

## Electromagnetic field and electromagnetic wave experiment two report

$$adj = \frac{\text{序号}}{10}$$

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序号：22

$$adj = \frac{\text{序号}}{10} = 2.2$$

### A. Problem one

#### a. Theoretical Calculation

Question: 内外半径分别为  $a$  和  $b$  的无限长空心圆柱中均匀分布轴向电流  $I$ , 求柱内外的磁感应强度。

1. When  $r < a$

$$\oint B dl = \mu_0 I = 0 \text{ then } B = 0$$

2. When  $a \leq r \leq b$

$$\oint B dl = \mu_0 I = \mu_0 \frac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)} I$$

$$B 2\pi r = \mu_0 \frac{r^2 - a^2}{b^2 - a^2} I$$

$$B = \frac{\mu_0 I (r^2 - a^2)}{2\pi r (b^2 - a^2)}$$

3. When  $r > b$

$$\oint B dl = \mu_0 I$$

$$B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

#### b. Simulation model

simulation 无限长空心圆柱轴线为  $z$  轴, 内外半径分别为 1mm 和 1.5mm, 其上均匀分布轴向电流  $(1+adj)\text{A}$ , 求  $x$  轴上  $\mathbf{B}$  的大小。

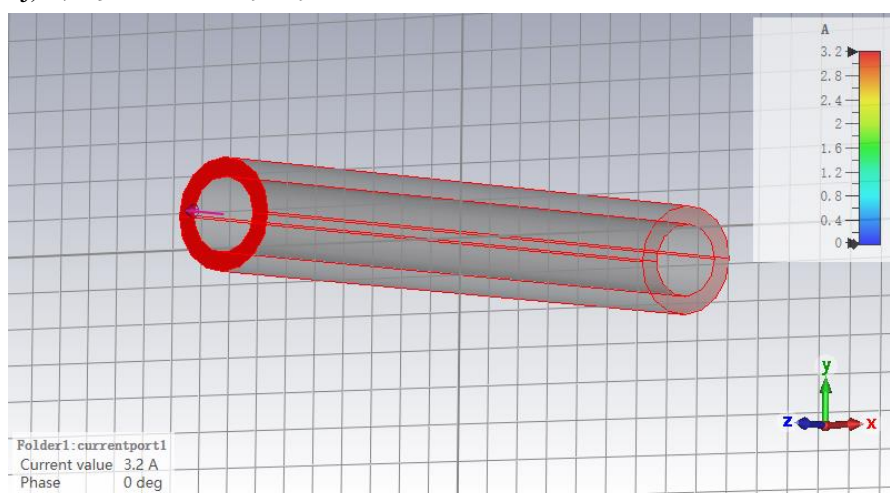


Figure A.1 Model cut graph

c. Comparison and analysis of simulation results and theoretical calculation results

1) The figure of simulation results and theoretical calculation results

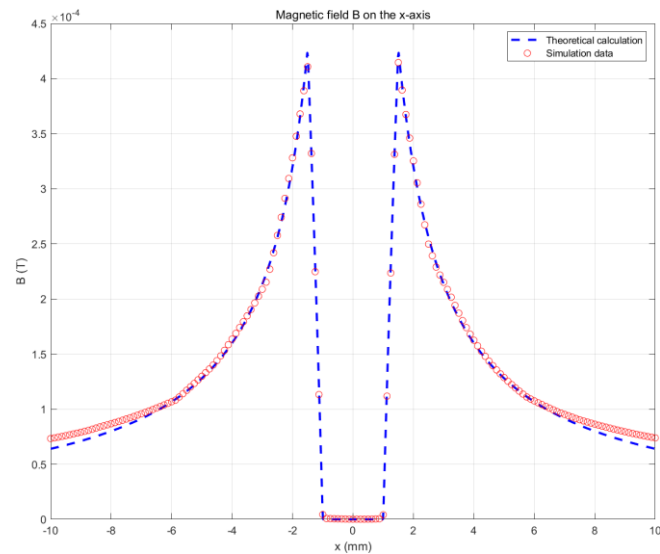


Figure A.2 simulation results and theoretical calculation results

2) The analysis of simulation results and theoretical calculation results

For  $0 \leq x < 1.5$  mm: The B distribution is zero

For  $1.5 \leq x < 2$  mm: The B distribution increases linearly.

For  $x \geq 2$  mm: The B distribution decreases according to an inverse proportional function.

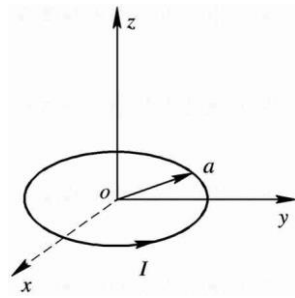
For  $0 \leq x \leq 2$  mm: The calculation and simulation values are in perfect agreement

For  $x > 2$  mm: Initially, the simulation values coincide with the calculation values. However, as  $x$  increases further, the simulation values gradually become larger than the calculation values. Although there exists tiny error, it is acceptable since it is impossible to absolutely simulate the real world condition.

## B. Problem two

### a. Theoretical Calculation

Question: 求载流的圆形导线回路在圆心处的  $\mathbf{B}$ 。



$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I dr \times \vec{\gamma}}{r^3} = \frac{\mu_0 I dl}{4\pi r^2}$$

since  $by = bz = 0$

$$B = \int B_x = \int \frac{\mu_0 I dl}{4\pi r^2} \sin \theta$$
$$B = 2\pi a \frac{\mu_0 I a}{4\pi r^3} = \frac{\mu_0 I a^2}{2} \left( \frac{1}{\sqrt{a^2 + z^2}} \right)^3$$

### b. Simulation model

Simulation: 载流的圆形导线半径为(20+adj)mm,求圆心处  $\mathbf{B}$  的大小。

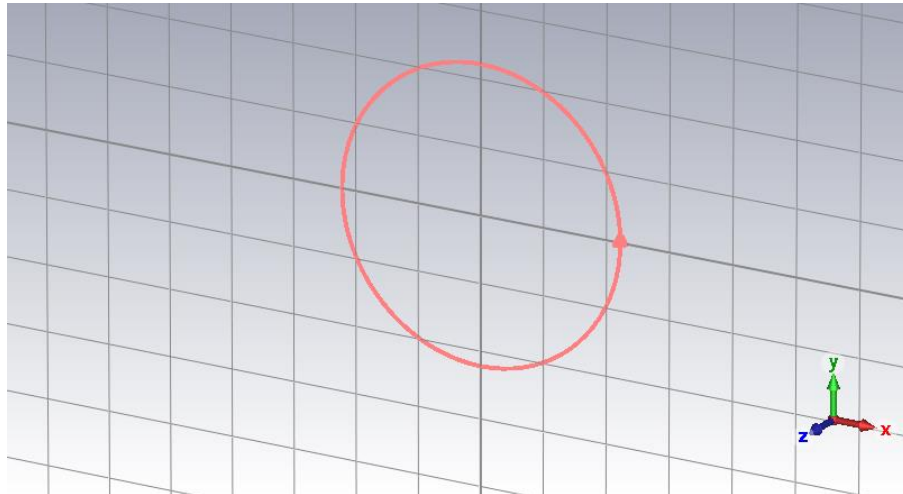


Figure B.1 Model cut graph

### c. Comparison and analysis of simulation results and theoretical calculation results

#### 1) The figure of simulation results and theoretical calculation results

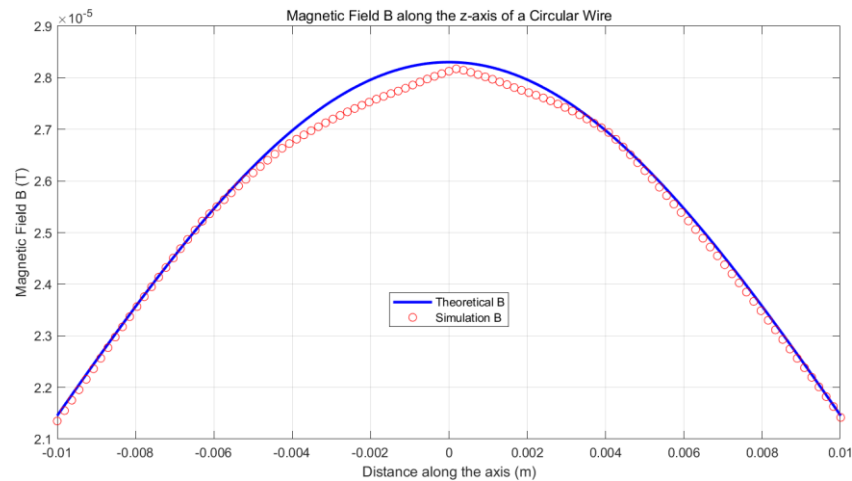


Figure B.2 simulation results and theoretical calculation results

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Simulation magnetic field B at z = 0: 2.8128e-05 T
Center magnetic field B (theory): 2.8303e-05 T
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Figure B.3 The B at the center of the circle

2) The analysis of simulation results and theoretical calculation results

At  $x=0$  mm: The B distribution reaches its maximum value. The results of simulation and theory are almost same.

For  $x>0$  mm: The B distribution decreases according to an inverse proportional function.

Possible Reasons for Discrepancy:

The observed discrepancies around  $x=0$  mm may be attributed to:

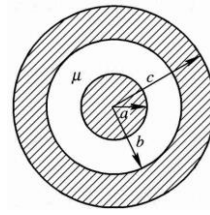
Solver Accuracy: The accuracy of the solver used in the simulation could also impact the results.

Despite these discrepancies, the errors are considered acceptable since it is inherently challenging to perfectly simulate real-world conditions.

### C. Problem Three

#### a. Theoretical Calculation

Question: 同轴线内外导体半径为  $a$ , 外导体的内半径为  $b$ , 外半径为  $c$ , 如下图。设内外导体分别流过反相的电流, 两导体之间介质的磁导率为  $\mu$ , 求各区域的  $\mathbf{H}$ 、 $\mathbf{B}$ 。若电流流向  $+z$  方向



When  $r < a$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \frac{\pi r^2}{\pi a^2} I$$

$$2\pi r B = \mu_0 I \frac{r^2}{a^2}$$

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

$$H = \frac{I r}{2\pi a^2}$$

When  $a \leq r \leq b$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

$$B = \frac{\mu I}{2\pi r}$$

$$H = \frac{I}{2\pi r}$$

When  $b \leq r \leq c$

$$\int \mathbf{B} \cdot d\mathbf{l} = 2\pi r B = \mu_0 \left( I - \frac{\pi (r^2 - b^2)}{\pi (c^2 - b^2)} I \right)$$

$$B = \frac{\mu_0 I}{2\pi (c^2 - b^2)} \frac{(c^2 - r^2)}{r}$$

$$H = \frac{I}{2\pi (c^2 - b^2)} \frac{(c^2 - r^2)}{r}$$

When  $r > c$ :

$$\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 I = 0$$

$$B = 0$$

$$H = 0$$

#### b. Simulation model

Simulation: 同轴线轴线为  $z$  轴,  $a=0.5\text{mm}$ ,  $b=1\text{mm}$ ,  $c=1.5\text{mm}$ , 两导体之间介质的相对磁导率为  $(4+\text{adj})$ , 求  $x$  轴上的  $\mathbf{H}$ 、 $\mathbf{B}$ 。

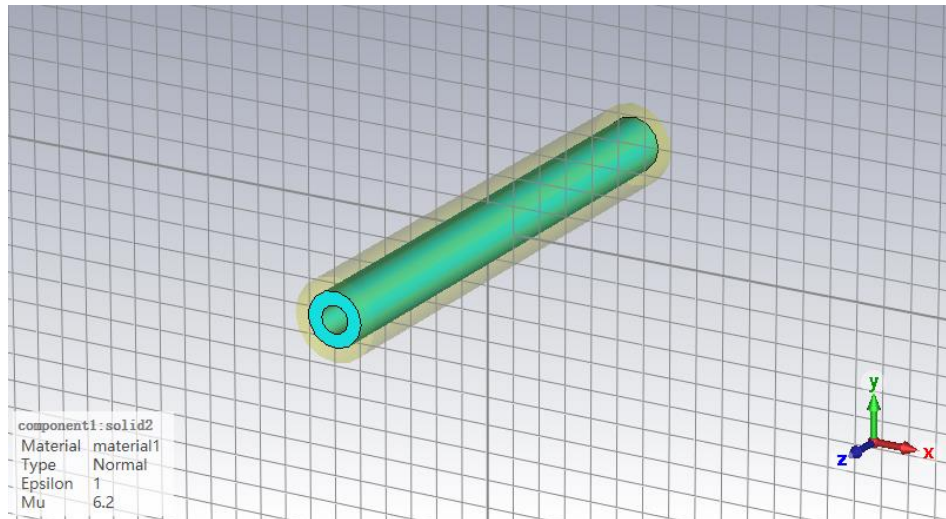


Figure C.1 Model cut graph

c. comparison and analysis of simulation results and theoretical calculation results

1) The figure of simulation results and theoretical calculation results

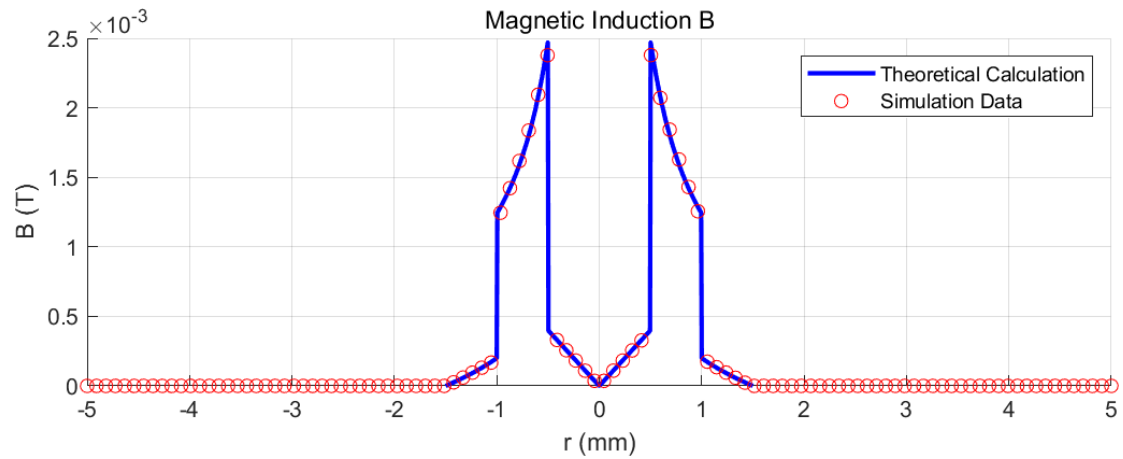


Figure C.2 The result of B

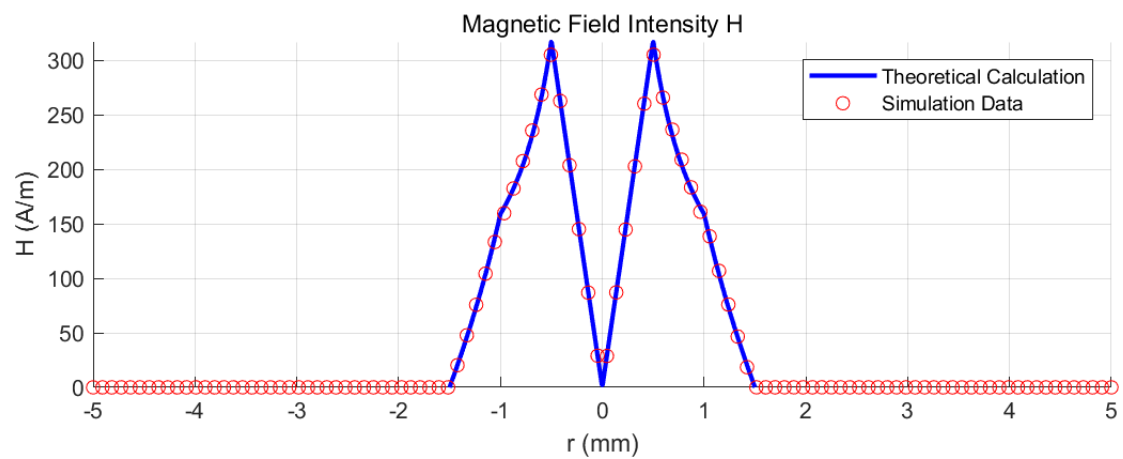


Figure C.3 The result of H

2) The analysis of simulation results and theoretical calculation results

At  $x=0$  mm:

The B and H distribution is 0

For  $0 < x \leq 0.5$  mm:

The B and H distribution increases linearly.

At  $x = 0.5$  mm:

The B and H distribution reaches its maximum

For  $0.5 < x < 1$  mm:

The B and H distribution decreases according to an inverse proportional function.

For  $1 < x < 1.5$  mm:

The B and H distribution decreases linearly.

And because of the change of  $\mu$  when  $x = 1$  mm. So B and H have different change when  $x = 1$  mm. The H drop suddenly when  $x = 1$  mm

For  $x \geq 1.5$  mm:

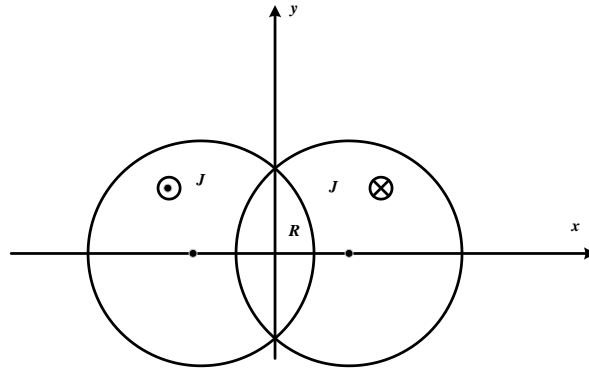
The B and H distribution is 0

Despite there are some minor discrepancies, the errors are considered acceptable because it is challenging to perfectly simulate real-world conditions.

#### D. Problem four

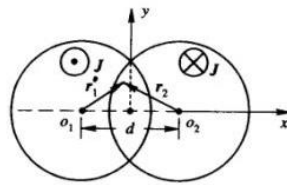
##### a. Theoretical calculation

Question: 两个半径都为  $a$  的圆柱体，轴间距为  $d$ ,  $d < a$ , 如下图。除两柱重叠部分  $R$  外，两柱上各有大小相等、方向相反的电流，密度为  $J$ , 求区域  $R$  的  $\mathbf{B}$ 。



By adding current distributions with equal values (density  $J$ ) and opposite directions in the overlapping areas, the current distribution of the original problem can be transformed into a uniformly distributed forward current within one cylinder and a uniformly distributed reverse current within another cylinder. The magnetic field generated by it can be calculated using the superposition principle.

The magnetic induction intensity generated by the current in the positive direction (left cylinder) in the overlapping area



$$\oint B_1 dL = 2\pi r_1 B_1 = \mu_0 \pi r_1^2 J$$

$$B_1 = \frac{\mu_0 J r_1}{2}$$

As the same reason, we can calculate the 2<sup>nd</sup> cylinder (right cylinder) in the overlapping area

$$B_2 = -\frac{\mu_0 J r_2}{2}$$

Then we can get the formula below through

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 J}{2} \vec{e}_y$$

So the value of  $B$  from  $-0.4$  mm to  $0.4$  mm:

$$B = \frac{\mu_0 J}{2}$$



### b. Simulation model

Simulation: 如下图所示, 两个圆柱半径为 1mm, 轴线与 z 轴平行, 轴线位置分别为  $x=0.5\text{mm}$  和  $x=-0.5\text{mm}$ , 除两柱重叠部分 R 外, 两柱上各有大小相等、方向相反的电流 1A, 求 x 轴上  $[-0.4\text{mm}, 0.4\text{mm}]$  范围上  $\mathbf{B}$  的大小。

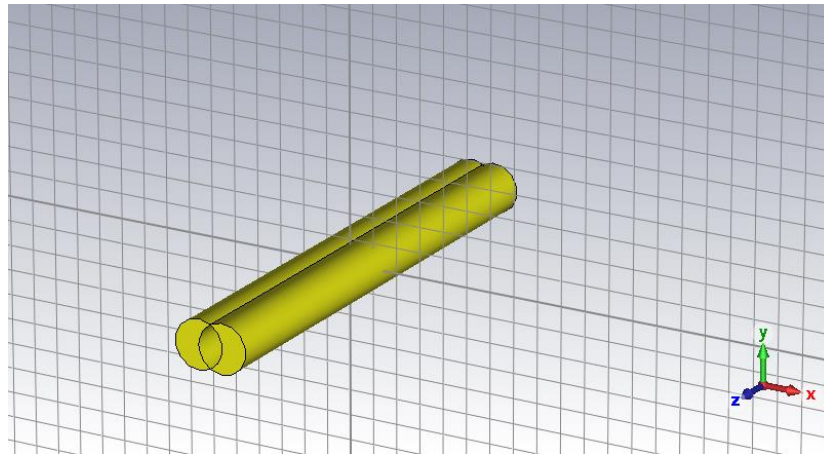


Figure D.1 Model cut graph

### c. Comparison and analysis of simulation results and theoretical calculation results

#### 1) The figure of simulation results and theoretical calculation results

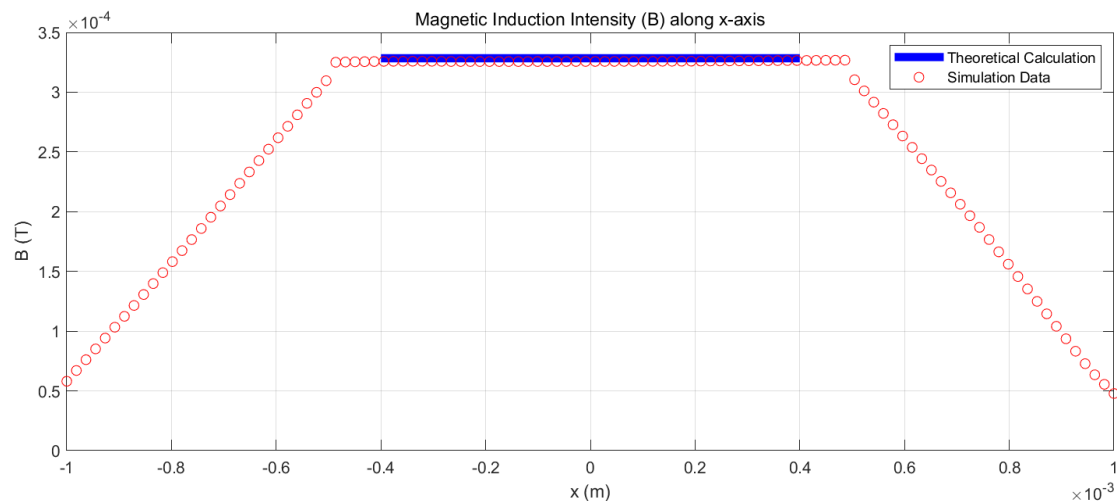


Figure D.2 The result of theory and simulation

#### 2) The analysis of simulation results and theoretical calculation results

Due to the high accuracy corresponding to setting  $-0.4\text{mm}$  to  $0.4\text{mm}$ , the displayed error is obvious and cannot fit well with MATLAB simulation.

So I set  $-1\text{mm}$  to  $1\text{mm}$ . We first analyze the tendency of the B distribution for calculation and simulation value. In the range of  $[-0.4, 0.4]\text{mm}$ , the B distribution is a constant that means the B distribution is the same value along these range.

From the figure, we observe that the calculation value and the simulation value is basically coincide which indicates that the setting of our experiment is reasonable.