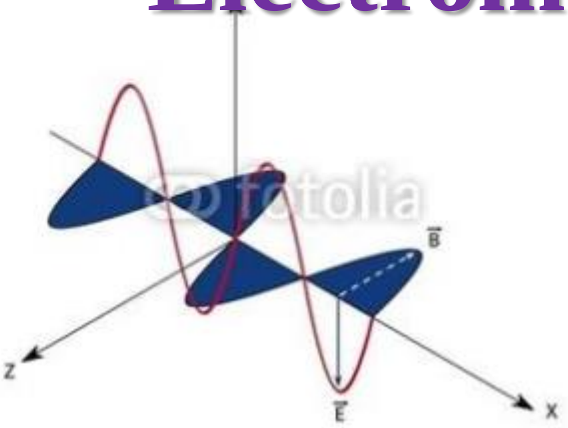
Electromagnetic Fields and Waves



Chapter 7

Electrostatic Field 1

-Electric Field in Substrate

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Electrostatic Field and Potential in Substrate

Content

- Auxiliary辅助 Function of the Electric Potential and Electrostatic Field
- Electrostatic Field in Substrate
- Basic Functions of Electrostatic Field

Electrostatic Field in Substrate物质

- ✓ The above discussions are the properties in the vacuum
- ✓ Properties of Electrostatic Field in the Conductor导体
- ✓ Properties of Electrostatic Field in the Dielectric 介质

Properties of Electrostatic Field in the Conductor

【 Basic Process】

➢ The valence electrons价电子 of the atom原子 in the conductor get rid of restraint of a single atom, and become an electric charge with negative charge, thus the atom turns into a positive ion, called Atomic Solid原子实.

➢ Atomic solids arrange regularly in the crystalline lattice结晶点阵, the free electrons moves irregularly without the external electric field, thus the conductor doesn’t have the electric properties to the outside world .

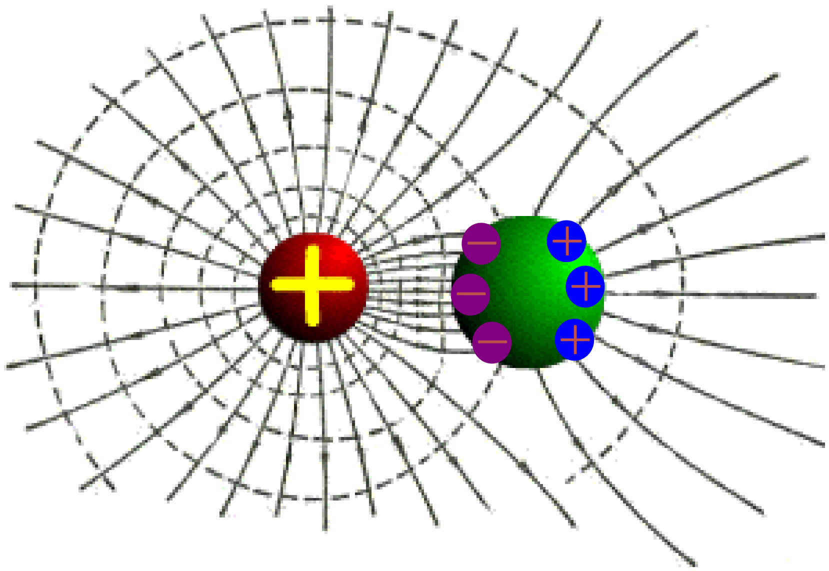
➢ When the external electric field exists, the free electrons in the conductor moves along the opposite direction of the electric field. When the electric field generated by the electric charges on the conductor surface is equal to the external electric field with the opposite directions, the combined electric field inner the conductor equals to zero, then the electrons won’t have the macroscopic motion.

Properties of Electrostatic Field in the Conductor

【 Charge Induction感应 】

➢ Affected by the electrostatic field, the free electrons accumulate on the conductor surface, which form a negative electric charge region. While the positive ions form a positive electric charge region, the phenomenon is called Charge Induction.

➢Furthermore, the isolated conductor doesn't have any net electric charge.

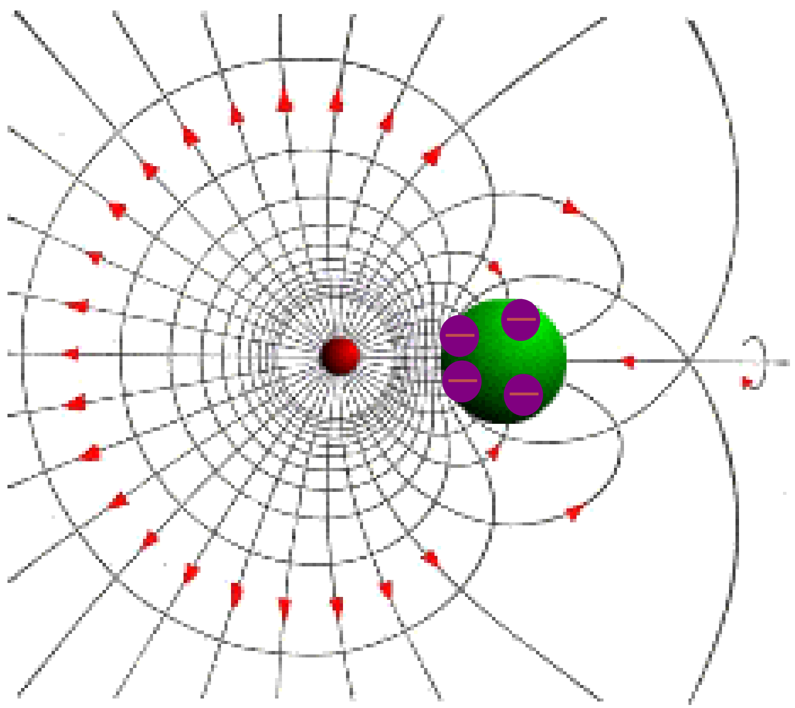


Electric field of isolated conductor

Properties of Electrostatic Field in the Conductor

【 Charge Induction 】

➢ If the conductor is connected to the ground, the free electrons move to the conductor, thus the conductor will accumulate some net electric charges.

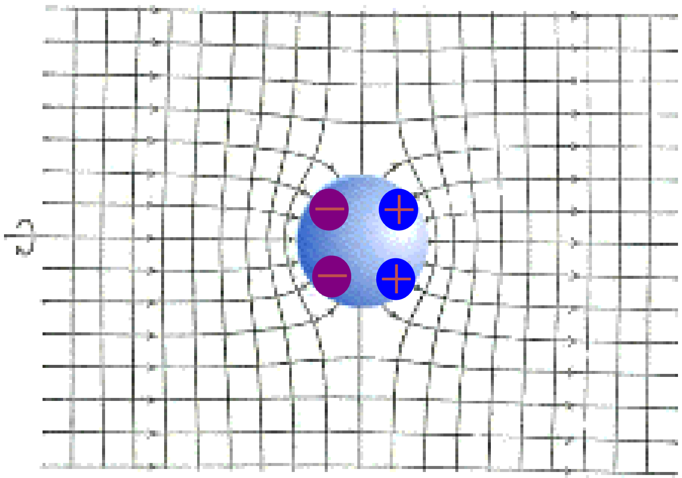


Electric field of ground conductor

Properties of Electrostatic Field in the Conductor

【 Equipotential等电位 Body】

➢ The total electric field in the conductor always remains at zero, thus the electric potential in the conductor is a constant, which implies that the conductor is an equipotential body.



Electric field of spherical conductor

Properties of Electrostatic Field in the Dielectric介质

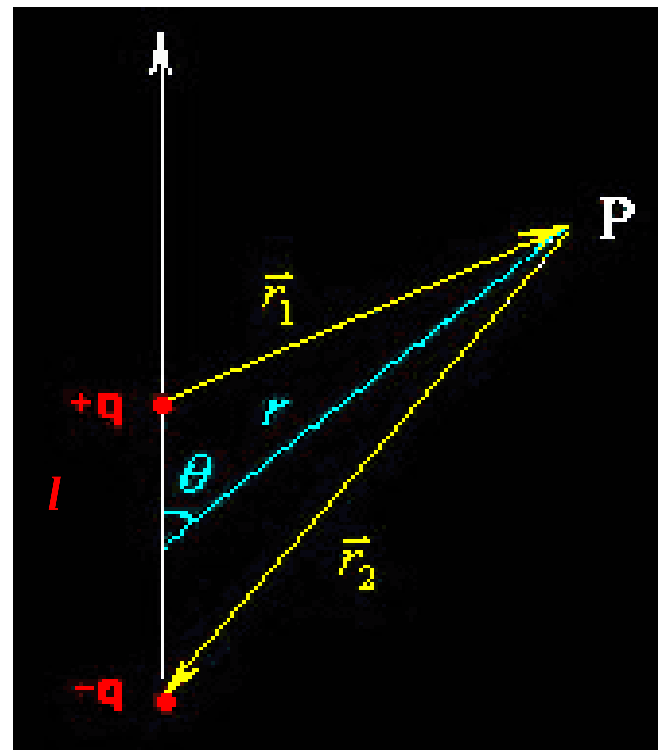
【 Electric Field of the Point Dipole电偶极子 】

- Point Dipole :Two electric charges closed to each other with equal charge but different sign.

- The electric potential of the point ***P*** which stays far away from the point dipole is obtained by:

$$\phi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_1 r_2}$$



Properties of Electrostatic Field in the Dielectric

- If r is large enough, and $l << r$, so :

$$r_2 - r_1 = l\cos\theta$$

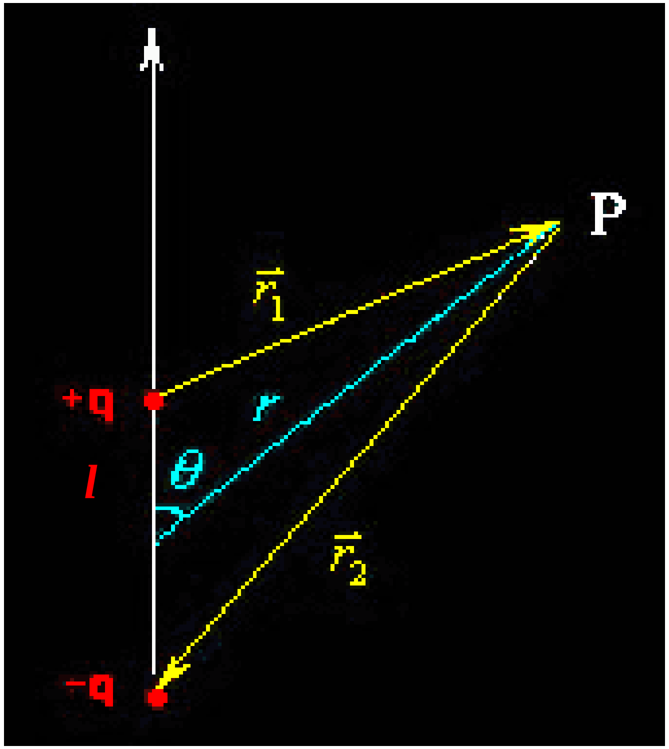
$$r_2 r_1 = \left(r - \frac{l}{2}\cos\theta\right)\left(r + \frac{l}{2}\cos\theta\right)$$

$$\approx r^2$$

- Then the electric potential :

$$\phi = \frac{ql\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{\vec{p}\bullet\hat{a}_r}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\bullet\vec{r}}{4\pi\varepsilon_0 r^3}$$



Where: $\vec{p} = q\vec{l}$ called **electric dipole moment**电矩, the positive direction is from $-q$ to $+q$.

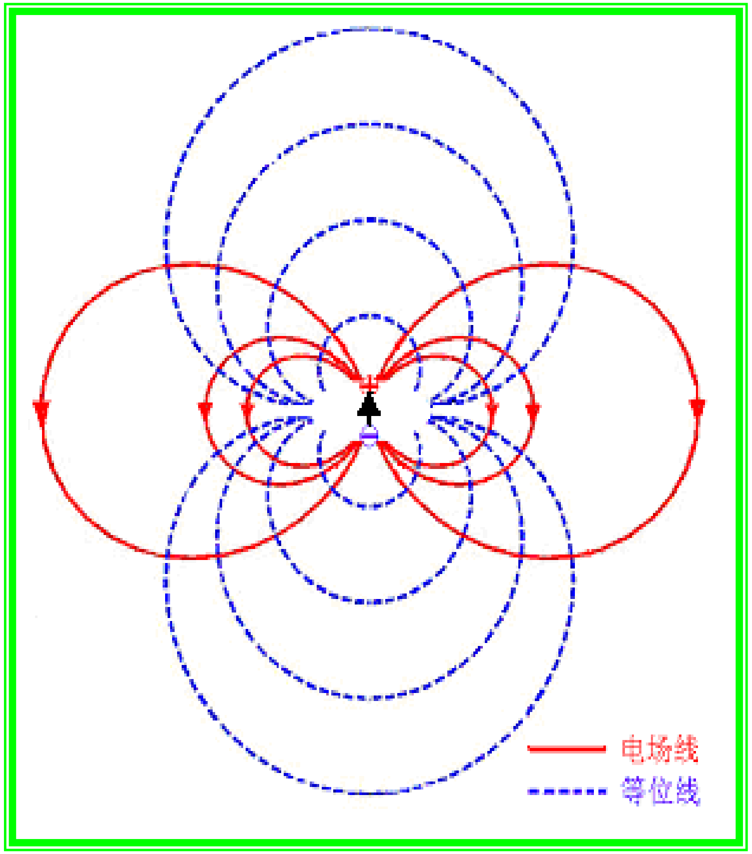
Properties of Electrostatic Field in the Dielectric

➢ Electric field of the point dipole can be derived from the electric potential ϕ as follow:

$$\vec{E} = -\nabla\phi$$

$$= -\left(\hat{a}_r \frac{\partial \phi}{\partial r} + \hat{a}_\theta \frac{1}{r}\frac{\partial \phi}{\partial \theta}\right)$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left(\hat{a}_r 2\cos\theta + \hat{a}_\theta \sin\theta\right)$$



Electric dipole moment电矩, the positive direction is from -q to +q.

Properties of Electrostatic Field in the Dielectric

【 Dielectric in the Electrostatic Field 】

- Two types of dielectric molecules

✓ Non-polar无极 molecules分子: The functional focus of the positive and negative electric charges are coincided. They doesn’t have the electric properties without the external electric field.

Nonpolar molecule

Polar molecule

Nonpolar molecule

Polar molecule

Properties of Electrostatic Field in the Dielectric

【 Dielectric in the Electrostatic Field 】

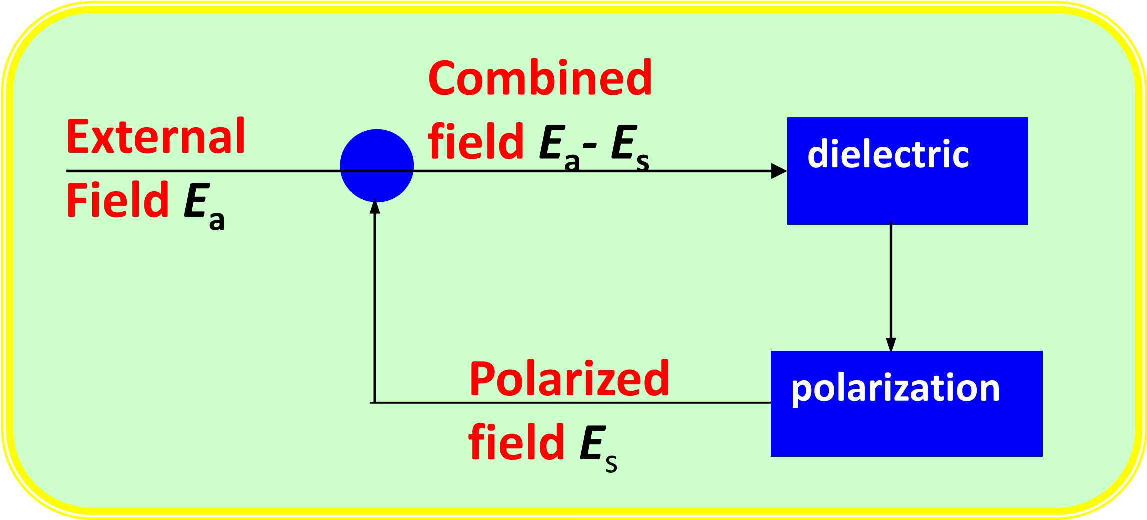
✓ Polar有极 molecules : The positive and negative electric charges inside the molecule form the point dipole, and the molecules move irregularly when there is no external electric field, thus the point dipoles in different molecules have different direction of the electric dipole moment, thus the molecules have no electric properties to the outside world .

✓ Polarization极化 : Under the effect of the external electric field, the function focus of the positive and negative electric charges of non-polar molecules have a relative displacement, while the point dipoles of the polar molecules tends to regular arrangement, both of them rendered to the distribution in the form of the electric dipole moment.

Properties of Electrostatic Field in the Dielectric

【 Dielectric in the Electrostatic Filed 】

✓ Polarization charge: Electric charges generated by the polarization. The polarization charge can also be called bound charge, because the polarization charge is bounded inside the point dipole. The extra electric field generated by the polarization charges just weaken the external field, not totally cancel it, thus the combined electric field isn't equal to zero in general.



Properties of Electrostatic Field in the Dielectric

【 Polarization intensity】

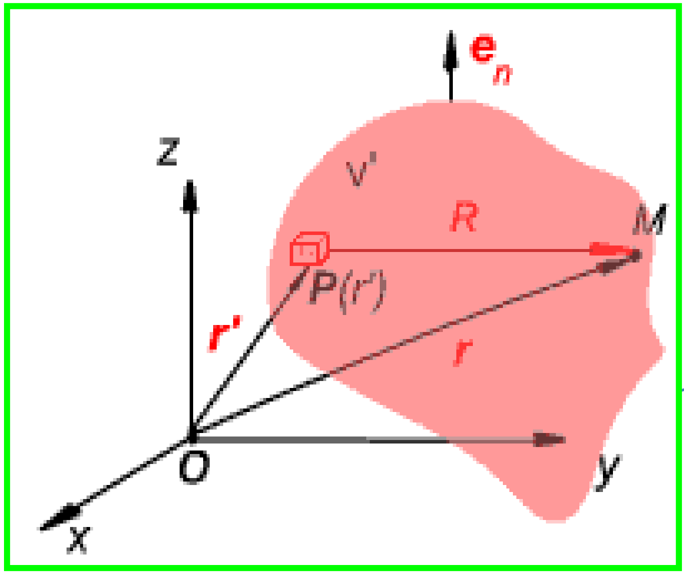
➢ After polarization, the electric dipole moment in an unit volume is :

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{N} \vec{p}_i}{\Delta V}$$

【 Density of the Polarization charge】

➢ Electric dipole moment in a volume element is :

$$\vec{p} = \vec{P} dV'$$



Properties of Electrostatic Field in the Dielectric

$$\phi = \frac{\vec{p} \bullet \vec{r}}{4\pi\varepsilon_0 r^3}$$

- Electric potential:

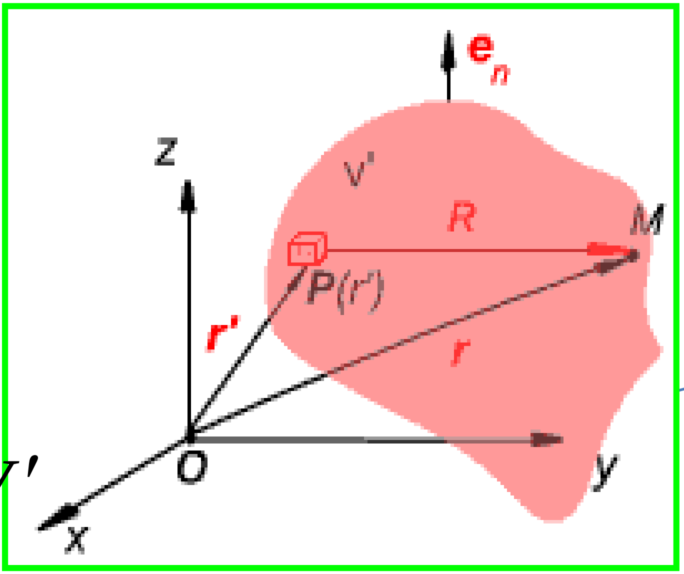
$$d\phi = \frac{\vec{P} \cdot \vec{R}}{R^3} dV'$$

$$= \vec{P} \bullet (-\nabla \frac{1}{R}) dV' = \vec{P} \bullet (\nabla' \frac{1}{R}) dV'$$

By using: $\frac{\vec{R}}{R^3} = -\nabla \frac{1}{R} = \nabla' \frac{1}{R}$

- Electric potential in a volume V of point dipoles:

$$\phi(\vec{r}) = \int d\phi = \frac{1}{4\pi\varepsilon_0} \int_V \vec{P} \cdot \left(\nabla' \frac{1}{R} \right) dV'$$



Properties of Electrostatic Field in the Dielectric

By using: $$\vec{P}\cdot\nabla'\frac{1}{R}=\nabla'\cdot\left(\vec{P}\frac{1}{R}\right)-\frac{1}{R}\nabla'\cdot\vec{P}$$

We get: $$\phi(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int_V\vec{P}\cdot\left(\nabla'\frac{1}{R}\right)dV'$$

$$\phi(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int_V\nabla'\cdot\left(\vec{P}\frac{1}{R}\right)\mathrm{d}V'+\frac{1}{4\pi\varepsilon_0}\int_V\frac{-\nabla'\cdot\vec{P}}{R}\mathrm{d}V'$$

$$\phi(\vec{r})=\frac{1}{4\pi\varepsilon_0}\oint_S\frac{\vec{P}\cdot\hat{n}\mathrm{d}S'}{R}+\frac{1}{4\pi\varepsilon_0}\int_V\frac{-\nabla'\cdot\vec{P}}{R}\mathrm{d}V'$$

- Then, we get the density of volume charge and surface charge:

$$\rho_p=-\nabla\cdot\vec{P} \qquad \sigma_{sp}=\vec{P}\cdot\hat{n}$$

Properties of Electrostatic Field in the Dielectric

【Isotropic各向同性 Dielectric】

- The electric dipole moment $\vec{P}$ always has the same direction with the external field $\vec{E}$, and has the relationship:

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

Where, χ_e (Kai) is dielectric polarization rate. It is used to measure how susceptible (or sensitive) a given dielectric is to electric fields.

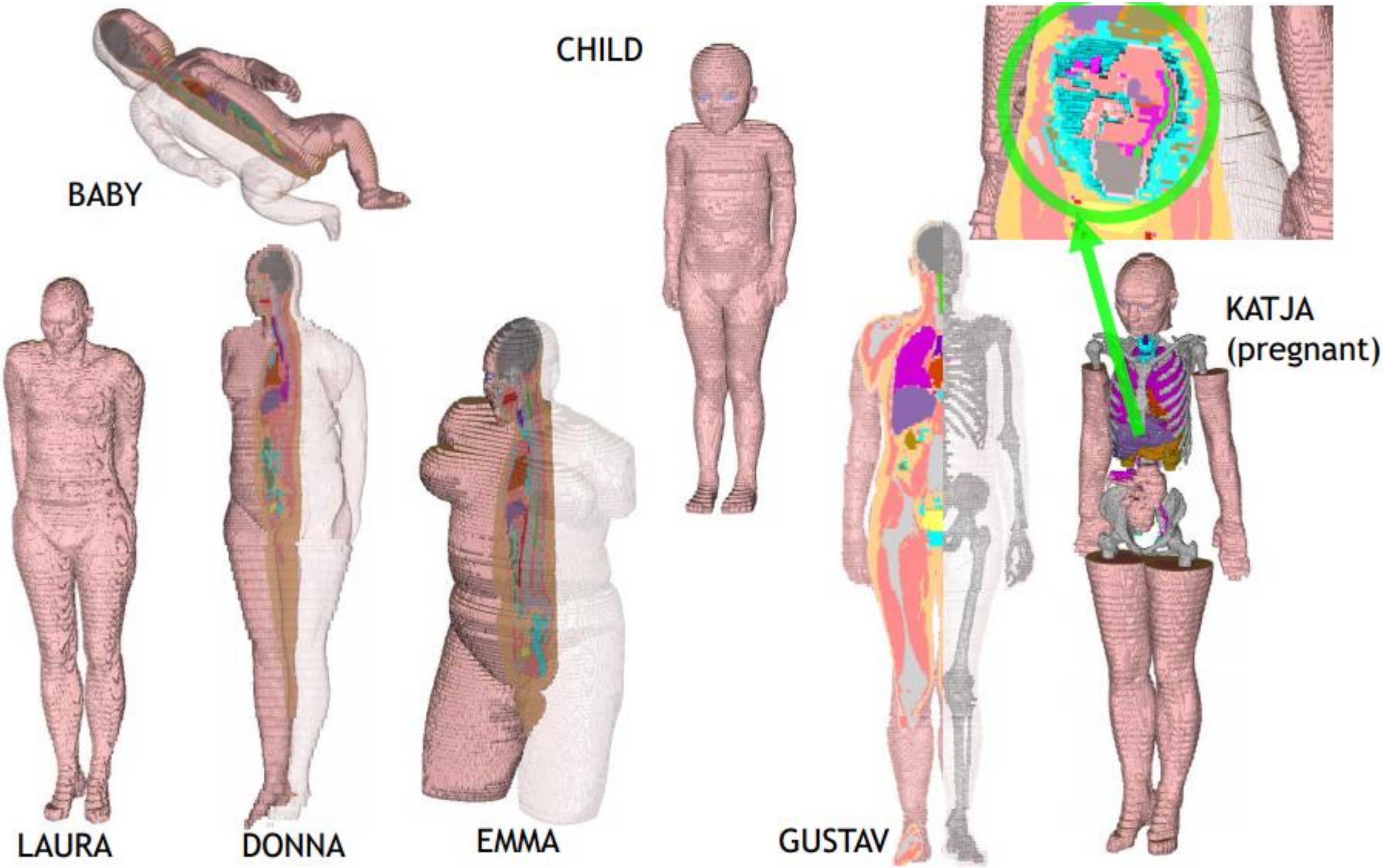
【Anisotropy各向异性 Dielectric】

- Polarization intensity is inconsistent in the dielectric, and has the relationship:

$$\vec{P} = [\chi_e] \varepsilon_0 \vec{E}$$

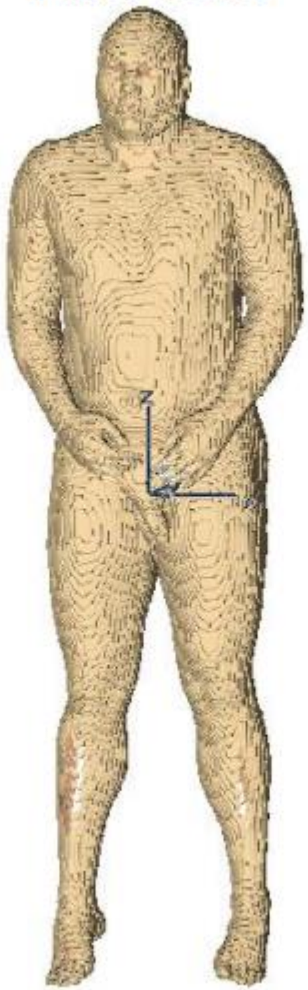
Where, $[\chi_e]$ is the dielectric polarization matrix.

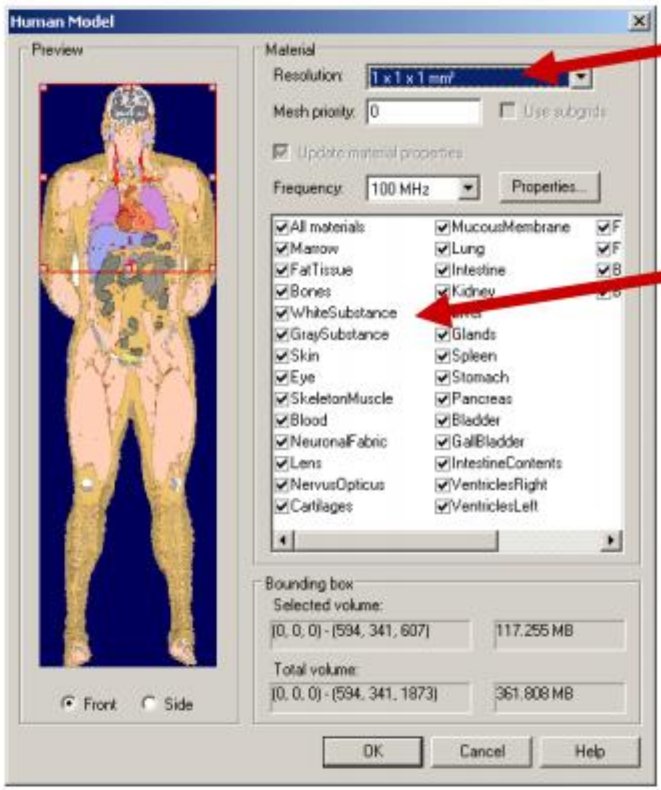
Examples of Dielectric



Examples of Dielectric

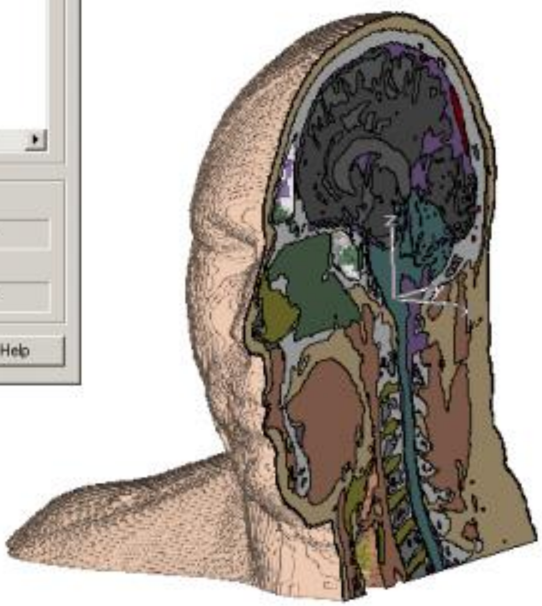
HUGO





Available in different resolutions

Materials of interest can be chosen



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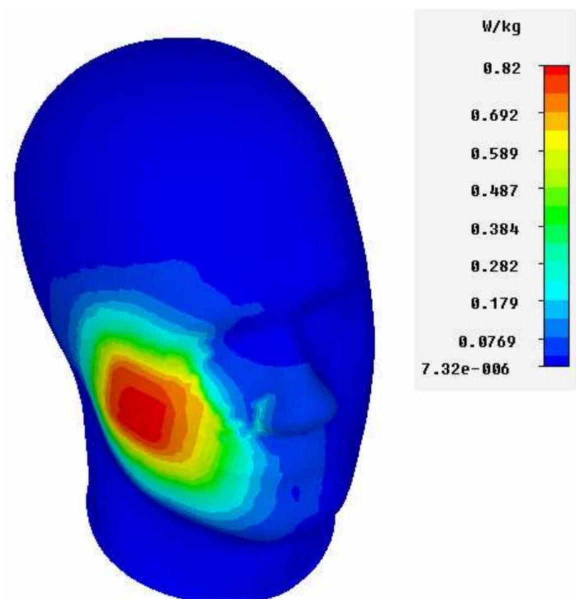
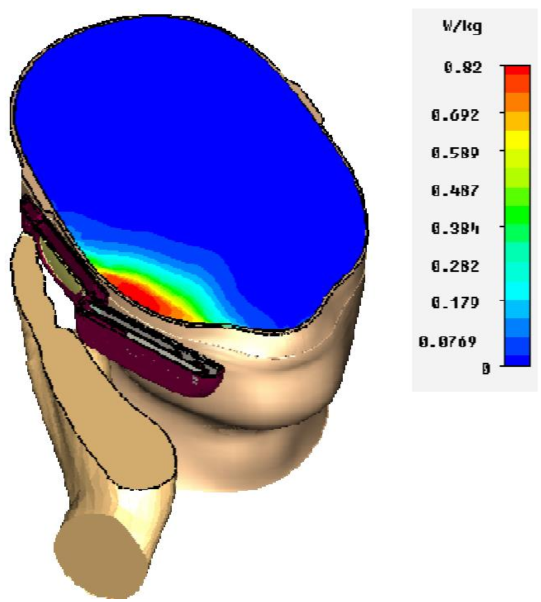
Properties of Electrostatic Field in the Dielectric

- Specific Absorption Rate (SAR)





International standard SAR value <2.0Watt/kg

Properties of Electrostatic Field in the Dielectric

【 Gauss Theorem for dielectric 】

➢ Assuming that polarized dielectric include free charges and bound charges, and the charge density are ρ and ρ_P , thus, we get:

$$\nabla \bullet \vec{E} = \frac{\rho + \rho_P}{\varepsilon_0}$$

$$\nabla \bullet (\varepsilon_0 \vec{E}) = \rho - \nabla \bullet \vec{P}$$

$$\nabla \bullet (\varepsilon_0 \vec{E} + \vec{P}) = \rho$$

【 Electric flux density, or Electric displacement vector】

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

Properties of Electrostatic Field in the Dielectric

Then:

$$\nabla \bullet \vec{D} = \rho$$

$$\oint_S \vec{D} \bullet d\vec{S} = Q$$

✓ Gauss theorem in dielectric has the same form with the Gauss theorem in the vacuum.

✓ The divergence of the electric displacement is related to the free charge.

Properties of Electrostatic Field in the Dielectric

For isotropic dielectric:

$$\vec{D} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E} = (1 + \chi_e)\varepsilon_0 \vec{E} = \varepsilon_r \varepsilon_0 \vec{E} = \varepsilon \vec{E}$$

where $\varepsilon_r = 1 + \chi_e$ is the dielectric relative permittivity (or dielectric constant), ε is the dielectric permittivity. Then:

$$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}$$

✓ Electric field in the dielectric is ε_r times smaller compared to the field in vacuum under the condition that they have the same source.

✓ The form of the field functions are consistent between dielectric and vacuum, just changing ε_0 into ε .

✓ The effect of the bound charges is reflected in ε_r.

Basic equation for electrostatic field

【 Differential form 】

$$\nabla \times \vec{E} = \vec{0} \quad \nabla \bullet \vec{D} = \rho$$

$$\vec{E} = -\nabla \phi \quad \vec{D} = \varepsilon \vec{E}$$

With source: $\nabla^2 \phi = -\dfrac{\rho}{\varepsilon}$ Possion equation

No source: $\nabla^2 \phi = 0$ Laplace equation

【 Integral form 】

$$\oint_l \vec{E} \bullet d\vec{l} = \vec{0} \qquad \oint_S \vec{D} \bullet d\vec{S} = Q$$

Conclusion

- Electric potential
- Properties of Electrostatic Field in the Conductor
- Properties of Electrostatic Field in the Dielectric
- Basic Equations for the Electrostatic Field

Assignment6，作业档案名请用格式：姓名+作业六

The electric field intensity in polystyrene (ε_r=2.55) filling the space between the plates of a parallel-plate capacitor is 10kV/m. The distance between the plates is 1.5 mm. Calculate:

(a) $\vec{D}$

(b) $\vec{P}$

(c) The potential difference between the plates.