# A generalized thermal conductivity model for soils and construction materials

**Jean Côté and Jean-Marie Konrad**

**Abstract:** This paper intends to develop a generalized thermal conductivity model for moist soils that is based on the concept of normalized thermal conductivity with respect to dry and saturated states. This model integrates well the effects of porosity, degree of saturation, mineral content, grain-size distribution, and particle shape on the thermal conductivity of unfrozen and frozen soils. The thermal conductivity for saturated soils is computed with the use of a well-known geometric model that includes the unfrozen water content in frozen fine-grained soils. Nearly 220 experimental results available from the literature were analysed to develop a generalized empirical relationship to assess the thermal conductivity of dry soils. A general relationship between the normalized thermal conductivity of soils and the degree of saturation using a soil-type dependent factor was used to correlate the normalized thermal conductivity for more than 650 test results for unfrozen and frozen moist soils, such as gravels, sands, silts, clays, peat, and crushed rocks.

*Key words:* heat transfer, soils, degree of saturation, mineral content, unfrozen–frozen, thermal conductivity.

**Résumé :** Dans cet article, on veut développer un modèle de conductivité thermique généralisé pour les sols humides basé sur le concept de conductivité thermique normalisé pour les états secs et saturés. Ce modèle intègre bien l'effet de la porosité, du degré de saturation, de la minéralogie, de la distribution granulométrique, et de la forme des particules sur la conductivité thermique des sols gelés et non gelés. La conductivité thermique pour les sols saturés est calculée au moyen du modèle géométrique bien connu qui comprend la teneur en eau non gelée des sols gelés à grains fins. Près de 220 résultats expérimentaux disponibles dans la littérature ont été analysés afin de développer une relation empirique généralisée pour évaluer la conductivité thermique des sols secs. On a utilisé une corrélation générale entre la conductivité thermique normalisée des sols et le degré de saturation en introduisant un facteur qui dépend du type de sol pour corréler la conductivité thermique normalisée de plus de 650 résultats d'essais sur des sols humides gelés et non gelés tels que des graviers, des sables, des silts, des argiles et des tourbes et roches broyées.

*Mots clés :* tranfert de chaleur, sols, degré de saturation, minéralogie, gelé–nongelé, conductivité thermique.

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## Introduction

Only a few methods have been developed for the estimation of thermal conductivity of partially saturated soils. Empirical models, such as that proposed by Kersten (1949) and those later proposed by Van Rooyen and Winterkorn (1959), Salomone and Kovacs (1984), and Gangadhara Rao and Singh (1999), predict the thermal conductivity of soils as logarithmic functions of water content and as power functions of dry density. Although grain-size distribution of soil is considered through fitting parameters, the mineral content is not taken into account. These methods, which were developed mostly for fine-grained soils and sands, are generally not suitable for dry or nearly dry soils (Farouki 1982).

The effect of mineral content has been considered in analytical prediction methods using the thermal conductivity of solid particles (Mickley 1951; De Vries 1963). The effect of soil texture was also introduced in a model developed by De Vries (1963) that uses particle shape factors. According to Johansen (1975), the values of the shape factors used by De Vries can hardly represent sand particles and vary, depending on whether or not the soil is dry. Moreover, the models developed by Mickley (1951) and De Vries (1963) were not originally intended to apply to frozen soils (Farouki 1982), and they cannot reproduce the fact that the thermal conductivity of frozen soils may be lower than that of unfrozen soils at low degrees of saturation, as observed by Kersten (1949), Penner et al. (1975), and Côté and Konrad (2005).

A comprehensive review of thermal properties, with emphasis on the factors influencing the thermal conductivity of soils, was presented by Farouki (1981). For a given soil or construction material, thermal conductivity is mostly influenced by its dry density and water content, owing to contrasting values of its basic components. For instance, the thermal conductivity of solid particles generally varies from 1 to 5 W/m°C, whereas values for water, ice, and air are, respectively, 0.6, 2.24, and 0.024 W/m°C. However, the mineral content and the fabric (which refers to the size and

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arrangement of particles, as well as pore space distribution, in a soil (Mitchell 1993)), have an undisputable influence on the thermal conductivity of soil. Farouki (1981, 1982) also reviewed and evaluated some of the commonly used prediction models for thermal conductivity of soils. It appeared that any of the evaluated models gave satisfactory results for the entire range of dry density and degree of saturation for fine-grained soils and medium to fine sands of various mineral compositions. However, it came out that the model developed by Johansen (1975) gave the best overall prediction results for sands and fine-grained soils available in the literature. This model normalizes the actual thermal conductivity data with respect to the thermal conductivity of dry and saturated fine soils and sands, handling the effect of mineral content through the thermal conductivity of solid particles and the effect of the degree of saturation and the grain-size distribution through empirical relationships.

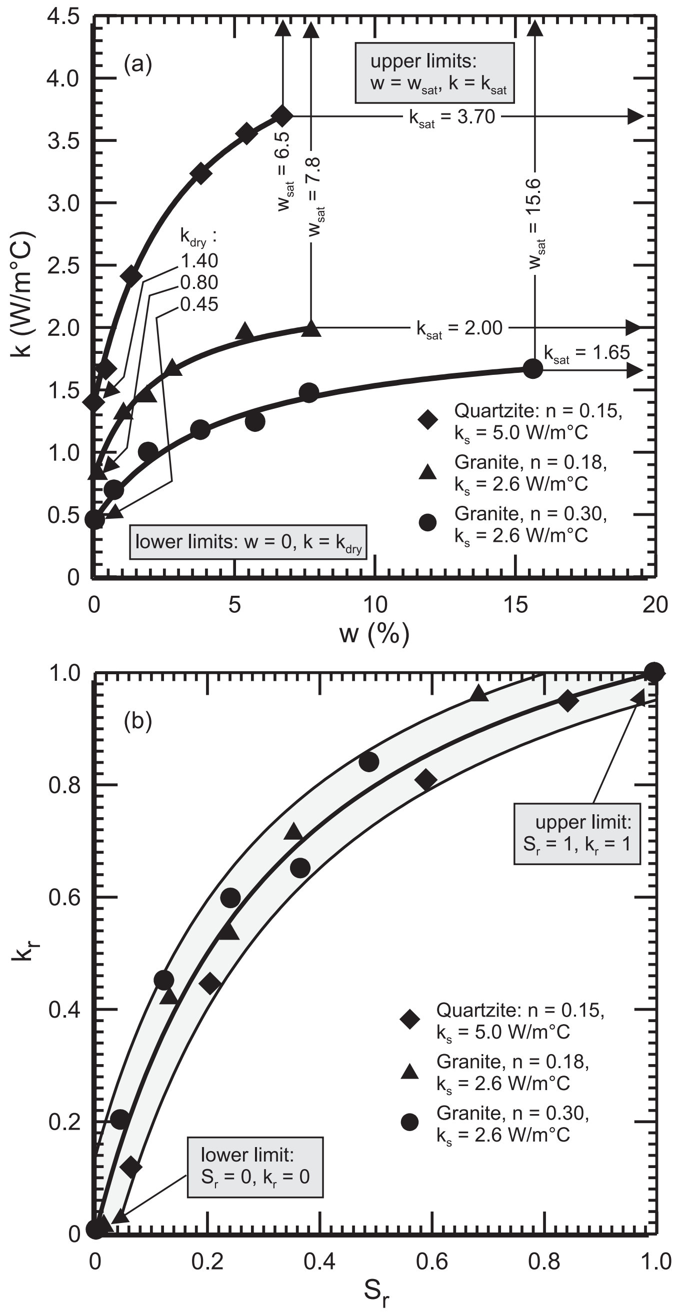
The objective of this paper is to use the normalized thermal conductivity method, first proposed by Johansen (1975), to develop a generalized model for estimating the thermal conductivity of natural soils and construction materials such as crushed rock; coarse, densely graded gravels and sands; medium and fine natural sands, silty and clayey soils, and peat. The paper presents the normalized thermal conductivity concept, in conjunction with a new generalized relationship, as a framework to predict the thermal conductivity of soils. The methodology employed in this study is then described for the analysis of experimental data available in the literature on the thermal conductivity of soils. Results of nearly 1000 analyses of thermal conductivity allowed the development of new generalized relationships between thermal conductivity and porosity for various dry soils of different particle shapes and between normalized thermal conductivity and the degree of saturation for a wide range of soil types. Finally, the paper summarizes the improved sets of equations proposed for this new generalized thermal conductivity model, which is illustrated with running examples.

## The normalized thermal conductivity concept

Using some of the data provided by Kersten (1949) for medium and fine sands and for fine-grained soils, Johansen (1975) developed the normalized thermal conductivity concept to predict the thermal conductivity of soils. Given the fairly good performance of the model noted by Farouki (1981, 1982), Côté and Konrad (2005) adapted it for the prediction of the thermal conductivity of pavement base-course materials. However, it still needs to be improved to cover a wider range of soil types, porosity, and degrees of saturations.

Figure 1 illustrates the normalized thermal conductivity concept. The thermal conductivity relationships with water content are shown in Fig. 1*a* for three unfrozen pavement base-course materials: crushed quartzite with thermal conductivity of solid particles ($k_s$) of 5.0 W/m°C and a porosity ($n$) of 0.15; and two crushed granites with $k_s$ of 2.6 W/m°C and porosities of 0.18 and 0.30 (Côté and Konrad 2005). The thermal conductivity ($k$) of a given material increases with water content ($w$) from the dry state to the saturated state. For example, for crushed quartzite at $w$ = 0%, $k$ = 1.4 W/m°C; and at $w = w_{sat}$ = 6.5%, $k$ = 3.7 W/m°C. The lower and upper limit conditions of the thermal conductivity functions can thus be expressed as follows:

**Fig. 1.** Thermal conductivity of unfrozen granular pavement materials, (*a*) usual form, (*b*) normalized form. (Data from Côté and Konrad 2005.)




$$
[1] \quad \begin{aligned} &\text{lower limit condition: } w = 0 \Rightarrow k = k_{dry} \\ &\text{upper limit condition: } w = w_{sat} \Rightarrow k = k_{sat} \end{aligned}
$$

where $w$ and $w_{sat}$ are the actual water content and the water content of saturated soils (% by weight), respectively; and $k$, $k_{dry}$, and $k_{sat}$ are the actual thermal conductivity and the thermal conductivity of dry and saturated soils, respectively (W/m°C).

Figure 1*a* shows that for given water contents, decreasing porosities (increasing dry densities) lead to increasing thermal conductivities, as with the two granite materials in Fig. 1, which have respective porosities of 0.18 and 0.30. Also, increasing thermal conductivities of solid particles lead to increasing thermal conductivities for a given water content and constant porosity, as observed with the quartzite material ($n$ = 0.15) and the granite material ($n$ = 0.18).

The thermal conductivity functions obtained for various soils can be expressed with the normalized thermal conductivity concept proposed by Johansen (1975):

$$[2]\qquad k_r = \frac{k - k_{dry}}{k_{sat} - k_{dry}}$$

Figure 1*b* shows the normalized thermal conductivities ($k_r$) of data shown in Fig. 1*a* as a function of the degree of saturation ($S_r$) (normalized water content). Clearly, a unique $k_r$–$S_r$ relationship is obtained for the three materials, which had three different $k$–$w$ relationships. This concept thus handles the effect of porosity, water content, and mineral content in a unique way for a given type of soil, with lower and upper limit conditions being as follows:

$$[3]\qquad \begin{array}{l} \text{lower limit condition: } S_r = 0 \Rightarrow k_r = 0 \\ \text{upper limit condition: } S_r = 1 \Rightarrow k_r = 1 \end{array}$$

## Framework for a generalized thermal conductivity model

Johansen (1975) proposed the following empirical relationships for soils studied by Kersten (1949):

$$[4]\qquad k_r = 0.7\log(S_r) + 1$$

unfrozen medium and fine sands

$$[5]\qquad k_r = \log(S_r) + 1$$

unfrozen fine-grained soils

$$[6]\qquad k_r = S_r$$

frozen medium and fine sands and fine-grained soils

where $k_r$ and $S_r$ are the normalized thermal conductivity and the degree of saturation, respectively.

It should be noted that eqs. [4] and [5] do not satisfy the first condition of eq. [3], because log (0) = –∞.

For peat, Johansen (1975) proposed direct mathematical equations (eqs. [7] and [8]) to fit Kersten's (1949) data:

$$[7]\qquad \sqrt{k} - \sqrt{k_{dry}} = S_r(\sqrt{k_{sat(u)}} - \sqrt{k_{dry}})$$

unfrozen peat

$$[8]\qquad k = k_{dry}\left(\frac{k_{sat(f)}}{k_{dry}}\right)^{S_r} \qquad \text{frozen peat}$$

where $k_u$ and $k_f$ are the thermal conductivity of unfrozen and frozen soils, respectively; and values of $k_{dry}$, $k_{sat(u)}$, and $k_{sat(f)}$ were set equal to 0.05, 0.55, and 1.8 W/m°C, respectively. It should be noted that the translation of Johansen's (1975) doctoral thesis contains an erroneous equation for the unfrozen peat, $(k - k_{dry})^{1/2} = S_r \times (k_{sat(u)} - k_{dry})^{1/2}$, which was also transcribed in Farouki (1981) and is corrected here (eq. [7]).

These equations can readily be normalized by using eq. [2]:

$$[9]\qquad k_r = 0.54S_r^2 + 0.46S_r \qquad \text{unfrozen peat}$$

$$[10]\qquad k_r = 0.029(36^{S_r} - 1) \qquad \text{frozen peat}$$

More recently, $k_r$–$S_r$ relationships were obtained by Côté and Konrad (2005) for nearly 200 samples of unfrozen and frozen pavement base-course and subbase materials, as shown in Fig. 2. Two mathematical equations were proposed to relate $k_r$ to $S_r$:

**Fig. 2.** Normalized thermal conductivity for (*a*) unfrozen and (*b*) frozen base-course materials (Côté and Konrad 2005).

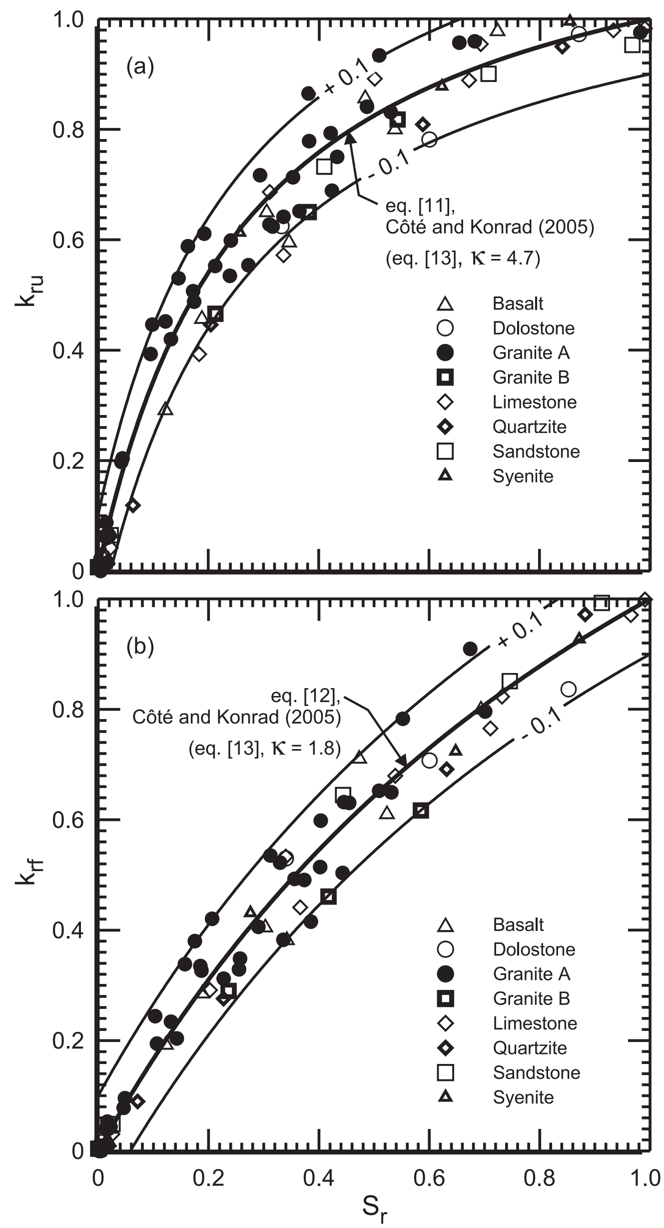
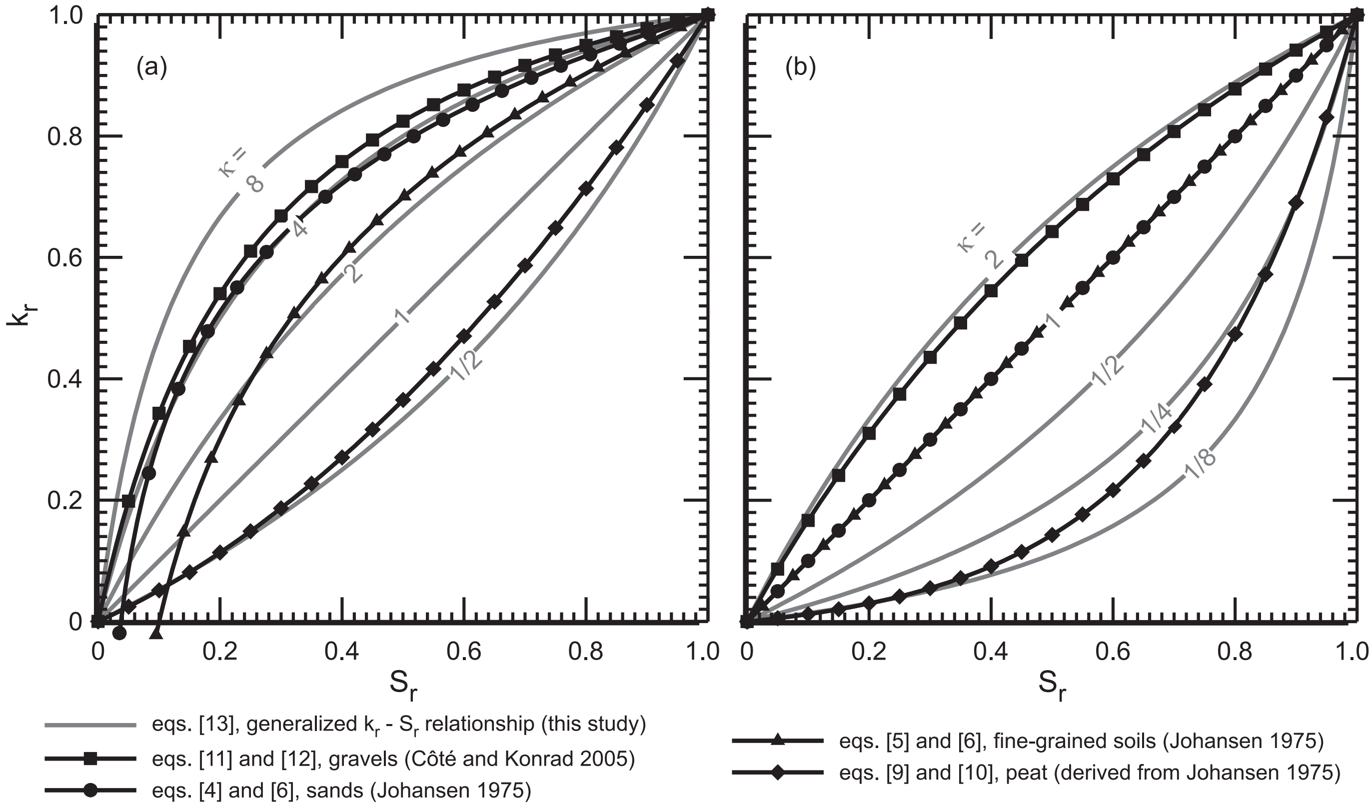


**Fig. 3.** The $k_r$–$S_r$ relationships for (*a*) unfrozen and (*b*) frozen states obtained from the literature, compared with $k_r$–$S_r$ curves obtained with eq. [13] for different values of κ.




$$[11] \qquad k_r = \frac{4.7S_r}{1+3.7S_r}$$

unfrozen base-course and subbase materials

$$[12] \qquad k_r = \frac{1.8S_r}{1+0.8S_r}$$

frozen base-course and subbase materials

Given the wide variety of mathematical expressions actually used to describe the normalized thermal conductivity for different soil types, a generalized equation valid for all soils would be useful for design engineers and thermal modellers. As the $k_r$–$S_r$ relationships given by Côté and Konrad (2005) for unfrozen (eq. [11]) and frozen (eq. [12]) base-course and subbase materials respect both conditions of eq. [3], the following generalized equation is proposed:

$$[13] \qquad k_r = \frac{\kappa S_r}{1+(\kappa-1)S_r}$$

where κ is an empirical parameter used to account for the different soil types in the unfrozen and the frozen states. In the case of pavement base-course and subbase materials, κ = 4.7 for the unfrozen state (Fig. 2*a*) and 1.8 for the frozen state (Fig. 2*b*).

As shown in Fig. 3, eq. [13] can be potentially used to fit experimental data, as the unique parameter κ describes well the thermal behaviour of soils of different types, both in the unfrozen and in the frozen states. The existing $k_r$–$S_r$ relationships (eqs. [4]–[6] and [9]–[12]) for unfrozen soils in Fig. 3*a* and for frozen soils in Fig. 3*b* are shown in conjunction with curves obtained with eq. [13] (grey lines) for different values of κ. For frozen soils, values of κ varying from 0.5 to 8 give good agreement between existing relationships and eq. [13], whereas values between 0.25 and 2 fit well for frozen soils. Indeed, it will be established in this paper with the reanalysis of Kersten's (1949) data that eq. [13] correlates normalized thermal conductivity fairly well to the degree of saturation for different types of soil with values of κ that can be associated to the soil's fabric.

## Methodology used for the analysis of experimental data

For determining the normalized thermal conductivities for soils from the literature with eq. [2] that allow the determination of the soil-type factor, κ, in eq. [13], the thermal conductivities for dry and saturated soils are needed. As most of the data in the literature do not provide such values, they have to be estimated. The $k_{sat}$ values can be computed with the well-accepted geometric mean method presented in this paper. Analyses of data from the literature will be conducted to estimate the $k_{dry}$ values for soils and construction materials. A method for estimating the volume fraction of unfrozen water content directly from experimental data on thermal conductivity will also be described. The thermal conductivity data from Kersten (1949) will thus be normalized by using the computed values of $k_{sat}$ and estimated values $k_{dry}$ to obtain the $k_r$–$S_r$ relationships and to assess the values of κ for different types of soils.

**Computation of the thermal conductivity model for saturated soils**

Many theoretical models have been proposed for the computation of the thermal conductivity of saturated soils. However, when the thermal conductivities of constituents do not contrast by more than one order of magnitude, the geometric mean method is as successful as any other, and it is clearly the simplest (Sass et al. 1971). Consequently, the computation of the thermal conductivity of saturated unfrozen soils is done with the following equations:

$$[14] \quad k_{\mathrm{sat(u)}} = k_{\mathrm{s}}^{1-n} k_{\mathrm{w}}^{n}$$

where $k_{\mathrm{sat(u)}}$ is the thermal conductivity of saturated unfrozen soil; $k_{\mathrm{s}}$ and $k_{\mathrm{w}}$ are the thermal conductivities of solid particles and water (W/m°C), respectively; and $n$ is the porosity of the soil, which can be obtained from

$$[15] \quad n = 1 - \frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{s}}}$$

where $\rho_{\mathrm{s}}$, and $\rho_{\mathrm{d}}$ are the density of solid particles and the dry density of soils (kg/m³), respectively.

The thermal conductivity of saturated frozen soils can be computed with

$$[16] \quad k_{\mathrm{sat(f)}} = k_{\mathrm{s}}^{1-n_{\mathrm{f}}} k_{\mathrm{i}}^{n_{\mathrm{f}}}$$

where $k_{\mathrm{sat(f)}}$ is the thermal conductivity of saturated frozen soil; $k_{\mathrm{i}}$ is the thermal conductivity of ice; and $n_{\mathrm{f}}$ is the porosity of frozen saturated soils.

For saturated soils freezing in a closed system, as in the experimental setup used by Kersten (1949) and by Côté and Konrad (2005), the volume of voids will increase by 9%, thus resulting in a change of porosity that can be estimated with eq. [17] according to Côté and Konrad (2005):

$$[17] \quad n_{\mathrm{f}} = 1.09 n_{\mathrm{u}}/(1 + 0.09 n_{\mathrm{u}})$$

where $n_{\mathrm{u}}$ is the porosity of unfrozen soil, as computed with eq. [15].

The thermal conductivity of saturated frozen fine-grained soils can be affected by the presence of unfrozen water. Equation [16] can be adapted as

$$[18] \quad k_{\mathrm{sat(f)}} = k_{\mathrm{s}}^{1-n_{\mathrm{f}}} k_{\mathrm{i}}^{n_{\mathrm{f}}-\theta_{\mathrm{u}}} k_{\mathrm{w}}^{\theta_{\mathrm{u}}}$$

where $\theta_{\mathrm{u}}$ represents the volume fraction of unfrozen water in frozen fine-grained soils.

When $\theta_{\mathrm{u}} = 0$, eq. [18] reduces to eq. [16]. The presence of unfrozen water can also be considered in the computation of porosity, $n_{\mathrm{f}}$, in a generalized form of eq. [17]:

$$[19] \quad \begin{aligned} n_{\mathrm{f}} &= n_{\mathrm{u}} && \Rightarrow \quad S_{\mathrm{ru}} n_{\mathrm{u}} \le \theta_{\mathrm{u}} \\ n_{\mathrm{f}} &= \frac{n_{\mathrm{u}} + 0.09(S_{\mathrm{ru}} n_{\mathrm{u}} - \theta_{\mathrm{u}})}{1 + 0.09(S_{\mathrm{ru}} n_{\mathrm{u}} - \theta_{\mathrm{u}})} && \Rightarrow \quad S_{\mathrm{ru}} n_{\mathrm{u}} > \theta_{\mathrm{u}} \end{aligned}$$

where $S_{\mathrm{ru}}$ is the degree of saturation in the unfrozen state, obtained from

$$[20] \quad S_{\mathrm{ru}} = \frac{w}{100} \frac{\rho_{\mathrm{d}}}{n_{\mathrm{u}} \rho_{\mathrm{w}}}$$

where $w$ is the water content (% by weight); and $\rho_{\mathrm{w}}$ is the density of water (kg/m³).

In the same manner, to consider the effect of volume change as water turns into ice on the degree of saturation, Côté and Konrad (2005) proposed the following equation:

$$[21] \quad S_{\mathrm{rf}} = \frac{1.09 S_{\mathrm{ru}}}{1 + 0.09 S_{\mathrm{ru}}}$$

In a generalized form that considers the presence of unfrozen water content in frozen soils, eq. [21] becomes

$$[22] \quad \begin{aligned} S_{\mathrm{rf}} &= S_{\mathrm{ru}} && \Rightarrow \quad S_{\mathrm{ru}} n_{\mathrm{u}} \le \theta_{\mathrm{u}} \\ S_{\mathrm{rf}} &= \frac{1.09 S_{\mathrm{ru}} n_{\mathrm{u}} - 0.09\theta_{\mathrm{u}}}{n_{\mathrm{u}} + 0.09(S_{\mathrm{ru}} n_{\mathrm{u}} - \theta_{\mathrm{u}})} && \Rightarrow \quad S_{\mathrm{ru}} n_{\mathrm{u}} > \theta_{\mathrm{u}} \end{aligned}$$

The unfrozen water content in frozen soils can be estimated for a given temperature by an empirical relationship proposed by Anderson and Tice (1972) that requires the specific surface area of solid particles:

$$[23] \quad \ln(w_{\mathrm{u}}) = 0.2618 + 0.5519 \ln(S_{\mathrm{s}}) - 1.449 \ln(-T)\, S_{\mathrm{s}}^{-0.264}$$

where $w_{\mathrm{u}}$ is the unfrozen water content (% by weight) in frozen soils; $S_{\mathrm{s}}$ is the specific surface area (m$^2$/g); and $T$ is the temperature (°C).

The volume fraction of unfrozen water can be approximated with

$$[24] \quad \theta_{\mathrm{u}} = \frac{w_{\mathrm{u}} \rho_{\mathrm{d}}}{100 \rho_{\mathrm{w}}}$$

Given that the density of water is 1000 kg/m³, the volume fraction of unfrozen water, $\theta_{\mathrm{u}}$, is obtained directly by rearranging eq. [23] with eq. [24]:

$$[25] \quad \ln(\theta_{\mathrm{u}}) = \ln(\rho_{\mathrm{d}}) + 0.5519 \ln(S_{\mathrm{s}}) - 1.449 \ln(-T)\, S_{\mathrm{s}}^{-0.264} - 11.251$$

Johansen (1975) successfully used eq. [14] to predict the thermal conductivity of saturated ocean sediments with “normal” quartz content (data from Ratcliffe 1960) and with high quartz content (data from Kasameyer et al. 1972), and Farouki (1981) reported that eq. [18] gave a good fit to Penner’s (1970) data on frozen fine-grained soils. However, the geometric mean model should not be applied to naturally occurring ice-rich permafrost soils, because of the presence of entrapped air bubbles and discontinuities that tend to lower their effective thermal conductivity, compared with soils with interparticle voids filled with pure ice only. Empirical relationships between the thermal conductivity of permafrost soils and dry density were provided by Slusarchuk and Watson (1975).

The values of $k_{\mathrm{w}}$ (eqs. [14] and [18]) and $k_{\mathrm{i}}$ (eq. [18]) are equal to 0.60 and 2.24 W/m°C, respectively, whereas that of $k_{\mathrm{s}}$ generally ranges from 1 to 5 W/m°C. Average values of $k_{\mathrm{s}}$ obtained by Andersland and Anderson (1978), Birch and Clark (1940), Côté and Konrad (2005), De Vries (1963), Goguel (1975), Johnston (1981), Jumikis (1977), Lide (1995), Missenard (1965), and Sass et al. (1971) for different types of rocks and soils are listed in Table 1. When the complete mineral composition is known, the thermal conductivity of solid particles can be assessed by using the generalized geometric mean method with data for the thermal conductivity of forming minerals:

**Table 1.** Average values of thermal conductivities of solid particles computed from various sources.

| Material | $\rho_s$ (g/cm³) | $k_s$ (W/m°C) |
|---|---|---|
| **Rocks** | | |
| Anorthosite | 2.73 | 1.8 |
| Basalt | 2.90 | 1.7 |
| Diabase | 2.98 | 2.3 |
| Dolostone | 2.90 | 3.8 |
| Gabbro | 2.92 | 2.2 |
| Gneiss | 2.75 | 2.6 |
| Granite | 2.75 | 2.5 |
| Limestone | 2.70 | 2.5 |
| Marble | 2.80 | 3.2 |
| Quartzite | 2.65 | 5.0 |
| Sandstone | 2.80 | 3.0 |
| Schist | 2.65 | 1.5 |
| Shale | 2.65 | 2.0 |
| Syenite | 2.80 | 2.0 |
| Trap rock | 2.90 | 2.0 |
| **Soil and organic matter** | | |
| Coal | 1.35 | 0.26 |
| Peat | 1.50 | 0.25 |
| Silt and clay | 2.75 | 2.90 |

$$[26] \qquad k_s = \prod_j k_{\mathrm{m}_j}^{x_j} \qquad \text{with} \qquad \sum_j x_j = 1$$

where $k_{\mathrm{m}}$ is the thermal conductivity of rock-forming mineral $j$ (W/m°C); and $x$ is the volumetric proportion of rock-forming mineral $j$.

Equation [26] was used by Sass et al. (1971) and Woodside and Messmer (1961) for predicting the thermal conductivity of rocks. It should be noted that the geometric mean method is suitable when the thermal conductivity contrast is less than one order of magnitude. Values for selected rock-forming minerals are given in Table 2 (Horai 1971). The minerals are grouped according to chemical composition: (1–6) silicates; (7) carbonates; (8) oxides; (9) hydroxides; (10) sulphates, (11) halides, (12) sulphides, and (13) phosphates. For example, hornblende is an amphibole in the inosilicate class (4); its density is 3.18 g/cm³; and its thermal conductivity is 2.81 W/m°C.

Johansen (1975) proposed a simplified version of eq. [26] for the assessment of the thermal conductivity of solid particles, considering quartz only, which has a thermal conductivity of 7.69 W/m°C, and a value of 2 W/m°C is assigned to all the other forming minerals. For quartz content lower than 20%, the thermal conductivity of forming minerals other than quartz is raised to a value of 3 W/m°C (Johansen 1975):

$$[27] \qquad k_{\mathrm{s}} = \begin{cases} 2.0^{1-q} \times 7.7^{q} \Rightarrow q > 0.2 \\ 3.0^{1-q} \times 7.7^{q} \Rightarrow q \le 0.2 \end{cases} \qquad \text{Johansen (1975)}$$

where $q$ is the volume fraction of quartz in the solid particles.

Johansen's (1975) method for solid particles proved to be quite reliable for fine-grained soils and is particularly useful when only quartz content is known. However, Côté and Konrad (2005) demonstrated that the generalized geometric mean method (eq. [26]) should be used for rock materials, especially when the quartz content is lower than 20%.

## Estimation of the thermal conductivity for dry soils

Very few models have been developed to estimate the $k_{\mathrm{dry}}$ values, and their narrow applicability range (soil type and porosity) only offers a reduced possibility for the generalized model this study attempts to develop. However, previous studies established that the thermal conductivity of dry soils is very sensitive to porosity and to a lesser extent to particle shape of natural mineral soils and crushed rocks (Smith 1942; Johansen 1975). Côté and Konrad (2005) also noticed that mineral content affects the thermal conductivity of dry base-course and subbase materials in the range of porosity of 0.13–0.4. The values of $k_{\mathrm{dry}}$ will be estimated by means of an empirical approach that will be developed from nearly 220 test results available from the literature for a wide variety of soils.

## Determination of the volume fraction of unfrozen water from experimental data

The thermal conductivity of frozen saturated fine-grained soils can be computed with eq. [18], which considers the presence of unfrozen water. Unfortunately, Kersten (1949) did not provide any information on the unfrozen water content of the fine-grained soils he studied. Furthermore, because the specific surface area was not determined, eq. [25] cannot be used to estimate $\theta_{\mathrm{u}}$ values. However, it is possible to extrapolate the thermal conductivity of saturated frozen soils directly from experimental data. This extrapolated $k_{\mathrm{sat(f)}}$ value is then used to calculate the volume fraction of unfrozen water, as illustrated by the following example.

Figure 4 shows the thermal conductivity relationships with the degree of saturation measured by Kersten (1949) for Fairbanks sand samples compacted at an average porosity of 0.29 ($n_{\mathrm{f}} = 0.31$) and for Healy clay samples compacted at an average porosity of 0.42 ($n_{\mathrm{f}} = 0.44$). The thermal conductivities of solid particles were estimated to be 5.3 W/m°C for Fairbanks sand and 2.8 W/m°C for Healy clay. For instance, according to Kersten (1949), the Fairbanks sand is composed of 59.4% quartz ($k_{\mathrm{m}}$ = 7.69 W/m°C); 3.6% orthoclase ($k_{\mathrm{m}}$ = 2.25 W/m°C); 5.0% feldspar (orthoclase, $k_{\mathrm{m}}$ = 2.25 W/m°C); 6.4% plagioclase (mean value for five minerals of the plagioclase series, $k_{\mathrm{m}}$ = 1.84 W/m°C); 8.0% olivine, amphibole, and pyroxene (mean value, $k_{\mathrm{m}}$ = 4.52 W/m°C); 10.0% igneous rock-forming minerals (mean value, $k_{\mathrm{m}}$ = 3 W/m°C); 2.5% hematite ($k_{\mathrm{m}}$ = 11.28 W/m°C); 0.1% mica ($k_{\mathrm{m}}$ = 2.03 W/m°C); and 5.0% minerals of unknown origin (mean value, $k_{\mathrm{m}}$ = 3 W/m°C). Equation [26] gives the following:

$$k_{\mathrm{s}} = 7.69^{0.549} \times 2.25^{0.036} \times 2.25^{0.05} \times 1.84^{0.063} \times 4.52^{0.08} \times 3^{0.1} \times 11.28^{0.025} \times 2.03^{0.001} \times 3^{0.05} = 5.29 \text{ W/m°C}$$

The thermal conductivities of Fairbanks sand and Healy clay, both in the saturated unfrozen state, were computed with eq. [14] and found to be 2.8 and 1.5 W/m°C, respectively. As shown by the trend lines that fit the experimental data in Fig. 4*a*, the computed values of $k_{\mathrm{sat(u)}}$ for both soils

**Table 2.** Thermal conductivities of selected minerals (converted to SI from Horai 1971).

| Mineral | $\rho_s$ (g/cm³) | $k$ (W/m°C) |
|---|---|---|
| **1. Nesosilicates** (single tetrahedron) | | |
| Olivine | | |
| Chrysolite | 3.47 | 4.81 |
| Fayalite | 4.36 | 3.16 |
| Forsterite | 3.28 | 5.03 |
| Hyalosiderite | 3.77 | 3.45 |
| Garnet | | |
| Almandine | 3.93 | 3.31 |
| Andradite | 3.75 | 3.09 |
| Grossularite | 3.49 | 5.48 |
| Pyrope | 3.75 | 3.18 |
| Spessartite | 3.99 | 3.40 |
| **2. Sorosilicates** (double tetrahedron) | | |
| Epidote | | |
| Clinozoisite | 3.36 | 2.40 |
| Epidote | 2.19 | 2.83 |
| Zoisite | 3.27 | 2.15 |
| Idocrase | 3.34 | 2.41 |
| **3. Cyclosilicates** (rings of tetrahedrons) | | |
| Beryl | 2.7 | 3.99 |
| Cordierite | 2.59 | 2.72 |
| Tourmaline | 3.13 | 5.28 |
| **4. Inosilicates** (chains of tetrahedrons) | | |
| Amphibole (double chains) | | |
| Actinolite | 3.06 | 3.48 |
| Grunerite | 3.40 | 3.32 |
| Hornblende | 3.18 | 2.81 |
| Tremolite | 2.99 | 4.78 |
| Pyroxene (single chains) | | |
| Acmite | 3.54 | 3.49 |
| Augite | 3.27 | 3.82 |
| Bronzite | 3.36 | 4.16 |
| Diopside | 3.33 | 4.93 |
| Enstatite | 3.27 | 4.47 |
| Spodumene | 3.16 | 5.65 |
| **5. Phyllosilicates** (sheets) | | |
| Chlorite | 2.75 | 5.15 |
| Mica | | |
| Biotite | 2.98 | 2.02 |
| Glauconite | 2.85 | 1.63 |
| Lepidomelane | 2.86 | 1.57 |
| Muscovite | 2.85 | 3.48 |
| Phlogopite | 2.85 | 2.13 |
| Pyrophyllite | 2.83 | 8.14 |
| Serpentine | | |
| Antigorite | 2.66 | 2.95 |
| Chrysotile | 2.60 | 5.30 |
| Lizardite | 2.60 | 2.34 |
| Talc | 2.81 | 6.10 |
| **6. Tectosilicates** (framework) | | |
| Feldspar | | |
| Celsian | 3.09 | 1.44 |
| Microline | 2.56 | 2.49 |

**Table 2** (*concluded*).

| Mineral | $\rho_s$ (g/cm³) | $k$ (W/m°C) |
|---|---|---|
| Orthoclase | 2.58 | 2.32 |
| Sanidine | 2.57 | 1.65 |
| Feldspathoid | | |
| Nepheline | 2.62 | 1.73 |
| Sodalite | 2.33 | 2.51 |
| Plagioclase | | |
| Albite | 2.63 | 1.96 |
| Anorthite | 2.77 | 1.68 |
| Bytownite | 2.72 | 1.56 |
| Labradorite | 2.70 | 1.53 |
| Oligoclase | 2.64 | 1.98 |
| Silica | | |
| Hyalite | 2.08 | 1.21 |
| Novaculite | 2.66 | 7.21 |
| Quartz | 2.65 | 7.69 |
| Vitreous silica | 2.21 | 1.35 |
| **7. Carbonates** | | |
| Calcite | 2.71 | 3.59 |
| Dolomite | 2.90 | 5.51 |
| Siderite | 3.81 | 3.01 |
| **8. Oxides** | | |
| Hematite | 5.14 | 11.28 |
| Ilmenite | 4.55 | 2.38 |
| Magnetite | 5.15 | 5.10 |
| Magnesiochromite | 4.23 | 2.54 |
| **9. Hydroxides** | | |
| Brucite | 2.39 | 8.44 |
| Gibbsite | 1.88 | 2.60 |
| **10. Sulphates** | | |
| Anhydrite | 2.98 | 4.76 |
| Barite | 4.41 | 1.34 |
| Gypse | 2.30 | 1.26 |
| Selenite | 2.32 | 1.26 |
| **11. Halides** | | |
| Fluorite | 3.10 | 9.51 |
| Halite | 2.10 | 6.11 |
| **12. Sulphides** | | |
| Chalcopyrite | 4.09 | 8.20 |
| Pyrite | 5.1 | 19.21 |
| Pyrrhotite | 4.6 | 1.89 |
| Sphalerite | 4.00 | 12.73 |
| **13. Phosphates** | | |
| Amblygonite | 3.03 | 4.99 |
| Chlorapatite | 3.15 | 1.39 |
| Fluorapatite | 3.22 | 1.37 |

agree fairly well with the experimental $k$–$S_r$ relationships, thus confirming the validity of eqs. [14] and [26] and the adequacy of the computed values of $k_s$.

Figure 4*b* shows the experimental data for the same soils in the frozen state. The extrapolated value of $k_{sat(f)}$ is 4.1 W/m°C for Fairbanks sand and 2.1 W/m°C for Healy clay. The volume fraction of unfrozen water corresponding

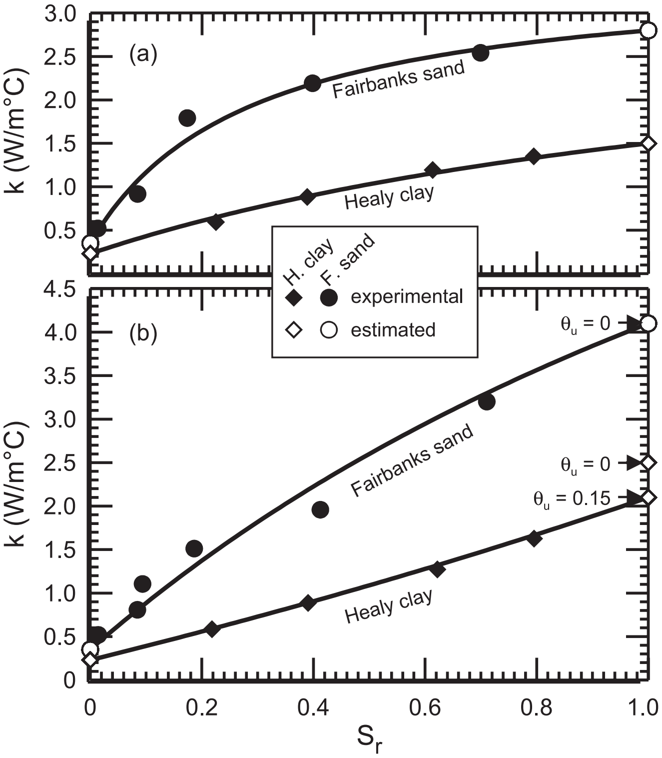



**Fig. 4.** Estimation of the thermal conductivity for (*a*) unfrozen and (*b*) frozen saturated soils (data from Kersten 1949).

to these values of $k_{sat(f)}$ was calculated with eq. [18]. As expected for Fairbanks sand, the calculated volume fraction of unfrozen water is approximately zero:

$$4.1 = 5.3^{1-0.31} \times 2.24^{0.31-\theta_u} \times 0.6^{\theta_u} \quad \Rightarrow \quad \theta_u = \frac{\log[4.1/(5.3^{1-0.31} \times 2.24^{0.31})]}{\log(0.6/2.24)} \approx 0$$

However, the volume fraction of unfrozen water that corresponds to the experimental data for Healy clay is approximately 0.15, as shown below:

$$2.1 = 2.8^{1-0.44} \times 2.24^{0.44-\theta_u} \times 0.6^{\theta_u} \quad \Rightarrow \quad \theta_u = \frac{\log[2.1/(2.8^{1-0.44} \times 2.24^{0.44})]}{\log(0.6/2.24)} \approx 0.15$$

It should be stressed that neglecting to consider the existence of unfrozen water content in this Healy clay soil would lead to a computed thermal conductivity of 2.5 W/m°C, which is about 20% higher than the experimental value, as shown in Fig. 4*b*.

## Results of analysis

### Thermal conductivity of dry soils

The high ratio of thermal conductivity between air and solid particles (>100) generally implies a higher sensitivity of thermal conductivity to the microstructure of dry soils (Johansen 1975). To date, no theoretical or analytical approach has been able to quantify this effect on $k_{dry}$ for a wide variety of dry soils. An empirical approach has thus been used in this paper to analyse the thermal conductivity data for dry soils. The analysis results are grouped according to the type of dry soil as (*i*) crushed rocks and gravels, (*ii*) natural mineral soils, and (*iii*) organic fibrous soils (peat). It should be noted that in the following, $k_{dry}$ values for soils studied by Kersten (1949) were extrapolated from $k$–$S_r$ relationships, because completely dry samples were not tested.

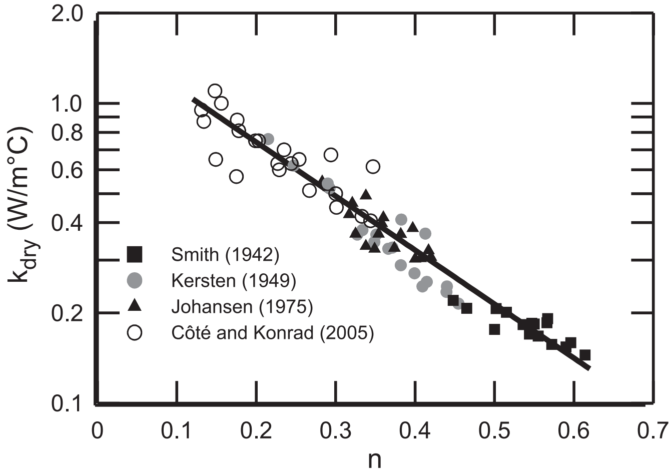



**Fig. 5.** Relationship between thermal conductivity and porosity for dry crushed rocks.

Figure 5 shows the relationship between the porosity and the thermal conductivity of dry crushed rocks, gravels, and fragmented soils, from Smith (1942), Kersten (1949), Johansen (1975), and Côté and Konrad (2005). The thermal conductivity of dry crushed rocks decreases with increasing porosity and ranges from about 1.4 to 0.13 W/m°C for porosities between 0.13 and 0.62. It is noted that the thermal conductivity of solid particles, $k_s$, reported by Smith (1942) ranges from 2.5 to 3.0 W/m°C, and Côté and Konrad (2005) reported thermal conductivities of solid particles of 1.6–3.4 W/m°C for seven types of crushed rocks. The thermal conductivity of solid particles, $k_s$, of Kersten's (1949) and Johansen's (1975) soils were computed from mineral composition and ranged from 2.7 to 4.3 W/m°C.

The relationship between the thermal conductivity of dry natural soils and porosity is presented in Fig. 6 for data from Johansen (1975), with estimated values of $k_s$ = 2.7–5.9 W/m°C; Kersten (1949), with estimated values of $k_s$ = 2–5 W/m°C; Slusarchuk and Watson (1975), with $k_s$ unknown; Smith (1942), with reported values of $k_s$ = 2.5–3.0 W/m°C; and Smith and Byers (1938) ($k_s$ unknown). The thermal conductivity of dry natural mineral soils ranges from about 0.4 to 0.12 W/m°C for porosities ranging from 0.25 to 0.67.

Figure 7 shows data for dry peat from Johansen (1975, Watzinger's data), Kersten (1949), and Smith and Byers (1938). It should be noted that an average value of 0.25 W/m°C is generally assigned to peat solid particles. The thermal conductivity of dry peat varies from 0.07 to 0.04 W/m°C for porosities ranging from 0.72 to 0.96. At a porosity of 0.96, the $k_{dry}$ values between 0.04 and 0.05 W/m°C are still about twice as high as that of air (0.024 W/m°C). This is probably caused by the existence of preferential heat flow along continuous peat fibres, which

**Fig. 6.** Relationship between thermal conductivity and porosity for dry natural mineral soils.

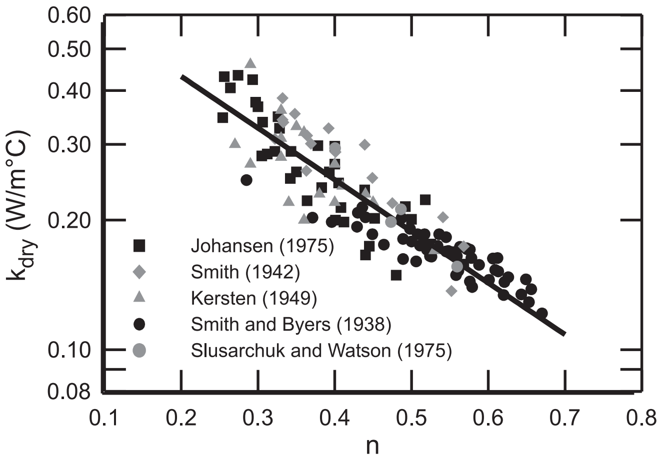
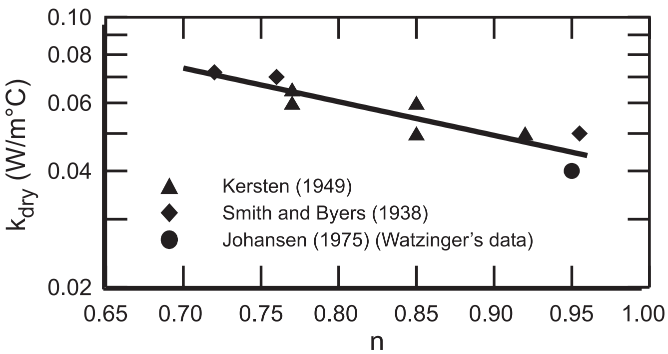



**Fig. 7.** Relationship between thermal conductivity and porosity for dry peat.




contributes to maintaining the thermal conductivity at a fairly high level, given that air occupies 96% of the total volume.

As the mineral content of solid particles has a small influence on the thermal conductivity of dry soils, Figs. 5–7 show that $k_{dry}$ can be related directly to the porosity $n$ for different types of soils. A generalized exponential relationship is thus proposed to estimate the $k_{dry}$ values for soils of different particle shapes:

$$[28] \quad k_{dry} = \chi \times 10^{-\eta n}$$

where $\chi$ (W/m°C) and $\eta$ (no units) are material parameters accounting for the particle shape effect; and $n$ is the porosity of soils (eq. [15]). The values of $\chi$ and $\eta$ are, respectively, 1.7 W/m°C and 1.8 for crushed rocks, 0.75 W/m°C and 1.2 for natural mineral soils, and 0.3 W/m°C and 0.87 for organic fibrous soil (peat).

Although not fully consistent with thermal conductivity limits and the volume fractions of constituents ($n = 0$, $k_{dry} = k_s$; and $n = 1$, $k_{dry} = k_a$), eq. [28] at least offers good fits to a very wide range of soils from the literature for porosities from $n = 0.1$ in coarse, broadly graded granular materials to $n = 0.95$ in peaty soils. This equation uses two soil-type-dependent empirical parameters ($\chi$ and $\eta$) that set reasonable limits for $k_{dry}$. At a porosity of $n = 0$, the upper thermal conductivity limits are given by the values of $\chi$, and at a porosity of $n = 1$, the lower thermal conductivity limits given by eq. [28] stand between values for $k_{dry}$ of 0.026 and 0.046 W/m°C, as shown in Fig. 8, where the solid lines represent the domains of porosity for the various soil types studied, and the dotted lines give the entire range of thermal conductivity predicted by eq. [28].

**Fig. 8.** Influence of soil type on $k_{dry}$.

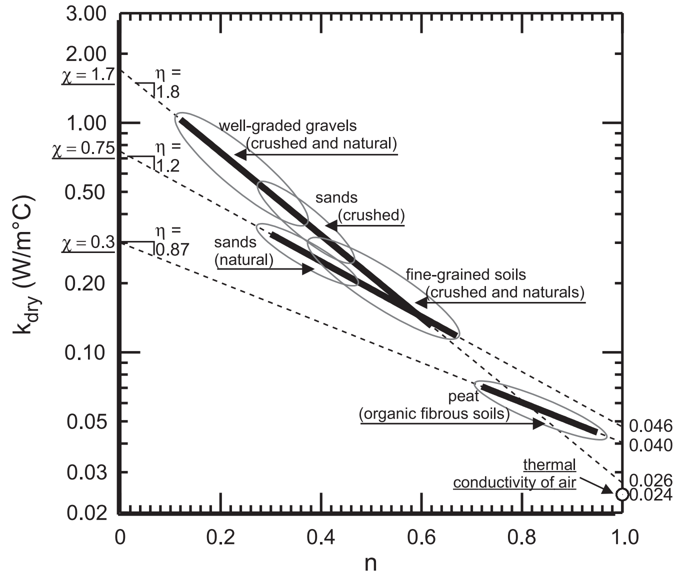



Figure 8 also shows the effects of grain-size distribution and particle shape (natural: subrounded; crushed: angular; and peat: fibrous) on the $k_{dry}$ relationships for the general domains of porosity for gravelly soils, sandy soils, fine-grained soils (silty and clayey soils), and peaty soils. For instance, in the sandy soils range of porosity, the difference between the relationships for natural sands and those for sands processed from crushed rock is pronounced. This difference tends to decrease as porosity increases, and both relationships merge in the porosity range of fine-grained soils ($n \approx 0.6$). For a sand sample with a given porosity, the improved interparticle contact in soils with angular particles, compared with soils with subrounded particles, probably explains the difference in thermal conductivity observed with experimental data. As seen for fine-grained soils, the influence of particle shape on the interparticle contacts decreases with increasing porosity, thus leading to similar values of $k_{dry}$. In the case of well-graded gravels, it seems that particle shape does not influence thermal conductivity. Referring back to Fig. 5, one can see that the data for natural gravel from Kersten (1949) actually follow the same $k_{dry}$–$n$ relationship as the data for crushed rocks from Côté and Konrad (2005). This is probably due to the fact that grain-size distributions of well-graded gravels contribute to obtaining lower porosities, thus improving interparticle contact, irrespective of particle shape.

## The $k_r$–$S_r$ relationships and determination of $\kappa$ for different types of soils

Actually, only Kersten's (1949) work, which still remains the most extensive study of its kind, provided enough information on the materials he studied to enable the estimation of the $k_{dry}$ and $k_{sat}$ values for the soil he studied and to develop the $k_r$–$S_r$ relationships. The thermal conductivity data from Kersten (1949) were thus normalized by eq. [2] for every unfrozen sample tested at a mean temperature of 4 °C and for every frozen sample tested at a mean temperature of –4 °C. Nearly 500 test results were analysed; the soils covered were crushed rocks, natural gravels, sands, fine-grained soils, and peat.

It is well known that the thermal conductivity functions, expressed with respect to the degree of saturation (or water content), are influenced by the grain-size distribution of soils (Kersten 1949; Van Rooyen and Winterkorn 1957; Johansen 1975). The normalized thermal conductivity relationships obtained in this study are grouped according to the grain-size distribution of the soils studied: (*i*) well-graded gravels and coarse sands; (*ii*) medium and fine uniform sands; (*iii*) silty and clayey soils; and (*iv*) peat, as discussed in the following paragraphs. Regressions using eq. [13] were made to determine the values of κ for each type of soil in the unfrozen and frozen states. The few negative $k_r$ values obtained at low degrees of saturation shown in Figs. 9–11 reveal that eq. [28] might slightly overestimate the values of $k_{dry}$ for some of the samples studied by Kersten (1949).

Results of the analyses of Kersten's (1949) data for Chena River gravel (broadly graded natural gravel), Fairbanks sand (broadly graded natural coarse sand), crushed quartz, crushed trap rock, crushed feldspar, and crushed granite (broadly graded coarse sands) reported by Kersten (1949) are presented in Fig. 9. The computed thermal conductivities of solid particles, $k_s$ (W/m°C), from the generalized geometric mean method, in conjunction with the mineral composition data, are 4.5 for Chena River gravel, 5.3 for Fairbanks sand, 7.7 for crushed quartz, 2.9 for crushed trap rock, 2.6 for crushed feldspar, and 2.7 for crushed granite. The porosities of these materials ranged from 0.22 to 0.45, and the testing water content varied from 0% to 11.8%. No unfrozen water content is expected to exist in these coarse materials at a temperature of –4 °C. The $k_r$–$S_r$ relationships obtained for gravels and coarse sands studied by Kersten (1949) lead to κ values of 4.5 in the unfrozen state (Fig. 9*a*) and 1.5 in the frozen state (Fig. 9*b*), which are pretty close to the values of 4.7 and 1.8 obtained by Côté and Konrad (2005) from 180 test results for materials with slightly coarser grain-size distributions and lower porosities.

The medium and fine sands studied by Kersten (1949) were standard and graded Ottawa sand ($k_s$ = 7.7 W/m°C), Lowell sand ($k_s$ = 5.3 W/m°C), Northway sand ($k_s$ = 2 W/m°C), and Northway fine sand ($k_s$ = 2.3 W/m°C). The porosity of these sands varied from 0.25 to 0.45, and testing water content varied from 0.2% to 18%. No unfrozen water content is expected to exist for these sands at a temperature of –4 °C. The $k_r$–$S_r$ relationships obtained in this study for medium and fine sands are shown in Fig. 10. The values of κ obtained with eq. [13] are 3.55 for the unfrozen state (Fig. 10*a*) and 0.95 for the frozen state (Fig. 10*b*).

The fine-grained soils studied by Kersten (1949) were Healy clay ($k_s$ = 2.8 W/m°C), Fairbanks silty clay loam ($k_s$ = 2.8 W/m°C), Fairbanks silt loam ($k_s$ = 3.3 W/m°C), Northway silt loam ($k_s$ = 2.0 W/m°C), Ramsey sandy loam ($k_s$ = 3.9 W/m°C), and Dakota sandy loam ($k_s$ = 5.0 W/m°C). The porosity of these soils ranged from 0.3 to 0.65 for water content varying from 1.5% to 39%. In general, the estimated volume fraction of unfrozen water, $\theta_u$, was 0.1 for all soils except Healy clay and Dakota sandy loam, for which mean values of 0.15 and 0.05, respectively, were obtained. The results of the computation of the normalized thermal conductivity obtained from the analyses of the fine-grained soils are shown in Fig. 11. The mean $k_r$–$S_r$ relationships are best described by eq. [13], with κ = 1.9 for the unfrozen state (Fig. 11*a*) and κ = 0.85 for the frozen state (Fig. 11*b*).

Kersten (1949) measured the thermal conductivity of some peat samples with porosities ranging from 0.73 to 0.92 and with water contents ranging from 10% to 285%. Some

**Fig. 9.** Normalized thermal conductivity for (*a*) unfrozen and (*b*) frozen broadly graded gravels and coarse sands (analyses of data from Kersten 1949).

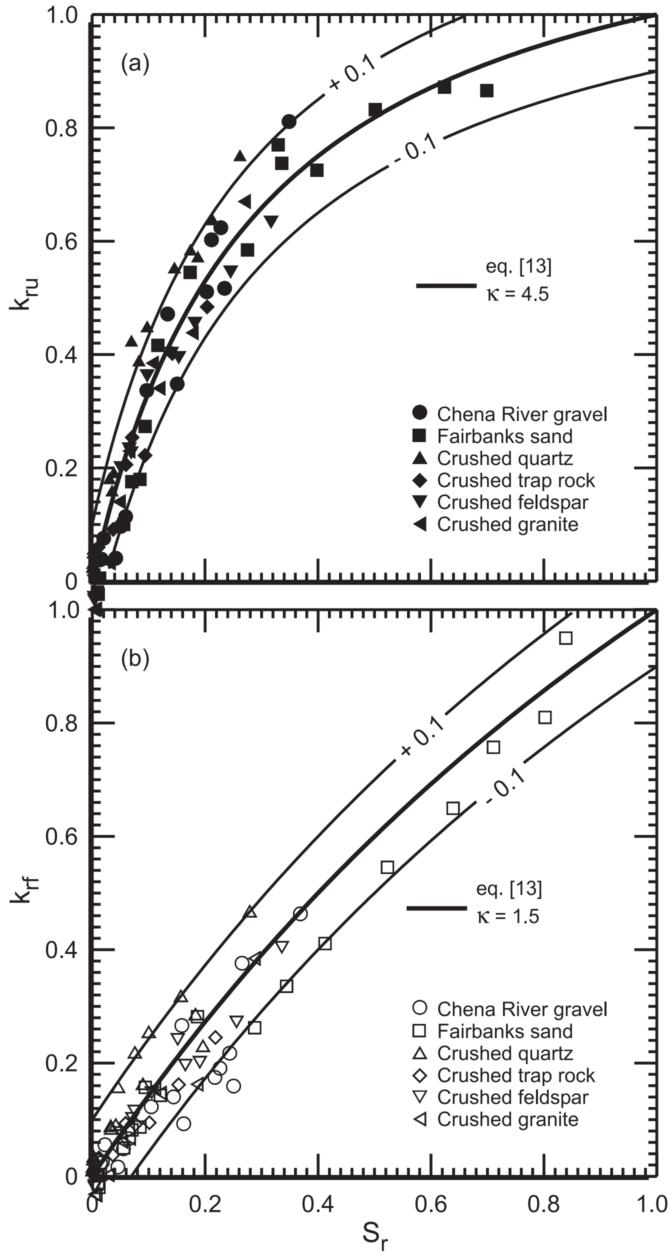


**Fig. 10.** Normalized thermal conductivity for (*a*) unfrozen and (*b*) frozen medium and fine sands (analyses of data from Kersten 1949).

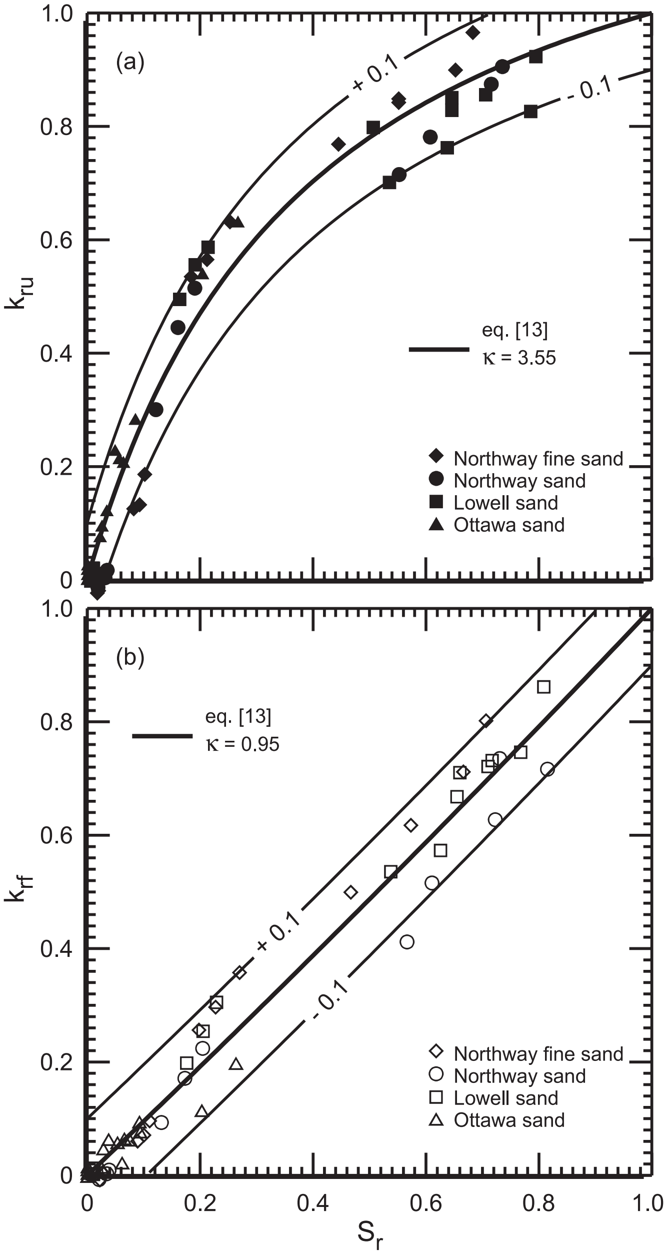
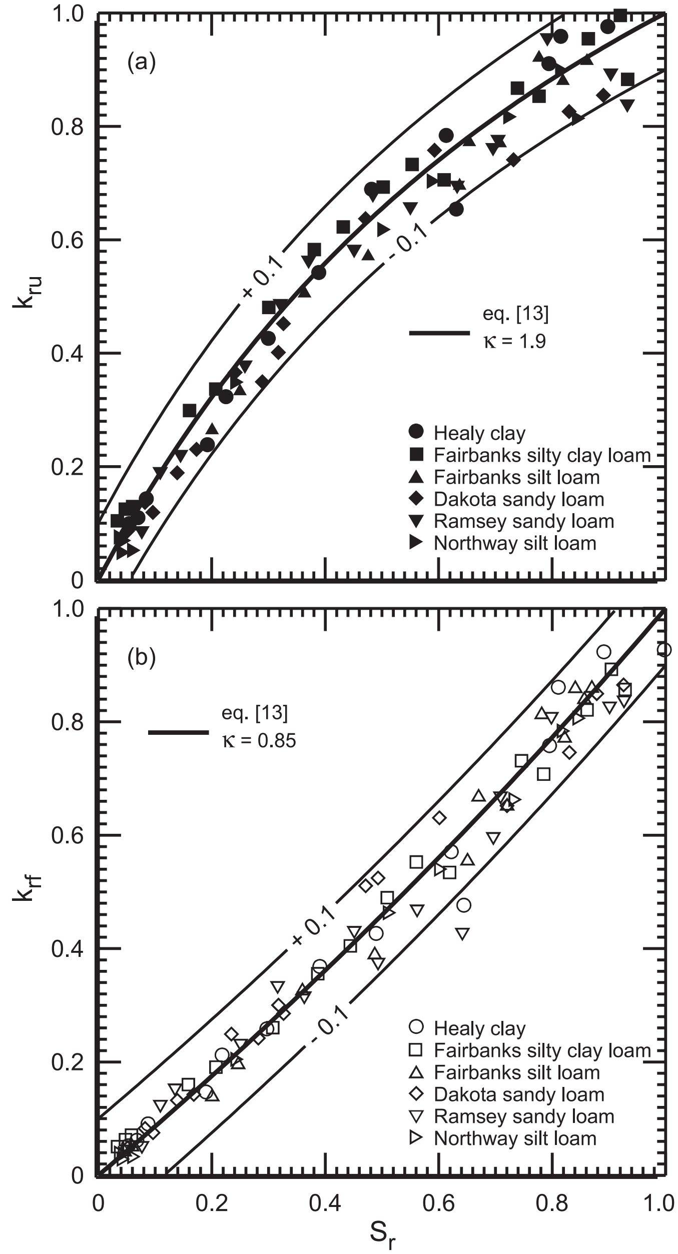



**Fig. 11.** Normalized conductivity for (*a*) unfrozen and (*b*) frozen thermal silty soils, clayey soils, silts, and clays (analyses of data from Kersten 1949).




data from Watzinger reported by Johansen (1975) for a porosity of 0.95 and water contents ranging from 34% to almost 900% were also analysed in this study. The mean density of peat solid particles was set at 1500 kg/m³ (Smith and Byers 1938), and the thermal conductivity of peat particles was taken to be equal to 0.25 (De Vries 1963), as these properties were not provided by the authors. Because of its very high porosity and fibrous texture, peat is not expected to contain significant amounts of unfrozen water at temperatures below 0 °C. The normalized thermal conductivities obtained for peat are shown in Fig. 12*a* for the unfrozen case and in Fig. 12*b* for the frozen case. Values of κ obtained for these highly organic soils are 0.6 for the unfrozen case and 0.25 for the frozen case.

As shown in Figs. 2 and 9–12, more than 95% of all the $k_r$ data analysed here lie within ±0.1 standard deviations of the mean $k_r$–$S_r$ relationships obtained in this study.

Relating the soil-type factor κ to other soil parameters requires minimal knowledge of geotechnical properties. The grain-size distribution would be a good indicator to characterize each soil analysed here, but only partial distributions were provided by Kersten (1949), so parameters such as mean particle diameter ($d_{50}$), effective particle diameter ($d_{10}$), uniformity coefficient ($C_u$), and curvature coefficient ($C_c$) cannot be determined precisely. Also, the diameters of peat solid particles cannot be characterized in the same manner as those of conventional soils (gravel, sand, silt, and clay), as a result of its highly fibrous texture. However, the

**Fig. 12.** Normalized thermal conductivity for (*a*) unfrozen and (*b*) frozen organic fibrous soils (analyses of data from Kersten 1949 and Johansen 1975).

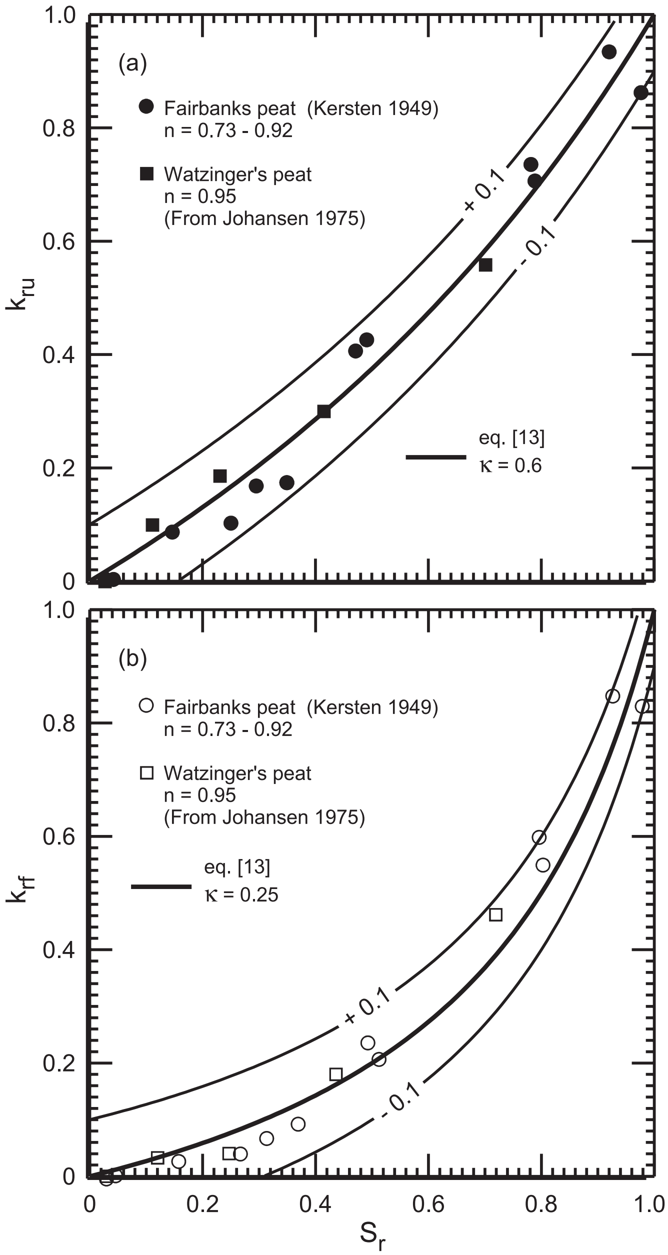
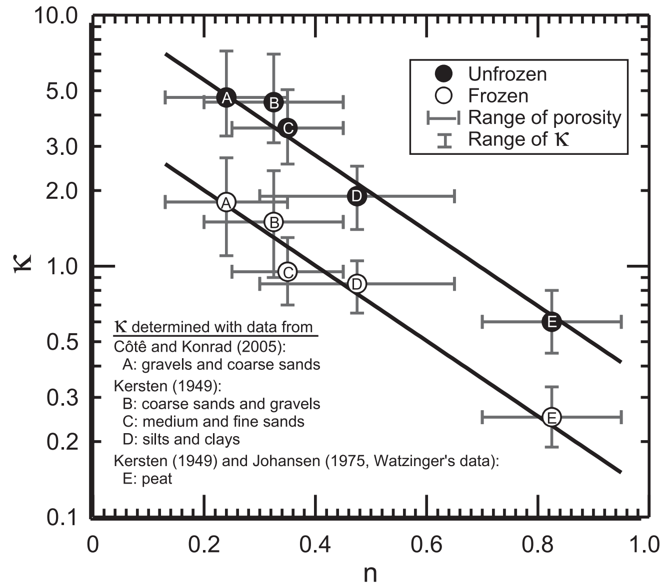



**Fig. 13.** Influence of type of soil and porosity of unfrozen and frozen soils on soil-type factor κ.




grain-size distribution, together with the shape of particles, will affect the packing geometry and the soil density. For example, gravel with widely distributed particle sizes can be packed to lower porosities than fine sand with particles of equal size. The porosity, which was computed for every soil analysed here, can thus be used to qualitatively characterize the soil fabric, as shown in Fig. 13, where the values of the soil-type factor κ for unfrozen and frozen conditions are plotted as functions of the median porosity for each type of soil studied. The full ranges of porosity and κ are also given for each type of soil studied (grey lines). In a semilogarithmic representation, the parallel linear relationships obtained in the unfrozen and frozen states show the strong influence of type of soil and porosity on the variation of κ.

Given the strong correlations obtained in this paper for a wide variety of soils, it is thus proposed that eq. [13] be used to describe the variation of normalized thermal conductivity with the degree of saturation for unfrozen and frozen soils. Typical values of the soil-type factor κ for the different types of soil studied in this paper are as follows: (*i*) for broadly graded gravels and coarse sands, $\kappa_u = 4.60$ and $\kappa_f = 1.70$, which are the means of the values obtained for materials from Côté and Konrad (2005) and Kersten (1949); (*ii*) for medium and fine sand, $\kappa_u = 3.55$ and $\kappa_f = 0.95$; (*iii*) for silty and clayey soils, silt, and clay, $\kappa_u = 1.90$ and $\kappa_f = 0.85$; and (*iv*) for organic fibrous soils (peat), $\kappa_u = 0.60$ and $\kappa_f = 0.25$.

## Predicting the thermal conductivity of soils

The normalized thermal conductivity concept, as proposed by Johansen (1975), normalizes the thermal conductivity of soils with respect to the thermal conductivity in dry and saturated states. It allows the modelling of the thermal conductivity as a function of porosity, degree of saturation, mineral content, grain-size distribution, and particle shape of unfrozen and frozen soils, with eq. [2] rewritten as

$$[29] \quad k = (k_{sat} - k_{dry})\, k_r + k_{dry}$$

Improvements to the original model have been reported in this paper: (*i*) A generalized empirical $k_r$–$S_r$ relationship (eq. [13]) using a soil-type-dependent empirical parameter has been developed to assess the normalized thermal conductivity with respect to the type of soil. (*ii*) The thermal conductivity of solid particles is chosen from average values in Table 1 or is computed using eq. [26] with values of thermal conductivity of rock-forming minerals from Table 2. The value of $k_s$ is then used in eqs. [14] and [18] to compute thermal conductivity of soils in the unfrozen and frozen saturated states. (*iii*) A generalized empirical relationship (eq. [28]) has been developed to compute the thermal conductivity of dry soils with respect to grain-size distribution and shape of particles. The new empirical equations pro-

**Table 3.** Parameters for the estimation of the thermal conductivities of four soils from Kersten (1949).

| | | | Crushed granite | Lowell sand | Healy clay | Fairbanks peat |
|---|---|---|---|---|---|---|
| $\rho_d$ | | (kg/m³) | 1926 | 1691 | 1293 | 332 |
| $\rho_s$ | | (kg/m³) | 2700 | 2670 | 2590 | 1500 |
| $w$ | | (%) | 3.9 | 11,0 | 34.8 | 185.0 |
| $n$ | eq. [15] | | 0.29 | 0.37 | 0.50 | 0.78 |
| $S_r$ | eq. [20] | | 0.26 | 0.50 | 0.90 | 0.79 |
| $\theta_u$ | eq. [25] | | 0.00 | 0.00 | 0.15[a] | 0.00 |
| $k_s$ | eq. [26] | (W/m°C) | 2.70 | 5.50 | 2.80 | 0.25 |
| $k_{sat(u)}$ | eq. [14] | (W/m°C) | 1.75 | 2.42 | 1.30 | 0.49 |
| $k_{sat(f)}$ | eq. [18] | (W/m°C) | 2.56 | 3.94 | 2.06 | 1.38 |
| $\chi$ | | (W/m°C) | 1.70 | 0.75 | 0.75 | 0.30 |
| $\eta$ | | | 1.80 | 1.20 | 1.20 | 0.87 |
| $k_{dry}$ | eq. [28] | (W/m°C) | 0.51 | 0.27 | 0.19 | 0.06 |
| $\kappa_u$ | | | 4.60 | 3.55 | 1.90 | 0.60 |
| $\kappa_f$ | | | 1.70 | 0.95 | 0.85 | 0.25 |
| $k_{ru}$ | eq. [13] | | 0.62 | 0.78 | 0.94 | 0.69 |
| $k_{rf}$ | eq. [13] | | 0.37 | 0.49 | 0.88 | 0.48 |
| $k_u$ | eq. [29] | (W/m°C) | 1.28 (1.38) | 1.95 (1.96) | 1.23 (1.27) | 0.36 (0.37) |
| $k_f$ | eq. [29] | (W/m°C) | 1.27 (1.31) | 2.07 (2.17) | 1.84 (1.90) | 0.70 (0.81) |

**Note:** Values in parenthesis represent measured parameters.
[a]Estimated from experimental $k$–$S_r$ relationships.

posed in this paper were developed using the most cited data banks in the literature and provide values of thermal conductivity that respect the bounding limits of thermal conductivity values for most soils.

Figure 14 summarizes the mathematical equations and parameters proposed to predict the thermal conductivity of soil using eq. [29]. To simplify calculations in this prediction method, the effect of volume change as water turns into ice below 0 °C is neglected, and the porosity and the degree of saturation are computed simply by using eqs. [15] and [20].

## Working examples

The use of this new model can be illustrated with examples for four soils from Kersten (1949). The basic geotechnical parameters and thermal conductivity parameters are grouped in Table 3. The soils are crushed granite (well-graded coarse sand, $\rho_d$ = 1926 kg/m³, $w$ = 3.9%), Lowell sand (natural medium sand, $\rho_d$ = 1691 kg/m³, $w$ = 11.0%), Healy clay (natural soil, $\rho_d$ = 1293 kg/m³, $w$ = 34.8%, $\theta_u$ = 0.15), and Fairbanks peat (organic fibrous soil, $\rho_d$ = 332 kg/m³, $w$ = 185.0%). The following paragraphs present the detailed computation for the crushed granite only.

The mean thermal conductivity of granite, $k_s$, is 2.5 W/m°C, as given in Table 1. However, for a more accurate analysis, it can be computed from mineral composition, as the solid particles of Kersten's (1949) crushed granite are 20% quartz, 30% orthoclase, 40% plagioclase, and 10% minerals of unknown origin. From Table 2, the thermal conductivity, $k_m$ (W/m°C), for forming minerals is 7.69 for quartz, 2.32 for orthoclase, and 1.74 for plagioclase (mean value for five minerals of the plagioclase series); a mean value of $k_m$ = 3 W/m°C is assigned to the unknown minerals. Equation [26] gives

**Fig. 14.** Method for calculating the thermal conductivity of unfrozen and frozen soils. Note: cr, crushed; org., organic.

*(a) basic geotechnical parameters :*

1) $n_f \approx n_u = n = 1 - \frac{\rho_d}{\rho_s}$   2) $S_{rf} \approx S_{ru} = S_r = \frac{w}{100}\frac{\rho_d}{n\rho_w}$

3) $\ln(\theta_u) = \ln(\rho_d) + 0.5519\ln(S_s) - 1.449\ln(-T)S_s^{-0.264} - 11.251$

*(b) thermal conductivity parameters:*

$$k = (k_{sat} - k_{dry}) \times k_r + k_{dry}$$

*unfrozen :* $k_s^{1-n} \times 0.6^n$

*frozen :* $k_s^{1-n} \times 2.24^{n-\theta_u} \times 0.6^{\theta_u}$

*with values of $k_s$:*
*- from Table 1*
*- or from* $\prod_j k_{mj}^{x_j}$ *with values of $k_m$ from Table 2*

$\chi \times 10^{-\eta n}$

| | $\chi$ | $\eta$ |
|---|---|---|
| *cr. rocks and gravels:* | *1.70* | *1.80* |
| *natural mineral soils:* | *0.75* | *1.20* |
| *org. fibrous soils (peat):* | *0.30* | *0.87* |

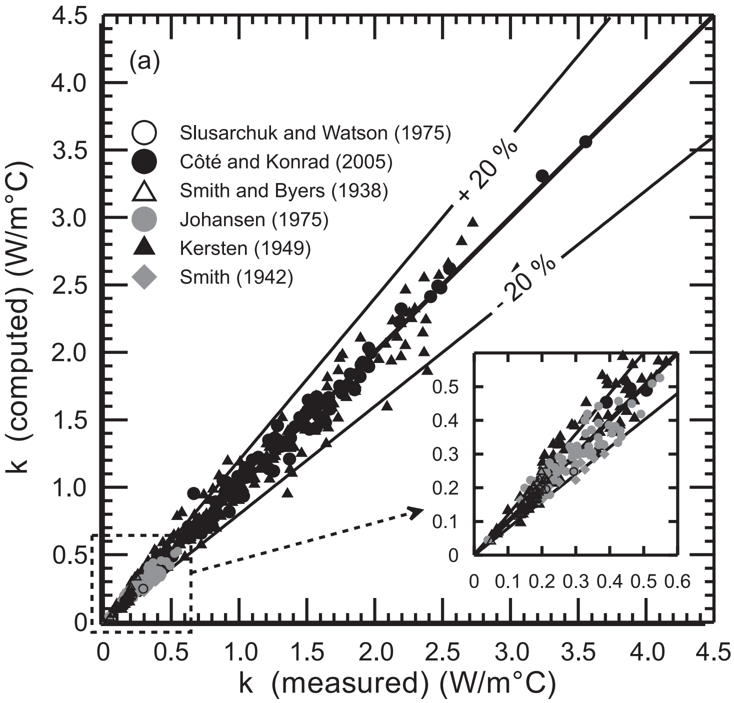
$\frac{\kappa S_r}{1 + (\kappa - 1)S_r}$

| | $\kappa$ unfrozen | frozen |
|---|---|---|
| *gravels and coarse sands:* | *4.60* | *1.70* |
| *medium and fine sands:* | *3.55* | *0.95* |
| *silty and clayey soils:* | *1.90* | *0.85* |
| *organic fibrous soils (peat):* | *0.60* | *0.25* |

$$k_s = 7.69^{0.2} \times 2.32^{0.3} \times 1.74^{0.4} \times 3.00^{0.1} = 2.69 \text{ W/m°C}$$

The thermal conductivities in the saturated unfrozen and frozen states are computed with eqs. [14] and [18]. As no unfrozen water content is expected to exist in this coarse sand, $\theta_u$ = 0, giving

**Fig. 15.** Predicted thermal conductivity values compared with measured values for (*a*) unfrozen moist sands and dry soils and (*b*) frozen moist soils.




$$k_{sat(u)} = 2.69^{0.71} \times 0.6^{0.29} = 1.75 \text{ W/m°C}$$

$$k_{sat(f)} = 2.69^{0.71} \times 2.24^{0.29-0.00} \times 0.6^{0.00} = 2.56 \text{ W/m°C}$$

The thermal conductivity in the dry state is computed with eq. [28], with $\chi = 1.7$ and $\eta = 1.8$ for crushed rocks:

$$k_{dry} = 1.7 \times 10^{-1.8\times0.29} = 0.51 \text{ W/m°C}$$

The normalized thermal conductivities in the unfrozen and frozen states are computed with eq. [13] and $\kappa_u = 4.6$ and $\kappa_f = 1.7$ for gravels and coarse sands:

$$k_{ru} = \frac{4.6 \times 0.26}{1 + (4.6 - 1) \times 0.26} = 0.62$$

$$k_{rf} = \frac{1.7 \times 0.26}{1 + (1.7 - 1) \times 0.26} = 0.37$$

Finally, the actual thermal conductivities in the unfrozen and frozen states are computed with eq. [29]:

$$k_u = (1.75 - 0.51) \times 0.62 + 0.51 = 1.28 \text{ W/m°C}$$

$$k_f = (2.56 - 0.51) \times 0.37 + 0.51 = 1.27 \text{ W/m°C}$$

Kersten (1949) reported measured values of thermal conductivity of 1.38 W/m°C for soils in the unfrozen state and of 1.31 W/m°C for soils in the frozen state. For the other examples reported here, the computed values for Lowell sand are $k_u$ = 1.95 W/m°C and $k_f$ = 2.07 W/m°C, whereas Kersten measured values of 1.96 and 2.17 W/m°C, respectively. For Healy clay, the computed values are $k_u$ = 1.23 W/m°C and $k_f$ = 1.84 W/m°C, whereas the measured values were 1.27 and 1.90 W/m°C, respectively. And finally, the computed values for Fairbanks peat are $k_u$ = 0.36 W/m°C and $k_f$ = 0.70 W/m°C, whereas the measured ones were 0.37 and 0.81 W/m°C, respectively.

Overall, the model generally predicts the thermal conductivity within an error of 20% for the full range of saturation, as shown in Fig. 15*a* for unfrozen moist soils and dry soils and in Fig. 15*b* for frozen moist soils, which is consistent with values from the data banks used in this study.

## Conclusion

The thermal conductivity of soils is a key parameter in analyses of soil heat transfer. It has been extensively measured for various types of soil for many decades. Numerous analytical and empirical thermal conductivity models have been developed for dry, moist, and saturated soils as a function of combinations of soil parameters, such as the degree of compaction, degree of saturation, mineral content, and grain-size distributions in the frozen and unfrozen states.

The data from nearly 1000 test results from the literature were analysed in this paper to develop a generalized thermal conductivity model for soils and construction materials. The model, which applies to a wide variety of soils, such as pavement base-course and subbase materials, gravels, sands, silty and clayey soils, silts, clays, and peat, is based on the normalized thermal conductivity of soils with respect to values for saturated and dry states.

The parameters of the model are the following:

(*i*) The thermal conductivity of solid particles, $k_s$: This parameter can be determined either from typical values summarized in this paper or from the mineral composition and using the generalized geometric mean model and values of thermal conductivity of rock-forming minerals.

(*ii*) The thermal conductivity of saturated unfrozen or frozen soils, $k_{sat}$: The geometric mean method with consideration of unfrozen water content is used to determine this parameter.

(*iii*) The thermal conductivity of dry soil, $k_{dry}$: Analyses of data from the literature allowed us to develop a generalized empirical relationship to assess the thermal conductivity of dry soils with respect to porosity and the shape of the particles in the soils studied.

(*iv*) The normalized thermal conductivity of soils, $k_r$: A generalized empirical relationship was developed to relate the normalized thermal conductivity of soils to the degree of saturation by using a single parameter, referred to as the soil-type factor, $\kappa$. This relationship covers the entire range of degrees of saturation, which is not the case with the logarithmic functions generally used in existing empirical models. Correlations were obtained for unfrozen and frozen soils with different grain-size distributions reported in the literature: well-graded gravels and coarse sands; medium and fine sands; silty and clayey soils, silts, and clays; and organic fibrous soils (peat). It was shown that $\kappa$ is related to the fabric of unfrozen and frozen soils.

Owing to its generalized form, this method can easily be automated for the estimation of the thermal conductivity in numerical analyses of heat transfer and frost propagation in soils.

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## List of symbols

- $k$ thermal conductivity (W/m°C)
- $k_{dry}$ thermal conductivity of dry soil (W/m°C)
- $k_f$ thermal conductivity of frozen soil (W/m°C)
- $k_i$ thermal conductivity of ice (W/m°C)
- $k_m$ thermal conductivity of mineral (W/m°C)
- $k_r$ normalized thermal conductivity
- $k_{rf}$ normalized thermal conductivity of frozen soil
- $k_{ru}$ normalized thermal conductivity of unfrozen soil
- $k_s$ thermal conductivity of solid particles (W/m°C)
- $k_{sat}$ thermal conductivity of saturated soil (W/m°C)
- $k_{sat(f)}$ thermal conductivity of frozen saturated soil (W/m°C)
- $k_{sat(u)}$ thermal conductivity of unfrozen saturated soil (W/m°C)
- $k_u$ thermal conductivity of frozen soil (W/m°C)
- $k_w$ thermal conductivity of water (W/m°C)
- $n$ porosity
- $n_f$ porosity of frozen soil
- $n_u$ porosity of unfrozen soil
- $q$ volume fraction of quartz in solid particles
- $S_r$ degree of saturation
- $S_{rf}$ degree of saturation in frozen soil
- $S_{ru}$ degree of saturation in unfrozen soil
- $S_s$ specific surface area ($m^2$/g)
- $T$ temperature (°C)
- $w$ water content (% by weight)
- $w_{sat}$ saturation water content (% by weight)
- $w_u$ unfrozen water content (% by weight)
- $x$ volume fraction of minerals in solid particles
- $\eta$ material parameter to estimate $k_{dry}$
- $\theta_u$ volume fraction of unfrozen water
- $\kappa$ fabric factor to estimate $k_r$
- $\kappa_f$ fabric factor to estimate $k_r$ for frozen soils
- $\kappa_u$ fabric factor to estimate $k_r$ for unfrozen soils
- $\rho_d$ dry density (kg/$m^3$)
- $\rho_s$ density of solid particles (kg/$m^3$)
- $\rho_w$ density of water (kg/$m^3$)
- $\chi$ material parameter to estimate $k_{dry}$