

Stochastic Signal Processing

Experiment 3: Advance

Experimental Report 3 - Advance

- A chirp signal $X(t) = \cos(2\pi(f_0t + \frac{1}{2}kt^2))$, where $f_0 = 1000\text{Hz}$, $k = 12000\text{ Hz}$, and the signal starts from 0 to 0.1s ($t \in [t_{min} = 0, t_{max} = 0.1]$). The frequency of chirp signal will change with time.
- Quick Question: What is the frequency now?
- Answer: it is not a point, it is an area.
 - Introductions can be found online. For example:
<https://zhuanlan.zhihu.com/p/671652386>

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对这里的参数进行解释

瞬时频率： $f(t)$

c 为瞬时chirp度(斜率)

f_1 ：终止频率

f_0 ：扫频起始频率 T ：扫频周期

实验部分

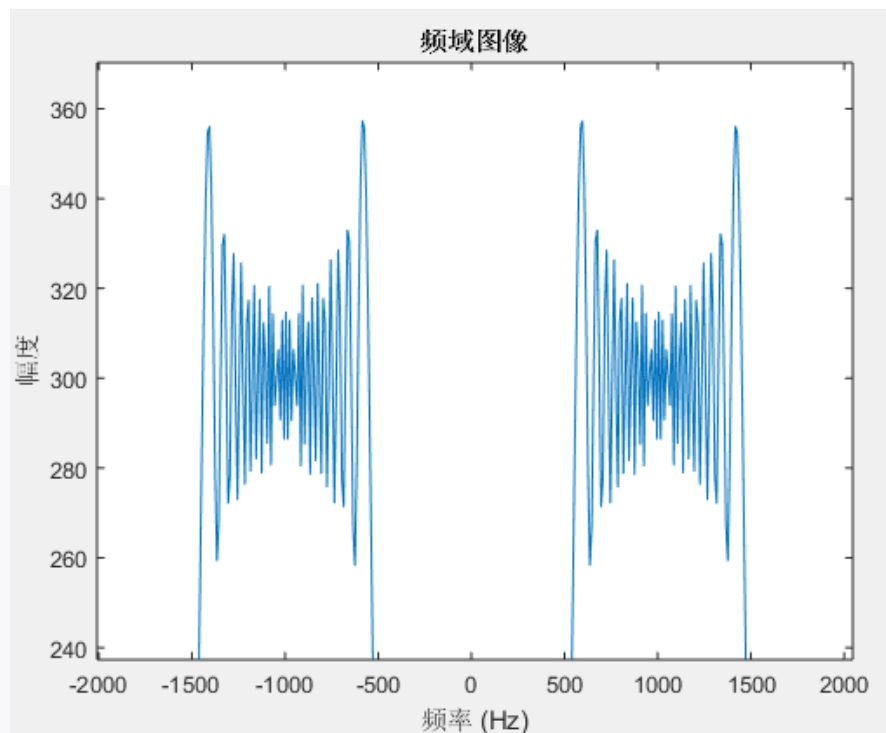
时域和频域分析

```
% 参数设置
A = 4;
phi_0 = 0;
f_0 = 500;
f_end = 1500;
T_period = 0.1;

% 采样参数
Fs = 15000; % 采样频率
T = 1/Fs; % 采样周期
t = 0:T:T_period-T; % 从0到周期进行采样

% 生成信号
c = (f_end - f_0) / T_period; % 计算线性调频斜率
x_t = A * cos(phi_0 + 2 * pi * (f_0 * t + 0.5 * c * t.^2));
```

这是一般的做法，即限定频率的上下限，然后依据信号的持续时间，去定义 k



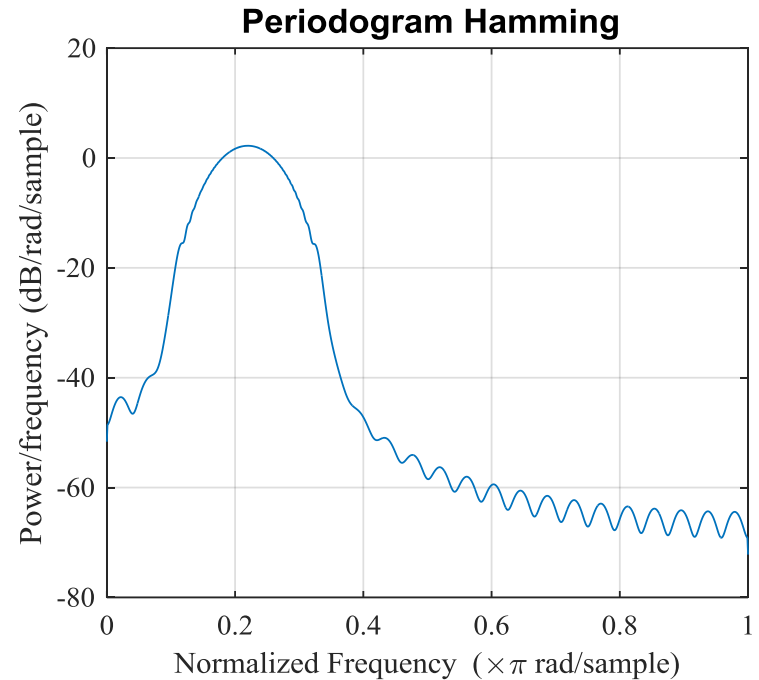
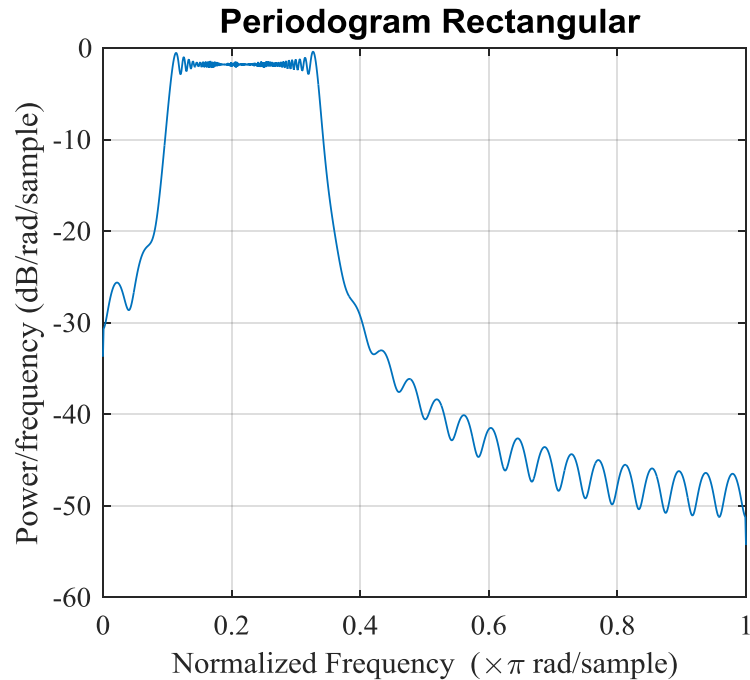
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- A chirp signal $X(t) = \cos(2\pi(f_0 t + \frac{1}{2} k t^2))$, where $f_0 = 1000\text{Hz}$, $k = 12000\text{ Hz}$, and the signal starts from 0 to 0.1s ($t \in [t_{min} = 0, t_{max} = 0.1]$). The frequency of chirp signal will change with time.
- We use the periodogram to see the power spectrum:

```
fs = 20000; % sample rate
f0 = 1000; %
% kf0 = 12000;
% kf0 = 20000;
T_end=0.1;
kf0 = 2000;
T_end=1;
T=1/fs;
t = 0:T:T_end-T; % sample point
x = sin(2*pi*(t.*f0+kf0.*t.^2));
% x = sin(2*pi*t.*(f0+kf0.*t)) + 0.1*randn(size(t));
subplot(1,2,1)
periodogram(x,rectwin(length(x))); % Periodogram with rectangular Window
set(gca,'fontsize',12,'fontname','times');
title('\fontname{} Periodogram Rectangular','fontsize',14);
subplot(1,2,2)
periodogram(x,hamming(length(x))); % Periodogram with Hamming Window
set(gca,'fontsize',12,'fontname','times');
title('\fontname{} Periodogram Hamming','fontsize',14);
set(gcf,'Units','centimeter','Position',[10 10 28 10]);
```

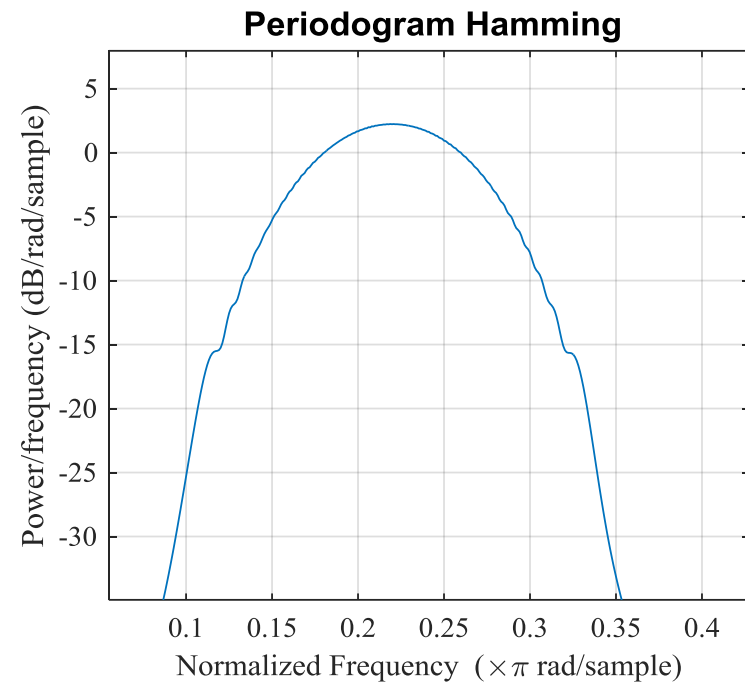
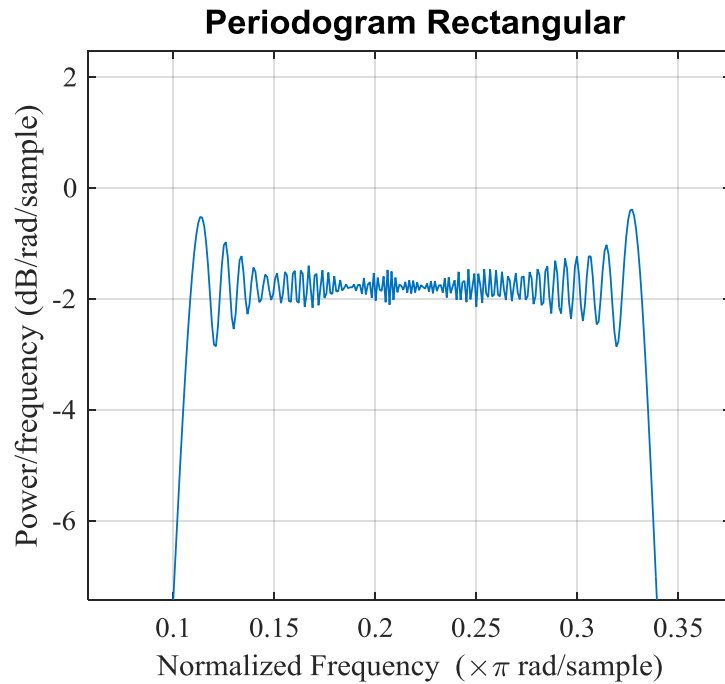
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- $t \in [t_{min} = 0, t_{max} = 0.1])$, $k = 12000 \text{ Hz}$



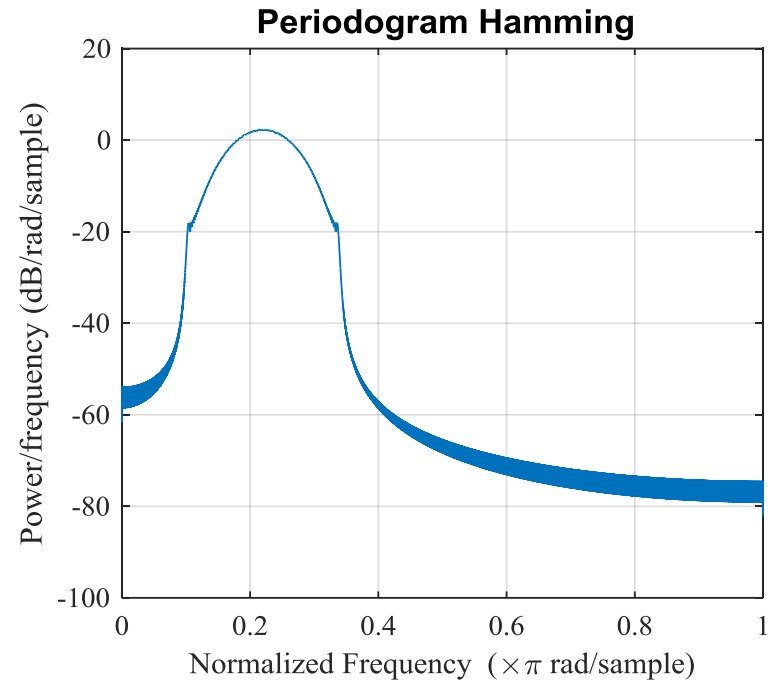
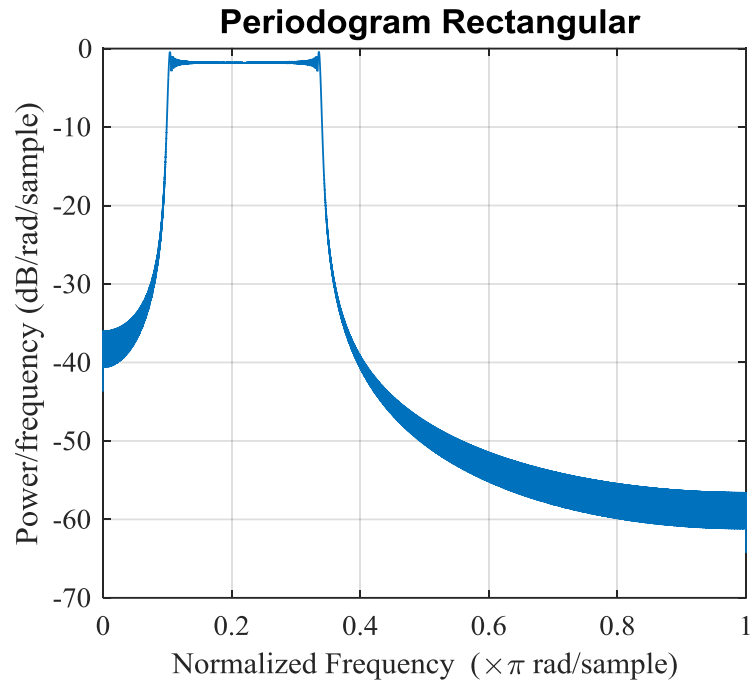
Experimental Report 3 - Advance

- $t \in [t_{min} = 0, t_{max} = 0.1])$, $k = 12000 \text{ Hz}$



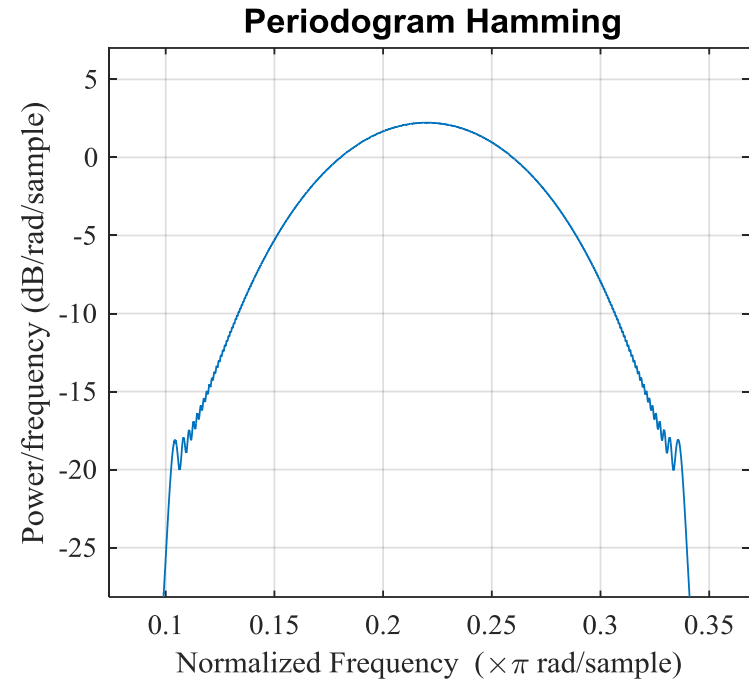
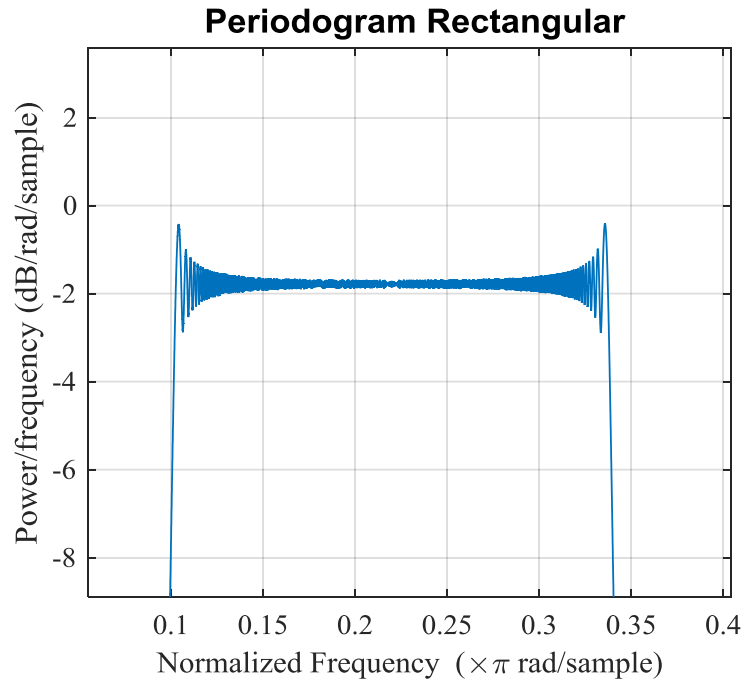
Experimental Report 3 - Advance

- $t \in [t_{min} = 0, t_{max} = 1])$, $k = 1200 \text{ Hz}$

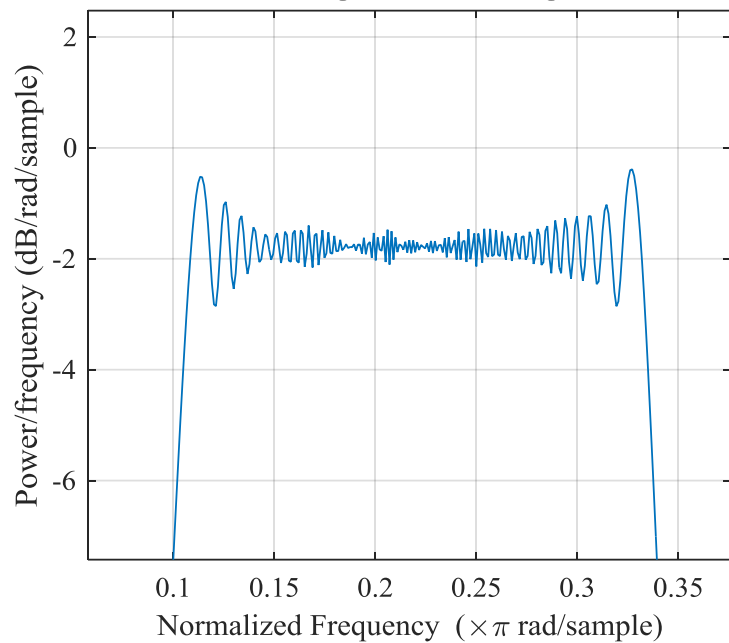


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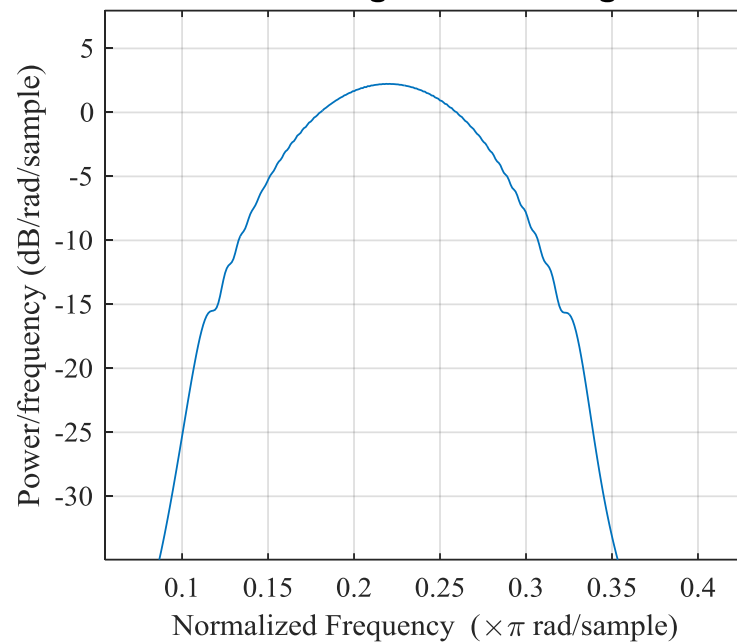
- $t \in [t_{min} = 0, t_{max} = 1])$, $k = 1200 \text{ Hz}$



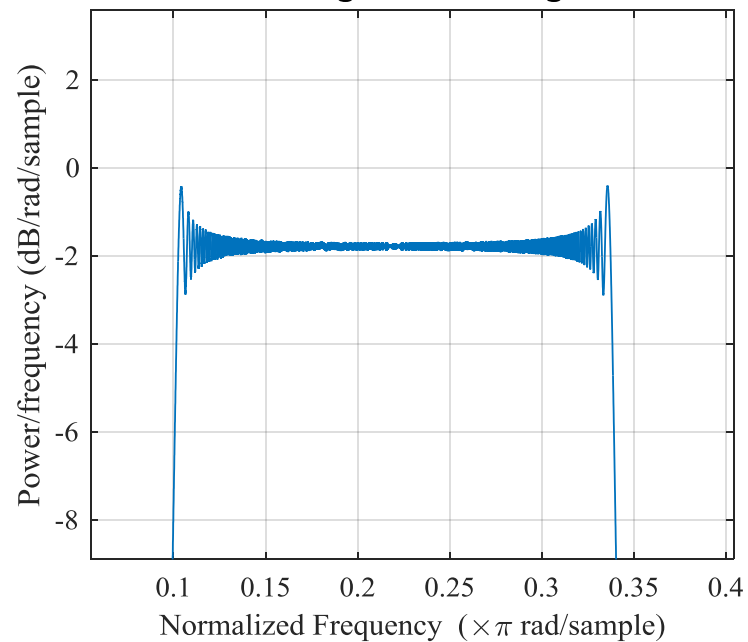
Periodogram Rectangular



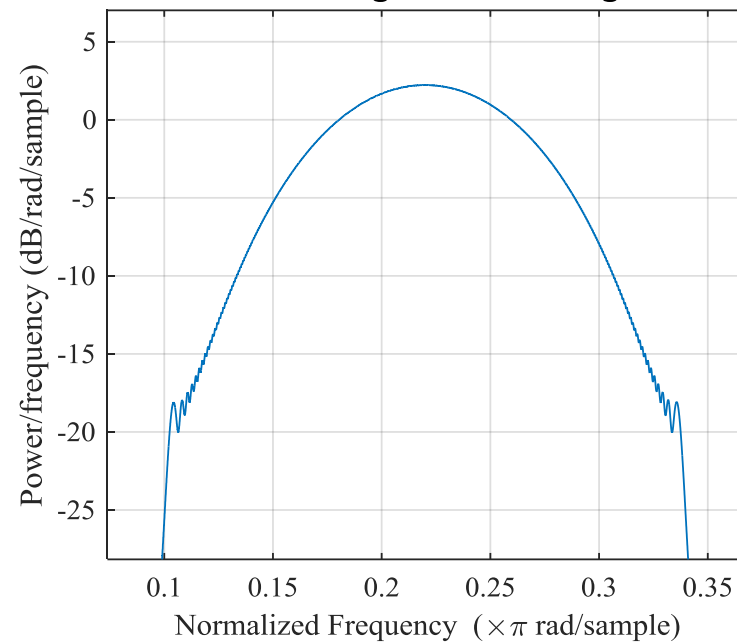
Periodogram Hamming



Periodogram Rectangular



Periodogram Hamming



Experimental Report 3 - Advance (40 points)

A chirp signal $X(t) = \cos(2\pi(f_0 t + \frac{1}{2}kt^2))$, where $f_0 = 1000\text{Hz}$, $k = 12000\text{ Hz}$, and the signal starts from 0 to 0.1s ($t \in [t_{min} = 0, t_{max} = 0.1]$). The frequency of chirp signal will change with time. Here we use a sampling frequency 50000Hz.

The chirp signal is sent out by a radar, and reflected by a target. We ignore any attenuation here, and assume that we receive a signal $Y(t) = X_1(t) + N(t)$, where $X_1(t)$ is a shift of $X(t)$. The duration of $X_1(t)$ is also 0.1s, but the end point is located between 0.11s and 1s in receiving system, or says, following a uniform distribution between [0.11, 1]. (in many radar, a ultra close target cannot be detected)

You are required to:

Experimental Report 3 - Advance (40 points)

1) Design a matched filter for chirp signal in Matlab. Use this matched filter to estimate the end time of this signal; and test it under different signal-to-noise ratios ($\text{SNR} = 10 * \log_{10}(P_s/\sigma^2)$ dB), where P_s is the average power spectrum. SNR should be designed by yourself. Finally, calculate the MSE and success rate of this system. (25 points)

Formula Definition

a. $\text{MSE} = \frac{1}{N} \sum_{i=1}^N (t_e - t_r)^2$, t_e is estimated time, t_r is true time

b. Success rate $P = K/N$, when $|t_e - t_r| < 0.03\text{s}$, it is successful, otherwise, it is failed. K represent the number of successes.

N is the number of tests (independent runs), you can use $N \geq 500$.

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1) Design a matched filter for chirp signal in Matlab. Use this matched filter to estimate the end time of this signal; and **test it under different signal-to-noise ratios** ($\text{SNR} = 10 * \log_{10}(P_s/\sigma^2)$ dB), where P_s is the average power spectrum. SNR should be designed by yourself. And, **calculate the MSE and success rate under different SNR**. (25 points)

Requirement:

1. Please plot your system flow chart in your report.
2. Give your MSE and success rate results, and analysis, under different SNR. (Hint: use table or figure, and you should choose an SNR range that can at least see '100% success' and '100% fail')

You are required to submit your code, and your code should directly give all the tables or figures in your report.

Experimental Report 3 - Advance (40 points)

2) Note that 1) is based on the 'match filter'. Now, assume that you did not send out any signal, but a target sent out such a signal. And, you do not know what signal the target sent, but you know that the duration is 0.1s. The end time of the signal is still in $[0.11, 1]$. Please use the periodogram (can use the Matlab default one or yours) to detect the end time of the signal, test it under different signal-to-noise ratios, and calculate the MSE and success rate of this new method. (15 points)

Hint:

- 1) if the signal is in $[0.35, 0.45]$, and what will happen if you do periodogram on time $[0.3, 0.4]$? $[0.35, 0.45]$? $[0.6, 0.7]$?
- 2) How to improve the accuracy?

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Requirement:

1. Please plot your algorithm flow chart in your report.
2. Give your MSE and success rate results, and analysis, under different SNR, and compare the results with 1). (Hint: use table or figure, and you should choose an SNR range that can at least see '100% success' and '100% fail')

You are required to submit your code, and your code should directly give all the tables or figures in your report.