Stochastic Signal Processing

Lesson 10

Filters I

Weize Sun

Examples from last week

1: find the mean and autocorrelation of $Y(t)$ if it is an Poisson impact process (泊松冲击过程) inputted to the Differentiator.

Hint: for Poisson impact process $X(t)$:

$$E[X(t)] = \lambda t \qquad\qquad R_X(t_1, t_2) = \lambda^2 t_1 t_2 + \lambda \min(t_1, t_2)$$

2: given a process $X(t) = Acos(\omega_0 t + \varphi)$ where A and ω_0 are constants, φ uniformly distributed in $(0,2\pi)$. Input the $X(t)$ to an differentiator and get $X'(t)$, calculate the mean, autocorrelation and power spectrum of $X'(t)$.

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Solution:

$$m_Y(t) = \frac{d}{dt} m_X(t) = \lambda \qquad R_Y(t_1,t_2) = \frac{\partial^2 R_X(t_1,t_2)}{\partial t_1 \partial t_2}$$

$$\frac{\partial R_X(t_1,t_2)}{\partial t_2} = \begin{cases} \lambda^2 t_1 & t_1 \le t_2 \\ \lambda + \lambda^2 t_1 & t_1 > t_2 \end{cases} = \lambda^2 t_1 + \lambda U(t_1 - t_2)$$

$$\frac{\partial^2 R_X(t_1,t_2)}{\partial t_1 \partial t_2} = \lambda^2 + \lambda \delta(t_1 - t_2)$$

$$R_Y(\tau) = \lambda^2 + \lambda \delta(\tau); \tau = t_1 - t_2$$

- The derivative of $U(t)$ is $\delta(t)$.
- In this example, the $X(t)$ is not stationary, however, its Differentiator is stationary!
- In fact, for non-stationary input, the output of a Differentiator can be stationary or non-stationary

Examples from last week

2: given a process $X(t) = Acos(\omega_0 t + \varphi)$ where A and ω_0 are constants, φ uniformly distributed in $(0,2\pi)$. Input the $X(t)$ to an differentiator and get $X'(t)$, calculate the mean, autocorrelation and power spectrum of $X'(t)$.

Solution:

$$E[X(t)] = \int_0^{2\pi} Acos(\omega_0 t + \varphi)\frac{1}{2\pi}d\varphi = 0$$

$$R_X(t_1, t_2) = A^2 \int_0^{2\pi} \cos(\omega_0 t_1 + \varphi)\cos(\omega_0 t_2 + \varphi)\frac{1}{2\pi}d\varphi \quad \tau = t_1 - t_2$$

$$= \frac{A^2}{4\pi}\int_0^{2\pi} \cos(\omega_0[t_1 - t_2]) + \cos(\omega_0[t_1 + t_2] + 2\varphi)\, d\varphi = \frac{A^2}{4\pi}\int_0^{2\pi} \cos(\omega_0 \tau)\, d\varphi$$

$$= \frac{A^2}{2}\cos(\omega_0 \tau)$$

$$S_X(\omega) = \frac{A^2\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$E[X'(t)] = \frac{dm_X}{dt} = 0 \qquad R_{X'}(\tau) = -\frac{\partial^2}{\partial\tau^2}R_X(\tau) = \frac{d}{d\tau}\{\frac{A^2}{2}sin(\omega_0\tau)\omega_0\} = \frac{A^2}{2}cos(\omega_0\tau)\omega_0^2$$

$$S_{X'}(\omega) = \omega^2 S_X(\omega) = \frac{A^2\omega^2\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

Note:

1. Fourier transform of $\cos(\omega_0\tau)$ is $\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$, see previous ppt.
2. The Differentiator is written as $\dot{X}(t)$ in some books, and written as $X'(t)$ in some other books.

Outline

- Band limiting process
  - Low-pass
  - Band-pass
  - Noise equivalent pass-band

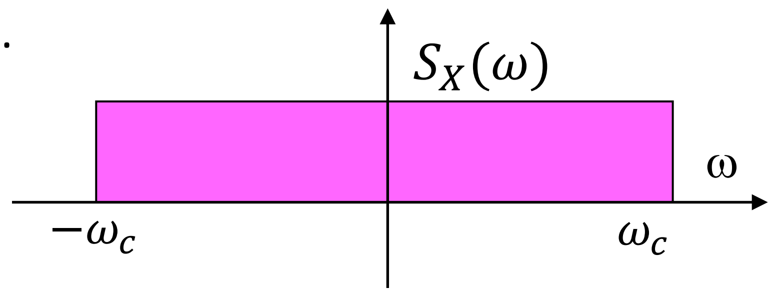
Band limiting process

- Band limiting process:
  - For a stochastic process, its power spectrum is a non-zero value at a certain frequency and 0 at other frequencies.
  - Low-pass
  - Band-pass

- Obviously, when a white noise passes through a band limiting system, it changes from a "full band non-zero" process to a (output) band limiting stochastic process.

Ideal low-pass process

- Low-pass process: the power spectrum of the stochastic process is not zero in $|\omega| <= \omega_c$, but zero otherwise.
- The figure shows an ideal low-pass process.



- The power spectrum of this ideal low-pass process is:

$$S_X(\omega) = \begin{cases} q, & |\omega| <= \omega_c \\ 0, & others \end{cases}$$

- Given the power spectrum $S_X(\omega)$, the autocorrelation of the low-pass process is:

$$R_X(\tau) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} S_X(\omega)e^{j\omega\tau}\,d\omega$$

- Note that any n-order derivative (n阶导数) of the autocorrelation (derivative of $R_X(\tau)$) exists:

$$R_X^{(n)}(\tau) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} (j\omega)^n S_X(\omega)e^{j\omega\tau}\,d\omega$$

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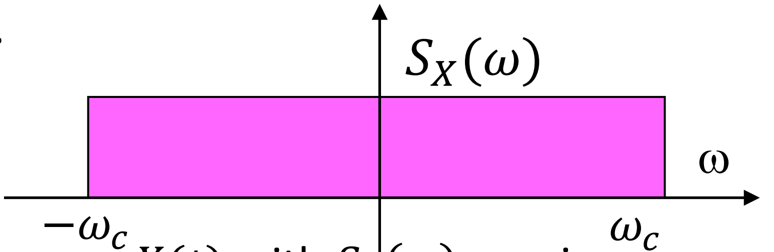

- The autocorrelation of this ideal low-pass process $X(t)$ with $S_X(\omega) = q$ is

$$R_X(\tau) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} q e^{j\omega\tau}\, d\omega = q\,\frac{\omega_c}{\pi}\frac{\sin\omega_c\tau}{\omega_c\tau} = q\,\frac{\omega_c}{\pi}\operatorname{sinc}(\omega_c\tau)$$

The total average power is $R_X(0) = q\,\frac{\omega_c}{\pi}$

注（上一节课也有提到）：$\int_{-a}^{a} e^{j\omega\tau}\, d\omega = 2a\frac{\sin a\tau}{a\tau}$

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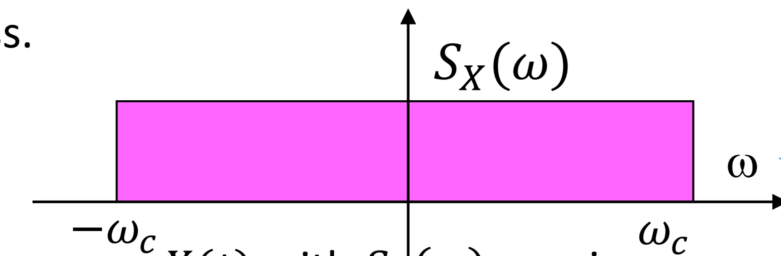

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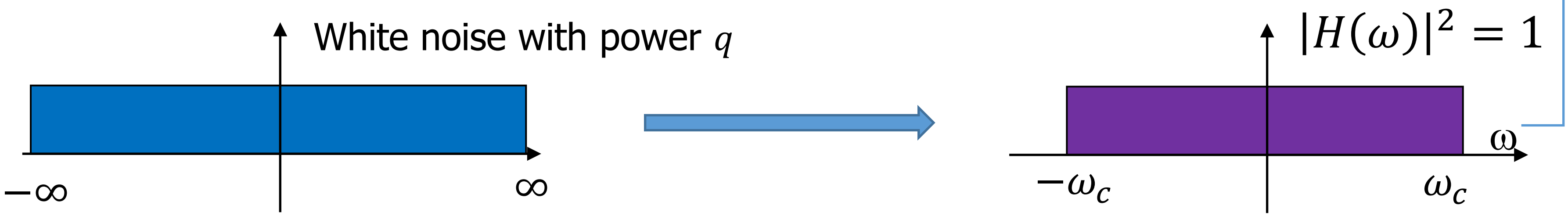
The ideal low-pass process can be generated by passing a white noise to an ideal low pass filter $H(\omega)$:



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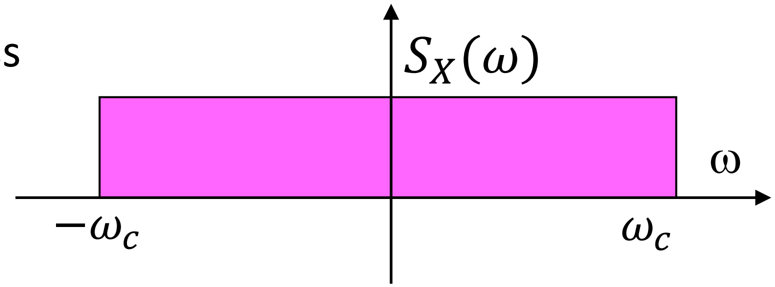
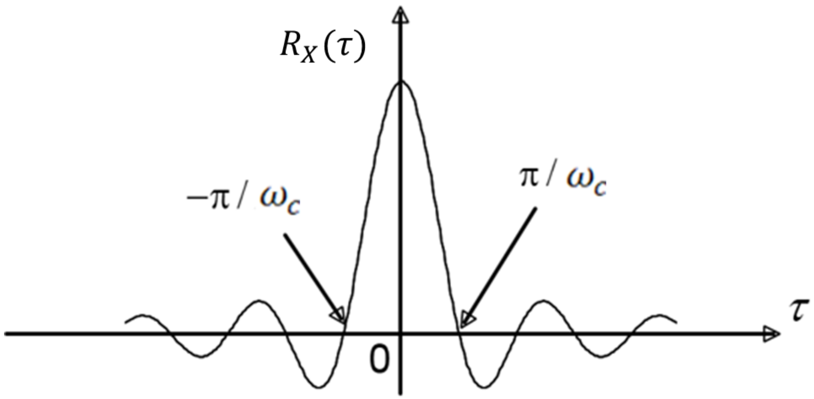


- The autocorrelation:

$$R_X(\tau) = q\frac{\omega_c}{\pi}\frac{\sin \omega_c \tau}{\omega_c \tau} = q\frac{\omega_c}{\pi}\sin c(\omega_c \tau)$$



When $\tau = k\pi/\omega_c (k = \pm1, \pm2, \dots)$:

$$R_X(\tau = k\pi/\omega_c) = 0$$

So, $X(t)$ and $X(t + k\pi/\omega_c)$ are orthogonal.

- For the ideal low-pass process, if the time interval of $\Delta = \pi/\omega_c$ is used for sampling, the samples (discrete data) obtained after sampling are orthogonal to each other.
- Thus we can generate discrete-time orthogonal signals

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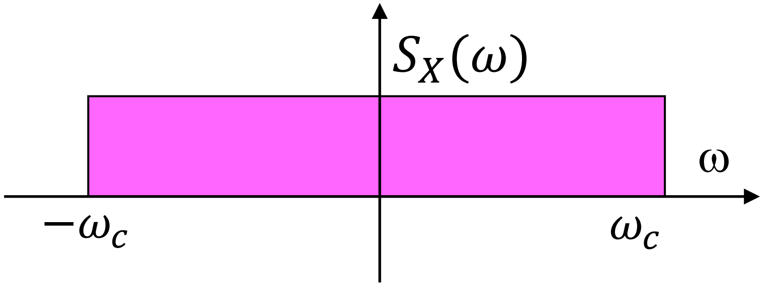
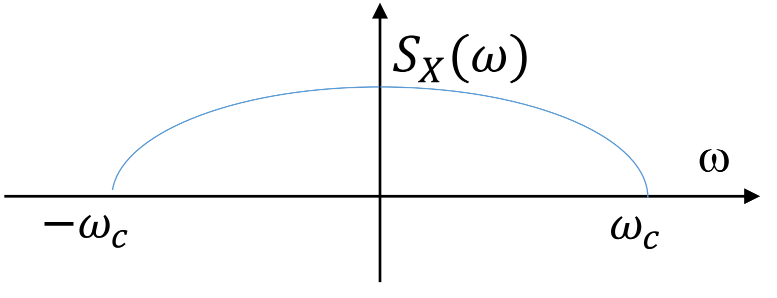
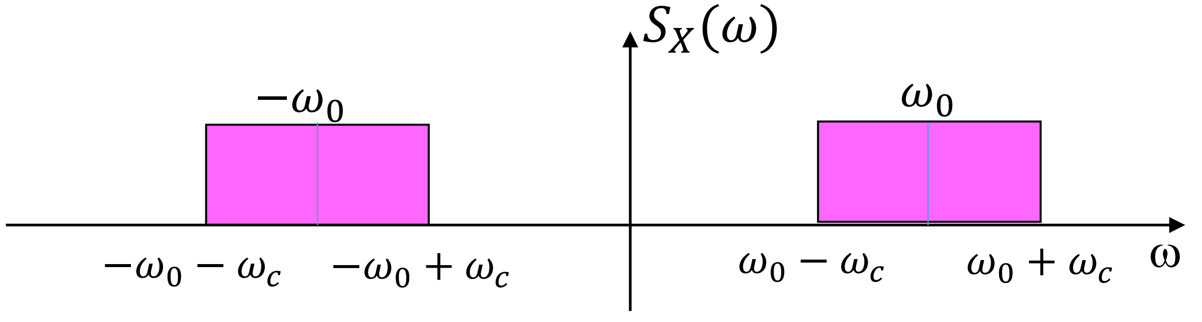


- A non-ideal low pass process might look like this (the amplitude gradually decrease from 0 to ω_c)

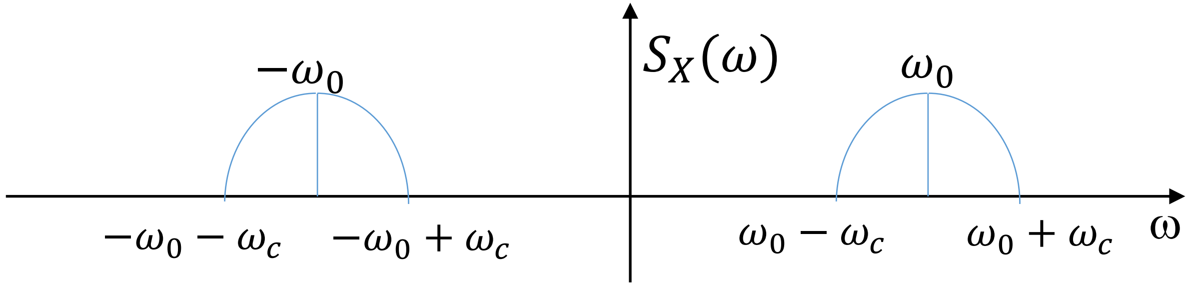


Ideal band-pass process

- Band-pass process: the power spectrum of stochastic process is not zero in the frequency band centered on ω_0, but zero otherwise.
- The ideal band-pass process looks like



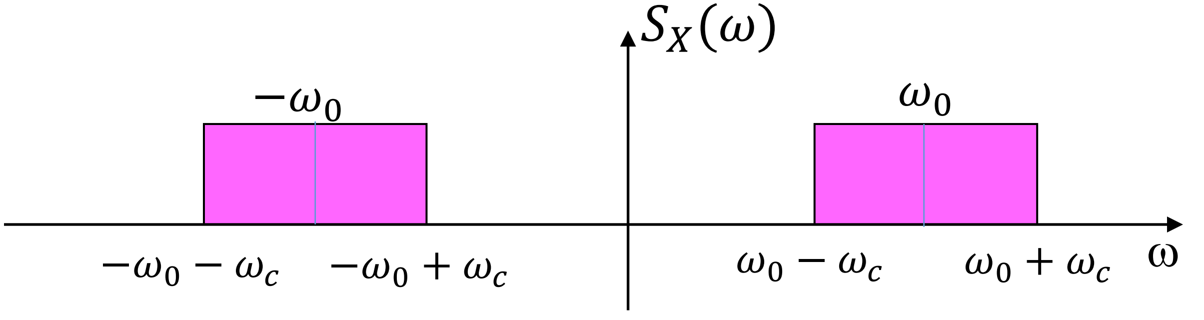
- The non-ideal band-pass process might looks like (the amplitude gradually decrease from $\pm\omega_0$ to $\pm\omega_0 \pm \omega_c$)



Ideal band-pass process

- The power spectrum of an ideal band-pass process is assumed to be ($\omega_0 \geq \omega_c$):

$$S_X(\omega) = \begin{cases} q, & -\omega_c \leq |\omega| - \omega_0 \leq \omega_c \\ 0, & others \end{cases}$$



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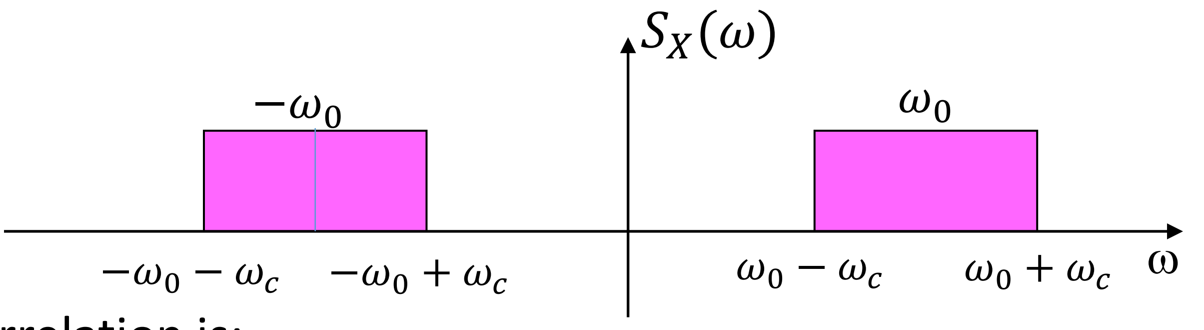

The autocorrelation is:

$$R_X(\tau) = \frac{1}{\pi}\int_0^{+\infty} S_X(\omega)\cos\omega\tau\, d\omega = \frac{1}{\pi}\int_{\omega_0-\omega_c}^{\omega_0+\omega_c} q\cos\omega\tau\, d\omega$$

$$= 2q\frac{\omega_c}{\pi}\frac{\sin\omega_c\tau}{\omega_c\tau}\cos\omega_0\tau$$

- The total average power is $R_X(0) = 2q\omega_C/\pi$

Examples

- Example 1: a white noise with power spectrum A passes through a filter with the transfer function $H(\omega) = \frac{1}{1+j\frac{\omega}{\omega_0}}$:

a) Is the filter like a low-pass filter? Why?

b) Find the autocorrelation $R_Y(\tau)$ and variance σ_Y^2 of the output

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- Example 1: a white noise with power spectrum A passes through a filter with the transfer function $H(\omega)=\frac{1}{1+j\frac{\omega}{\omega_0}}$:

a) Is the filter like a low-pass filter? Why?

b) Find the autocorrelation $R_Y(\tau)$ and variance σ_Y^2 of the output

Solution:

a) For this filter, when $\omega=0$, $|H(\omega)|=1$ is the maximum; when $\omega=\pm\infty$, $|H(\pm\infty)|=0$; the amplitude of this filter decreases from 0 to $\pm\infty$, it is a non-ideal low-pass filter

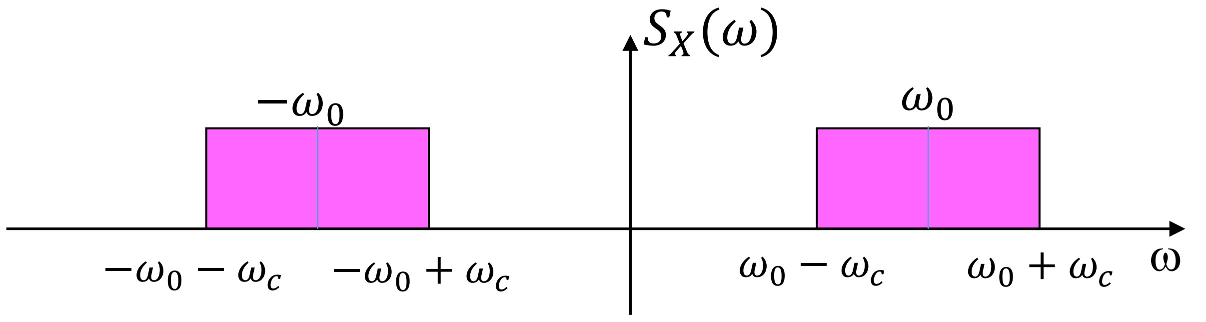
b) We have $S_Y(\omega)=A|H(\omega)|^2=A\frac{\omega_0^2}{\omega_0^2+\omega^2}$, thus

$$R_Y(\tau)=F^{-1}\{S_Y(\omega)\}=\frac{A\omega_0}{2}e^{-\omega_0|\tau|}\text{ (逆傅里叶变换)}$$

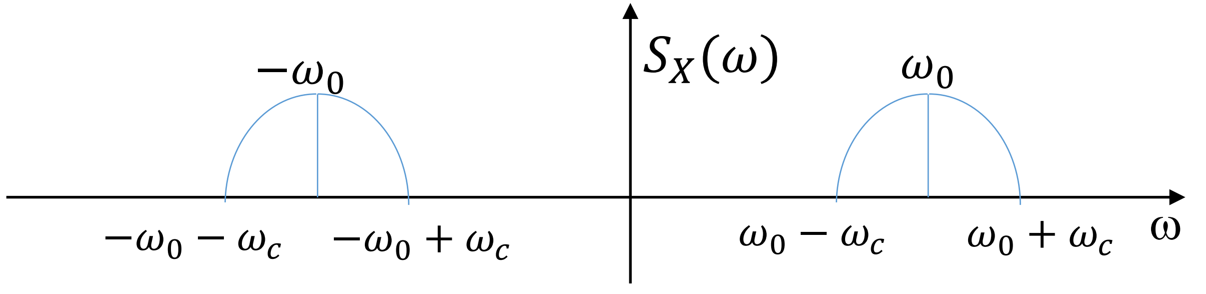
$$\text{The mean is } 0,\ R_Y(\infty)=0,\ \rightarrow\ \sigma_Y^2=R_Y(0)-R_Y(\infty)=\frac{A\omega_0}{2}$$

Noise equivalent pass-band (噪声等效通能带)

- In general, linear systems are non ideal.
  - For example, a white noise passes through an ideal band-pass filter will get



  - But when it passes through a non-ideal band-pass filter might get



- How to establish a connection between the above two?

Noise equivalent pass-band !

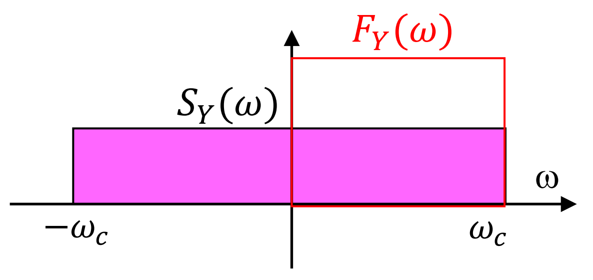
Noise equivalent pass-band

- Definition:

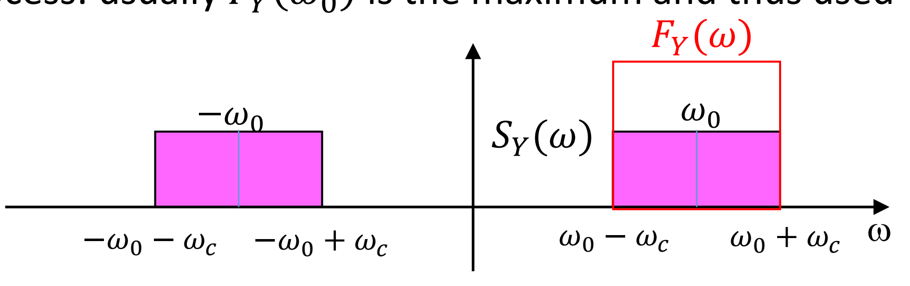
$$F_Y(\omega_0)\Delta\omega_e = \int_0^\infty F_Y(\omega)\ d\omega$$

Where $F_Y(\omega) = 2S_Y(\omega)$ for $\omega \geq 0$ and $F_Y(\omega) = 0$ otherwise, it is referred to as physical spectrum (物理谱), $F_Y(\omega_0)$ is usually the maximum point of the $F_Y(\omega)$ in $[0, \infty]$

- For low-pass process: $F_Y(0)$ is the maximum and thus used



- For band-pass process: usually $F_Y(\omega_0)$ is the maximum and thus used



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- Therefore, for band-pass process, the noise equivalent pass-band is:

$$\Delta\omega_e = \frac{\int_0^{\infty} F_Y(\omega)\, d\omega}{F_Y(\omega_0)} = \frac{\int_0^{\infty} |\, H(\omega)|^2 d\omega}{|H(\omega_0)|^2}$$

Note: for band-pass, it is assumed that $|H(\omega)|$ obtains the maximum value at $|H(\omega_0)|$!

Noise equivalent pass-band

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- Similarly, for the low-pass process, the noise equivalent pass-band is:

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  - Note that $\Delta\omega_e$ is the angular frequency(角频率), $\Delta\omega_e = 2\pi\Delta f_e$ where Δf_e is frequency

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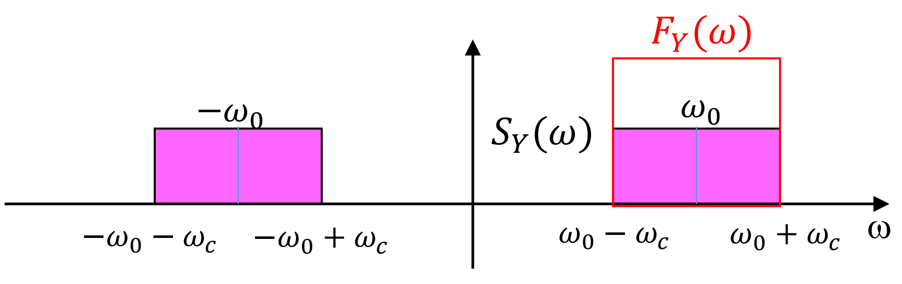
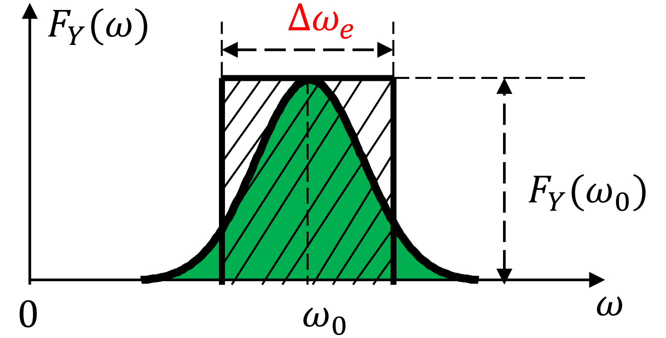
- **The noise equivalent passband is only determined by the linear system, not related to the input process!**

Noise equivalent pass-band

- Physical meaning:
  - By equating the area of the green part (non ideal band-pass system, or says, actual system) with the area of the shadow part, an ‘ideal system’ is obtained. The height of the “ideal power spectrum” is the height of the highest point of the “non ideal power spectrum”, and the width is the noise equivalent pass-band $\Delta\omega_e$ （$\Delta\omega_e = 2\pi\Delta f_e$）
  - Then, we can use this ‘ideal system’ to substitute the ‘non-ideal system’, and also, if a process passes through the ‘non-ideal system’ (as in the figure) we will get a ‘non-ideal band-pass process’, and this ‘non-ideal band-pass process’ can be replaced by the ‘ideal band-pass process’ with center frequency ω_0 and pass-band $2\omega_c = \Delta\omega_e$





Noise equivalent pass-band

- Recall that:
  - A low-pass process (ideal or non-ideal) is usually generated by passing a white noise to a low-pass filter (linear system)
  - A band-pass process (ideal or non-ideal) is usually generated by passing a white noise to a band-pass filter (linear system)
- Assume such situation: a white noise $S_X(\omega) = q$ passes through a system, the system is very complicated, or says, the $H(\omega)$ is very complicated, making the output $S_Y(\omega) = S_X(\omega)|H(\omega)|^2$ very complicated.
- But we want to get the average power of the output process, which is $R_Y(0)$, to get this, we need

$$R_Y(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Y(\omega)e^{j\omega\tau}\, d\omega \qquad \text{or} \qquad R_Y(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Y(\omega)e^{j\omega 0}\, d\omega$$

- But the $S_Y(\omega)$ is so complicated that the integral is hard to compute.
- However, good news is, we have the 'noise equivalent pass-band' $\Delta\omega_e$ of the complicated non-ideal system, can we calculate the $R_Y(0)$ based on that?

YES!

Noise equivalent pass-band

- For low-pass process (generated from passing a white noise $S_X(\omega) = q$ to a low-pass system):

$$\int_0^\infty F_Y(\omega)d\omega = \Delta\omega_e F_Y(0), \text{ therefore we have}$$

$$R_Y(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Y(\omega)e^{j\omega 0}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Y(\omega)\, d\omega = \frac{1}{2\pi}\int_0^{\infty} F_Y(\omega)\, d\omega$$

$$= \frac{1}{2\pi} F_Y(0)\Delta\omega_e$$

$$F_Y(0) = 2S_Y(0) = 2S_X(0)|H(0)|^2 = 2q|H(0)|^2$$

$$\rightarrow \quad R_Y(0) = q\Delta\omega_e|H(0)|^2/\pi = 2q\Delta f_e|H(0)|^2$$

Noise equivalent pass-band

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$$R_Y(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Y(\omega)e^{j\omega 0}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Y(\omega)\,d\omega = \frac{1}{2\pi}\int_0^{\infty} F_Y(\omega)\,d\omega$$

$$= \frac{1}{2\pi}F_Y(0)\Delta\omega_e$$

$$F_Y(0) = 2S_Y(0) = 2S_X(0)|H(0)|^2 = 2q|H(0)|^2$$

→ $$R_Y(0) = q\Delta\omega_e|H(0)|^2/\pi = 2q\Delta f_e|H(0)|^2$$

Average power or average output power of a system inputted by white noise

Average input power

System's Transfer function valued at 0

Noise equivalent pass-band (a angular frequency or frequency

Given any 3 out of the 4 parameters, we can get the last one

- Note that

$$\boxed{\begin{aligned} H(\omega) &= \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt \\ H(0) &= \int_{-\infty}^{\infty} h(t)dt \end{aligned}}$$

The impulse response $h(t)$

Sometimes, $h(t)$ is given instead of $H(0)$

Noise equivalent pass-band

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$$\int_0^\infty F_Y(\omega)d\omega = \Delta\omega_e F_Y(\omega_0)\text{, therefore we have}$$

$$R_Y(0) = \frac{1}{2\pi}\int_0^\infty F_Y(\omega)d\omega = \frac{1}{2\pi}\Delta\omega_e F_Y(\omega_0)$$

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Average power or average output power of a system inputted by white noise

Average input power

System's Transfer function valued at ω_0

Noise equivalent pass-band (at angular frequency or frequency)

Given any 3 out of the 4 parameters, we can get the last one

Noise equivalent pass-band

- Example 2: a white noise with power spectrum A pass through a low-pass system with $h(t) = U(t+2) - U(t-2)$, its noise equivalent pass-band is $\Delta\omega_e = 1 \times 10^6$Hz . If the average power of output on 1 Ω resistor（1欧姆电阻） is 1W（1瓦）, calculate A.

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$H(0) = \int_{-\infty}^{\infty} h(t) dt$$

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$$H(0) = \int_{-\infty}^{\infty} h(t)dt$$

Solution:

First we need to calculate: $H(0) = \int_{-\infty}^{\infty} h(t)dt = \int_{-2}^{2} 1dt = 4$

The formula is $R_Y(0) = A\Delta\omega_e|H(0)|^2/\pi$

Therefore, the average power output of the low-pass system is

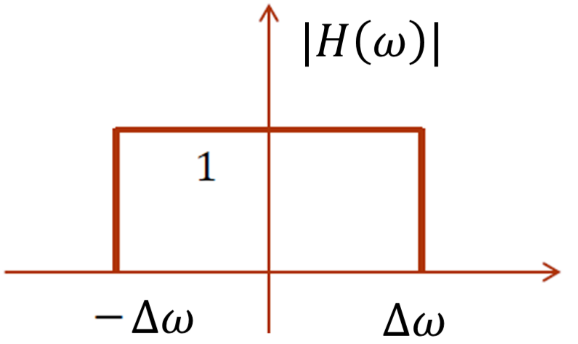
$$R_Y(0) = A\Delta\omega_e|H(0)|^2/\pi$$

→ $$R_Y(0) = 10^6 \times 4^{\,2} A/\pi = 1\text{W}$$

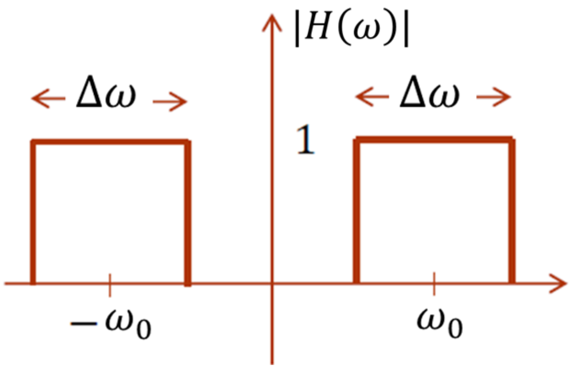
→ $$A = \frac{\pi}{16} \times 10^{-6}\ W/Hz \cdot \Omega$$

More examples

- 1: given an input stationary process $X(t)$ with power spectrum $S_X(\omega) = \frac{4\lambda}{4\lambda^2+\omega^2}$, and it is inputted to the ideal low-pass filter with amplitude $|H(\omega)| = 1$, calculate the average output power $R_Y(0)$ of the output $Y(t)$.



- 2: given an input white noise $X(t)$ with power spectrum $S_X(\omega) = N_0/2$, and it is inputted to the ideal band-pass filter as below, calculate the average output power of the output $Y(t)$.



Reading

- This week:
  - Text book: 7.4
  - Red book: 3.3 (限带过程) 注：限带过程的讲解将以此书为主导
- Next 2 weeks:
  - discrete time process + Spectrum Estimation basic + Optimal linear filter (& Matched filter)
  - Text book: 10.1, 10.2
  - Red book: 3.5 (最佳线性滤波器) 注：最佳线性滤波器的讲解将以此书为主导
  - And more examples

Experiment

- Go on with Experiment 2