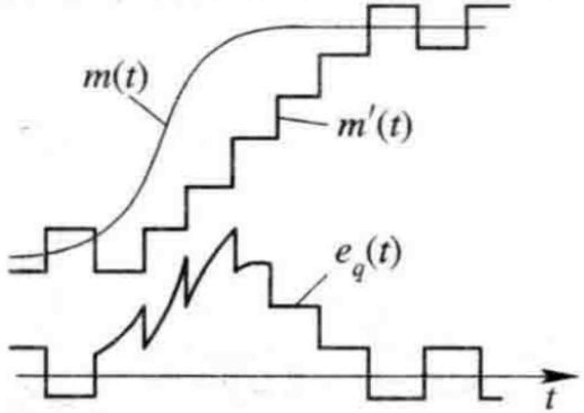
Delta Modulation (ΔM)

Basic quantization noise



The same as the quantization noise as PCM (the error between quantization value and the sample)

Overload quantization noise



The signal slope is too large, which exceed the increasing Δ.

Encoding and Decoding

Delta Modulation (ΔM)

Define the slope of a step: $K = \dfrac{\sigma}{\Delta t} = \sigma \cdot f_s$

Guarantee no overload noise: $\left|\dfrac{\mathrm{d}m(t)}{\mathrm{d}t}\right|_{\max} \leqslant \sigma \cdot f_s$

Assume a signal is $m(t) = A\sin\omega_k t$ The slope of the signal: $\dfrac{\mathrm{d}m(t)}{\mathrm{d}t} = A\omega_k \cos\omega_k t$

no overload noise condition: $A\omega_k \leqslant \sigma \cdot f_s$

The maximum amplitude requirement: $A_{\max} = \dfrac{\sigma \cdot f_f}{\omega_k} = \dfrac{\sigma \cdot f_s}{2\pi f_k}$

The minimum amplitude requirement (if not satisfied, the variations of the signal can't be represented by the ΔM signal): $A_{\min} = \dfrac{\sigma}{2}$

Encoding and Decoding

Delta Modulation (ΔM)

The average power of quantization noise: $$E\left[e_q^2(t)\right] = \int_{-\sigma}^{\sigma} \frac{e^2}{2\sigma} \mathrm{d}e = \frac{\sigma^2}{3}$$

The power spectral of quantization noise: $$P(f) \approx \frac{E\left[e_q^2(t)\right]}{f_s} = \frac{\sigma^2}{3 f_s}$$

The power of quantization noise after the LPF: $$N_q = P(f) \cdot f_m = \frac{\sigma^2 f_m}{3 f_s}$$

The power of signal without overload noise: $$S_0 = \frac{A_{\max}^2}{2} = \frac{\sigma^2 f_s^2}{8\pi^2 f_k^2}$$

The signal to quantization noise ratio: $$\frac{S_o}{N_q} = \frac{3}{8\pi^2} \cdot \frac{f_s^3}{f_k^2 f_m} \approx 0.04 \frac{f_s^3}{f_k^2 f_m}$$

Thank you!

Exercise

Answer briefly:

(1) Discuss the relationship and difference between PCM, DPCM and delta modulation?

(2) In ΔM system, if the signal is a constant, what is the output of the encoder?

(3) What is the advantage of ΔM system compared to PCM?

(4) Why is fold code often used for coding in PCM system?

(5) What is there is a 7/11 transformation? Why we need 7/12 transformation in the receiver instead of 7/11

(6) Draw a block diagram of the PCM system, and qualitatively draw the waveforms of each point in the diagram. Briefly explain the function of each part in the figure.

Exercise

Ex: Adopt 13 broken line A-law coding, set the minimum quantization level as Δ, and the known sampling pulse value is +635 Δ

(1) find the 8 bits non-linear PCM code.

(2) find the corresponding 11 bit linear code and 12 bit code.

(3) What is the quantization error of the 11 bit linear code and the 12 bit code.

MATLAB

1、Read the PCM code and write the explanatory note for the code (the note rate should not be less than 50%)

2、Use the PCM code to encode an audio signal (only need to process the first channel). Plot the encoded result of the first and the last 100 samples in 2 figures (use stem plot)

3、Write a function of 7/11 transformation based on the PCM. (Give the 11-bit encoded results of the first 10 samples.)

MATLAB

Recommendation reading and coding: ΔM modulation and practices

https://blog.csdn.net/weixin_41476562/article/details/106314308