第一次作业

2024.04.08 课堂上交

1. Multiple choices。(5*2=10 points)
   1) If $X = c$， where c is a constant, which is correct?
      A. $f_x(x) = U(x - c)$
      B. $f_x(x) = \delta(x - c)$
      C. $f_x(x) = \frac{1}{2}\delta(x - c) + \frac{1}{2}\delta(x + c)$
      D. $f_x(x) = \delta(x)$
      E. *None of the above*

   2) *If* $Y = 2X$， which is correct?
      A. $P_Y(y) = \frac{1}{2}P_X\left(\frac{1}{2}y\right)$
      B. $P_Y(y) = 2P_X(2y)$
      C. $P_Y(y) = P_X(2y)$
      D. $P_Y(y) = P_X\left(\frac{1}{2}y\right)$
      E. *None of the above*

2. (30 points) the pdf of the r.v X is：

$$f(x) = \begin{cases} c(x^2 + 2x - 2), & 0 \le x \le 1 \\ 0, & \text{其他} \end{cases}$$

   1) Find c。
   2) Calculate the CDF of X, which is $F_X(x)$.

3. (20 points) Given the r.v X uniformly distributed in $[a, b]$, $a \ge 0$, and r.v Y uniformly distributed in $[0, X]$, find:
   1) $E(Y|X = x), a \le x \le b$
   2) $E(Y)$

   Hint: $E(Y) = E(E(Y|X))$

4. (20 points) Two boxes B_1 and B_2 both contain 100 balls. The first box (B_1) has 70 red balls and 30 blue balls, while the second box (B_2) has 80 red balls and 20 blue balls. Suppose a box is selected randomly (50% B_1 and 50% B_2), and one ball is picked out:
   1) What is the probability that it is a red ball?
   2) Suppose that the ball picked out is red, what is the probability that it come from box B_1?

5. (20 points) Sssume that the Θ is a r.v X uniformly distributed in $(0,2\pi)$, and $X = cos2\Theta$， $Y = sin2\Theta$, then:
   1) Is X and Y orthogonal? Prove it
   2) Calculate the correlation coefficient of X and Y