第三次作业

2024.06.03 课上提交

1. For a zero mean Gaussian stationary process $X(t)$, whose power spectrum is:

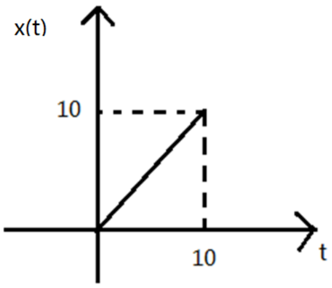
$$S_X(\omega) = \begin{cases} A, & \left||\omega| - \omega_0\right| < \dfrac{\Delta\omega}{2} \\ 0, & \text{others} \end{cases}$$

Where $\omega_0 > \Delta\omega$. Calculate the 1-D pdf of it. (10 points)

2. Assuming that a stationary Gaussian white noise with power spectrum A is fed into a filter $H(\omega) = \frac{2}{j\omega+1}$, calculate the one-dimensional pdf of the output. (20 points)

3. A stationary stochastic process $X(t)$ is fed into a low-pass filter $h(t) = \alpha e^{-\alpha t}U(t)$. The autocorrelation of $X(t)$ is $\delta(\tau)$. Calculate the output autocorrelation $R_Y(\tau)$. (20 points)

4. Given an input $z(t) = x(t) + n(t)$, where $x(t)$ is a deterministic signal as follows (triangular wave), and $n(t)$ is stationary Gaussian white noise with power spectrum of q. (30 points)



a) Calculate the maximum signal-to-noise ratio of the output if this $z(t)$ is fed into its matched filter. (10 points)

b) There is another input $z_1(t) = \frac{1}{3}x(t) + n(t)$, calculate the maximum signal-to-noise ratio of the output of this input if it is fed into its matched filter. (10 points)

c) Calculate the matched filter $H(\omega)$ of the signal. (10 points)

5. The stochastic process $Y(t) = Xcos(\omega_0 t + \theta)$, where ω_0 is a constant, X and θ are independent random variables, X is zero mean white Gaussian variable with variance c^2 where c is a constant, and θ follows uniformly distributed in $(-\pi, \pi)$. (20 points)

a) Calculate the power spectrum of $Y(t)$.

b) Is $Y(t)$ an ergodicity process? Prove it.