Hash-Based Indexes

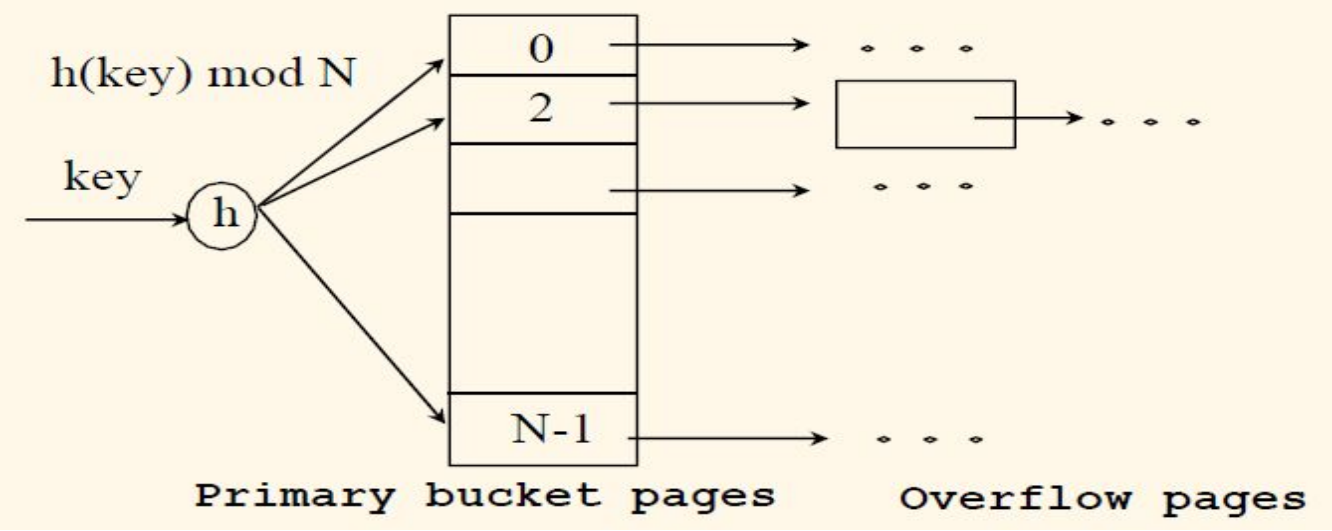
Chapter 11.

Introduction

- As for any index, 3 alternatives for data entries k*:
  - Data record with key value k
  - <k, rid of data record with search key valuek>
  - <k, list of rids of data records with search key k>
  - Choice orthogonal to the indexing technique
- Hash-based indexes are best for equality selections. Cannot support range searches.
- Static and dynamic hashing techniques exist; trade-offs similar to ISAM vs. B+ trees.

Static Hashing

- The number of primary pages is fixed.
- Primary pages are allocated sequentially, never de-allocated;
  - overflow pages if needed.
- h(*k*) *mod M* = bucket to which data entry with key k belongs. (M = number of buckets)



Static Hashing (Contd.)

- Buckets contain *data entries*.
- Hash function works on *search key field of record r. Must distribute values over range 0 … M-1.*
  - h(*key) = (a * key+ b) usually works well.*
  - *a* and *b* are constants; lots known about how to tune **h.**
- Long overflow chains can develop and degrade performance.
  - *Extendible and Linear Hashing: Dynamic techniques to fix this problem.*

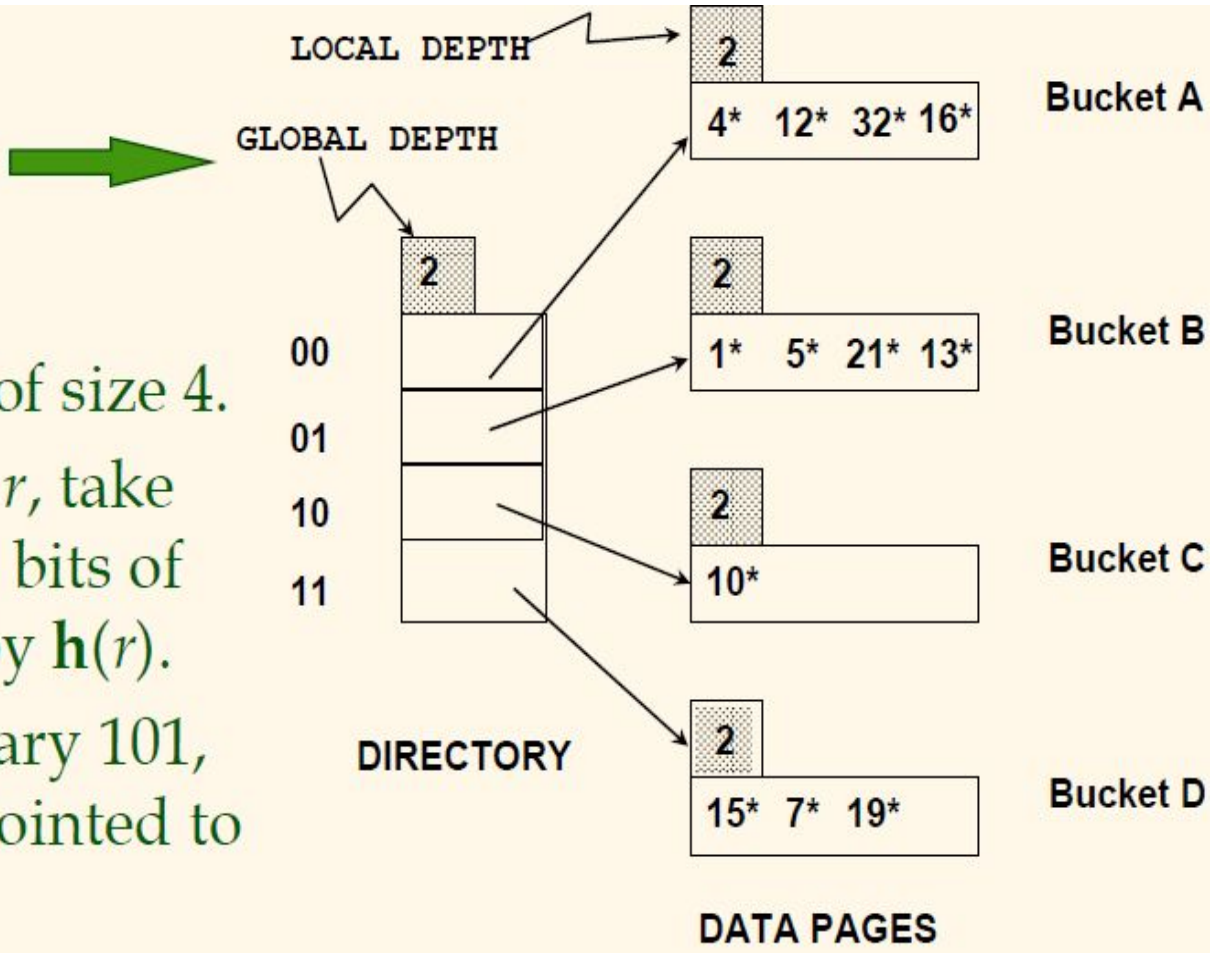
Extendible Hashing

- Situation: Bucket (primary page) becomes full. Why not re-organize file by *doubling the number of buckets?*
  - Reading and writing all pages is expensive!
- *Idea of Extendible Hashing:*
  - *Use directory of pointers to buckets, double the number of buckets by doubling the directory,*
  - *splitting just the bucket that overflowed!*

Extendible Hashing (Contd.)

- Directory is much smaller than file, so doubling it is much cheaper.
- Only one page of data entries is split. *No overflow page!*
- Trick lies in how hash function is adjusted!

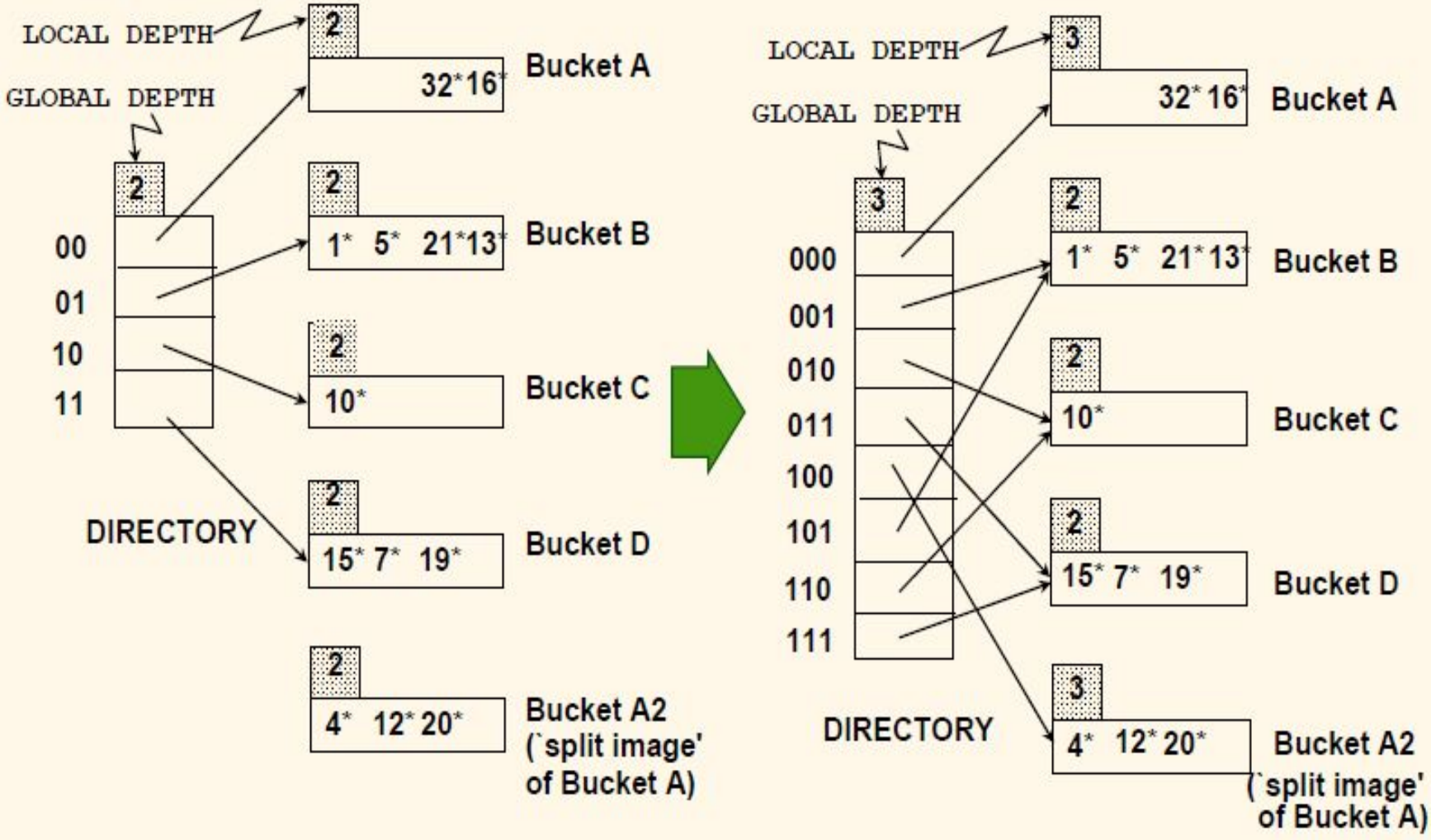
Example



- Directory is array of size 4.
- To find bucket for *r*, take last `*global depth*' # bits of **h**(*r*); we denote *r* by **h**(*r*).
  - If **h**(*r*) = 5 = binary 101, it is in bucket pointed to by 01.

- **Insert**: If bucket is full, *split* it (*allocate new page, re-distribute*).
- *If necessary*, double the directory. (As we will see, splitting a bucket does not always require doubling; we can tell by comparing *global depth* with *local depth* for the split bucket.)

*Insert **h**(r)=20 (Causes Doubling)*



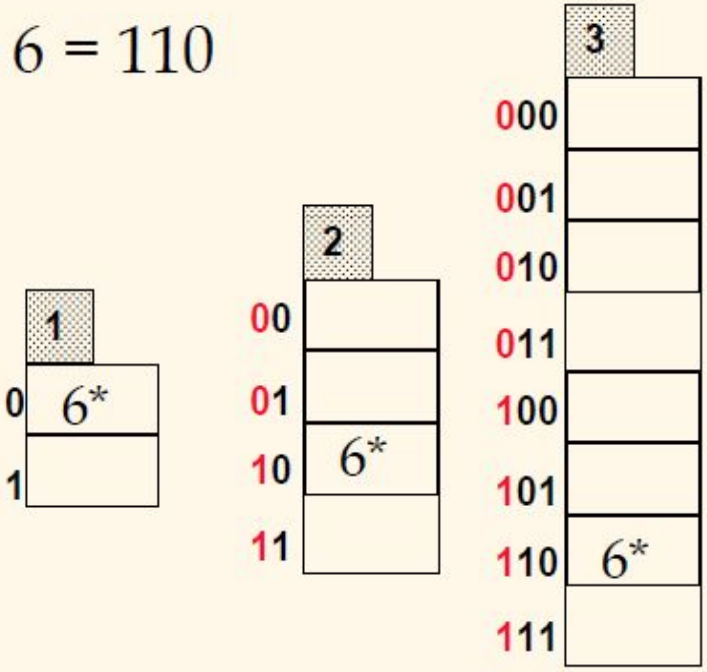
Points to Note

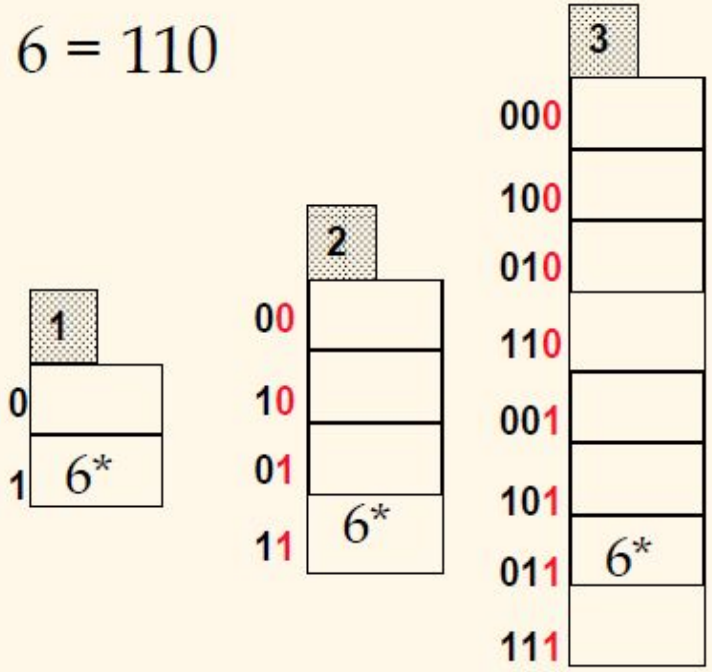
- 20 = binary 10100. Last 2bits (00) tell us *r belongs in A or A2. Last 3bits needed to tell which.*
  - *Global depth of directory: Max # of bits needed to tell which bucket an entry belongs to.*
  - *Local depth of a bucket: # of bits used to determine if an entry belongs to this bucket.*

- When does bucket split cause directory doubling?
  - Before insert, *local depth of bucket = global depth. Insert causes local depth to become > global depth.*

Directory Doubling

Why use least significant bits in directory?
⇔ Allows for doubling via copying!





Least Significant vs. Most Significant

Equality Search in Extendible Hashing

- If directory fits in memory, equality search answered with one disk access; else two.
  - 100MB file, 100 bytes/rec, 4KB/page, contains 1,000,000 records (as data entries) and 25,000 directory elements; chances are high that directory will fit in memory.

Delete in Extendible Hashing

- If removal of data entry makes a bucket empty, the bucket can be merged with its `split image'.

- If each directory element points to same bucket as its split image, we can halve the directory.

Linear Hashing (LH)

- This is another dynamic hashing scheme, an alternative to Extendible Hashing.

- LH handles the problem of long overflow chains without using a directory, and handles duplicates.
  - What problem will duplicates cause in Extendible Hashing?

The Idea of Linear Hashing

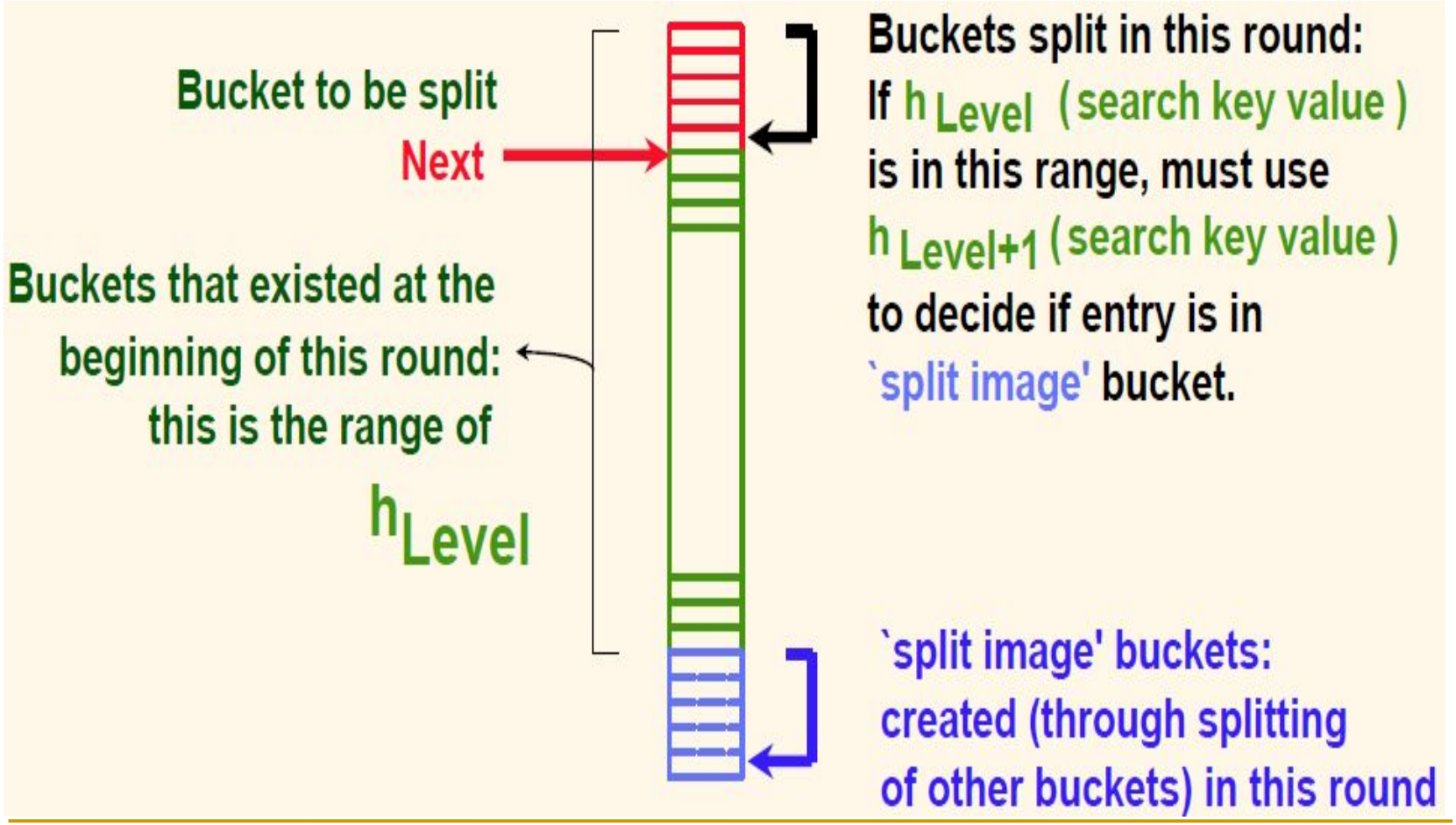
- Use a family of hash functions h_0, h_1, h_2, ..., where h_{i+1} doubles the range of h_i (similar to directory doubling)
  - $h_i(key) = h(key) \bmod (2^i N)$; *N = initial # buckets*
  - *h is some hash function (range is not 0 to N-1)*
  - *If* $N = 2^{d0}$, *for some d0,* h_i *consists of applying h and looking at the last di bits, where di = d0 + i.*

The Idea of Linear Hashing (Contd.)

- Directory avoided in LH by using overflow pages, and choosing bucket to split round-robin.
  - Splitting proceeds in `rounds'. Round ends when all N_R initial (for round *R*) buckets are split. Buckets 0 to *Next-1* have been split; *Next* to N_R yet to be split.
  - Current round number is *Level*.

Overview of LH File:

in the Middle of the *Level* –th Round



Search in Linear Hashing

- To find bucket for data entry *r, find* $h_{Level}(r)$:
  - If $h_{Level}(r)$ in range `*Next to* N_R', *r* belongs here.
  - Else, *r* could belong to bucket $h_{Level}(r)$ or bucket $h_{Level}(r)$ + N_R; must apply $h_{Level+1}$(r) to find out.

Inserting a Data Entry in LH

- Find bucket by applying $h_{Level} / h_{Level+1}$:
  - If the bucket to insert into is full:
    - Add overflow page and insert data entry.
    - (Maybe) split *Next* bucket and increment *Next*.
  - Else simply insert the data entry into the bucket.

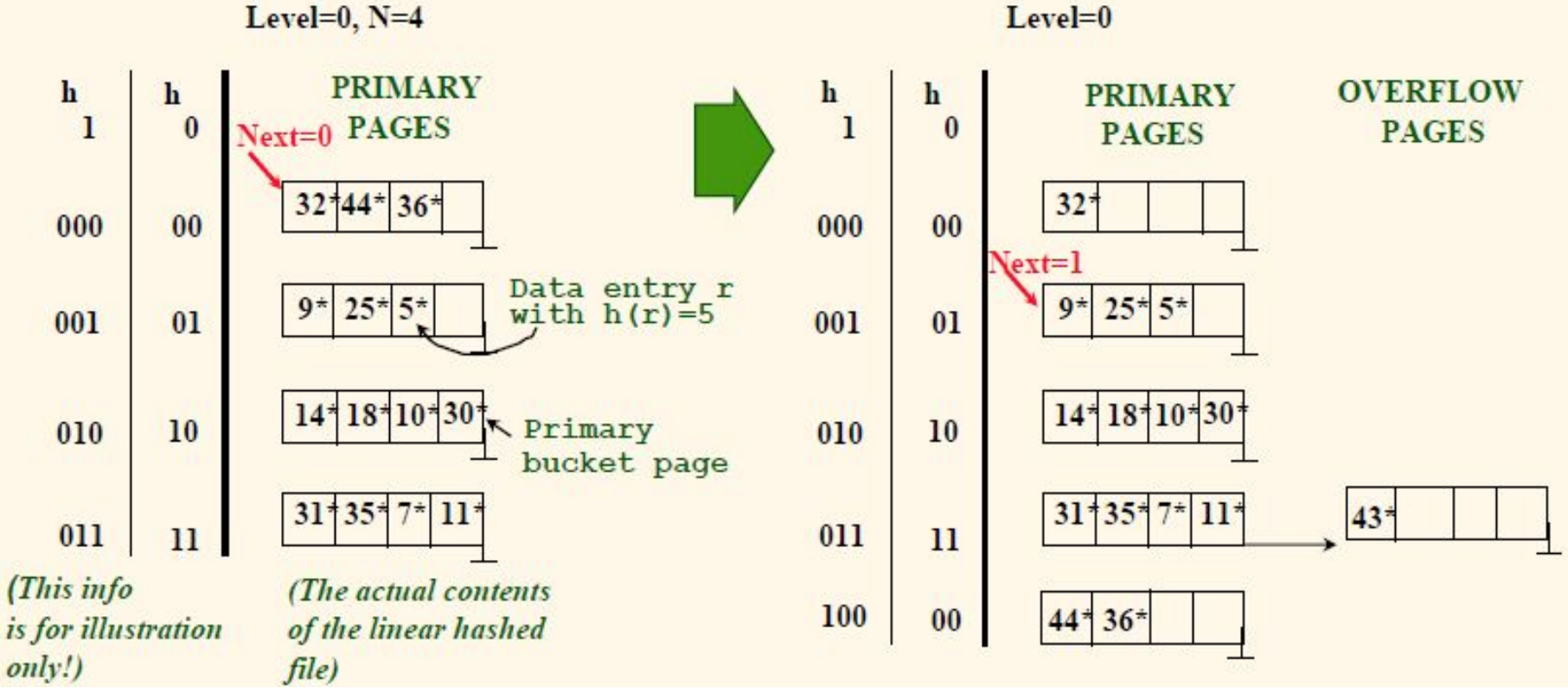
Bucket Split

- A split can be triggered by
  - the addition of a new overflow page
  - conditions such as space utilization
- Whenever a split is triggered,
  - the *Next* bucket is split,
  - and hash function $h_{Level+1}$ redistributes entries between this bucket (say bucket number b) and its split image;
  - the split image is therefore bucket number $b+N_{Level}$.
  - *Next* ← *Next* + 1.

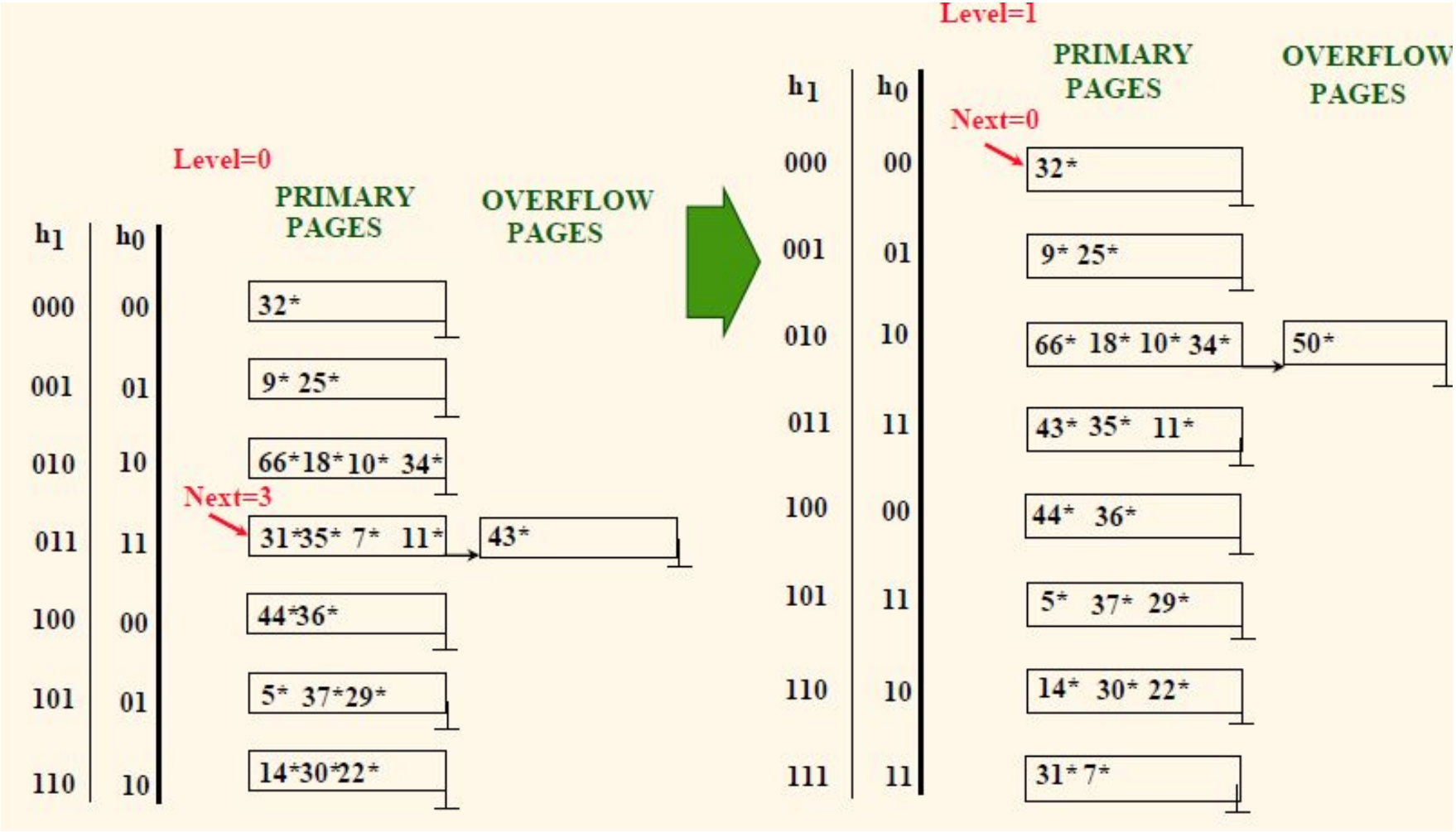
Example of Linear Hashing

- On split, $h_{Level+1}$ is used to re-distribute entries.

Insert data entry of 43*



Example: End of a Round



Extendible VS. Linear Hashing

- Imagine that we also have a directory in LH with elements 0 to *N*-1.
  - The first split is at bucket 0, and so we add directory element *N*.
  - Imagine directory being doubled at this point, but elements <1,*N*+1>, <2,*N*+2>, ... *are the same. So, we can avoid copying elements from 1 to N*-1.
  - We process subsequent splits in the same way,
  - And at the end of the round, all the orginal *N* buckets are split, and the directory is doubled in size.
- i.e., LH doubles the imaginary directory gradually.

Summary

- Hash-based indexes: best for equality searches, cannot support range searches.
- Static Hashing can lead to long overflow chains.
- Extendible Hashing avoids overflow pages by splitting a full bucket when a new data entry is to be added to it.
- Linear Hashing avoids directory by splitting buckets round-robin, and using overflow pages.