第二次作业
2024.05.20 课堂上交

一、 Fill in the blanks.（10 points）

1. The autocorrelation of a real stationary stochastic process is $R_X(\tau) = \frac{40+72\tau^2}{1+2\tau^2}$, then for this process, the mean is ________, the variance is ________________. （3 points each）
2. Mean value of white noise is_____________，The autocorrelation function is an impulse function (or δ Function). （4 points）

二、 Calculation. （90 points）

1. (20 points) If $Y(t) = X(t-\alpha)$, and the system input $X(t)$ is a stationary stochastic process with autocorrelation $R_X(\tau)$ and power spectrum is $S_X(\omega)$, find the autocorrelation $R_Y(\tau)$ and power spectrum $S_Y(\omega)$. (written as $R_X(\tau)$ and $S_X(\omega)$)

2. (20 points) The power spectrum of a stationary stochastic process is $S_x(\omega) = \frac{\omega^2+17}{\omega^4+34\omega^2+225}$: Calculate the autocorrelation, mean, variance and correlation coefficient of the stochastic process.

3. (20 points) The stochastic process $X(t) = A\cos(\omega_0 t + \Phi)$, where ω_0 is a constant, and A and Φ are independent random variables. Φ is uniformly distributed in $(-3.5\pi, 2.5\pi)$, and A is a zero mean Gaussian random variable with variance 1.

(1) Is $X(t)$ Wide Sense Stationary (WSS)? Prove it. (10 points)
(2) Calculate the power spectrum of this process $X(t)$. (10 points)

4. (30 points) Given real joint stationary processes $X(t)$ and $Y(t)$: $\alpha Y(t) + \frac{d^3Y(t)}{d^3t} = X(t) - \beta\frac{d^2X(t)}{d^2t}$, and the power spectrum of $X(t)$ is $S_X(\omega)$,

(1) Calculate the transfer function $H_Y(\omega)$ and the cross-power spectrum of $S_{XY}(\omega)$ and $S_{YX}(\omega)$ (represented by α, β and $S_X(\omega)$). (15 points)

(2) if the input $X(t)$ is a white noise with power spectrum q, and $S_Y(\omega) = \frac{2\beta\omega^2+2}{\omega^4-\omega^2+1}$,

calculate α，β，q (value only, do not consider the units). (15 points)