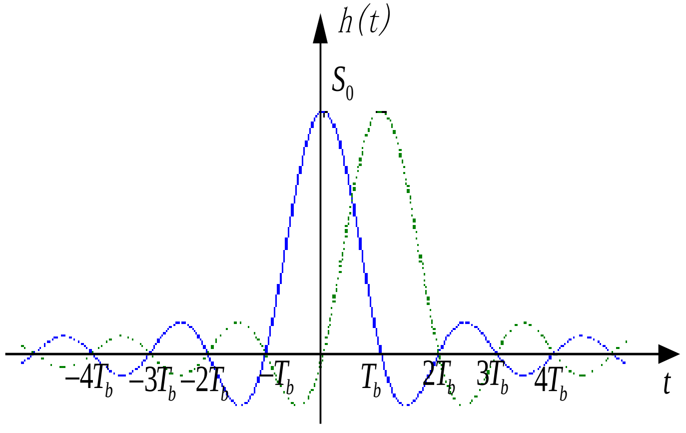
Principles of Communications

Chapter 5 — Presentation and Transmission of Baseband Signal

Zhen Chen

Review

Partial Response System



(b)

The received signal symbol: $$C_k = a_k + a_{k-1}$$

Symbol recovery: $$a_k = C_k - a_{k-1}$$

Error propagation phenomenon: If ak-1 is wrong, the following symbol decisions from ak will all be wrong

Solution to error propagation phenomenon: Precoding

Get rid of the correlation coding influence

Review

Optimal receiver

Optimal receiver

Matched filter: Aim to maximize the SNR of receiving signal

Frequency domain expression

$$H(\omega) = KS^*(\omega)e^{-j\omega t_0}$$

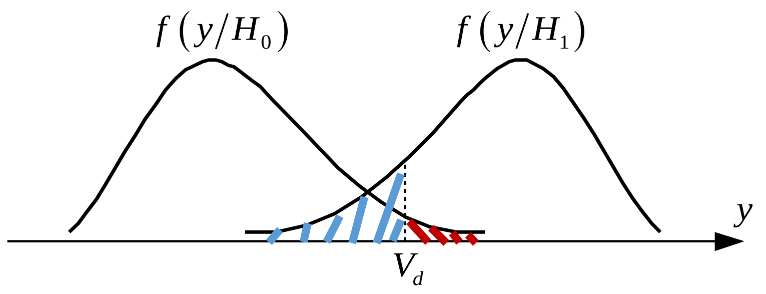
Time domain expression

$$h(t) = Ks(t_0 - t)$$

Performance limitation

$$r_0 \le \frac{2E}{n_0}$$

Optimal detector : Find the decision threshold that has minimal symbol error rate



Optimal baseband transmission: Eliminate the intersymbol interference and have the minimal probability of error

Eye pattern

What's the problem in reality?

- In practice, impossible for the baseband system to fully avoid intersymbol interference

Factors that affect intersymbol interference

transmitter filter

channel

receiver filter

difficult to calculate or analyze the bit error rate caused by these factors

- In practice, it is necessary to qualitatively estimate and adjust the performance of the system via experiments to minimize the impact of intersymbol interference

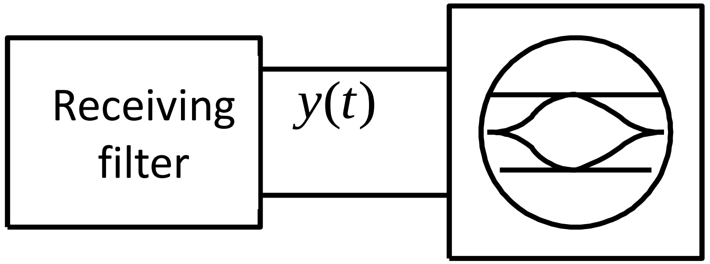
Optimal receiver

What is eye pattern?

Eye pattern provides a good way to qualitatively analyze the sytem performance

The eye pattern is a human eye-like graph observed on an oscilloscope to estimate and improve the performance of a transmission system.

Observe the signal after the transmission system.



Use an oscilloscope to observe the output signal of the system

Optimal receiver

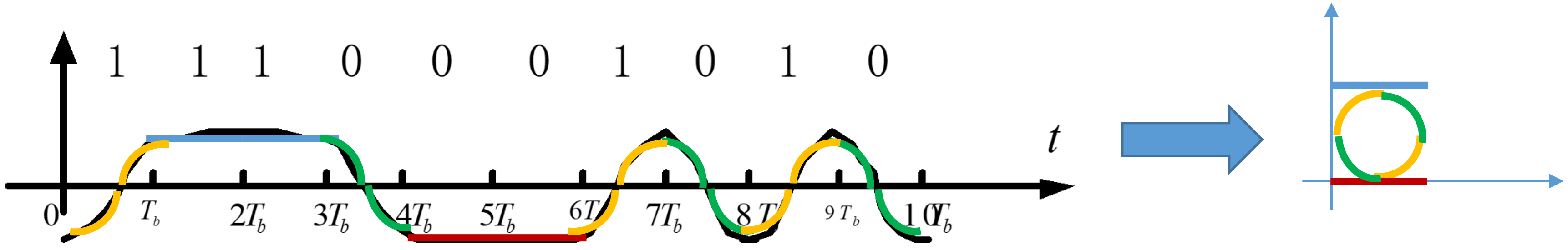
What is special of eye pattern?

- The output signal of the receive filter is added to the vertical axis of the oscilloscope
- The horizontal scanning period of the oscilloscope is synchronized with signal period
- A graph displayed on the oscilloscope helps estimate the system performance (intersymbol interference and noise)
- By observing the eye pattern, the receiving filter can be adjusted to reduce the intersymbol interference and improve the transmission performance of the system.

Intersymbol interference

1 . The eye pattern without noise or intersymbol interference

- Assume the binary code is 1110001010
- Receive continuous “1”: a positive voltage level for time duration of τ
- Receive continuous “0”: a negative voltage level for time duration of τ
- Iterative “1”“0”: voltage level fluctuate between positive level and negative level
- τ=(k–1)Tb, k is the number of the continuous same symbols



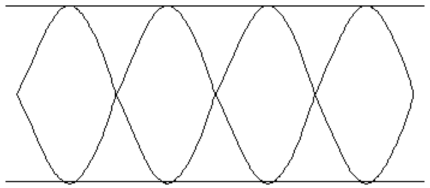
Matched filter

The output signal y(t) of the receive filter is added to the vertical axis of the oscilloscope

Adjust the horizontal scanning period of the oscilloscope to be nTb （n＝4）

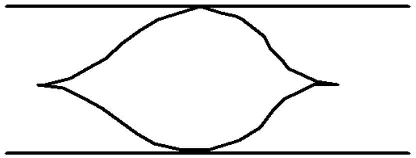
Bipolar raised cosine:

Symbol changes



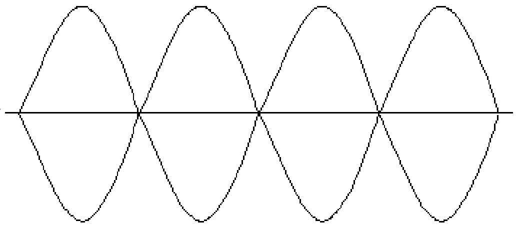
Continuous 1

Continuous 0



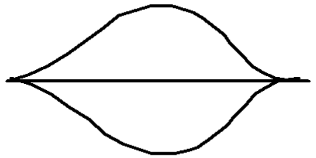
AMI code: iterative +1 and -1 for “1”, 0 for “0”

Caused by “0”



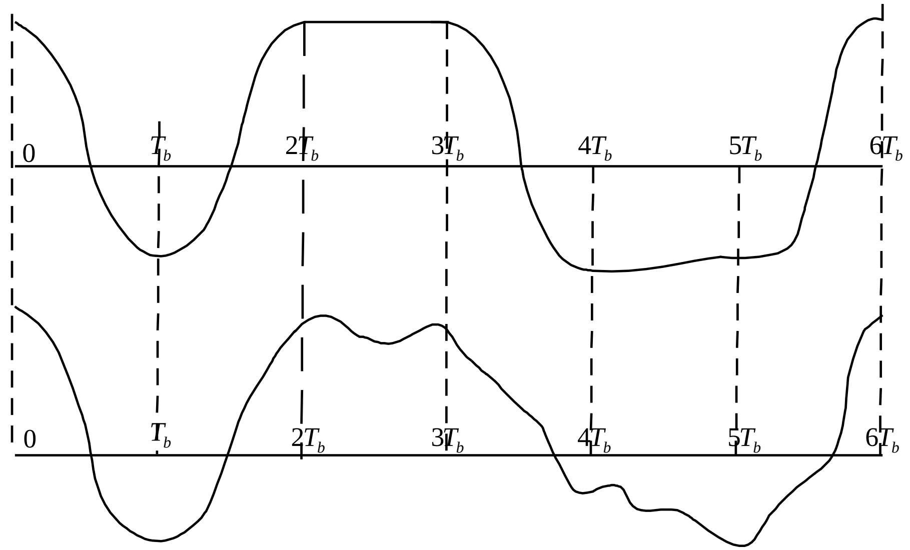
Caused by

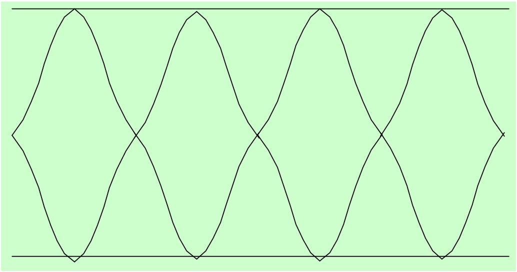
iterative ±1



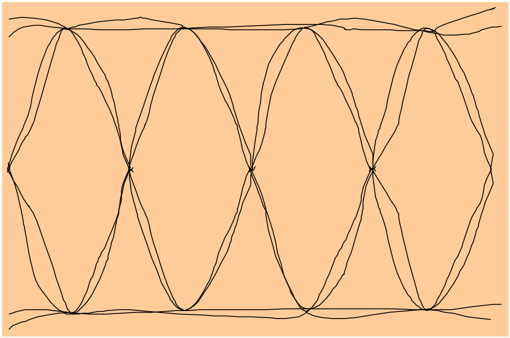
Matched filter

Eye pattern with intersymbol interference but without noise





without intersymbol interference



with intersymbol interference

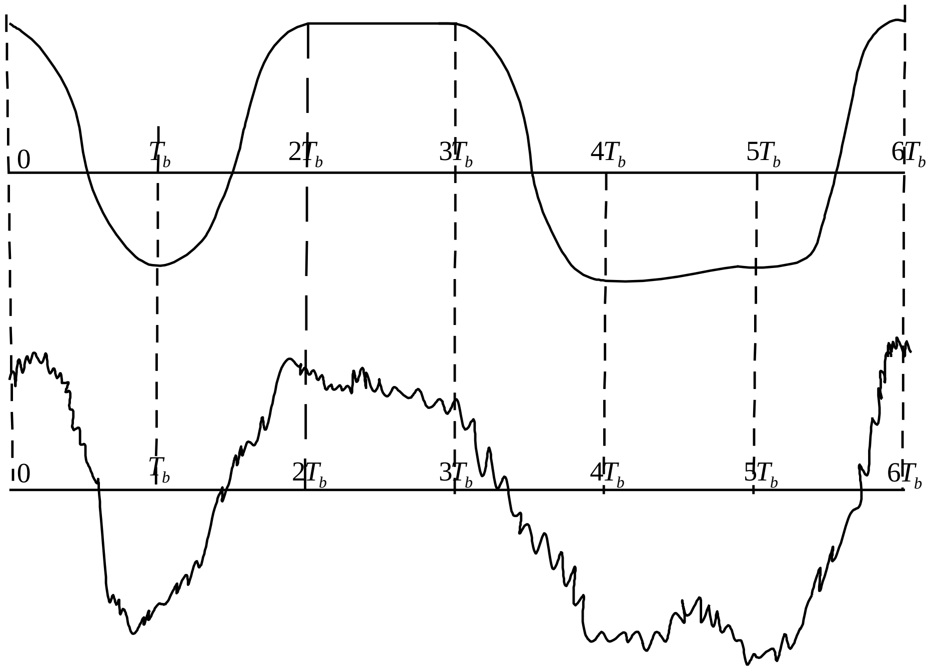
Lines intertwined with each other

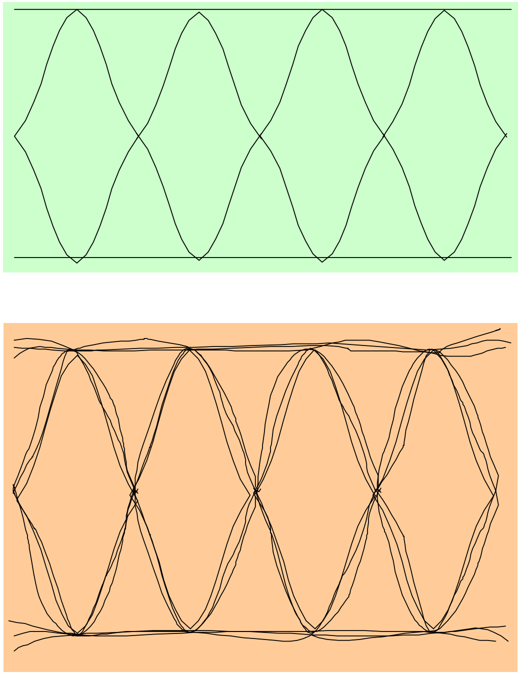
- The closer the lines are, the larger the eye is, and the smaller the interference is.
- When the open eye is small, the intersymbol interference is large.

Matched filter

Eye pattern with intersymbol interference and noise

The original thin lines that were clear





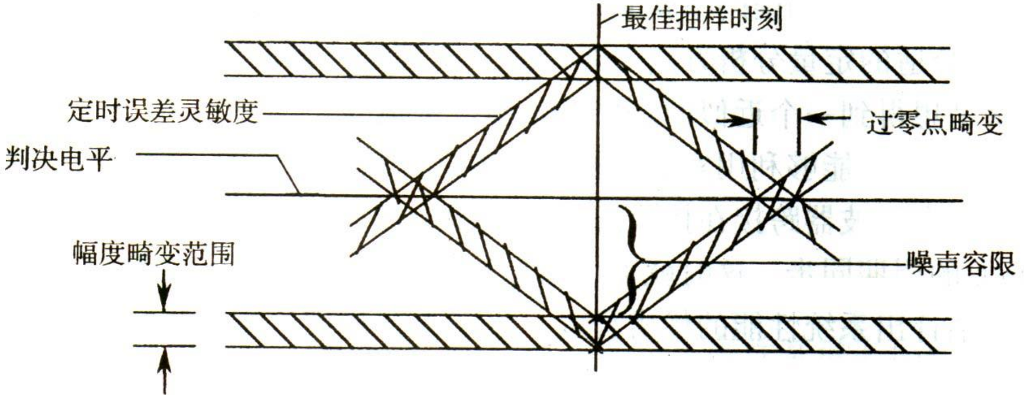
Ideal case

with intersymbol interference and noise

The contaminated signal eye pattern has blurred band-like thick lines

Matched filter

- The best sampling moment should be the moment when the "eye" is most open
- Timing error sensitivity is determined by the hypotenuse slope, the larger slope, the more sensitive
- The vertical height of the shaded area indicates the range of signal amplitude distortion
- Half of the distance of the eye is the noise tolerance (if the instantaneous noise exceeds it, a wrong judgment may occur)
- The horizontal axis in the center corresponds to the decision threshold level



Time-domain equalizer

What's the problem in reality?

In practice

Filter design errors and changes in channel characteristics are unavoidable

→ System performance degradation

Solution: Have an equalizer between the receiving filter and the decider

→ To correct or compensate system characteristics so as to reduce the impact of intersymbol interference

Time-domain equalizer

The classification of equalization

Equalization

Frequency domain equalization

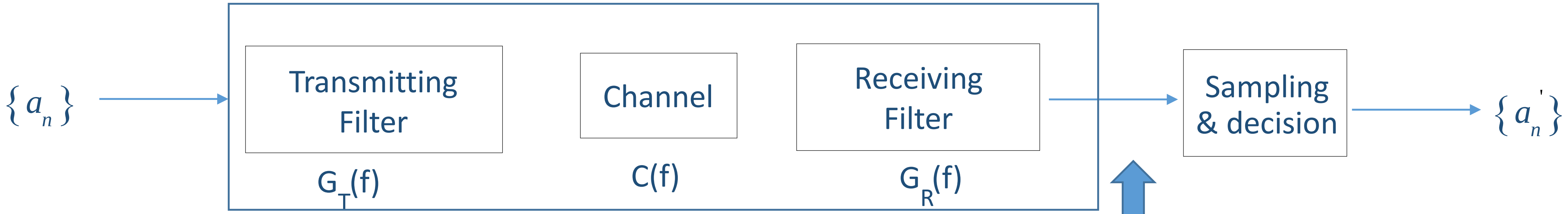
- Definition: Calibrate the system frequency characteristic to meet the “no intersymbol interference” requirement (A compensation of frequency distortion)
- Applicable conditions: channel characteristics remain unchanged & low-speed data transmission

Time domain equalization

- Definition: Equalizer generate time waveform to directly correct the distorted waveform, so as to meet the condition of no intersymbol interference (correction of the received waveform)
- Advantage: Adjusted based on channel changes to effectively reduce interference, and widely used in high-speed data transmission systems.

Time-domain equalizer

The principle of time equalization



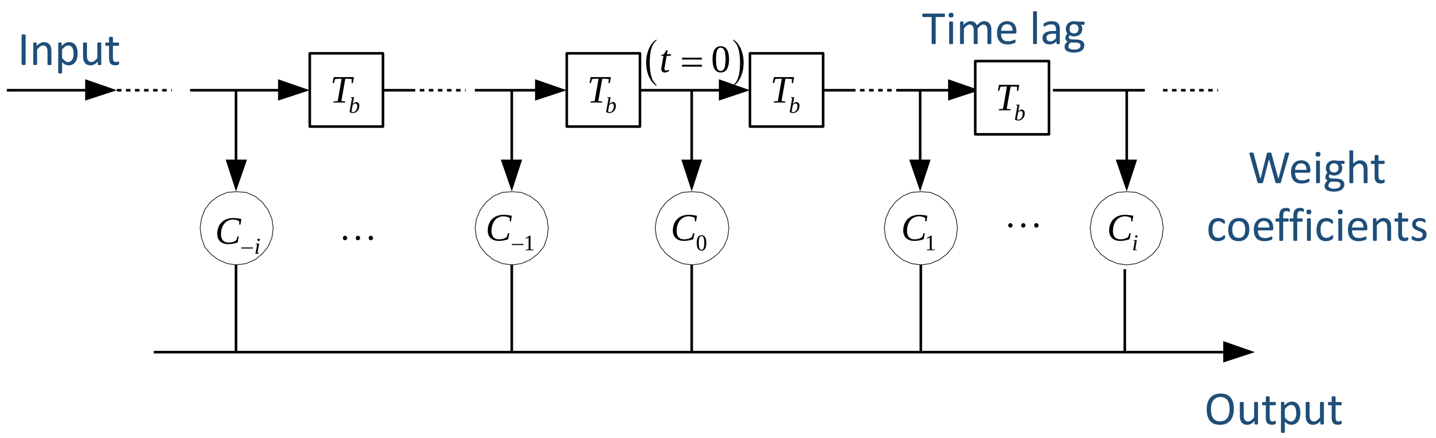
$$H(f)=G_T(f)\cdot C(f)\cdot G_R(f)$$

When $\sum_{m=-\infty}^{\infty} H(f+m/T_b)=T_b$ can't be satisfied

Equalization (adjustable transversal filter)

Proposition: If an adjustable transversal filter is inserted between the receiving filter and the decider, the intersymbol interference can be completely eliminated theoretically.

Time-domain equalizer



Transversal filter impulse response:

$$h_T(t)=\sum_{n=-\infty}^{\infty} C_n\delta(t-nT_b) \Longleftrightarrow T(f)$$

$$T(f)H(f)=H'(f) \Longrightarrow \sum_{m=-\infty}^{\infty} H'(f+m/T_b)=T_b \qquad |f|\le\frac{f_b}{2}$$

No intersymbol interference

Time-domain equalizer

$$\sum_m H(f+\frac{m}{T_b})T(f+\frac{m}{T_b})=T_b, |f|\le\frac{f_b}{2}$$

T [f+(m/Tb)]=T(f)

$$T(f)=\frac{T_b}{\sum_m H(f+\frac{m}{T_b})}, \quad |f|\le\frac{f_b}{2}$$

If T(f) is a periodic function with period of 1/Tb, T(f) is not related to m

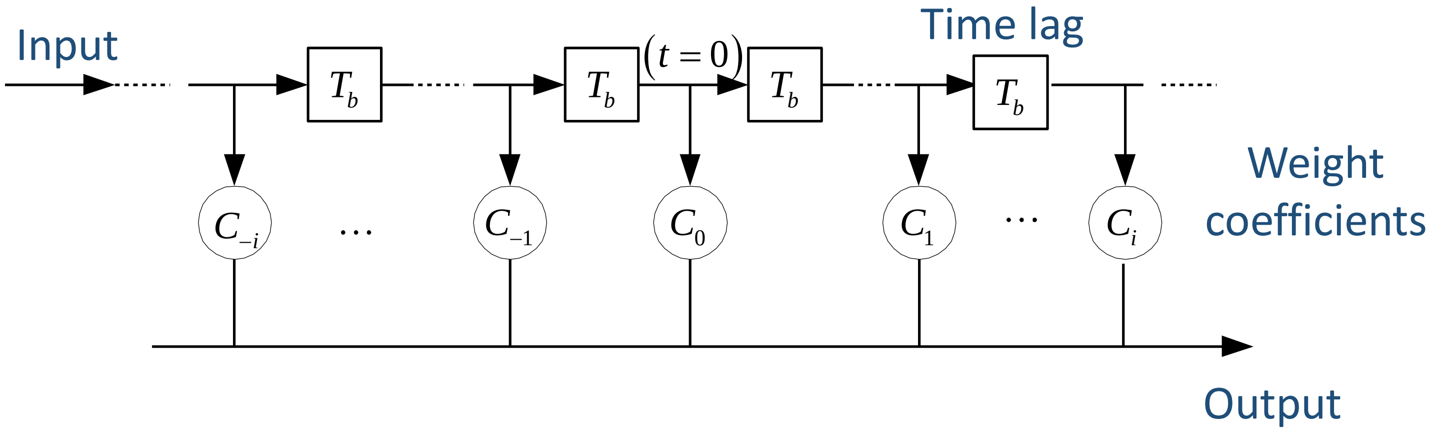
$$T(f)=\sum_{n=-\infty}^{\infty} C_n e^{jn2\pi fT_b}$$

$$C_n=T_b\int_{-1/2T_b}^{1/2T_b} T(f)e^{-j2\pi nfT_b}df$$

The Fourier coefficients Cn is determined by H (f)

$$h_T(t)=F^{-1}[T(f)]=\sum_{n=-\infty}^{\infty} C_n\delta(t-nT_b)$$

Time-domain equalizer

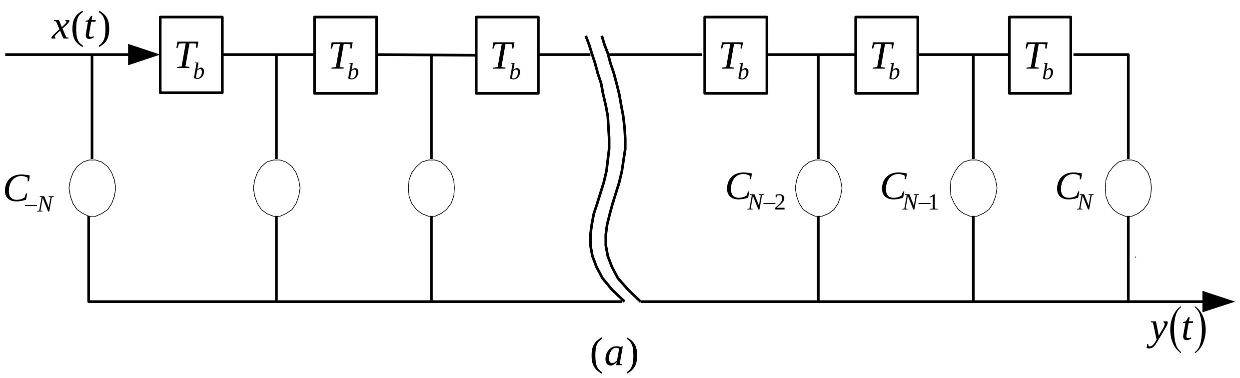


$$h_T(t) = \sum_{n=-\infty}^{\infty} C_n \delta(t - nT_b)$$

- Function: Transform the response waveform with intersymbol interference (the output end of the receiving filter) into a response waveform without intersymbol interference
- Infinite traversal filter: Theoretically, the intersymbol interference can be completely eliminated, but it is not practical
- Need further discussion on tap adjustment of finite transversal filters

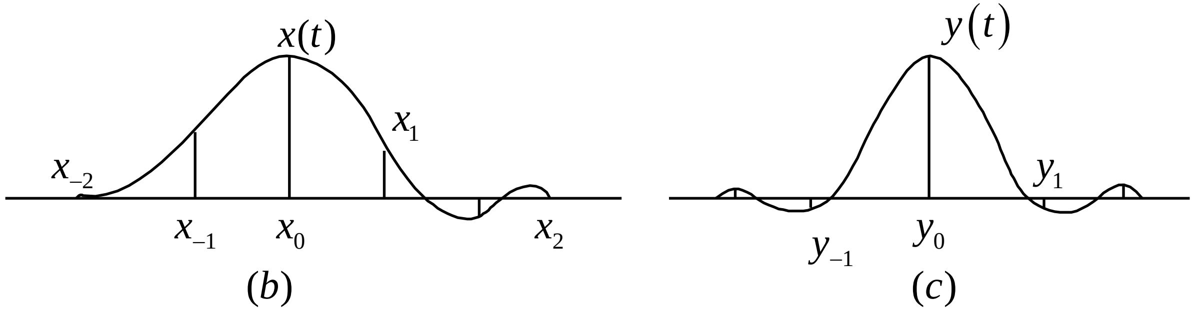
Time-domain equalizer

Signal from receiving filter



To decider

Waveform before equalizer



Waveform after equalizer

Impulse response: e(t)

$$e(t) = \sum_{i=-N}^{N} C_i \delta(t - iT_b)$$

Tap number: 2N+1

Frequency response: E(f)

$$E(f) = \sum_{i=-N}^{N} C_i e^{-j2\pi f T_b}$$

Time-domain equalizer

The output of the traversal filter

$$y(t) = x(t) * e(t) = \sum_{i=-N}^{N} C_i x(t - iT_b)$$

K-th sampling moment

$$y(kT_b + t_0) = \sum_{i=-N}^{N} C_i x(kT_b + t_0 - iT_b) = \sum_{i=-N}^{N} C_i x[(k-i)T_b + t_0]$$

$$y_k = \sum_{i=-N}^{N} C_i x_{k-i}$$

Composed of 2N+1 products of C_i and x_{k-i}

Apart from y_0, all y_k is the intersymbol interference of waveform distortion

Problem: When given an input waveform x(t) (i.e. all possible x_{k-i} is fixed), how to adjust C_i to make specified y_k be zeros?

Time-domain equalizer

Example: Assume that a traversal filter with 3 taps, C_{-1}=–1/4， C_0=1， C_{+1}=–1/2, the input of equalizer (t) at each sampling moment is： x_{-1}=1/4， x_0=1， x_{+1}=1/2， others are 0。 Please find the value of the equalizer output y(t) at each sampling moment

Solution:

$$y_k = \sum_{i=-N}^{N} C_i x_{k-i}$$

k=0 $$y_0 = \sum_{i=-1}^{1} C_i x_{-i} = C_{-1}x_1 + C_0 x_0 + C_{+1}x_{-1} = 3/4$$

k=1 $$y_{+1} = \sum_{i=-1}^{1} C_i x_{1-i} = C_{-1}x_2 + C_0 x_1 + C_1 x_0 = 0$$

k= − 1 $$y_{-1} = \sum_{i=-1}^{1} C_i x_{-1-i} = C_{-1}x_0 + C_0 x_{-1} + C_1 x_{-2} = 0$$

y_{-2}=–1/16， y_{+2}=–1/4

It is possible to reduce intersymbol interference by using a finite transversal filter, but it is impossible to completely eliminate it

Problem: how to adjust the weighting coefficients to obtain the optimal equalizer？

Time-domain equalizer

Two criteria

Peak Distortion Criterion

Minimize the ratio of the maximum value of the intersymbol interference to the sample value of the useful signal

min $D = \frac{1}{y_0} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} |y_k|$

Sum of absolute values of sample values except k=0

y0 is the sample value of the useful signal

maximum value of the intersymbol interference

Mean square distortion criterion: min $e^2 = \frac{1}{y_0^2} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y_k^2$

Determine the coefficients of equalizer based on 2 criteria can minimize distortion and obtain the best equalization effect

Time-domain equalizer

Peak Distortion Criterion

Input peak distortion before equalization - called initial distortion

$$D_0 = \frac{1}{x_0} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} |x_k| \xrightarrow{\text{If xk is normalized, then x0=1}} D_0 = \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} |x_k|$$

$$y_k = \sum_{i=-N}^{N} C_i x_{k-i} \xrightarrow{\text{yk is also normalized and y0=1}} y_0 = \sum_{i=-N}^{N} C_i x_{-i} = 1 \longrightarrow C_0 x_0 + \sum_{\substack{i=-N \\ i \neq 0}}^{N} C_i x_{-i} = 1$$

$$C_0 = 1 - \sum_{\substack{i=-N \\ i \neq 0}}^{N} C_i x_{-i}$$

$$y_k = \sum_{\substack{i=-N \\ i \neq 0}}^{N} C_i (x_{k-i} - x_k x_{-i}) + x_k$$

Question: D is a function of Ci, how to design Ci to minimize D?

$$D = \sum_{\substack{i=-N \\ i \neq 0}}^{N} \left| \sum_{\substack{i=-N \\ i \neq 0}}^{N} C_i (x_{k-i} - x_k x_{-i}) + x_k \right|$$

Time-domain equalizer

Proved: if the initial distortion $D_0<1$， then the minimal value of D must occur when y_k ($|k|\le N$, $k\ne 0$) that in front of or behind y_0 are all 0

Mathematical meaning: All the weighting coefficients $\{C_i\}$ should be $y_k=\begin{cases}0, & 1\le |k|\le N\\ 1, & k=0\end{cases}$

The solution of 2N+1 simultaneous equations

$$\begin{cases}\sum_{i=-N}^{N} C_i x_{k-i}=0, & k=\pm1,\pm2,\cdots,\pm N\\ \sum_{i=-N}^{N} C_i x_{-i}=1, & k=0\end{cases} \Rightarrow \begin{bmatrix} x_0 & x_{-1} & \cdots & x_{-2N}\\ \vdots & \vdots & \cdots & \vdots\\ x_N & x_{N-1} & \cdots & x_{-N}\\ \vdots & \vdots & \cdots & \vdots\\ x_{2N} & x_{2N-1} & \cdots & x_0 \end{bmatrix}\begin{bmatrix} C_{-N}\\ C_{-N+1}\\ \vdots\\ C_0\\ \vdots\\ C_{N-1}\\ C_N\end{bmatrix}=\begin{bmatrix}0\\ \vdots\\ 0\\ 1\\ 0\\ \vdots\\ 0\end{bmatrix}$$

The designed weighting coefficients C_i, can force N samples in front of or behind y_0 to be 0

The designed equalizer is also called "zero-forcing" equalizer

Time-domain equalizer

Example: Design a 3 taps zero-forcing equalizer to reduce intersymbol interference. Knowing that $x_{-2}=0$, $x_{-1}=0.1$, $x_0=1$, $x_1=-0.2$, $x_2=0.1$, please find the coefficients of the 3 taps and the peak distortion before and after the equalizer.

Solution:

2N+1=3 $\Rightarrow$ $$\begin{bmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$ $\Rightarrow$ $$\begin{cases} C_{-1} + 0.1C_0 = 0 \\ -0.2C_{-1} + C_0 + 0.1C_1 = 1 \\ 0.1C_{-1} - 0.2C_0 + C_1 = 0 \end{cases}$$

$C_{-1}=-0.09606$, $C_0=0.9606$, $C_1=0.201$

$y_{-1}=0$, $y_0=1$, $y_1=0$, $y_{-3}=0$

$y_{-2}=0.0096$, $y_2=0.0557$, $y_3=0.02016$

Input peak distortion: $D_0=0.4$

Output peak distortion: $D=0.0869$

4.6 times less peak distortion after equalization

When the taps are limited, intersymbol interference cannot be completely eliminated, but can reduce interference to a relatively small degree

Time-domain equalizer

Realization and adjustment of equalizer

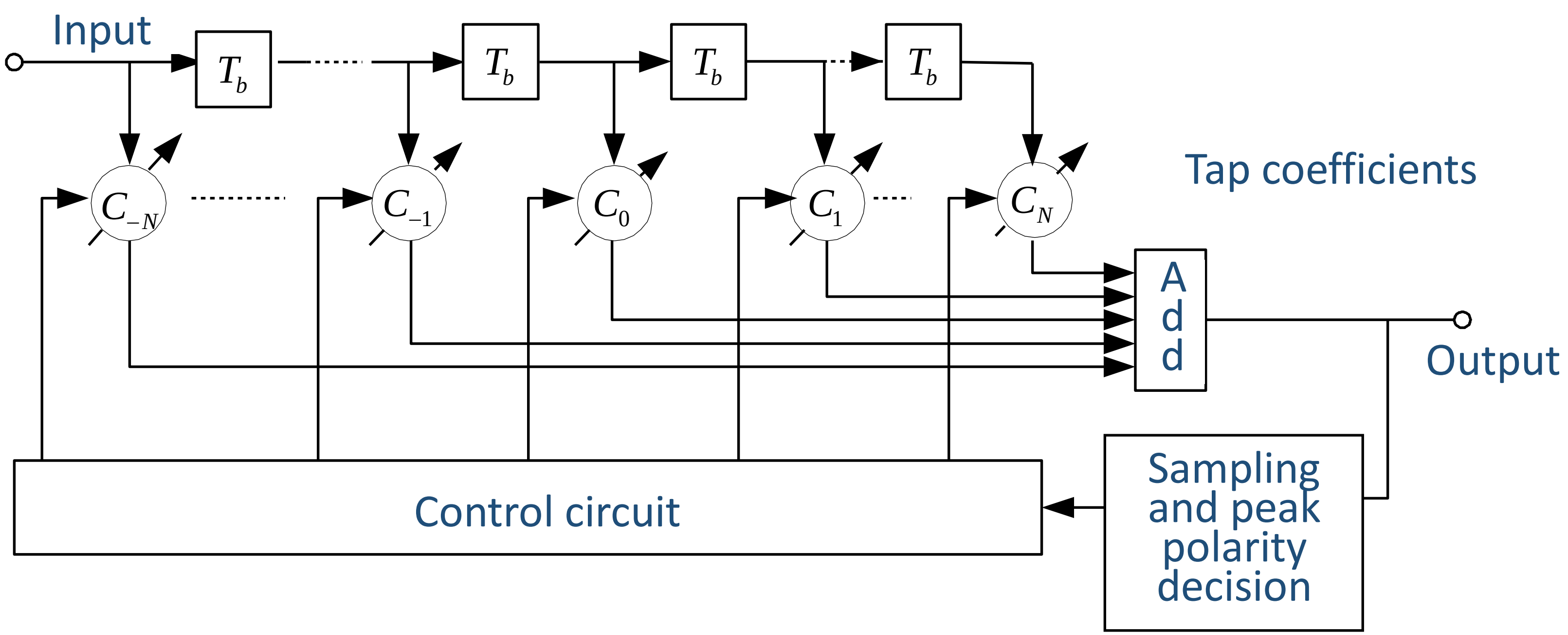
equalizer
- manual equalizer
- automatic equalizer
  - Preset equalizer
    - Sends a pre-defined test sequence before actual transmission
    - Adjust tap coefficient based on sample value $\{x_k\}$ of the test pulse until the error is less than a certain allowable range
    - After completing adjustment, transmit data and no adjustment will be made during the data transmission.
  - Adaptive equalizer
    - Adaptive equalization can continuously adjust the tap coefficient according to a certain algorithm during data transmission

Time-domain equalizer

Preset equalizer

Transmitter sends a test waveform regularly (this waveform is the waveform from receiving filter with intersymbol interference)

Input waveform every Tb



y_k (k=–N, –N+1, …, N–1, N)

Time-domain equalizer

Preset equalizer

Zero-forcing：

- If y_k is positive, the corresponding C_k should be decreased by an increment Δ;
- If y_k is negative, the corresponding C_k should be increased by an increment Δ

Adjusting methods：

- yk is sampled at the output and polarity judgment is made. Two possible judgment results are represented by "polarity pulse" and added to the control circuit
- The control circuit will apply all "polarity pulses" to the taps so that they will change by increasing or decreasing Δ
- After several adjustments, the equalization can be achieved
- The accuracy of equalizer relates to increment Δ and the adjustment time. The smaller Δ, the higher accuracy, but the longer adjustment time

Time-domain equalizer

Adaptive equalizer

	Preset equalizer	Adaptive equalizer
Similarity	Equalization is achieved by adjusting the tap gain of the transversal filter	
Difference	Use a specified test single pulse to adjust the error	Adjust the gain with the help of the signal itself during data transfer

- The output waveform is no longer a single pulse response, but an actual data signal
- Generally, it is designed based on minimum mean square error criterion

Time-domain equalizer

Adaptive equalizer

Assume
- Transmitted sequence: $\{a_k\}$
- Input of the equalizer: x(t)
- Output of the equalizer: $\{y_k\}$

- Error： $e_k = y_k - a_k$
- Mean square error： $\overline{e^2} = E(y_k - a_k)^2$

Purpose: minimize $\overline{e^2} = E(y_k - a_k)^2$

$$y_k = \sum_{i=-N}^{N} C_i x_{k-i}$$

$$\overline{e^2} = E\left(\sum_{i=-N}^{N} C_i x_{k-i} - a_k\right)^2$$

minimize $\overline{e^2}$

$$\frac{\partial \overline{e^2}}{\partial C_i} = 2E[e_k x_{k-i}]$$

$$e_k = y_k - a_k = \sum_{i=-N}^{N} C_i x_{k-i} - a_k$$

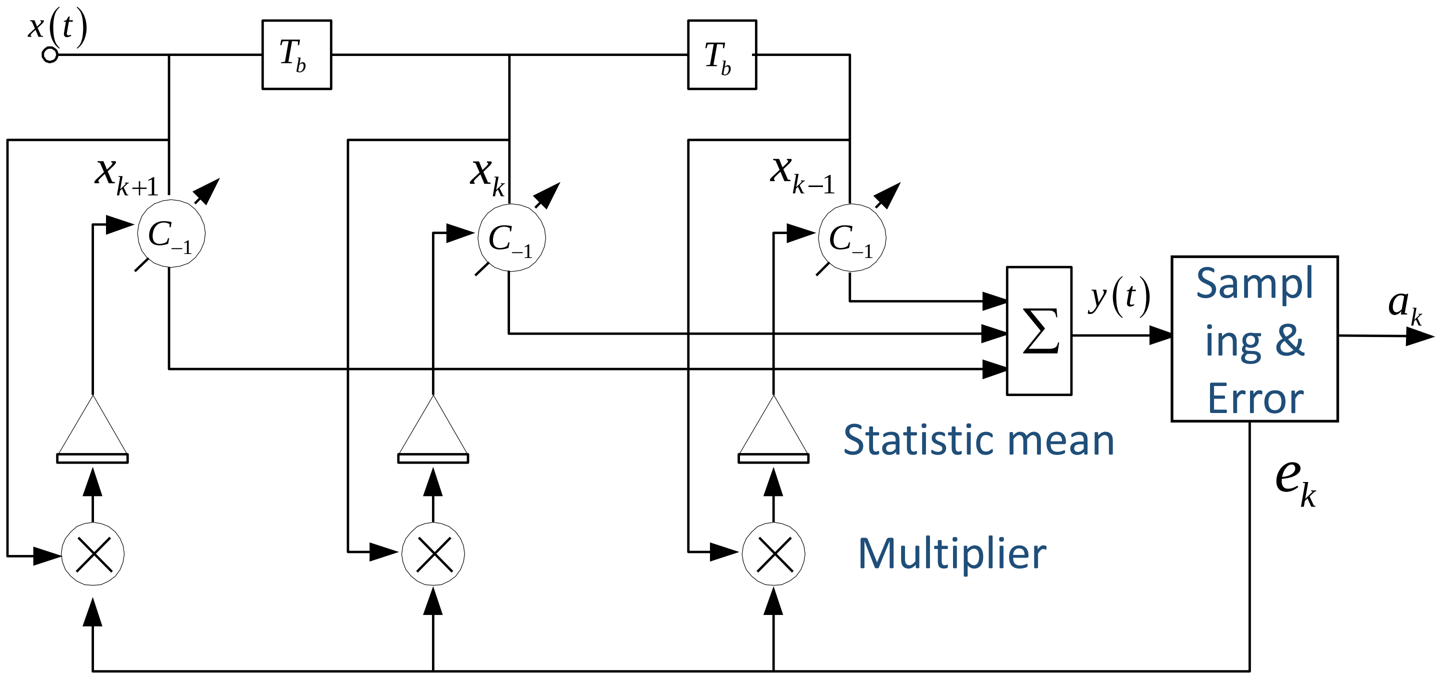
$$E[e_k x_{k-i}] = 0$$

Error e_k and the input of equalizer xk-i should not be related

Adjustment: by adjusting the tap gain to make it change to zero until it is equal to zero

Time-domain equalizer

3-taps Adaptive equalizer: minimal mean square error



The tap coefficients adjustment:

- Adjusted adaptively with the time-varying channel characteristics
- High adjustment accuracy and no pre-adjustment time

Adaptive equalizers are commonly used in high-speed digital transmission systems

Time-domain equalizer

Classic adaptive equalizer algorithm

- Zero-forcing（ZF）
- stochastic gradient algorithm
- Recursive Least Squares (RLS)
- Kalman algorithm
- Minimal mean square error algorithm (MMSE)

1. Compared with the ZF, the MMSE has better convergence and shorter adjustment time.
2. To overcome the difficulty of initial equalization, a random sequence known to the receiver must be sent before data transmission to "train" the equalizer.
3. The equalization technique based on the initial adjustment coefficients without using the training sequence is called self-recovery or blind equalization

Time-domain equalizer

Linear equalizer

The above equalizer is a linear equalizer (since the transversal filter is a type of linear filter) and it works well for channels like telephone wires

Nonlinear equalizer

If the intersymbol interference caused by serious channel distortion makes it difficult for a linear equalizer, need a nonlinear equalizer.

3 effective nonlinear equalization algorithms

- decision feedback equalization (DFE)
- maximum likelihood symbol detection
- maximum likelihood sequence estimation

Conclusion

4 basic waveform

- Unipolar NRZ, Bipolar NRZ, Unipolar RZ, Bipolar RZ。
- The unipolar code waveform has DC, and the judgment level at the receiving end is not fixed, so the application is limited; the bipolar waveform has no DC, and the decision level is fixed (zero), so it is widely used
- Compared with NRZ, the main disadvantage of RZ is the large bandwidth, and the main advantage is that it is easy to distinguish symbols, especially the unipolar RZ waveform for bit timing.
- Important： Draw the waveform when giving the sequence or the opposite

Conclusion

Symbol code types transformation

- Symbol code types transformation helps the match to the channel
- Commonly used symbol code types: AMI, HDB3(*), biphase code
- Important: how to turn the binary sequence to the specific code type or recover the sequence, and draw the waveform

Power Spectral Density of Digital Baseband Signal

- Includes continuous spectrum (determine bandwidth) and discrete spectrum (determine the DC and the timing)
- Remember the spectrum formula and learn to analyze whether there is DC or timing component
- The bandwidth of RZ with half duty ratio has twice width of bandwidth of NRZ

Conclusion

Intersymbol interference (ISI)

- Very important. It combines the transmitting filter, channel and receiving filter into a network H(w) without considering the channel noise, and studies the conditions of eliminating intersymbol interference. The conclusion is:
- Time domain condition：

$$h(kT_b) \equiv h_k = \begin{cases} 常数 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

- Frequency domain condition：

$$H_{eq}(f) = \sum_{m=-\infty}^{\infty} H(f + m/T_b) = 常数 \qquad |f| \leq \frac{f_b}{2}$$

- Based on the conditions, there are 3 types of system for eliminating ISI

Conclusion

Ideal low-pass filter system (LPF)

- If the frequency upper bound of LPF is f_N, then as long as the symbol rate is R_B ($R_B=f_s=1/T_s$) satisfying $R_B=2f_N$, or $R_B=2f_N/K$ (K is positive integer), ISI can be eliminated. Efficiency reaches maximum 2Bd/Hz;
- Disadvantage: Hard to achieve, and high synchronization accuracy

Roll off system

- Compared with LPF, it is easy to achieve and low synchronization accuracy but has reduced bandwidth efficiency
- Both LPF and roll off are based on Nyquist first criterion
- There is a relationship between the bandwidth, roll off coefficient and efficiency

Conclusion

Partial response system

- Based on Nyquist second criterion
- It retains the advantage of the highest frequency band utilization of LPF, and uses correlation coding to reduce tail, and the part intersymbol interference caused by correlation coding is overcome by precoding.
- Advantages: Highest efficiency while require low synchronization accuracy, it is widely used.
- Disadvantages: After correlation coding, the number of transmission signal levels increases, thereby reducing reliability . Thus, partial response systems trade off reliability for increased effectiveness.

Conclusion

Anti-noise performance

- Not considering ISI here. The channel noise is still considered to be AWGN, and derive error rate formulas and the optimal decision threshold.
- Grasp the analysis of match filter
- Grasp the structure and the performance of optimal receiver
- Know how to derive the minimal error rate

Conclusion

Eye pattern

- In practice, an oscilloscope is often used to observe the received baseband signal waveform to qualitatively see the system performance; because the image displayed by the oscilloscope is very similar to the human eye, it is called an eye pattern;
- the clearer the eye line, the greater the opening degree , the better the system performance.
- Requirement: get familiar with the eye pattern model

Conclusion

Equalization

- Although the method of eliminating ISI has been theoretically obtained, due to design errors and channel characteristics, there will always be ISI in the actual system, so it needs equalization

Requirements:

- Know the principle of the time domain equalizer
- Grasp weighting coefficients of zero-force equalizer
- Know how to realize and adjust the equalizer

Thank you！

Exercise

Answer briefly

1、What is eye pattern and what system performance can be estimated from the eye pattern?

2、What is time domain equalization? Why traversal filter can achieve time domain equalization?

3、How to measure the effect of the equalization? Explain the criteria.

Exercise

Ex1: design a 3-tap zero-force equalizer. We know that $x_{-2}=0$, $x_{-1}=0.2$, $x_0=1$, $x_1=-0.3$, $x_2=0.1$, other $x_k=0$, please find the coefficients of the 3 taps and calculate the peak distortion before and after the equalizer.

MATLAB

1、Generate binary bipolar random sequence

2、Let the signal traverse a AWGN channel (2 different SNR setup SNR = 5/SNR = 15)

3、Design zero-forcing equalizer and plot the samples before equalization and after equalization

4、Use cosine roll-off waveform and plot the eye pattern before and after the equalization.