# Artificial Intelligence & Machine Learning and Pattern Recognition ——kNN and NB


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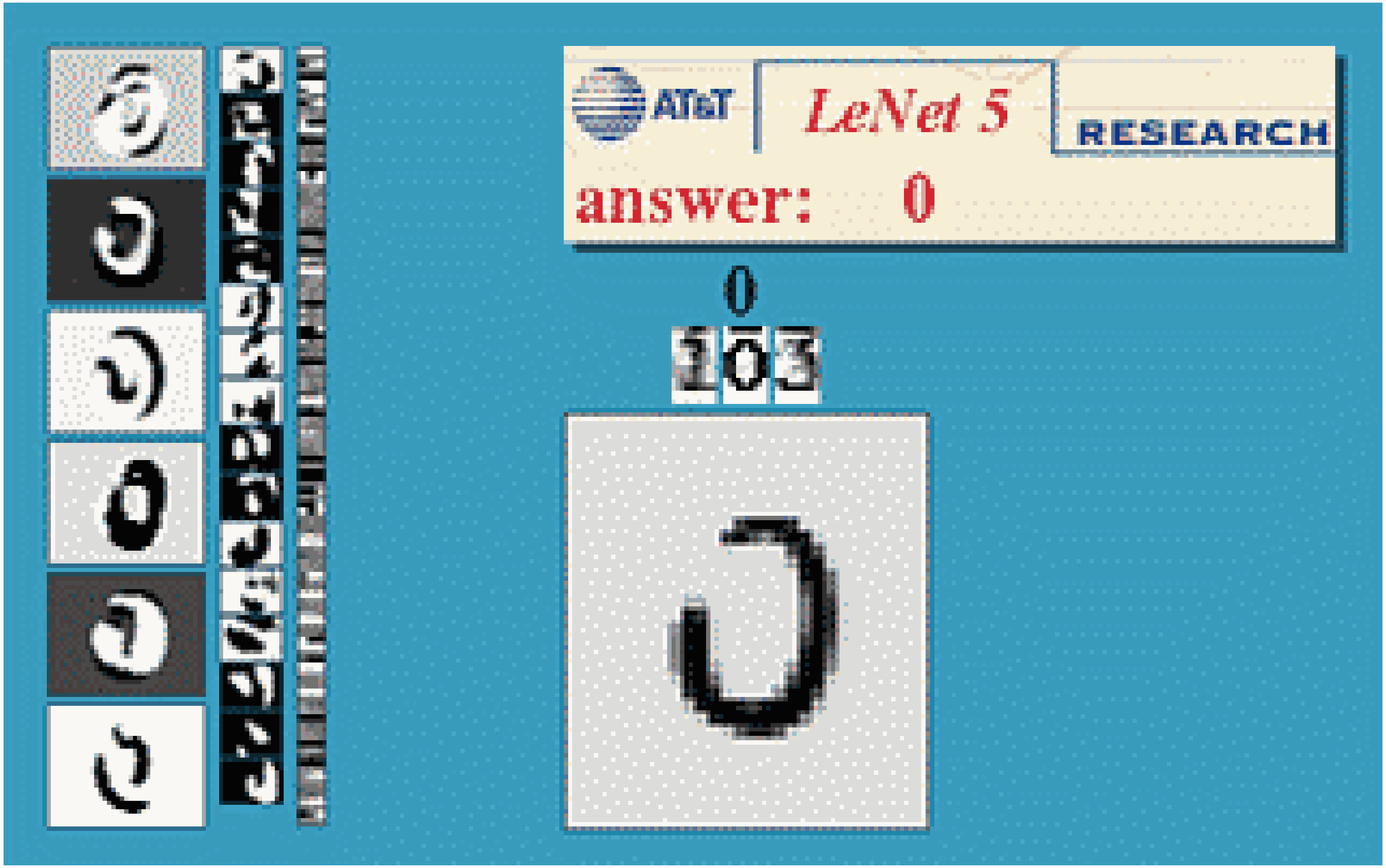
# Regression vs Classification

| DocumentID | Words (split by space) | joy |
|---|---|---|
| train1 | sheva delight us | 0.6 |
| train2 | goal delight for sheva | 0.7 |
| test1 | sheva goal | ? |

| DocumentID | Words (split by space) | emotion |
|---|---|---|
| train1 | sheva know us | not joy |
| train2 | goal delight for sheva | joy |
| test1 | sheva goal | ? |

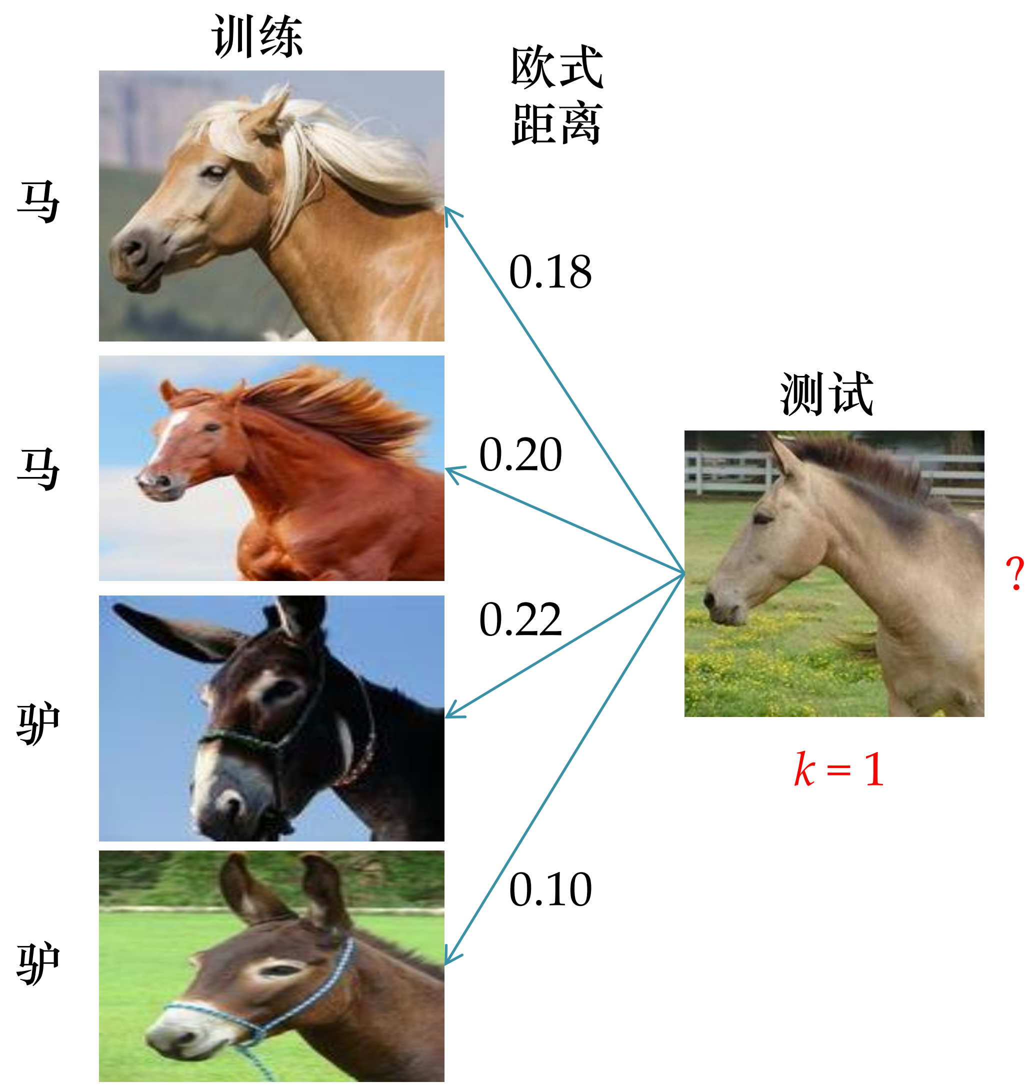
# Regression vs Classification



# *k*-Nearest Neighbor

- All data/objects correspond to vectors in the *n*-D space (*n*维空间的向量)
- The nearest neighbor could be defined in terms of Euclidean distance, etc.
- Target (目标) vector could be discrete- or real- valued
- For discrete-valued, *k*-NN returns the most common value (众数) among the *k* training examples nearest to *X*(测试)

# $k$-Nearest Neighbor



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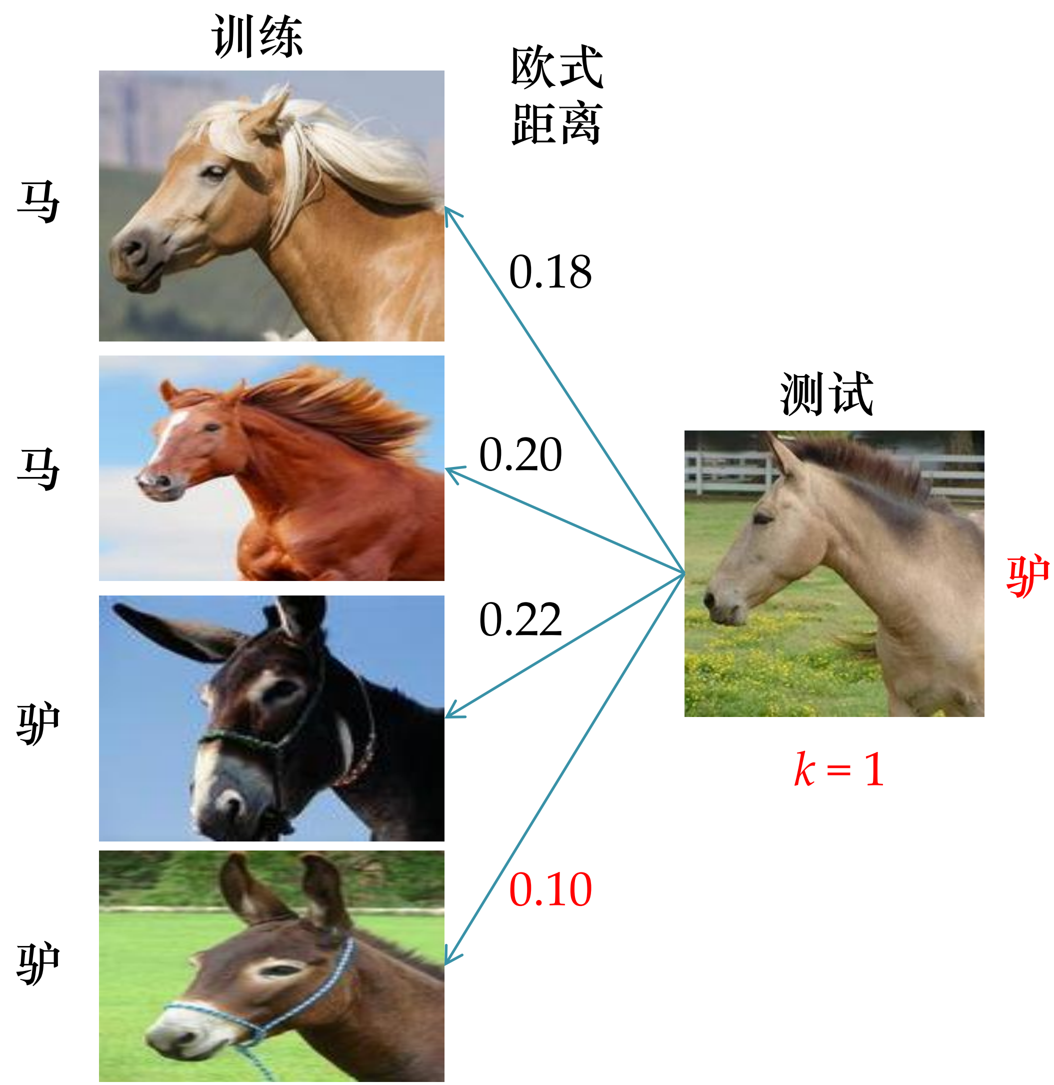

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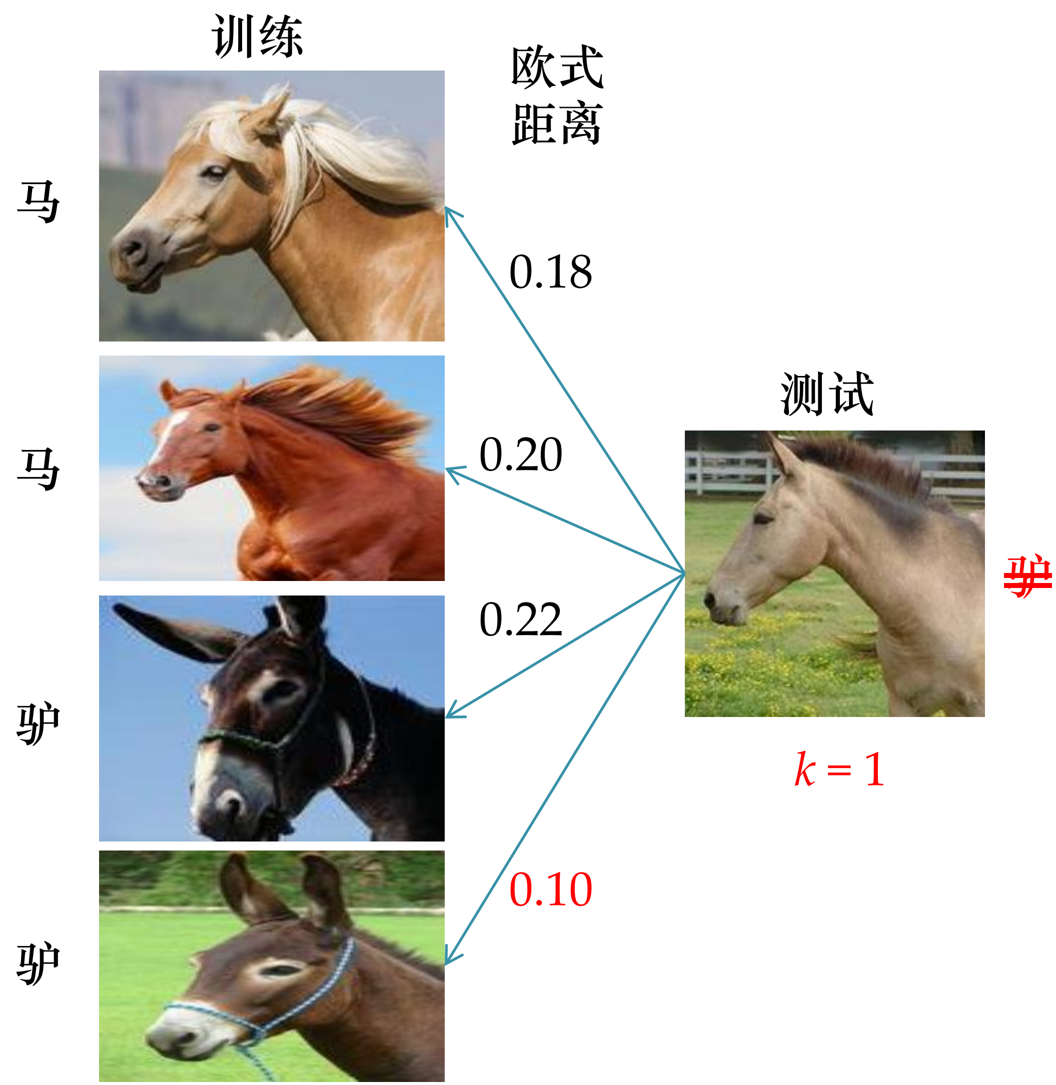

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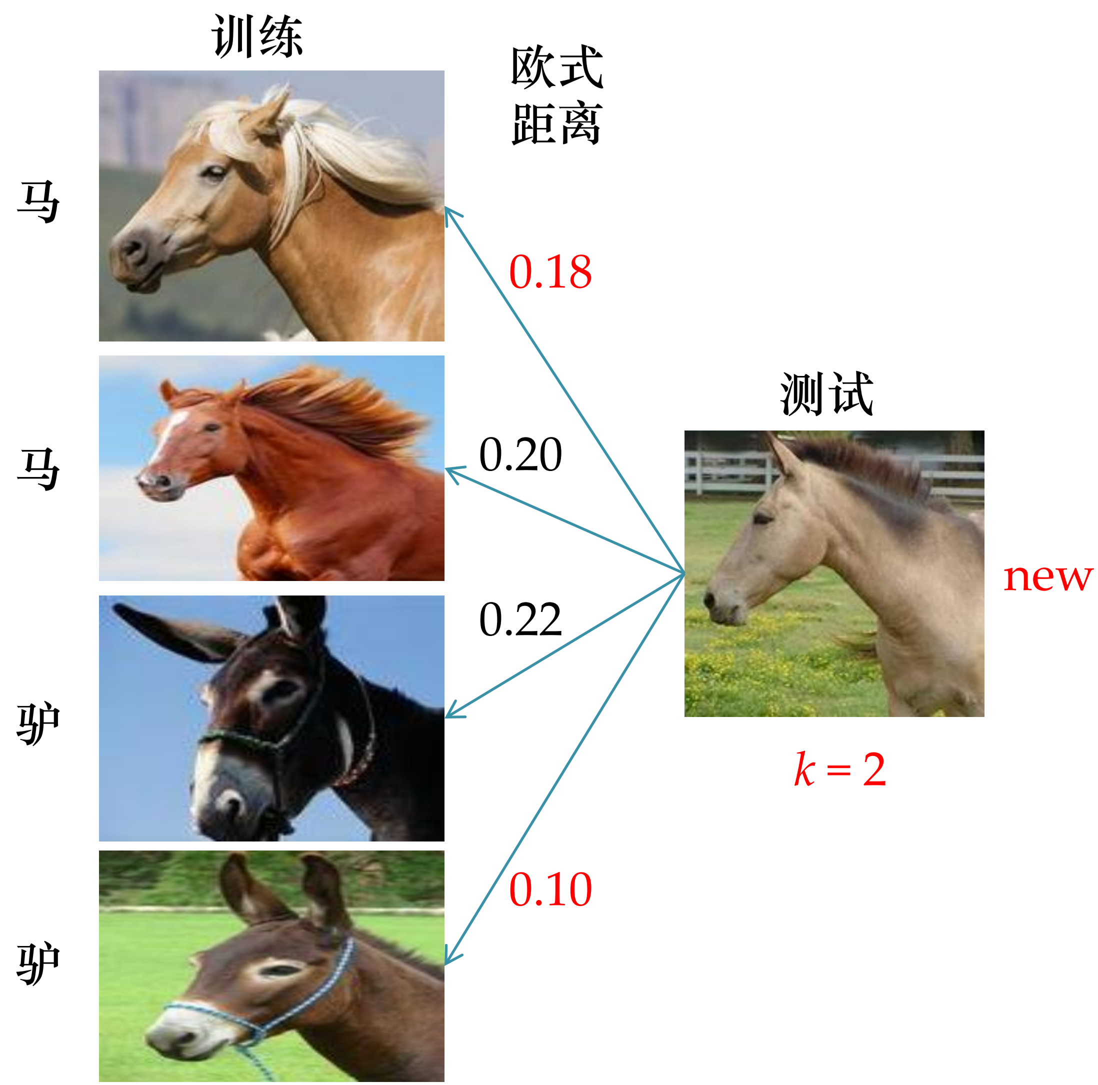

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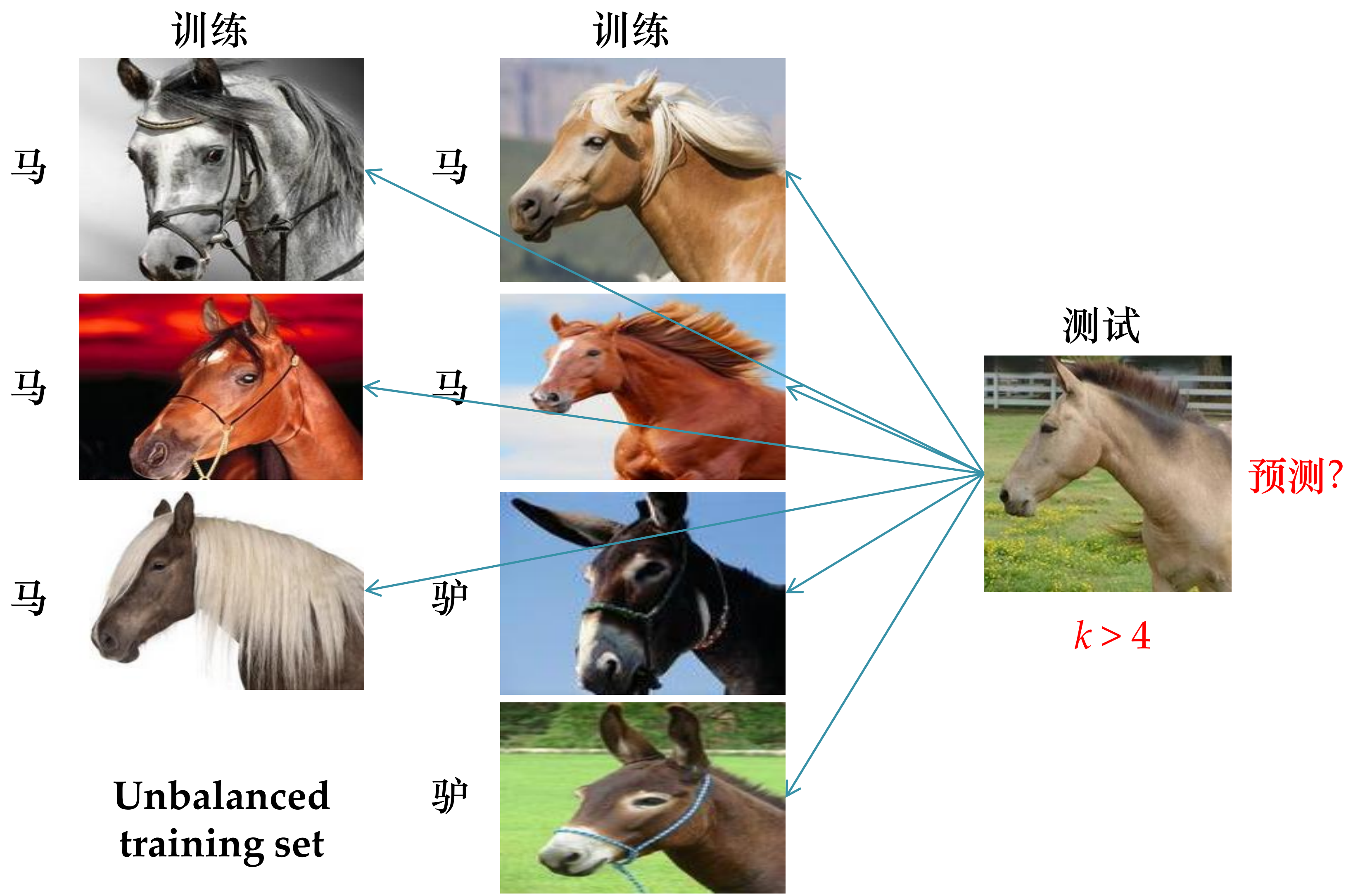
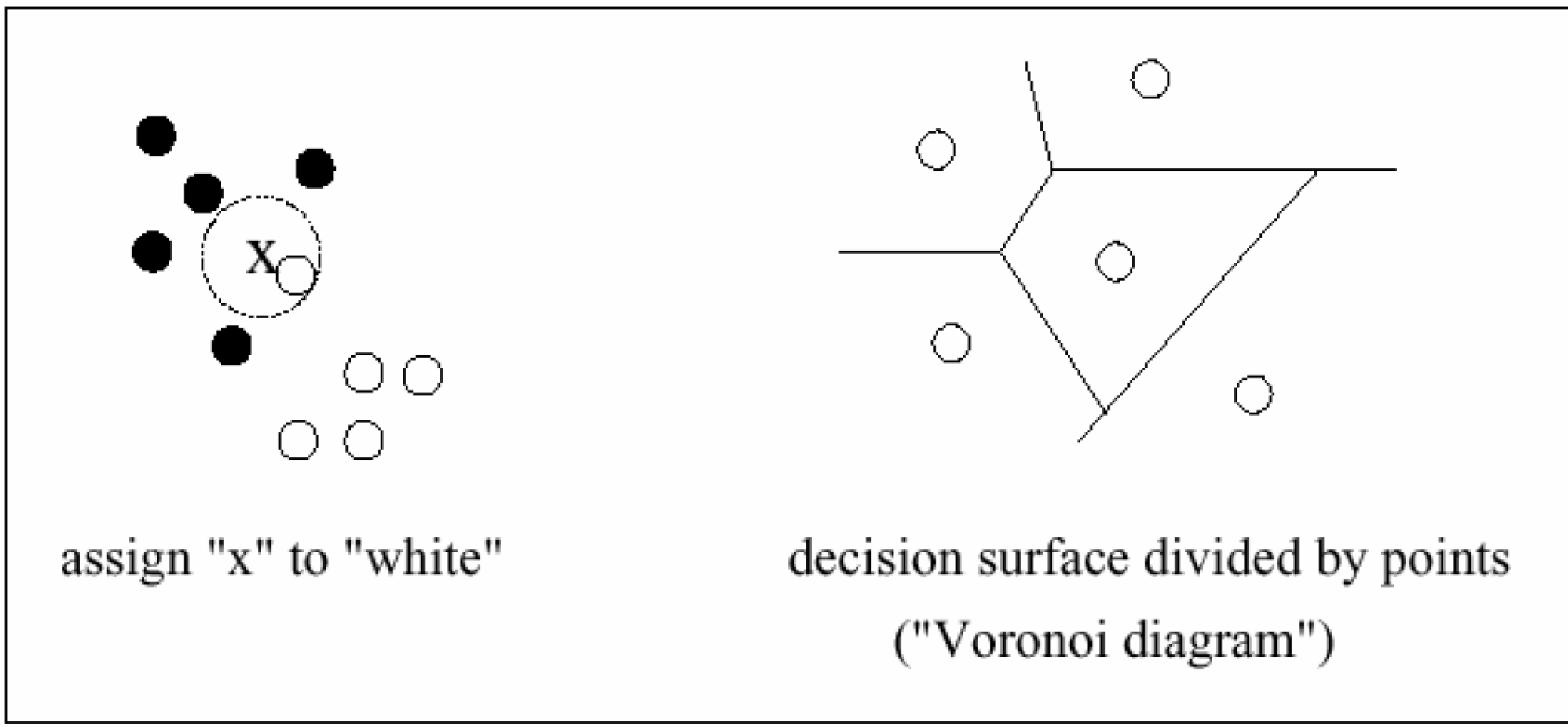


# *k*-Nearest Neighbor

- *k*-NN for real-valued prediction for a given unknown *X* (测试)
  - Returns the mean / median gold values of the *k* nearest neighbors
- Instance-based learning/Lazy-learning
  - initially by Fix and Hodges (1951)
  - theoretical error bound analysis by Duda & Hart (1957)
  - store all the training samples
  - high computational cost for each new object if using the original *k*-NN algorithm

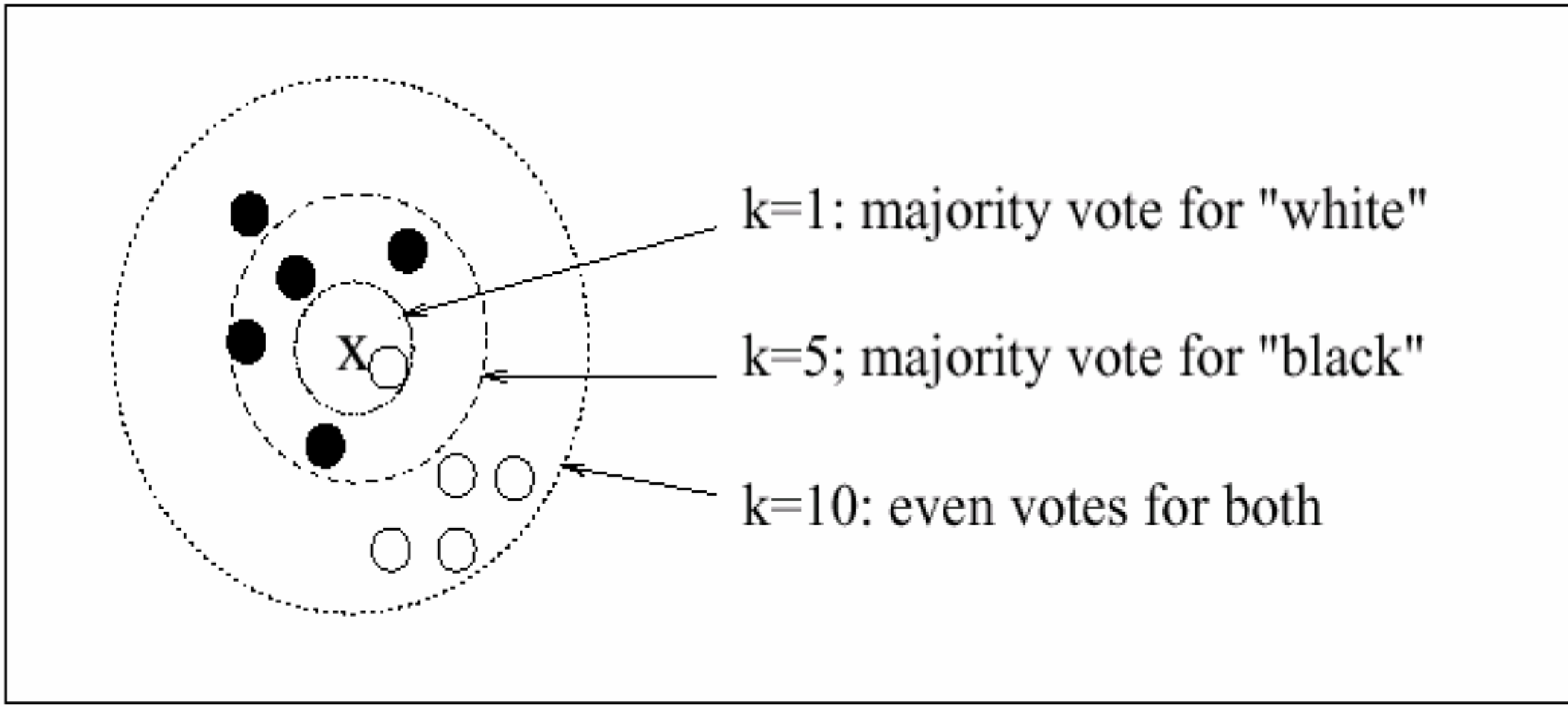
# *k*-Nearest Neighbor

1-NN: assign "x" (new point) to the class of it nearest neighbor



# *k*-Nearest Neighbor

*k*-NN using a majority voting scheme



# *k*-Nearest Neighbor

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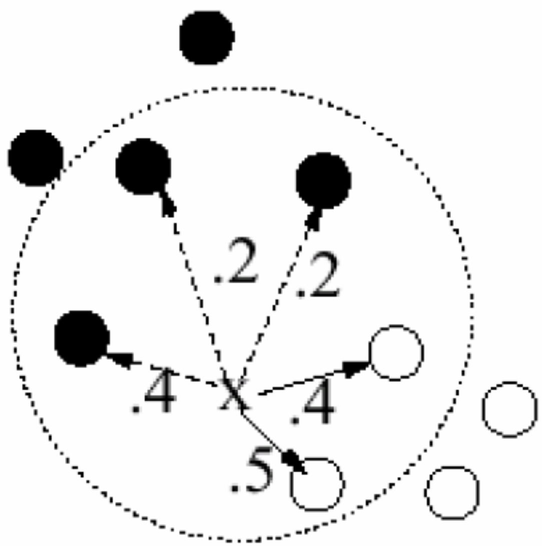
- Key aspects (影响因素) of *k*-NN
  - Similar item: We need a functional definition of similarity if we want to apply this automatically.
  - How many neighbors? (the value of *k*)
  - Does each neighbor get the same weight?
  - Count all classes for all neighbors? Or, use the frequently-occurred classes to make decisions?

# *k*-Nearest Neighbor

*k*-NN using a weighted-sum voting scheme




**kNN (k = 5)**

Assign "white" to x because the weighted sum of "whites" is larger then the sum of "blacks".

Each neighbor is given a weight according to its nearness.

# Naïve Bayesian

- A statistical model
  - Use Bayes' (贝叶斯) Theorem to perform probabilistic prediction, e.g., predict class membership probabilities
- Assumption
  - The effect of an attribute on a given class is independent of other attributes
- Performance
  - Comparable with decision trees (决策树) and selected neural network classifiers

# Naïve Bayesian Classifier

- Given a training set of attributes and their associated class labels, and each object is represented by a *n*-D vector ($n$维向量) $\mathbf{X} = (x_1, x_2, \ldots, x_n)$
- Suppose there are *m* classes, *i.e.*, $C_1, C_2, \ldots, C_m$.
- Naïve Bayesian Classifier is to derive the maximum posteriori (后验概率), *i.e.*, the maximal $P(C_i|\mathbf{X})$

# Naïve Bayesian Classifier

- This can be derived from Bayes' theorem

$$P(C_i \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid C_i)P(C_i)}{P(\mathbf{X})}$$

- Since $P(\mathbf{X})$ is constant for all classes, only

$$P(C_i \mid \mathbf{X}) \propto P(\mathbf{X} \mid C_i)P(C_i)$$

  needs to be maximized
- $P(C_i)$ can be obtained from training set $s_i/s$

# Derivation

- **Assumption**: attributes are conditionally independent (i.e., no dependence relation between attributes): $$P(\mathbf{X} \mid C_i) = \prod_{k=1}^{n} P(x_k \mid C_i)$$
- This greatly reduces the computation cost: Only counts the class distribution
- If $A_k$ is categorical, $P(x_k | C_i) = s_{ik}/s_i$, count the distribution
- If $A_k$ is continuous-valued, $P(x_k | C_i)$ can be computed based on Gaussian distribution

# Example

| age | income | student | credit_rating | buys_computer |
|---|---|---|---|---|
| <=30 | high | no | fair | no |
| <=30 | high | no | excellent | no |
| 31…40 | high | no | fair | yes |
| >40 | medium | no | fair | yes |
| >40 | low | yes | fair | yes |
| >40 | low | yes | excellent | no |
| 31…40 | low | yes | excellent | yes |
| <=30 | medium | no | fair | no |
| <=30 | low | yes | fair | yes |
| >40 | medium | yes | fair | yes |
| <=30 | medium | yes | excellent | yes |
| 31…40 | medium | no | excellent | yes |
| 31…40 | high | yes | fair | yes |
| >40 | medium | no | excellent | no |
| <=30 | medium | yes | fair | ? |

# Example

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357

- Compute $P(X|C_i)$ for each class
  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6
  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4
  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2
  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

**$P(X|C_i)$ :** P(X|buys_computer = "yes") = 0.222 x 0.444 x 0.667 x 0.667 = 0.044
P(X|buys_computer = "no") = 0.6 x 0.4 x 0.2 x 0.4 = 0.019
**$P(X|C_i)$*$P(C_i)$ :** P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028
P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

# Comments

- **Advantages**
  - Easy to implement
  - Good results obtained in most of the cases
- **Disadvantages**
  - Assumption: class conditional independence (给定每个类别, 条件独立), therefore loss of accuracy
    - Practically, dependencies do exist among variables, e.g., Symptoms: fever, cough, etc.
    - Dependencies among these cannot be modeled by Naïve Bayesian Classifier
  - 0 probability value: for example, new words in the testing document. Laplace smoothing factor?