

STOCHASTIC ANALYSIS OF SOIL WATER REGIME IN A WATERSHED¹

J.W. HOPMANS² and J.N.M. STRICKER

Department of Hydraulics and Catchment Hydrology, Agricultural University, Nieuwe Kanaal 11, 6709 PA Wageningen (The Netherlands)

(Received February 15, 1988; accepted after revision June 1, 1988)

ABSTRACT

Hopmans, J.W. and Stricker, J.N.M., 1989. Stochastic analysis of soil water regime in a watershed. *J. Hydrol.*, 105: 57–84.

A stochastic-deterministic one-dimensional model is proposed that simulates soil water flow for variable soil hydraulic properties and variable lower boundary conditions. These variations were determined by a normalization procedure which yields a scaled mean or reference curve and a set of scale factor values. Knowledge of the distribution type and mean and standard deviation of scale factor values allowed new scale factor values to be randomly generated. Using Monte Carlo (MC) simulations, two sets of random scale factor values for both the soil hydraulic properties and the lower boundary condition yielded a set of output variables. From calculation of the statistical properties of each such set of output variables, it was then possible to divide the Hupselse Beek watershed into a number of soil classes, each of which yielded distinct values for average actual transpiration during the growing seasons of 1976 and 1982. Computer simulated cumulative transpiration, watershed discharge and groundwater levels compared favourably with measured values in most cases. From the results of the MC analysis it was furthermore possible to predict reduction in actual transpiration (*RED*) from initial groundwater level and scale factor values for the horizons. With multiple regression analysis, *RED* values could be predicted with an estimated standard deviation of approximately 10 mm in 1976 and 7 mm in 1982.

INTRODUCTION

The deterministic solution of the unsaturated water flow equation has proven to be adequate at a scale of a field plot or laboratory experiment. However, on a field or watershed scale, variations in soil hydraulic properties and boundary conditions are such that statistical techniques to treat these variations are essential.

The fundamental question then arises of how to deal with the heterogeneous reality as one attempts to develop quantitative descriptions of water flow in a large-scale soil water system. A possible choice could be to construct a detailed deterministic model, representing the actual heterogeneity. These models,

¹This study was supported by the Netherlands Foundation of Earth Science Research (AWON) with funds from the Dutch Organization for the Advancement of Pure Research (ZWO).

²Currently employed at the Department of Land, Air and Water Resources, University of California, Veihmeyer Hall, Davis, CA 95616 (U.S.A.).

however, will require too much computational effort and are impractical in terms of the amount of data required. In addition, presuming that such a detailed model exists, the level of detail in output would most often be excessive in relation to the application requirements. To reduce the level of detail, one might differentiate between subareas, identified by a soil survey and find a deterministic solution for each subarea (Wösten et al., 1985).

Another possibility requires the definition of an equivalent uniform porous medium. Assuming that it would be possible to characterize the hydraulic properties of the soil medium by a representative function, a single deterministic run would be sufficient to obtain the overall behavior of the considered soil-water system. However, analyses for one-dimensional transient groundwater flow (Freeze, 1975) as well as one-dimensional transient unsaturated water flow (Russo and Bresler, 1981a; Bresler and Dagan, 1983) have shown that such an equivalent uniform soil cannot be defined by simply averaging the parameters that describe the soil hydraulic properties. In addition, application of the equivalent uniform medium concept would not give insight in the magnitude of the variability of output variables.

In the stochastic approach the input parameters for a flow model are considered to be random variables with an associated probability density function. The majority of investigations using the stochastic approach entail studies of steady groundwater flow, where a random variability in saturated hydraulic conductivity is assumed. The statistical properties of the hydraulic conductivity being known and the hydraulic conductivity supposed to be taken from a stationary stochastic process, the objective of these groundwater flow studies is to determine the statistical properties of output variables, such as hydraulic head.

Pioneering work in the stochastic analysis of one-dimensional groundwater flow was done by Freeze (1975). In order to generate an input data field with the desired stochastic properties, Freeze (1975) assumed the hydraulic conductivity to be spatially uncorrelated, while Smith and Freeze (1979a) introduced spatial correlation with a nearest-neighbour generator. Stochastic analysis of groundwater flow was carried out by Monte Carlo simulation techniques of which the solution led to a probability density function of the hydraulic head. It was shown that the standard deviation in hydraulic head increases with either an increase in the standard deviation of hydraulic conductivity or the strength of correlation between neighbouring conductivity values.

Alternatively, Bakr et al. (1978) characterized the spatial variation of the saturated hydraulic conductivity by a covariance function. Under the assumptions of statistical homogeneity, spectral analysis theory is used to solve perturbed forms of the stochastic partial differential equation describing groundwater flow through a porous medium with randomly varying hydraulic conductivity values. The final result is a closed-form solution, expressing the variability in hydraulic head as a function of the variability of the conductivity field. Gelhar (1986), however, questions whether this approach will be adequate in cases of more complicated phenomena, such as unsaturated flow.

To analyse variability in unsaturated flow, one needs to define and determine the variation in both the hydraulic conductivity and the soil-water characteristic curve. Russo and Bresler (1980a, b, 1981a) used five parameters to characterize the spatial variability of the soil hydraulic properties in a heterogeneous field. To generate random values of these parameters, they needed the distribution functions and their moments for each parameter as well as the correlation coefficients between the parameters. Scaling is an alternative approach to express the variability of soil hydraulic properties. If the sampled soils behave approximately as geometrically similar media, then the distribution of only one scaling parameter may express both the variability of the soil water characteristic and the unsaturated hydraulic conductivity curve. Contributions to the scaling theory include work by Miller and Miller (1956) and Warrick et al. (1977a).

Relatively simple infiltration and drainage problems were solved by Warrick et al. (1977b) and Warrick and Amoozegar-Fard (1979). They used Monte Carlo analysis, while generating random values for scale factor values from a known distribution. One-dimensional steady-state unsaturated flow with an autocorrelated stationary hydraulic conductivity function was examined by Andersson and Shapiro (1983). In their study they compared the stochastic output obtained with the Monte Carlo technique in combination with the nearest-neighbour of Smith and Freeze (1977a) and the perturbation technique of Bakr et al. (1978) and Gutjahr and Gelhar (1981). In four examples a relatively good comparison was obtained between the results of the two techniques.

Although the present study will treat one-dimensional vertical flow only, the influence of increasing the spatial dimension of the flow domain must be understood. Contracting flow to one dimension exaggerates the effects of the lower tail of the conductivity distribution in determining the standard deviation in hydraulic head values. Regions of low hydraulic conductivity will to a large extent control the hydraulic head values everywhere along the flow line. In two dimensions, water can bypass such low conductivity regions, which will generally lead to a decrease in the standard deviation of hydraulic head (Smith and Freeze, 1979b; Gutjahr and Gelhar, 1981). For unsaturated flow, similar effects may be expected. Yeh et al. (1985a) compared soil water pressure head variances of one- and three-dimensional steady unsaturated flow. They concluded that, especially for finer textured soils, extrapolation of one-dimensional to three-dimensional results could lead to wrong conclusions about the effects of field heterogeneity. Anisotropy of unsaturated hydraulic conductivity may well have a significant effect on the lateral flow component and influences vertical water movement to the groundwater (Yeh et al., 1985b).

When one-dimensional water flow is considered, it is usually assumed that the input parameters are statistically independent in the other directions. However, work by Smith and Freeze (1979b), Bakr et al. (1978), Andersson and Shapiro (1983) and others shows that in more-dimensional analysis the spatial correlation structure of input parameters plays a key role in the stochastic

solution. In stochastic analysis of saturated as well as unsaturated water flow, it has been shown that the variance of output variables increases with an increase in distance over which parameter values are autocorrelated. When Russo and Bresler (1981b) found spatial structure in the parameter values that describe the soil hydraulic properties, they concluded that two-dimensional analysis of unsaturated water flow must include the spatial structure of soil hydraulic properties.

The use of scale factors as a means to express the spatial variability of soil hydraulic properties has successfully been applied by Peck et al. (1977) to determine the variability of water balance components in a watershed. In a similar study, Clapp et al. (1983) showed that scale heterogeneity in soil hydraulic properties may account for approximately 75% of the observed standard deviation in water content. In the study by Peck et al. (1977), however, it was tacitly assumed that: (1) only five sampled locations were enough to represent the variability in soil hydraulic properties in a 100 ha area; (2) scale factor values for different horizons were equal; and (3) that the boundary conditions were uniform for the whole watershed.

The present study intends to be closer to the real situation. In a 650 ha watershed, soil hydraulic properties of different soil horizons, identified by a soil survey comprising a total of 1100 borings, were intensively measured. An extra complication was caused by the presence of a groundwater table within the first few meters from the soil surface. Since also the groundwater depth has a considerable spatial variation, a method is introduced to incorporate this variation of the lower boundary of the soil moisture domain in the stochastic analysis.

The first objective of this paper is to describe the measured variation in model parameters and input variables. The ultimate goal is to investigate the effects of spatially variable soil hydraulic properties on plant transpiration and to quantify the statistical properties of the waterbalance components in a watershed. The stochastic treatment of soil water flux is tested by comparing simulated with measured values of these components. Although Philip (1980) poses the question whether it is altogether feasible to comprehend unsaturated unsteady flows in heterogeneous systems, this study will try to describe a heterogeneous soil-moisture system.

MATERIALS AND METHODS

The stochastic analysis is applied to the watershed Hupselse Beek. This watershed is situated in the eastern part of the Netherlands and has been an experimental study area for almost 20 years. The study area covers 650 ha, and its altitude varies between 22 and 33 m above mean sea level. Land use is predominantly agricultural: 70% pasture and 20% arable land. The slightly loamy sands within the watershed area have groundwater tables within the first few meters below the soil surface. Aeolian sands were deposited over the entire area on top of impermeable Miocene clay sediments and low-permeable boulder clay layers, which are found at depths starting between 0.2 and 10.0 m

below the present land surface. A more detailed description of soils and geology can be found in Wösten et al. (1985).

One-dimensional unsaturated water flow is simulated with the model SWATRE (Belmans et al., 1981). This model was chosen for various reasons: (1) the soil-water domain can be differentiated into layers with different soil hydraulic properties; (2) various types of boundary and initial conditions can be used as input to the model; and (3) it also incorporates a sink term to simulate root water uptake. If a sink term is included, SWATRE numerically solves the following unsaturated flow equation:

$$\frac{\partial \Psi}{\partial t} = \frac{1}{C(\Psi)} \frac{\partial}{\partial z} \left[K(\Psi) \left(\frac{\partial \Psi}{\partial z} + 1 \right) \right] - \frac{ST(\Psi)}{C(\Psi)} \quad (1)$$

where; Ψ = soil water pressure head (cm, negative in unsaturated soil); z = vertical coordinate (cm) with origin at soil surface and directed positively upwards; t = time (days); K = hydraulic conductivity (cm d^{-1}); C = differential moisture capacity (cm^{-1}); and ST = sink term ($\text{cm}^3 \text{cm}^{-3} \text{d}^{-1}$).

SWATRE has been used extensively in other studies since its development, and has proven to be adequate when simulations over long periods are needed.

Soil hydraulic properties

Soil hydraulic properties were measured for various horizons at three different scales of observation. In the first sampling scheme seven profiles across the 650 ha study area were examined. These seven sites were chosen in such a way that they included most of the characteristic soil profiles and horizons in the watershed. The results of the soil physical measurements at these seven sites, as well as the soil survey of the watershed were reported by Wösten et al. (1983). The second sampling scheme comprised an area of 0.5 ha and was chosen such that the seven sampled sites within this area were all from the same and most important soil map unit. Duplicate samples were taken in the A and B horizons. Finally, the highest sampling density was achieved in the third sampling scheme, where at six sites triplicate samples were taken within 2m^2 in the A, B and C horizons. This sampled area was located in the second scheme.

Soil water retention data were obtained from simultaneous measurement of soil water pressure head and water content during slow evaporation of wet undisturbed samples in the laboratory or by using the sandbox apparatus (Stakman et al., 1969). Undisturbed soil samples were thereby put on a box filled with sand or clay, while suction was applied by a hanging water column or a suction pump. Hydraulic conductivity values (K) were measured with the crust-test (Bouma, 1977) for values of soil water pressure head (Ψ) larger than approximately -0.5 m , and by the sorptivity method (Dirksen, 1979) and the hot-air method (Arya et al., 1975) for lower values of Ψ .

Before analyzing the hydraulic data, both the soil water retention and unsaturated hydraulic conductivity data were fitted to Van Genuchten's (1978)

closed-form analytical expressions. In these expressions both hydraulic functions are described by the same parameters a_g , n and m .

$$\Theta = [1 + (a_g |\Psi|)^n]^{-m} \quad (2)$$

and:

$$K_{\text{rel}} = K/K_s = \Theta^{1/2} [1 - (1 - \Theta^{1/m})^n]^2 \quad (3)$$

where:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}, \quad \text{and } m = 1 - (1/n)$$

The subscripts s and r denote saturated and residual values of the volumetric water content θ , while the parameters a_g , n , and m determine the shape of the $\Theta(\Psi)$ curve. The relative conductivity, K_{rel} , is defined as the ratio of the actual unsaturated hydraulic conductivity and the saturated hydraulic conductivity, K_s . Van Genuchten (1978) developed a nonlinear least-squares curve-fitting procedure to estimate the parameters a_g , n , and m from measured $\Theta(\Psi)$ data. Equation (3) shows that the hydraulic conductivity function can solely be predicted by a measured K_s value and the parameter m , obtained from fitting water retention data to eqn. (2). However, since measured hydraulic conductivity data were available, an alternative K_s value was determined for each curve such that an optimal fit was found between the experimental data and eqn. (3) by minimizing the sum of squares of the residuals (Hopmans and Overmars, 1986).

The purpose of scaling is to simplify the description of statistical variation of soil hydraulic properties. By this simplification, the pattern of spatial variability can be described by a set of scale factors α_i , relating the soil hydraulic properties at each location i to a representative mean or reference curve. Based on the functional normalization approach by Warrick et al. (1977b) and Russo and Bresler (1980a, b), variations in the soil water retention curve and the hydraulic conductivity function are connected by the scale factor α .

A scaling parameter α_i is defined as the ratio of the microscopic characteristic length λ_i of a soil at location i and the characteristic length of a reference soil, or:

$$\alpha_i = \lambda_i / \lambda_{\text{ref}} \quad (4)$$

where $i = 1, \dots, L$ denotes locations. As a result of scaling, one can relate the soil water characteristics and hydraulic conductivity function at any location i to a reference Ψ_{ref} and K_{ref} curve, such that:

$$\Psi_i = \Psi_{\text{ref}} / \alpha_i \quad (5)$$

$$K_i = \alpha_i^2 K_{\text{ref}} \quad (6)$$

Since the porosities of samples from the same soil type are likely to be different in most cases, both hydraulic functions in eqns. (5) and (6) are written as a

function of degree of saturation S , where $S = \theta/\theta_s$. Then, any combination of Ψ_i and Ψ_{ref} or K_i and K_{ref} corresponds to the same S (or Θ , if $\theta_r = 0$ in eqn. (2)).

Using techniques, as described in Hopmans (1987a), one can determine a reference soil water retention and hydraulic conductivity function with a corresponding set of scale factor values for each function. Knowing the distribution of scale factor values and its moments allows other values to be generated. The modified distribution-free Kolmogorov-Smirnov statistic (Stephens, 1974) was used to determine the goodness-of-fit of a hypothesized theoretical distribution with an estimated mean and variance to the empirical distribution function. From soil physical data collected in the Hupselse Beek watershed (Hopmans and Stricker, 1986), it was found that scale factor values as determined from water retention data, eqn. (5), correspond fairly well with scale factor values calculated from conductivity data, eqn. (6), alone ($R^2 \approx 0.76$). Therefore, the distribution type and statistics from water retention data only are assumed to be sufficient to describe the variation of soil hydraulic properties. The scaled mean functions $\Theta(\Psi_{ref})$ and $K_{ref}(\Theta)$ will be described by the Van Genuchten model as well, eqns. (2) and (3). Since θ_s will vary between locations, also its distribution must be investigated.

The soil map as a result of an intensive soil survey of the watershed (Wösten et al., 1983) indicated that most soil types could be characterized by an A, B, and C horizon, all with a sandy or loamy sand texture, and a D horizon consisting of a sandy or silty clay. When characterizing the soil physical properties of these horizons, Wösten et al. (1985) showed that the B and C horizons of all types could be combined.

Lower boundary condition

Approximately 85 tubes and wells are used to monitor groundwater levels in the watershed. These groundwater levels are measured every two weeks or every three months. The groundwater levels for the latter tubes were correlated to a tube for which daily groundwater levels are recorded, to obtain biweekly groundwater levels for each tube. Groundwater data showed that at any time, the range of groundwater levels over all 85 tubes is approximately 2 m.

None of the already cited studies has indicated how to treat the lower boundary condition of the flow domain in stochastic analysis, if a relatively shallow groundwater table is present. Since groundwater levels are measured at many locations, one may choose to regard the position of the groundwater table with time as a stochastic process. However, a groundwater table is also governed by the soil hydraulic properties (of deeper layers), which will be assumed to be random functions. Highly conductive soils will drain faster while in soils which have potential for capillary rise, groundwater levels are lowered by upward water movement towards the rootzone. Therefore, unless a quantitative measure can be found to relate groundwater level with soil physical properties, this type of boundary condition cannot be used.

A lower boundary condition which is governed partly by the physical properties of the deeper layers is the relationship between groundwater level (h) at a specific location and the watershed discharge (q), taken negative for the unsaturated zone. A experimental relationship between these two variables was found by Ernst and Feddes (1979) for the sandy soils in the eastern part of The Netherlands. This expression is of the form:

$$q = a e^{b/h} \quad (7)$$

where a and b are parameters to be determined from measurements of q and h . This type of boundary condition suggests that there is at all times some downward flux leaving the soil water system at the bottom, although its magnitude can be very small as compared to the upward flux. The discharge is measured only at the outlet of the watershed, but since h is measured at 85 locations, the parameters a and b will differ for each location. However, if an average curve can be determined and if variation in the $q(h)$ relationship can be expressed by another set of scale factors (γ), then also the lower boundary condition can be viewed as a stochastic process.

From the logarithm of eqn. (7) a_i and b_i are determined for each tube i using linear regression, where $i = 1, \dots, N$ and N is the number of tubes measured. Since it is expected that all curves have a similar shape, it is reasonable to suggest that an average \bar{b} value can be calculated to describe all N curves. Using this common \bar{b} value, eqn. (7) becomes:

$$q = a_i e^{\bar{b}/h} \quad (8)$$

Equation (8) indicates that $q(h)$ curves differ only by the parameter a_i . Scale factors and the parameter a_{ref} for the reference curve can be determined from the a_i . The normalizing condition:

$$1/N \sum_{i=1}^N \gamma_i = 1 \quad (9)$$

where γ_i denotes the scale factor value for each tube i , and assigns a scale factor of 1.0 to the reference curve. A scale factor γ_i needs to be found such that the $q(h)$ data for each tube i coalesce to the reference curve, simply by multiplying the discharge by this scale factor. This can be accomplished by setting:

$$\gamma_i a_i = a_{\text{ref}} \quad (10)$$

The combination of eqns. (9) and (10) yields:

$$1/a_{\text{ref}} = 1/N \sum_{i=1}^N 1/a_i \quad (11)$$

which is used to determine a_{ref} , once the a_i have been calculated from eqn. (8). The scale factor value γ_i can be calculated from eqn. (10), so that the discharge-groundwater level relationship for each tube i is described by:

$$q = a_{\text{ref}/\gamma_i} e^{\bar{b}/h} \quad (12)$$

Once the distribution type of γ_i is found (normal or lognormal), eqn. (12) will be part of a Monte Carlo analysis, where values for γ_i are generated from the first and second moments of its distribution. More details on this procedure can be found in Hopmans (1987b).

Other input data

SWATRE requires daily values for precipitation and potential evapotranspiration (E_{pot}) to represent the boundary condition at the soil surface. E_{pot} was calculated for grassland from the Thom and Oliver (1977) expression, for which the relevant meteorological parameters are measured continuously and averaged over every twenty minutes at the meteorological station of the watershed. Precipitation and E_{pot} were assumed to be uniformly distributed.

All simulations started at April 1 and ended October 1. It was assumed that the soil profile was in hydraulic equilibrium at the start of the simulations, i.e., the initial pressure head profile is calculated from the groundwater level. This type of initial condition was chosen since the period around April 1 generally corresponds with a period of gradually lowering groundwater levels, when evaporation and transpiration are still low.

The thickness of rootzone was considered to be 0.3m during the whole growing season for the total area. Results are, therefore, only obtained for grass-grown fields which comprise approximately 70% of the watershed. The same applies to the models that are used to simulate the root water extraction rate. These models were calibrated for grass only. Belmans et al. (1981) defined the sinkterm ST per unit of depth:

$$ST = c(\Psi) ST_{\text{max}} \quad (13)$$

$$ST_{\text{max}} = \frac{E_{\text{pot}}}{D_r} \quad (14)$$

where: $c(\Psi)$ = dimensionless sinkterm variable ($0 \leq c \leq 1$); E_{pot} = potential evapotranspiration rate (cm d^{-1}); and D_r = depth of rootzone (cm).

Equation (13) implies that the prescribed potential transpiration demand is distributed uniformly over the various layers of the rootzone. and that the reduction in extraction occurs through the sinkterm variable $c(\Psi)$ which is a function of the soil water pressure head Ψ at each node within the rootzone. The actual transpiration rate (EA) is defined by E_{pot} minus the reduction in root water extraction. Various ST models differ in how $c(\Psi)$ is defined. The implemented sink-term model allows rootwater extraction rate not only to be influenced by Ψ , but also by the unsaturated hydraulic conductivity of the soil in the rootzone. Details can be found in Belmans et al. (1981) and Hopmans (1987c).

Stochastic analysis

Variation in soil hydraulic properties and in the lower boundary condition

was simulated through generation of scale factor values from the statistical properties of both random processes, while an ensemble of realizations was obtained from Monte Carlo analysis. Since thickness of the BC horizon varies with the starting depth of clay (D horizon), the Hupsel watershed was mapped into five classes of different profile types. Depth to clay and areal percentage for each profile type are listed in Table 1, while the map in Fig. 1 shows the distribution of the five profile types within the watershed. A series of 30 or 50 Monte Carlo simulations was carried out for each profile type, after which a probability density function (PDF) of output variables is obtained for each

TABLE 1

Areal percentage of profile types with different starting depths to clay

Depth to clay (m)	< 0.4	0.4-0.8	0.8-1.2	1.2-1.6	> 1.6
Areal percentage	4	23	27	12	34

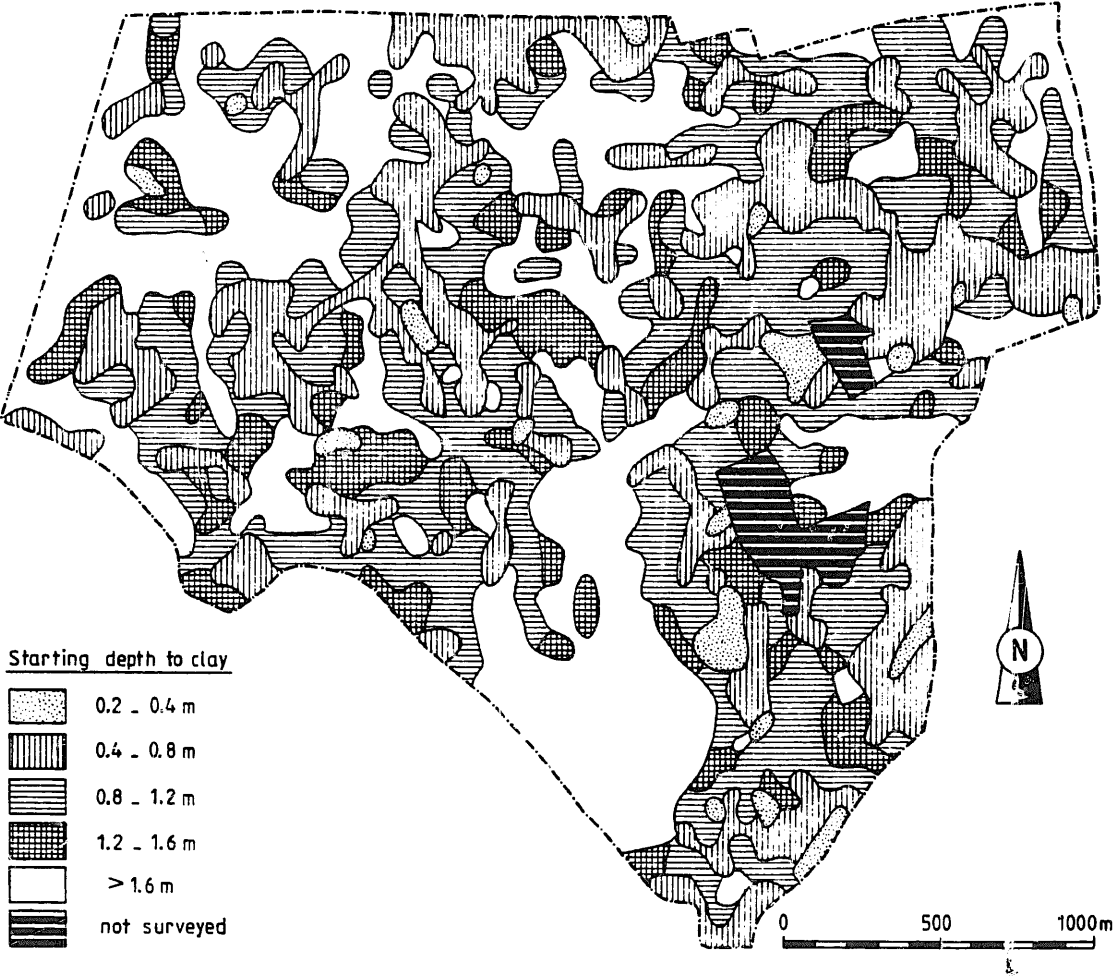


Fig. 1. Distribution of profile types in Hupselse Beek watershed.

profile type. An increase to 200 simulations for one particular profile type did not significantly affect the PDF and its moments. Output variables of special interest were the groundwater table at the end of the growing season (GWL1, m below soil surface), the cumulative flux through the bottom of the soil profile taken positive for downward flow (QB , mm), the cumulative reduction in plant transpiration (RED , mm), and the actual plant transpiration (EA , mm) during the simulated growing season.

Comparison of measured groundwater levels at the end of the growing season and GWL1 indicated how well the stochastic model performs. Since also the discharge of the watershed is monitored, its magnitude should compare favourably with the simulated outflow QB . Finally, values of RED and its variation resulted in a map, differentiating between regions that are vulnerable to drought stress. Such an analysis has been performed for 2 years, 1976 and 1982. In Dutch weather conditions, 1982 is a fairly dry year, while 1976 was extremely dry.

A sensitivity analysis preceded the Monte Carlo simulations. The influence of decreasing grid size (distance between two vertical grid points) on simulation results is shown. Furthermore, variation in scale factor values of soil hydraulic properties was introduced by assigning random scale factor values to each grid point within a horizon. The results of these simulations were compared with those for which the horizons only have different random scale factor values (means: 100% autocorrelation of hydraulic properties of grid points within the same horizon). To study effects of random variable soil hydraulic properties and $q(h)$ relations, the mean values of output variables of a series of Monte Carlo simulations were compared with those of a single deterministic run using the reference curves. All simulations with respect to the sensitivity analysis were done for a soil profile with A and BC horizon only, and for the growing season of 1982.

The proposed numerical model was implemented such that it could be run on a personal computer. Run time on an ATARI STF (1 MB) for simulation of a growing season of 183 days was approximately 1.0–2.0 h.

RESULTS AND DISCUSSION

Variability of soil hydraulic properties

Because replicate water retention curves were determined for sampling schemes 2 and 3, it was first investigated whether scale factor values between locations for a given sampling scheme and horizon were significantly different. After fitting eqn. (2) to the water retention data of each replicate sample, the water retention curves for each scheme and horizon were scaled independently. The modified Kolmogorov–Smirnov (KS) test showed that scale factor values were lognormally distributed. Analysis of variance (ANOVA) was used to test for significant difference of log-transformed scale factor values between locations (Hopmans and Stricker, 1986). In total five sets of scale factor values (scheme 2: A and B horizon; scheme 3; A, B and C horizon) were statistically

analyzed. In only one set a significant difference between locations was found. It should be noted, however, that ANOVA generally assumes that the analyzed data are statistically independent. Construction of semivariograms showed that scale factors are spatially dependent up till a range of approximately 10 m. This distance is larger than the distance between locations in sampling scheme 3, so that the ANOVA results of scheme 3 can be biased by the independence assumption.

ANOVA was also used to test whether the means of the log-transformed scale factor values of the sampled horizons were statistically different. For this purpose, all available water retention data were used in the scaling. When the B and C horizon of the third sampling scheme were combined, there was indeed a significant difference between scale factor values of the A and BC horizon. Therefore, water retention data of both the A and BC horizon were each scaled independently. Both sets of scale factor values were found to be lognormally distributed, so that the mean and standard deviation of the log-transformed scale factor sets were calculated (Table 2B). The resulting reference water retention and hydraulic conductivity curves for the two horizons, whose parameters of the Van Genuchten model are listed in Table 2B, are shown in Fig. 2.

When the PDF of scale factor values is used to generate scale factor values, it is important to notice that the soil water characteristic and conductivity curves are written as a function of $S(=\theta/\theta_s)$ or Θ . Therefore, also θ_s was subjected to a closer statistical analysis. The KS test indicated that θ_s was normally distributed. After determination of the mean and standard deviation for each horizon (Table 2B), ANOVA showed that also the mean θ_s of the A horizon was significantly different from its mean value of the BC horizon. Random hydraulic functions can thus be generated from the statistical properties of scale factor and θ_s - values. However, if the scale factor value and θ_s for each horizon are correlated, the two variables cannot be generated independently from each other. Coefficients of determination (R^2) between θ_s and $\log(\alpha)$ were 0.0017 and 0.0713 for the A and BC horizon, respectively. Therefore, θ_s can be drawn independently of the α -value of both horizons.

The statistical analysis excluded the D horizon (boulder clay), since only a limited number of samples (from sampling scheme 1 only) were taken from the D horizon. However, also water retention and conductivity data of the D horizon were scaled and are shown in Fig. 2. The parameters of the scaled mean function and the statistical properties of the scale factor and θ_s of this horizon are also shown in Table 2.

Stochastic description of $q(h)$ relationship

Because it was expected that groundwater levels were influenced by the occurrence of clay within the first 2 m of the soil profile, the locations of the monitoring tubes were separated into classes of depth to clay. When comparing mean and standard deviation of groundwater levels for various classes for the

TABLE 2

Input parameters for stochastic water flow model SWATRE

Parameter	Horizon		
	A	BC	D
<i>A. Parameters Van Genuchten model</i>			
a_g	0.01924	0.02043	0.0388
n	1.5931	1.8187	1.4491
θ_s	0.4024	0.3195	0.4397
θ_r	0.0	0.0	0.1881
K_s (cm d ⁻¹)	33.70	40.55	10.93
<i>B. Statistics hydraulic functions</i>			
α (lognormal distribution)			
μ	-0.1155	-0.0706	-0.4377
σ	0.3430	0.2567	0.9917
θ_s (normal distribution)			
μ	0.4060	0.3197	0.4366
σ	0.0354	0.0395	0.050
<i>C. Parameters + statistics $q(h)$ relation</i>			
	1976		1982
	Class 1	Class 2	Class 1
\bar{b} (cm ⁻¹)	-0.03463	-0.03838	-0.03696
a_{ref} (cm d ⁻¹)	-0.3173	-0.9734	-0.4376
μ_r	-0.05144	1.0	-0.06124
σ_r	0.2268	0.6938	0.2516
distribution type	Lognormal	Normal	Lognormal
h_{init} (cm)	51	72	56
			78

years 1976 and 1982, it was found that the groundwater level tubes could be divided into two major classes. Groundwater levels at locations where clay was present within the first 1.2 m of the profile (class-1 tubes) were 30–40 cm higher for both years than for the locations where clay occurred at depths larger than 1.2 m (class-2 tubes). Therefore, before the discharge–groundwater level relationships, eqn. (7), were scaled, the groundwater level monitoring tubes were separated into these two classes. Groundwater levels were measured at 55 class-1 tubes (starting depth to clay < 1.2 m below soil surface) and 28 class-2 tubes (starting depth to clay > 1.2 m below soil surface). For each class and year an average \bar{b} value, eqn. (8), was calculated. Minimization of the sum of squares of the difference between the measured and predicted q values yields a_i , according to:

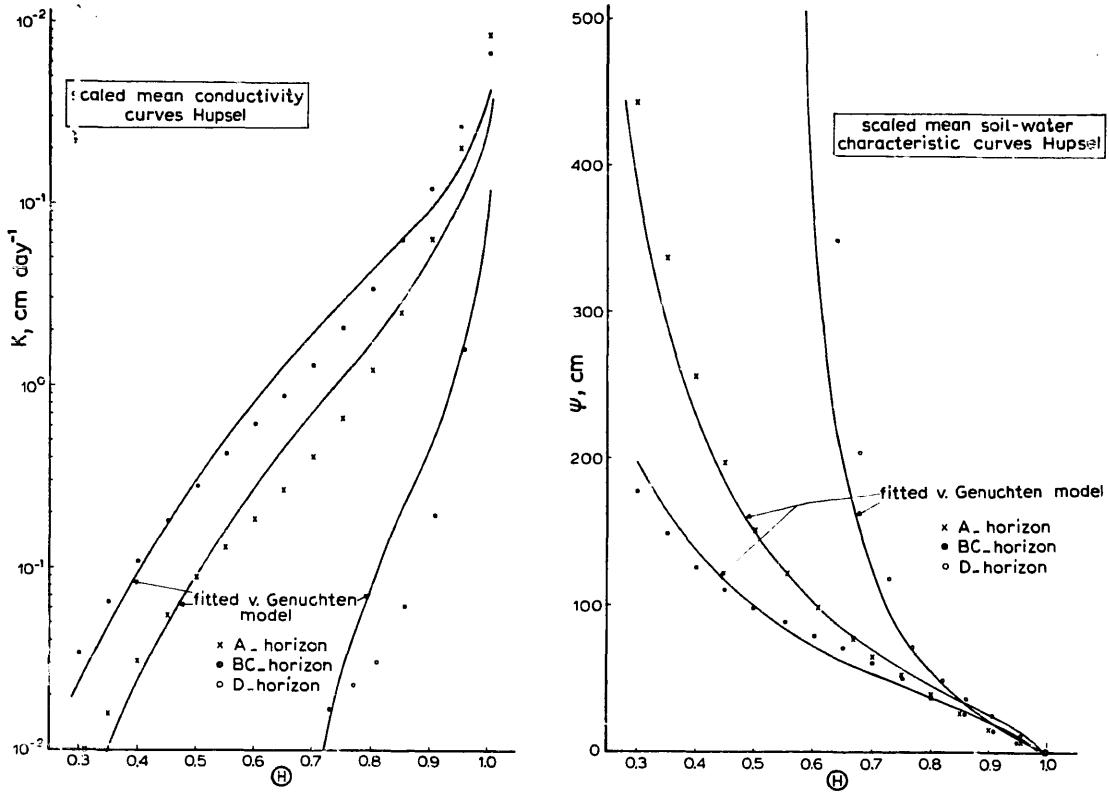


Fig. 2. Reference curves of A, BC horizon and D horizon.

$$a_i = \frac{\sum_{j=1}^M q_j e^{\delta_{h,j}}}{\sum_{j=1}^M (e^{\delta_{h,j}})^2} \quad (15)$$

where M denotes the total number of observed discharge and groundwater level data in one year for each tube i . Therefore, a_{ref} and γ_i were simply calculated from eqns. (11) and (10), respectively.

In order to use Monte Carlo analysis, the distribution of γ_i must be known. Therefore, the Kolmogorov-Smirnov test was also used to test whether the set of γ_i values for each class and year fitted a normal or lognormal distribution. However, none of these described the set of scale factor values of the class-2 tubes. To obtain a better fit, eqn. (10) was changed to:

$$\gamma_i^2 a_i = a_{\text{ref}} \quad (16)$$

While using eqn. (9), a_{ref} can then be determined from:

$$\left(\frac{1}{a_{\text{ref}}}\right)^{1/2} = \frac{1}{N} \sum_{i=1}^N \left(\frac{1}{a_i}\right)^{1/2} \quad (17)$$

and the flux at each tube i is calculated from:

$$q = (a_{\text{ref}}/\gamma_i^2) e^{\bar{b}|h|} \quad (18)$$

For the class-1 tubes the set of γ_i values now fitted a lognormal, while a normal distribution described the set of γ_i values for the class-2 tubes.

The parameters a_{ref} and \bar{b} for the reference $q(h)$ relationship, as well as the mean, standard deviation and distribution type of the set of γ values for both classes and years are listed in Table 2C. The reference curves [$\gamma_i = 1$ in eqn. (18)] are shown in Fig. 3. As expected, the two classes yielded different reference curves. However, the two years have similar curves, with approximately the same statistical properties, indicating that both years might have been combined before scaling.

Due to the described procedure of generating a random $q(h)$ relationship, the initial groundwater level (GWL_0) will depend on the value for γ . As input serves the initial groundwater level ($h = h_{\text{init}}$ in Table 2C), at a location where the $q(h)$ relation coalesces approximately with the reference $q(h)$ relation ($\gamma = 1$). The actual initial groundwater level is then calculated from the random generated γ value and the discharge as calculated from eqn. (18) where $\gamma = 1$ and $h = h_{\text{init}}$. Subsequent groundwater levels are calculated from the simulated mass balance of the unsaturated-saturated soil profile.

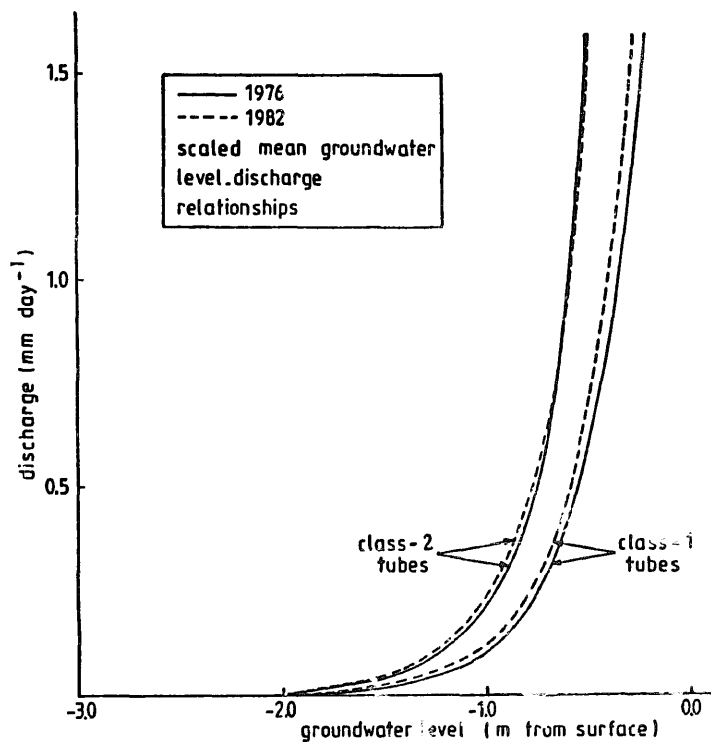


Fig. 3. Reference curves $q(h)$ relationship.

Sensitivity analysis

Between- versus within-horizon variation

Variation in soil hydraulic properties can be introduced in two ways. One can define equal scale factor values for all grid points within a horizon (cases I and II in Table 3), or define a different scale factor value for each grid point based upon the distribution of scale factor values for each horizon (cases III and IV, Table 3). The first option assumes the maximum possible spatial dependency in the vertical direction (all scale factor values within a horizon are equal), while the latter approach assumes no spatial dependency at all; i.e., scale factor values within a horizon are randomly distributed. The true variation in the vertical direction will generally lie between those two extremes, but is unknown since no data are available to investigate the vertical variation within a horizon.

Comparison of cases I with III and cases II with IV in Table 4 shows that the

TABLE 3

Cases of sensitivity analysis (1982)

Case	Variability with respect to		No. of grids	No. of simulations
	Soil hydraulic properties	$q(h)$ relationship		
I	Random horizons	Random	30	30
II	Random horizons	Fixed	30	30
III	Random grid points	Random	30	30
IV	Random grid points	Fixed	30	30
V	Fixed horizons	Random	30	30
VI	Fixed horizons	Fixed	30	1
VII	Fixed horizons	Fixed	60	1
VIII	Random horizons	Random	30	30
IX	Plant transpiration, watershed discharge and groundwater levels measured			

TABLE 4

Simulation results sensitivity analysis (1982)

Case	EA (mm)		RED (mm)		QB (mm)		GWL_0 (cm)		GWL_1 (cm)		Range
	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	
I	428.8	15.3	15.0	15.8	21.1	7.4	79.0	27.8	202.5	34.7	151.2–278.1
II	431.1	11.3	12.7	11.	21.1	5.7	78.0	–	199.3	29.7	150.1–270.4
III	427.3	13.8	16.6	13.8	20.4	6.5	79.8	31.5	191.5	11.4	168.6–214.3
IV	433.6	5.6	10.2	5.6	22.2	2.0	78.0	–	191.2	7.2	173.8–204.1
V	433.3	10.1	10.5	10.0	22.6	3.9	70.5	25.8	181.2	10.0	170.2–204.2
VI	435.8	–	8.0	–	23.9	–	78.0	–	184.0	–	–
VII	435.8	–	8.0	–	23.8	–	78.0	–	182.3	–	–
VIII	428.9	18.0	14.8	18.1	25.0	21.7	85.2	31.8	209.6	41.3	136.7–291.4
IX	419.8	–	24.0	–	22.0	–	77.0	41.0	174.0	57.0	50.0–280.0

variation in water balance components and groundwater levels at the end of the 1982 growing season (GWL_1) after 30 Monte Carlo (MC) simulations increases with increasing spatial dependency. Hence, also the range of simulated GWL_1 increased if all grid points within a horizon had equal soil physical characteristics. Note also that the mean generated groundwater level at the beginning of the growing season (GWL_0) is approximately equal to the h_{init} value in Table 2. Of all variables in Table 3, only the mean GWL_1 is somewhat affected by the choice of variation in soil hydraulic properties. Since the true variation within a horizon is unknown, it seems appropriate to consider both extremes in variation of soil hydraulic properties.

Influence of variability in soil hydraulic properties and in $q(h)$ relationship

The total variation in water balance components caused by a random variable $q(h)$ relationship has approximately the same magnitude as the variation caused by random variable soil hydraulic properties (compare cases II and V). However, the variation in GWL_1 is significantly larger for case II where the soil hydraulic properties only were varied.

The effects of variable soil hydraulic properties can be studied when the $q(h)$ relationship is assumed to be fixed ($\gamma = 1$, case II). The influence of α on the soil water regime can be understood from the results of the MC simulations in combination with the values of soil hydraulic properties as a function of α . Figure 4 shows the soil water characteristics, hydraulic conductivity and water capacity functions for a value of α smaller and larger than 1, based on the reference curves of the A horizon. The magnitude of scale factor values influences the transport of soil water in two ways. Firstly, in the surface horizon where soil water pressure head values are generally small, values of $\alpha > 1$ result in lower θ values for corresponding Ψ values and smaller amounts of available water to be extracted by plant roots than for α values < 1 , i.e., $C(\Psi)$ decreases for increasing α values. Therefore, increasing values of α for the surface horizon create increasing reduction in plant transpiration. Secondly, in the subsurface horizon where soil water pressure head values are generally large, values of $\alpha > 1$ result in an increasing amount of water available for upward water movement towards the rootzone without lowering the groundwater table, i.e., $C(\Psi)$ increases for increasing α values. Therefore, increasing values of α for the subsurface horizon result in higher groundwater tables.

The influence of groundwater level on plant transpiration can be investigated by fixing the soil hydraulic properties, while varying the $q(h)$ relationship (case V). From these simulations it appears that a direct relationship exists between initial groundwater level (GWL_0) and plant transpiration (EA). An initial higher groundwater table facilitates upward water movement and, hence, allows the plant roots to take up more water. A statistical analysis of the influence of scale factor values and groundwater table on the output variables will follow later.

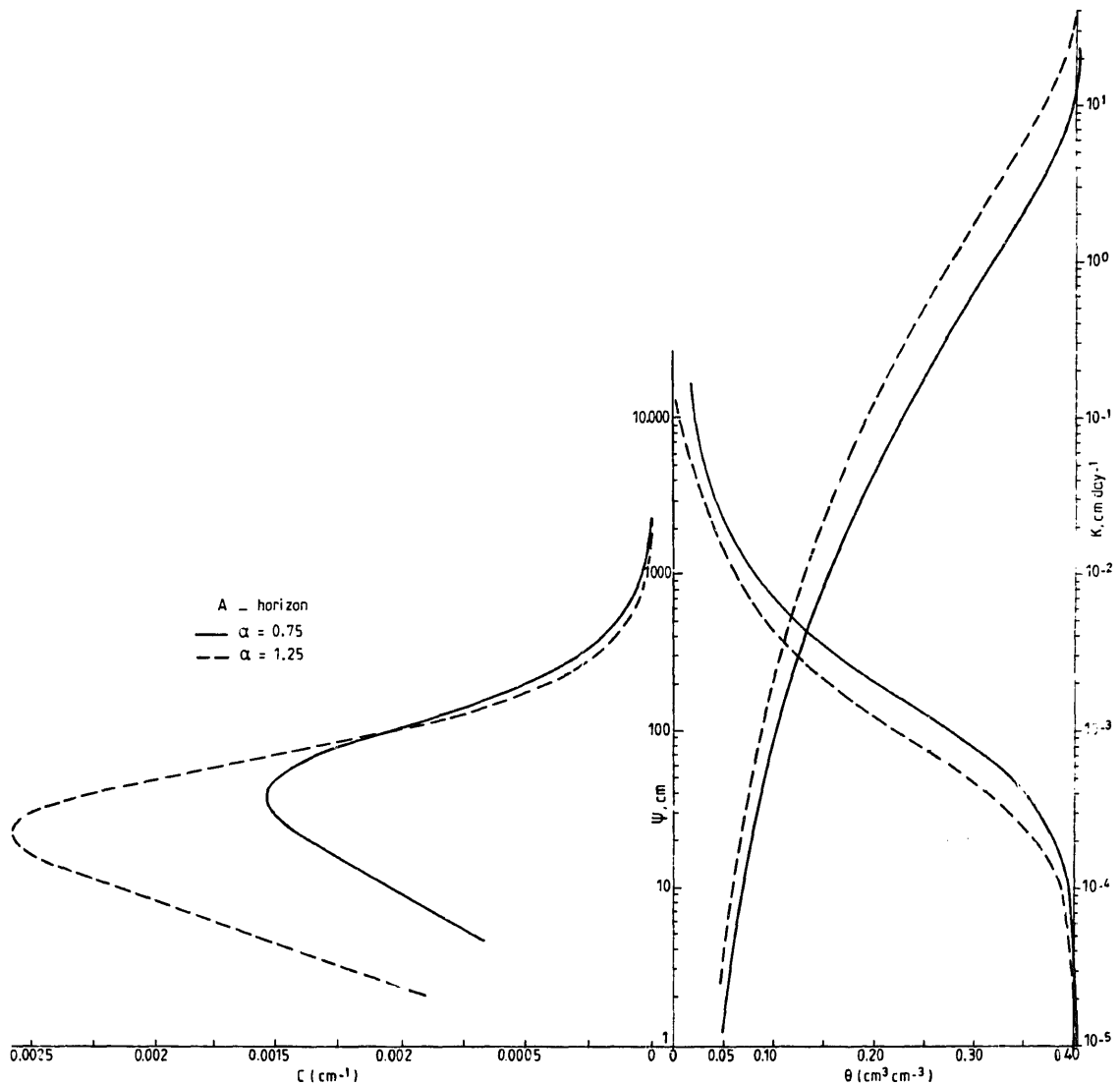


Fig. 4. Soil water characteristic, hydraulic conductivity, and water capacity curves for α values of 0.75 and 1.25, using the reference curves of the A horizon.

Further analysis

To verify that a vertical grid spacing of 10 cm results in sufficiently accurate results, simulations were performed with a grid spacing of 10 and 5 cm (cases VI and VII, Table 3). The water balance components of both simulations were close enough to allow a grid point distance of 10 cm.

It was furthermore investigated whether the combination of an A-BC profile yielded simulation results different from those when the whole profile would be considered as being characterized by the soil physical properties of the A horizon only (cases I and VIII, Table 3). The mean values for cases I and VIII in Table 4 are almost identical. The variation, however, increases if only one horizon is considered to be representative for the whole profile. The latter

result is in accordance with the larger standard deviation of scale factor values for the A than for the BC horizon (Table 2) and with the result of an increase of spatial dependency.

Finally, all simulation results of the sensitivity analysis (cases I to VIII, Table 4) were compared with the measured water balance components and groundwater levels (case IX). In doing so, it appears that the measured plant transpiration (EA) is smaller than any of its mean simulated values. No such straight comparison is possible for the discharge and groundwater levels, since those were obtained from measurements of the total watershed area, while the simulated results of the sensitivity analysis refer to A-BC profiles only.

Monte Carlo simulations

For each soil profile type, as specified in Table 1 and illustrated in Fig. 1, 30 MC simulations were performed for the growing season of 1982. Soil profile thickness was assumed to be 3.0m. This thickness was chosen, so that the groundwater table would never exceed this depth. Initial groundwater levels, obtained from eqn. (18), were randomly generated such that they would never be within the first 0.3m from the soil surface. Shallower groundwater tables could result in ponded water after precipitation, a condition unallowable for the numerical model. Also scale factor values for the soil hydraulic properties were truncated such that they were not smaller than 0.25 and not larger than 2.5. This maximum range of scale factor values was in accordance with the measured soil hydraulic properties. Values outside this range appeared to cause numerical instability in the simulations.

The statistics for the output variables of this series of MC simulations are listed in Table 5A and B. It shows the mean and standard deviation of the simulated cumulative actual plant transpiration (EA), cumulative reduction (RED) in transpiration from the potential transpiration and the cumulative discharge from the soil profile (QB), as well as the mean and standard deviation in initial and final groundwater levels (GWL_0 and GWL_1) at the beginning and end of the growing season, respectively, for each profile type. Table 5 also lists the measured values of EA (TNO Report, 1984), RED and precipitation (measured at meteorological station), and watershed discharge during the simulated growing season. Table 5A shows the simulation results if each grid point within a horizon is assigned a randomly generated scale factor value, while Table 5B shows the same results for randomly generated scale factor values for each horizon.

The output variables of primary interest are EA and RED . Table 5 shows that EA values are not affected by the presence of clay if the clay layer starts at a depth greater than 1.0m below the soil surface, i.e., the simulation results for profile types 1 and 3 can be combined to yield an average EA value for those soils where the starting depth to clay is larger than 1.0m. Decreasing EA values and thus increasing RED values were calculated for simulations of profile types 4 and 5. Therefore, the Hupsel watershed can be divided into three

TABLE 5

Results MC simulations 1982 (30 simulations for each soil type) and 1976 (50 simulations for each soil type)

Soil type	Starting depth to clay (cm)	EA (mm)		RED (mm)		QB (mm)		GWL ₀ (cm)		GWL ₁ (cm)		Range	
		$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$		
A. Spatial variable grid points (1982)													
1	no clay	427.3	13.8	16.6	13.8	20.4	6.5	79.8	31.5	191.5	11.4	168.6-214.3	
2	140	430.7	13.7	12.1	13.7	23.7	10.5	73.3	31.2	187.1	15.5	161.7-231.3	
3	100	430.1	12.4	12.8	12.4	21.0	6.5	69.4	29.6	181.4	9.8	163.1-198.1	
4	60	415.1	15.9	28.7	15.9	22.7	5.2	65.4	22.3	162.2	9.7	145.5-180.6	
5	30	375.9	15.5	67.1	15.5	23.2	4.8	9.0	20.2	142.9	9.9	123.8-165.3	
Measured		419.8		24.0		22.0		(Precip: 254.3 mm)					
B. Spatial variable horizons (1982)													
1	no clay	428.8	15.8	15.0	15.8	21.1	7.4	80.7	26.7	202.5	34.7	151.2-278.1	
2	140	428.8	16.7	15.0	16.7	21.8	17.8	74.7	32.8	198.6	17.9	166.9-241.1	
3	100	430.6	15.3	13.2	15.3	17.6	6.5	58.3	21.7	185.3	19.5	134.6-230.0	
4	60	419.9	16.9	23.9	16.9	19.7	8.2	62.3	25.9	169.6	19.3	130.7-212.4	
5	30	381.4	13.5	62.4	13.5	22.5	9.4	64.8	22.1	150.0	27.0	103.6-196.2	
Measured		419.8		24.0		22.0							
C. Spatial variable horizons (1976)													
1	no clay	429.8	36.2	74.6	36.2	18.7	9.9	77.6	30.1	206.6	28.6	152.8-266.1	
2	140	420.3	42.4	84.1	42.4	19.2	9.6	83.7	34.4	207.2	24.4	161.4-261.8	
3	100	440.6	24.1	63.8	24.1	14.0	5.4	60.6	26.1	194.7	19.7	152.5-242.9	
4	60	391.9	27.3	112.5	27.3	15.0	6.0	61.6	23.4	179.1	24.2	140.1-263.6	
5	30	346.3	21.0	158.1	21.0	15.9	8.2	58.5	24.6	163.6	28.3	118.3-243.8	
Measured		369.5		134.9		8.2		(Precip: 213.9 mm)					

classes, each class representing a characteristic value for the mean and standard deviation of the plant transpiration reduction (Fig. 5). As expected, the mean groundwater level at the end of the growing season (GWL_1) decreases with a decreasing starting depth to clay. The more clay present in the soil profile, the less water will be transported upwards and, therefore, the less fall in groundwater table is needed to compensate for the upward flow.

The proposed simulation model can be validated in various ways. Firstly, it is important to note the mean generated initial groundwater levels (GWL_0) are approximately equal to the mean measured groundwater levels (70 cm generated versus 76 cm measured). Secondly, the measured discharge of the watershed during the growing season of 1982 was 22 mm, a value very close to the simulated QB for any profile type. Since groundwater levels were measured during the whole year, a third validation can be obtained from the comparison of the mean and standard deviation of measured and simulated groundwater levels at the end of the growing season (GWL_1). This is done in Table 6, which again makes a distinction between simulations with random variable grid

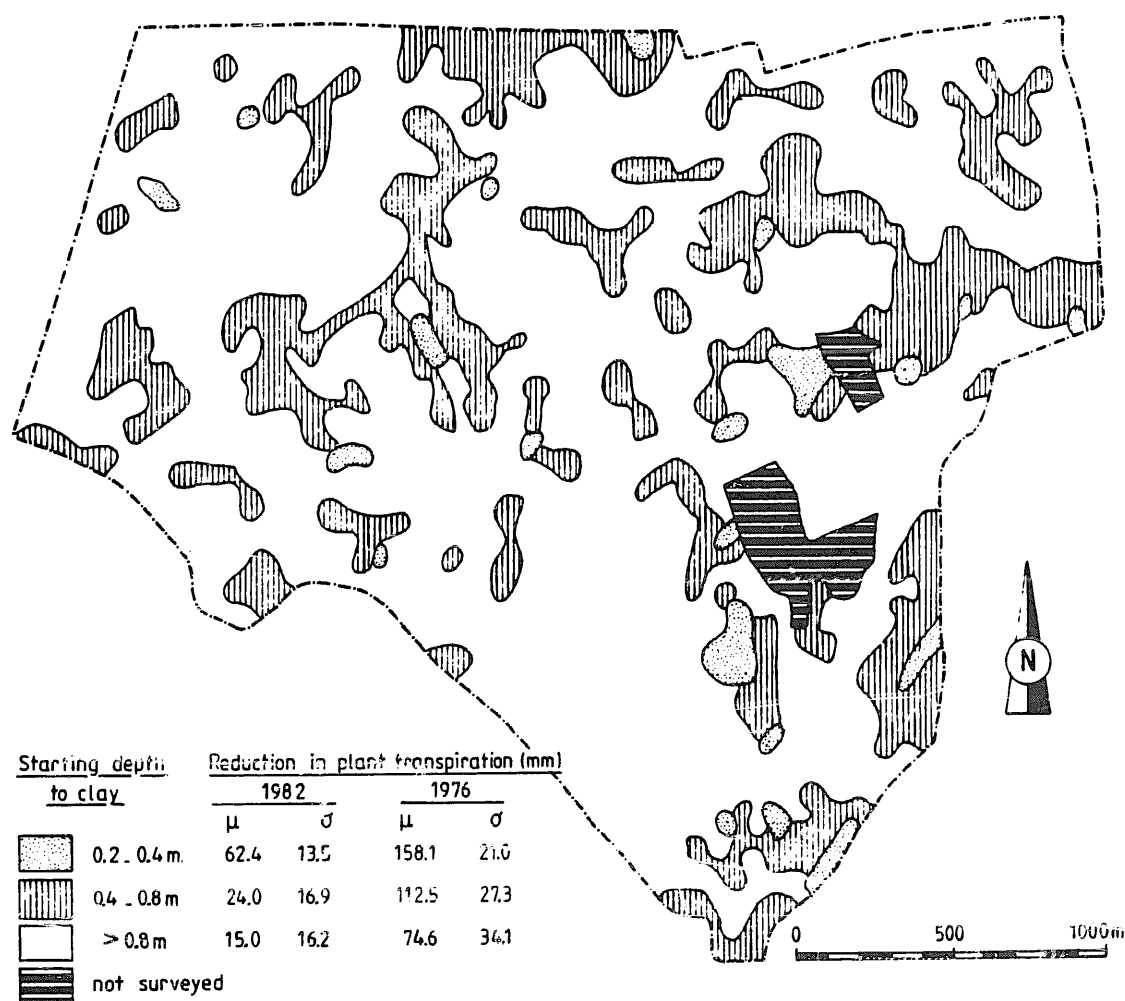


Fig. 5. Distribution of soil classes as a result of MC simulations.

TABLE 6

Groundwater (cm) levels at end of growing season: measured versus simulated (weighted by fraction of area representing each profile type, Table 1)

	Statistics determined from					
	MC-simulations				Measured ground-water levels	
	Variable grids		Variable horizons			
	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
<i>A. 1982</i>						
All profile types combined	179.6	16.5	188.0	30.3	174.0	50.0
Class-1 tubes only (start- ing depth to clay < 1.2 m)	170.4	16.2	176.6	20.9	165.0	41.0
Class-2 tubes only (start- ing depth to clay > 1.2 m)	190.4	12.0	201.5	31.8	193.0	56.0
<i>B. 1976</i>						
All profile types combined			195.4	27.6	191.2	60.0
Class-1 tubes only			185.8	24.0	183.2	58.0
Class-2 tubes only			206.8	27.4	208.0	61.0

points and horizons only. The comparison in Table 6 shows that the mean simulated values of GWL_1 are fairly close to the measured ones. However, as important as the mean is the simulated variation in GWL_1 as compared with its measured variation. Since one of the objectives of this study was to quantify the variation in water balance components, the simulated variation in GWL_1 should be of comparable magnitude of the measured variation. The results in Table 6 show that approximately 50–60% of the measured variation is explained, if it is assumed that the soil hydraulic properties within a horizon do not vary in the vertical direction, while only 25–35% of the measured variation is calculated if each grid point is assigned a random α value. Therefore, decreasing the spatial structure of scale factor values within a horizon (e.g. by randomly generating α values for each grid point) decreases the variation in simulated groundwater levels. The conclusion is in agreement with results from others (Smith and Freeze, 1979b; Andersson and Shapiro, 1983). It is probably more realistic to include within horizon variation only when two dimensional flow is considered. In two dimensions, variation in soil hydraulic properties may cause by-pass flow or create preferential flow paths and, therefore, decrease the ensemble variation in output variables. Finally, simulated EA values for each profile type were weighted by the fraction of the total watershed area represented by each profile type. This weighted average EA value was very close to the EA value measured at the meteorological station (425.3 versus 419.8 mm) in 1982.

For each simulation, α and θ_s values for each horizon and an initial groundwater level are randomly generated, resulting in many possible combinations from which the ensemble statistics are calculated. Since it was intuitively felt that 30 MC simulations were possibly not adequate to represent the variation within one profile type, the number of simulations in 1976 was increased to 50. Again, when comparing the *EA* values, profile types 1 to 3 can be combined to characterize one typical soil class (Table 5C). However, since the 1976 summer was extremely dry and warm, *RED* values were much larger than for 1982. In contrast to 1982, the weighted average *EA* value (419.5 mm) was considerable higher than its measured value of 369.5 mm; a result which will be discussed hereafter.

The shape of $c(\Psi)$ in eqn. (13) was calibrated in an earlier study (Dienst Informatie-verwerking, Rijkswaterstaat, 1984) in such a way that for the soil hydraulic properties and groundwater levels measured at the meteorological station, the simulated and measured *EA* values were approximately identical. However, a soil profile with different soil physical characteristics would yield different simulated *EA* values. Since it was concluded from a water balance study that the measured *EA* values were representative for the whole watershed, the calibration of $c(\Psi)$ would only be correct if the soil physical properties of the calibration site were representative for the watershed. There is no indication that this is true, and it may therefore be more appropriate to calibrate $c(\Psi)$ while taking into consideration the variation in soil hydraulic properties and groundwater levels.

When comparing measured and simulated GWL_1 values (Table 6), it must be concluded that the mean values are very close but that the simulated variation explains only 50–60% of the measured variation. Most likely, the measured output variables will depend on several other sources of variability than those included in the model. It was assumed that precipitation, infiltration, potential transpiration and rooting depth are uniformly distributed over the watershed area, while in reality these variables will be spatially variable. Moreover, the sink term, eqn. (1), was defined for grass fields only. It is expected that when incorporating other crops, which cover approximately 25% of the watershed, the variation in simulated output variables will increase. It was furthermore assumed that the soil profile consisted of only three distinct horizons, each having a fixed thickness for each profile type. Also the one-dimensional treatment of soil water flow excludes the possibility of lateral flow. Most of these assumptions will lead to a smaller than actual variation in the simulated output variables.

Statistical analysis of output variables

The X^2 goodness-of-fit test was used to determine the distribution function of the output variables *RED*, GWL_1 and *QB*. In general, *RED* and GWL_1 followed a normal distribution, while *QB* was of the normal or lognormal type.

It was further investigated whether the output variables could be predicted

from known input values of initial groundwater level and scale factor and saturated water content values for each horizon. The results of the multiple regression analysis used are shown in Table 7. The signs below each listed independent variable indicates whether a positive (+) or negative (-) correlation between the tabulated dependent and independent variables was significant. Most of the significant correlations in Table 7 were already found in the sensitivity analysis.

A deeper initial groundwater level limits upward movement and, hence, increases *RED* and decreases the fall in groundwater table during the growing season. The latter results, therefore, in a larger cumulative discharge *QB*. Of course, also *GWL*₀ and *GWL*₁ are positively correlated.

Larger scale factor values in the surface horizon will generally lead to smaller amounts of water available for the plants and, therefore increase *RED* and decrease *GWL*₁. However, a larger scale factor value in the subsurface horizons might increase upward water flow since the unsaturated hydraulic conductivity concomitantly increases, while *GWL*₁ will decrease as more water

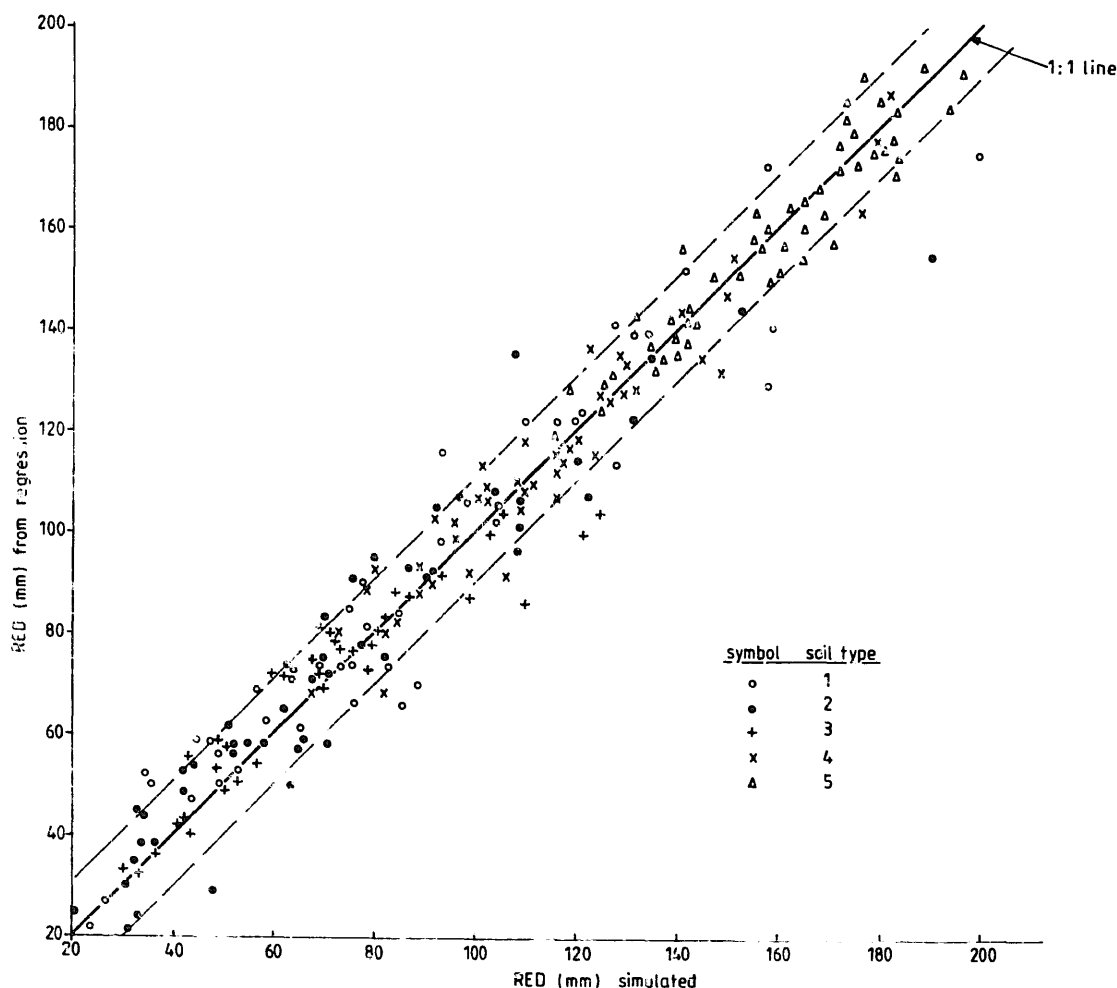


Fig. 6. *RED* values simulated versus estimated from multiple regression analysis.

can be extracted from the soil profile before the groundwater table needs to be lowered to meet the evaporative demand (potential transpiration minus precipitation).

The influence of θ_s on the output variables is identical to that of the scale factor values. A larger θ_s makes more water available for root uptake in the surface horizon and for upward water flow in the subsurface horizon, and will therefore decrease *RED* and *GWL₁* for reasons mentioned before. The regression coefficients make it furthermore possible to predict *RED* or *GWL₁* values in a specific year for any soil if the initial groundwater level is known. Figure 6, which shows a plot of *RED* values predicted versus simulated, illustrates that such predictions can be made with a standard deviation of approximately 10 mm in 1976. A similar analysis for 1982 yielded predicted *RED* values with a standard deviation of ca. 7 mm.

SUMMARY AND CONCLUSIONS

A stochastic-deterministic one-dimensional model is proposed that simulates soil water flow for variable soil hydraulic properties and a variable lower boundary condition. These variations were determined by a normalization procedure which yields a scaled mean or reference curve and a set of scale factor values. Knowledge of the distribution type, and the mean and standard deviation of scale factor values, allowed other scale factor values to be randomly generated. These random scale factor values, in combination with a Monte Carlo analysis and a numerical water flow model, yielded distribution

TABLE 7

Results multiple regression analysis (5% probability level, 1976)

Dependent variable	Soil type	<i>GWL₀</i>	Scale factor values of horizon			θ_s of horizon			<i>R</i> ² -value
			A	BC	D	A	BC	D	
<i>RED</i>	1	+	+	+	•	—	—	•	0.91
	2	+	+	+	+	•	—	•	0.98
	3	+	+	+	—	—	—	•	0.85
	4	+	—	+	—	—	—	—	0.93
	5	+	—	•	•	—	•	—	0.89
<i>QB</i>	1	+	•	+	•	+	•	•	0.85
	2	+	+	+	•	•	+	•	0.70
	3	+	+	+	+	+	•	•	0.87
	4	+	•	+	+	•	+	+	0.79
	5	+	•	•	+	•	•	+	0.83
<i>GWL₁</i>	1	+	—	—	•	•	—	•	0.85
	2	+	•	—	—	•	—	•	0.92
	3	+	—	—	—	•	•	—	0.91
	4	+	•	—	—	•	—	—	0.88
	5	+	•	•	—	•	•	—	0.87

functions of several water balance components for each of the two simulated growing seasons. From calculation of the statistical properties of the PDF of the calculated output variables, it was then possible to characterize the Hupselse Beek watershed by a number of soil classes that yielded distinct values for actual transpiration during the growing seasons of 1976 and 1982. The soil water regime in other years can be simulated in an identical way, however, provided that precipitation and potential evapotranspiration values are known on a daily basis.

From measurements of soil hydraulic properties at 20 sampling sites at various depths within the watershed and by statistical analysis of the scale factor values of the fitted soil water characteristic curves it was concluded that the representative soil profile in the watershed could be differentiated by A, BC, and D horizons. Monte Carlo analysis was performed for five profile types, each profile type being representative for a different starting depth to clay (D horizon). Based upon analysis of measured groundwater levels it was further concluded that also the discharge groundwater level relationships were different, depending on clay depth. Therefore, two normalized $q(h)$ relationships were defined, based upon whether starting depth to clay was above or below the 1.2m depth.

The simulation model was validated by comparing measured and simulated transpiration values. There was close agreement for 1982, but for 1976 the simulated values for actual evapotranspiration were significantly larger than measured. Further validation included the comparison of the statistical properties of groundwater levels at the end of the growing season. The mean simulated and measured groundwater levels were very close, but the simulated variation was only 50–60% of the measured variation. Most likely, the major part of this reduction was caused by the assumptions regarding the variability of input variables.

With the results from the MC analysis it was furthermore possible to predict reduction in actual transpiration (*RED*) from initial groundwater level and scale factor values for the horizons. By multiple regression analysis, *RED* values could be predicted with an estimated standard deviation of approximately 10 mm in 1976 and 7 mm in 1982.

Statistical analysis of the MC simulations with respect to plant transpiration showed that the soils of the Hupselse Beek watershed could be regarded as being composed of three different soil classes. Ongoing research may reveal how well the presented results and conclusions can be extrapolated to a two-dimensional analysis. For that purpose, the spatial structure of the soil hydraulic properties in the horizontal direction will be incorporated in a two-dimensional unsaturated-saturated water flow model.

REFERENCES

- Andersson, J. and Shapiro, A.M., 1983. Stochastic analysis of one-dimensional steady-state unsaturated flow: A comparison of Monte Carlo and perturbation methods. *Water Resour. Res.*, 19(1): 121–133.

- Arya, L.M., Farrell, D.A. and Blake, G.R., 1975. A field study of soil water depletion patterns in presence of growing soybean roots. I. Determination of hydraulic properties of the soil. *Soil Sci. Soc. Am., Proc.*, 39: 424-430.
- Bakr, A.A., Gelhar, L.W., Gutjahr, A.L. and MacMillan, J.R., 1978. Stochastic analysis of spatial variability in subsurface flows. 1. Comparison of one- and three-dimensional flows. *Water Resour. Res.*, 14(2): 263-271.
- Belmans, C., Wesseling, J.G. and Feddes, R.A., 1981. Simulation model of the water balance of a cropped soil providing different types of boundary conditions (SWATRE). *Nota 1257, ICW, Wageningen*.
- Bouma, J., 1977. Soil survey and the study of water movement in unsaturated soil. *Soil Surv. Inst., Wageningen, Soil Surv. Pap.*, 13: 47-52.
- Bresler, E. and Dagan, G., 1983. Unsaturated flow in spatially variable fields. 2. Application of water flow models to various fields. *Water Resour. Res.*, 19(2): 421-428.
- Clapp, R.B., Hornberger, G.M. and Cosby, B.J., 1983. Estimating spatial variability in soil moisture with a simplified dynamic model. *Water Resour. Res.*, 19(3): 739-745.
- Dirksen, C., 1979. Flux-controlled sorptivity measurements to determine soil hydraulic property functions. *Soil Sci. Soc. Am., J.*, 43: 827-834.
- Ernst, L.F. and Feddes, R.A., 1979. Invloed van grondwateronttrekking voor berekening en drinkwater op de grondwaterstand. *Nota 1116, ICW, Wageningen*.
- Freeze, R.A., 1975. A stochastic-conceptual analysis of one-dimensional groundwater flow in nonuniform homogeneous media. *Water Resour. Res.*, 11(5): 725-741.
- Gelhar, L.W., 1986. Stochastic subsurface hydrology from theory to applications. *Water Resour. Res.*, 22(9): 135S-145S.
- Gutjahr, A.L. and Gelhar, L.W., 1981. Stochastic models of subsurface flow: Infinite versus finite domains and stationarity. *Water Resour. Res.*, 17(2): 337-350.
- Hopmans, J., 1987a. A comparison of various techniques to scale soil hydraulic properties. *J. Hydrol.*, 93: 241-256.
- Hopmans, J., 1987b. Treatment of the groundwater table in the stochastic approach of unsaturated water flow modelling. *Agric. Water Manage.*, 13: 297-306.
- Hopmans, J., 1987c. Some major modifications of the simulation model SWATRE. Rep. 79 Dep. Hydraul. Catchment Hydrol., Agric. Univ., Wageningen.
- Hopmans, J. and Overmars, B., 1986. Presentation and application of an analytical model to describe soil hydraulic properties. *J. Hydrol.*, 87: 135-143.
- Hopmans, J. and Stricker, J.N.M., 1986. Soil hydraulic properties in the study area Hupselse Beek as obtained from three different scales of observation: An overview. Rep. 78, Dep. Hydraul. Catchment Hydrol., Agric. Univ., Wageningen.
- Miller, E.E., and Miller, R.D., 1956. Physical theory for capillary flow phenomena. *J. Appl. Phys.*, 27(4): 324-332.
- Peck, A.J., Luxmoore, R.J. and Stolzy, J.C., 1977. Effects of spatial variability of soil hydraulic properties in water budget modeling. *Water Resour. Res.*, 13(2): 348-354.
- Philip, J.R., 1980. Field heterogeneity: Some basic issues. *Water Resour. Res.*, 16(2): 433-448.
- Dienst Informatieverwerking, Rijkswaterstaat, 1984. Herziening van de berekening van de gewasverdamming in het hydrologisch model GELGAM. Rep. Ad Hoc Group Transpiration, Dienst Informatieverwerking, Rijkswaterstaat, Arnhem.
- Russo, D. and Bresler, E., 1980a. Field determinations of soil hydraulic properties for statistical analysis. *Soil Sci. Soc. Am., J.*, 44: 697-702.
- Russo, D., and Bresler, E., 1980b. Scaling soil hydraulic properties of a heterogeneous field. *Soil Sci. Soc. Am., J.*, 44: 681-684.
- Russo, D. and Bresler, E., 1981a. Effect of field variability in soil hydraulic properties on solutions of unsaturated water and salt flows. *Soil Sci. Soc. Am., J.*, 45(4): 675-681.
- Russo, D. and Bresler, E., 1981b. Soil hydraulic properties as stochastic processes: I. An analysis of field spatial variability. *Soil Sci. Soc. Am., J.*, 45(4): 682-687.
- Smith, L. and Freeze, R.A., 1979a. Stochastic analysis of steady state groundwater flow in a bounded domain. 1. One-dimensional simulations. *Water Resour. Res.*, 15(3): 521-528.

- Smith, L., and Freeze, R.A., 1979b. Stochastic analysis of steady state groundwater flow in a bounded domain. 2. Two-dimensional simulations. *Water Resour. Res.*, 15(6): 1543-1559.
- Stakman, W.P., Valk, G.V. and Van der Harst, G.G., 1969. Determination of soil moisture retention curves. I. Sandbox apparatus. ICW, Wageningen.
- Stephens, M.A., 1974. EDF-Statistics for goodness-of-fit and some comparisons. *J. Am. Stat. Assoc.*, 69: 730-737.
- Thom, A.S. and Oliver, H.R., 1977. On Penman's equation for estimating regional evaporation. *Q.J.R. Meteorol. Soc.*, 103: 345-357.
- TNO Report, 1984. Comparison of simulation models for unsaturated soil-water transport and plant transpiration. *Hydrol. Onderz. TNO, Ser., Rapp. Nota's No. 13* (in Dutch).
- Van Genuchten, R., 1978. Calculating the unsaturated hydraulic conductivity with a new closed-form analytical model. *Dep. Civ. Eng., Princeton Univ., N.J.*, 78-WR-08.
- Warrick, A.W. and Amoozegar-Fard, A., 1979. Infiltration and drainage calculations using spatially scaled hydraulic properties. *Water Resour. Res.*, 15(5): 1116-1120.
- Warrick, A.W., Mullen, G.J. and Nielsen, D.R., 1977a. Scaling field-measured soil hydraulic properties using a similar media concept. *Water Resour. Res.*, 13: 355-362.
- Warrick, A.W., Mullen, G.J. and Nielsen, D.R., 1977b. Predictions of the soil water flux based upon field-measured soil-water properties. *Soil Sci. Soc. Am., J.*, 41: 414-419.
- Wösten, J.H.M., Stoffelsen, G.H., Jeunissen, J.W.M., Holst, A.F. and Bouma, J., 1983: Proefgebied Hupselse Beek; regionaal bodemkundig en bodemfysisch onderzoek. *Rapp. No. 1706*, Sticht. Bodemkartering, Wageningen.
- Wösten, J.H.M., Bouma, J. and Stoffelsen, G.H., 1985. Use of soil survey data for regional soil water simulation models. *Soil Sci. Soc. Am., J.*, 49: 1238-1244.
- Yeh, T.-C., Gelhar, L.W. and Gutjahr, A.L., 1985a. Stochastic analysis of unsaturated flow in heterogeneous soils. 1. Statistically isotropic media. *Water Resour. Res.*, 21(4): 447-456.
- Yeh, T.-C., Gelhar, L.W. and Gutjahr, A.L., 1985b. Stochastic analysis of unsaturated flow in heterogeneous soils. 3. Observations and applications. *Water Resour. Res.*, 21(4): 465-481.