Sparse Erroneous Vehicular Trajectory Compression and Recovery Via Compressive Sensing

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***Abstract*—Vehicle tracking information is necessary to enable safety communication systems and intelligent transportation systems. Compression technologies with high efficiency and low complexity provide a promising approach to address the transmission and computing problems in vehicle tracking applications. Especially, vehicular trajectory with sparse errors that happened in the measurement sensing process poses a great challenge on traditional compression algorithms. In this paper, we analyze and design a compressive sensing (CS) based erroneous trajectory compression and recovery algorithm for vehicle tracking scenario. Moreover, some theoretical bounds for the proposed recovery optimization problem are analyzed and proved. The CS-based method proposed in this paper could not only achieve a fairly high compression rate and recovery accuracy, but fit the bandwidth mismatch between the road side unit (RSU) and on board unit (OBU). In another aspect, the Kalman filtering (KF) technology is applied for further optimizing the system performance, e.g. mean square error (MSE). Extensive simulations with real vehicular trajectories are carried out, which shows that CS-based compression algorithm achieves relatively high compression performance compared to some state-of-the-art trajectory compression algorithms.**

***Index Terms*—Erroneous Vehicular Trajectory, Compression, Recovery, Compressive Sensing**

I. Introduction

Trajectory collection and analysis has become a special and important issue for vehicular communication systems. With the increasing possession number of on board units (OBUs), various vehicular behavioral characteristics can be excavated via trajectory processing. Understanding these mobile inhabitants can help in dealing with many scientific and social issues, including having a deeper understanding of our life, providing valuable information for the business, and even finding out the trends of urban evolution etc. To achieve the above mentioned targets, the acquired data is needed as much as possible. However, enormous volumes of data bring challenges of storage,transmission,computation and display.

Obviously, there is an urgent need for compression technologies for vehicular global positioning system (GPS) data set. Let us assume that a vehicular trajectory data stream is a sequence of $\langle t, p^{(x)}, p^{(y)} \rangle$, in which $p^{(x)}, p^{(y)}$ represent coordinates of the vehicle at time t, respectively. If such data is collected every 10 seconds, 100 Mb of storage capacity is required to store the data for just over 400 objects for a single day [1], barring any data compression. We obviously want to be monitoring many more moving objects, and for much longer time duration.

To deal with the big data issues, compression technologies had been studied as a popular way for trajectory processing, which had attracted tremendous attentions [1]–[5]. Line simplification algorithms, including uniform sampling [2], Douglas-Peucker algorithm [3], and Bellman's algorithm [4] etc., are treated as a commonly used processing method because of its simplicity and relative accuracy. Moreover, trajectory can be reduced based on additional speed or direction information. Potamias *et. al* [5] showed that, in addition to spatial positions, changes in speed and direction are key factors for predicting locations in a trajectory. However, the performance of these algorithms in [1]–[5] degrade significantly.

One of the main problems when using GPS receivers in vehicular environments is the multiple electromagnetic interference (EMI) sources, due to numerous factors such as rolling stock, locomotives, onboard power-supply installations, catenaries, electrical devices, and signaling and telecommunication equipment. Therefore, the obtained vehicular trajectory data can never be completely error-free. Sometimes the error is acceptable, such as when using trajectory data to identify which district a vehicle is in. For other situations, various filtering technologies, including mean filter [6], median filter [7], and Kalman filter [8] etc., had been applied to smooth the noise and measurement errors. However, the filtering technologies are all post-processing means, which can only work after obtaining all required data set. Consequently, this characteristic will increase the calculating complexity and processing time for the front sampling device, which is inconsistent with the asymmetric processing ability between OBUs and road side units (RSUs). It is commonly admitted that the OBUs' front-possessing ability is weak, while the RSUs' post-processing ability is relatively strong.

Compressive sensing (CS) confirms that sparse signals can be recovered from far fewer samples than is predicted by the Nyquist-Shannon sampling theorem [9], [10]. The sampling process in CS can accomplish both detection and compression functions simultaneously. CS trades off an increase in the computational complexity of post-processing against the convenience of a smaller quantity of data acquisition and lower demands on the computational capability of OBU. CS directly acquires the compressed processed version while sampling and no explicit compression process is required.

In this paper, a compressive sensing (CS) based erroneous vehicular trajectory compression and recovery principle is proposed. Some works had utilized CS on processing the similar issues, but the previous works [11], [12] focused on

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compression and recovery problem on datasets without errors. In this paper, a CS-based erroneous trajectory compression and recovery algorithm is proposed and some theoretical bounds for the proposed recovery optimization problem are analyzed and proved. The CS-based method proposed in this paper can not only achieve a fairly high compression rate and recovery accuracy, but fit the bandwidth mismatch between the road side unit (RSU) and on board unit (OBU). The contributions of this paper are listed as follows:

- An efficient CS-based erroneous vehicular trajectory compression and recovery algorithm is proposed and solved by a modified optimization methodology. Moreover, some theoretical bounds for the proposed recovery optimization problem are analyzed and proved.
- The proposed algorithm is evaluated based on real vehicular trajectory data, which shows a confirmation for the relatively high accuracy of the proposed algorithm. Extensive simulations show that CS-based trajectory compression is resilient to erroneous data for its oriented distortion ability to information loss.
- The performance obtained from pure CS-based algorithm can be enhanced by adding Kalman filtering. Especially, performance evaluation shows that the Kalman filtering's performance with post-processing is relatively higher than that with pre-processing.

The remainder of this paper is organized as follows. section II presents the system model. The analytical models are proposed and discussed in section III. The numerical results obtained from our analytical models and simulations are compared in section IV. Last but not least, section V provides a discussion for the content of this paper, and section VI concludes this paper and gives a future proposal.

II. Preliminaries

A. Compressive Sensing Framework

Compressive sensing (CS) is widely used in data processing community, and is suitable to the sparse data compression in the sensing phase. The required sampling rate is much lower compared to the traditional Shannon-Nyquist sampling method. Instead of using more than double the highest frequency Nyquist sampling rate, the CS uses much lower sampling rate and randomly collects the samples from the entire sparse data space. Then it recovers the original data by solving an optimization equation and reconstruct the original data set with little error.

Design the measurement sensing matrix Φ is a challenging problem for CS. In [13], Candès and Tao introduced the *restricted isometry property* (RIP) of Φ and established its important role in CS, which can be shown in the following definition.

Definition II.1. (*Restricted Isometry Principle*) A matrix Φ satisfies the RIP of order k if there exists a constant δ_k within (0, 1), such that

$$(1-\delta_k)||x||_2^2 \leq ||\Phi x||_2^2 \leq (1+\delta_k)||x||_2^2 \tag{1}$$

holds for all k-sparse vectors x.

Based on the classic principle of CS, it is necessary for the measurement port to collect enough data samples for the recovery process, and the randomness is the basic requirement for the RIP of the CS matrix. Moreover, it has been shown that the solution x^* to the following optimization problem

$$\min_{x\in\mathbb{R}^n} ||x||_1 \quad \text{s.t.} \quad y = \Phi x. \tag{2}$$

recovers x exactly provided that Φ obeys RIP and x is sufficiently sparse. It should be noticed that the real vehicular trajectory data is sparse originally because of the inherent characteristics, including the drivers's routine steering behaviors, and the deterministic road map etc.

For noiseless compression and recovery, assuming that $\delta_{2s} < \sqrt{2}-1$, Candés *et. al* [14] proved that:

Definition II.2. (*Noiseless Recovery*) The solution x^* to problem shown in Equ. (6) obeys

$$||x^*-x||_1 \leq C_0||x-x_s||_1 \tag{3}$$

and

$$||x^*-x||_2 \leq C_0 s^{-1/2}||x-x_s||_1, \tag{4}$$

where $C_0 = 2\dfrac{1-(1-\sqrt{2})\delta_{2s}}{1-(1+\sqrt{2})\delta_{2s}}$. In particular, if x is s-sparse, the recovery is exact.

B. System Model

Generally speaking, there are two categories of data reduction techniques reported in the literature of moving object and trajectory management. The basic idea behind data reduction techniques in the first category, called *batched compression techniques*, is to first collect the full set of sampled location data points and then compress the data set by discarding redundant location points for transmission to the location server. Another category of trajectory data reduction techniques is named as *online data reduction techniques* since the sample points are achieved online and updated in real time. In this paper, the patched data compression process is adopted and the proposed algorithm can be modified into one adaptive to the online case.

In this paper, suppose the vehicular trajectory data is collected every τ seconds, and a total n points are taken into consideration. The data set is symbolized as $\mathbf{P} = \{p_1, p_2, \ldots, p_n\}$. After compressive sensing processing, m points are obtained as a compression version for the overall data set. The compression version can be represented as $\mathbf{C} = \{c_1, c_2, \ldots, c_m\}$.

Note that a position data point is typically represented using two geographic Cartesian coordinates, the x-coordinates (longitude data) and the y-coordinates (latitude data). Since the focused vehicular trajectory data is 2-dimensional for each sample, strategy proposed in this subsection is based on parallel transmission. That is, the latitude information and longitude information are compressed and recovered separately, which can be shown in Fig. 1.

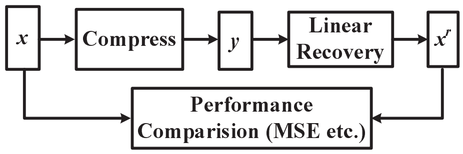


Fig. 1. Parallel Compression and Recovery Block Diagram

The compression process for error-free data (without both original sensor measurement errors and compressive sensing errors) can be represented as:

$$y = \Phi x = \Phi\Psi\alpha = A\alpha, \tag{5}$$

where $A = \Phi\Psi$, and there are not any errors in the front-measurement process. That is, $x = \Psi\alpha$, where Ψ represents the dictionary matrix, and α is corresponding sparse signal representation of x. Note that x and α are equivalent representation of the trajectory segment, with x in the time domain and α in the Φ domain.

The optimal solution for above under-determined problem can be obtained by solving the following optimization problem:

$$\min_{x_0 \in \mathbb{R}^n} ||\alpha||_1 \quad \text{s.t.} \quad y = A\alpha. \tag{6}$$

However, not all the errors happen in the measurement errors, and many errors happen in the original data acquisition part. Especially for the vehicular trajectory data, multiple EMI interference, including rolling stock, locomotives, onboard power-supply installations, catenaries, electrical devices, and signaling and telecommunication equipment, greatly influence on the data accuracy.

III. Erroneous Trajectory Compression and Recovery via Compressive Sensing

In this section, we define the problems of erroneous vehicular trajectory data compression, and find the optimal recovery solution. Under this topic, three sub-problems are discussed including:

- Parallel compression and recovery for erroneous data.
- Erroneous trajectory compression and recovery with filtering.

Based on the above mentioned problems, possibly effective and accuracy strategies are proposed be means of CS. In the following aspects, the problems and strategies are introduced and analyzed respectively.

A. Parallel Compression and Recovery for Erroneous Data

1) Problem Statement: The original data is supposed to have sparse errors, which can be divided into *original data error* and *compression measurement error*. The original vehicular trajectory GPS data accuracy can be influenced by multiple EMI interference, including rolling stock, locomotives, onboard power-supply installations, catenaries, electrical devices, and signaling and telecommunication equipment. This aspect, measurement process, can be represented as:

$$y = \Phi x + e, \tag{7}$$

where n-dimensional e denotes the measurement error, and n-dimensional x can represent either the latitude data or the longitude data. Moreover, there exist errors for the original data set, that is,

$$x = \Psi\alpha + \beta, \tag{8}$$

where b denotes the n-dimensional original data error.

The target of this subsection is to propose an efficient compression and recovery strategies for the erroneous vehicular trajectory GPS data based on CS. For the compression aspect, m-dimensional measurements y can be obtained from original n-dimensional original data x. For the recovery aspect, the goal is to recover n-dimensional data x^r from m-dimensional measurements y.

2) Compression and Recovery Strategy: Based on the equations (7) and (8), we can obtain the compression process as follows:

$$y = \Phi x + e = \Phi(\Psi\alpha + \beta) + e, \tag{9}$$

where the recovery problem lies in the solution of the following optimization problem:

$$\begin{aligned} \min_{\alpha,\beta \in \mathbb{R}^n} \quad & ||\alpha||_1 + \lambda||\beta||_1 \\ \text{s.t.} \quad & ||y - \Phi(\Psi\alpha + \beta)||_2 \leq \epsilon. \end{aligned} \tag{10}$$

However, the optimization problem in (10) is non-convex, which can hardly produce the optimal results. To overcome this drawback, we proposed one modified convex optimization problem, from which we can obtain the near-optimal recovery results from the compressed data.

Supposed that the n-dimensional vectors α and β are $2n$-dimentional s,s'-sparse respectively. We construct one expanded $(s + s')$-sparse vector as follows:

$$\gamma = [\alpha; \beta]. \tag{11}$$

Based on Equ. (10) and the newly constructed vector γ, the optimization problem can be modified into:

$$\begin{aligned} \min_{\gamma \in \mathbb{R}^{2n}} \quad & \mathbf{\Lambda}||\gamma||_1 \\ \text{s.t.} \quad & ||y - B\gamma||_2 \leq \epsilon. \end{aligned} \tag{12}$$

where $B = \Phi[\Psi\ \mathbf{I_n}]$, $\mathbf{\Lambda} = [\mathbf{I_n}\ \mathbf{O_n}; \mathbf{O_n}\ \lambda\mathbf{I_n}]$, in which $\mathbf{I_n}$ denotes the identity matrix, and $\mathbf{O_n}$ represents the zero matrix. For obtaining the upper and lower limits for the CS based strategy, three theorems are given and proven as follows.

Theorem III.1. *Given an unitary matrix Ψ, the matrix $B = \Phi[\Psi\ \mathbf{I_n}]$ satisfies*

$$\sqrt{2}\lambda_{min} \leq \frac{||B\gamma||_2^2}{||\gamma||_2^2} \leq \sqrt{2}\lambda_{max}, \tag{13}$$

for all $2n$-dimensional vectors γ, given that the matrix $\Phi\Psi$ satisfies

$$\lambda_{min} \leq \frac{||\Phi\Psi\alpha||_2^2}{||\alpha||_2^2} \leq \lambda_{max}, \tag{14}$$

for all n-dimensional vectors α.

Proof: Since that Ψ is an unitary matrix, all the singular values of the matrix $\left[\frac{\Psi}{\sqrt{2}}\ \frac{\mathbf{I_n}}{\sqrt{2}}\right]$ are 1 and $\lambda^{\Phi\Psi} = \lambda^{\Phi}$. According to the properties of the singular values of the product of two matrices, for $B = \sqrt{2}\Phi\tilde{\Psi}$, where $\tilde{\Psi} = \left[\frac{\Psi}{\sqrt{2}}\ \frac{\mathbf{I_n}}{\sqrt{2}}\right]$, it can be obtained that

$$\lambda^{B}_{min} \geq \sqrt{2}\lambda^{\Phi}_{min} \cdot \lambda^{\tilde{\Psi}}_{min}, \tag{15}$$

and

$$\lambda^{B}_{max} \leq \sqrt{2}\lambda^{\Phi}_{max} \cdot \lambda^{\tilde{\Psi}}_{max}, \tag{16}$$

where $\lambda^{\tilde{\Psi}}_{min} = \lambda^{\tilde{\Psi}}_{max} = 1$. According to the above derivations, it is easy to prove the results shown in Theorem III.1. ■

Theorem III.2. *Given an unitary matrix Ψ, the restricted isometry constant (RIC) δ^B of matrix $B = \Phi[\Psi\ \mathbf{I_n}]$ satisfies*

$$\delta^B = \min\{1-(1-\delta^{\Phi})(1-\delta^{\tilde{\Psi}_J}), (1+\delta^{\Phi})(1+\delta^{\tilde{\Psi}_J})-1\}, \tag{17}$$

where $\tilde{\Psi} = [\Psi\ \mathbf{I_n}]$, and J means the support of $\tilde{\Psi}$.

Proof: According to the singular value properties for the product of matrices,

$$1 - \delta^B = \lambda_{min}(\Phi\tilde{\Psi}_J) \geq \lambda_{min}(\Phi) \cdot \lambda_{min}(\tilde{\Psi}_J), \tag{18}$$

and

$$1 + \delta^B = \lambda_{max}(\Phi\tilde{\Psi}_J) \leq \lambda_{max}(\Phi) \cdot \lambda_{max}(\tilde{\Psi}_J). \tag{19}$$

Moreover, based on the RIC property of matrices Φ and $\tilde{\Psi}_J$, we can obtain that

$$\begin{aligned} \lambda_{min}(\Phi) &\geq 1 - \delta^{\Phi} \\ \lambda_{min}(\tilde{\Psi}_J) &= 1 - \delta^{\tilde{\Psi}_J} \\ \lambda_{max}(\Phi) &\leq 1 + \delta^{\Phi} \\ \lambda_{max}(\tilde{\Psi}_J) &= 1 + \delta^{\tilde{\Psi}_J} \end{aligned} \tag{20}$$

Applying (20) into (18) and (19), we can derive the results shown in Theorem III.2. ■

Theorem III.3. *The solution α^* to problem shown in Equ. (12) obeys*

$$||\alpha^* - \alpha||_1 \leq C_0'||\alpha - \alpha_s||_1 + C_0'||\beta - \beta_{s'}||_1, \tag{21}$$

and

$$||\alpha^* - \alpha||_2 \leq C_0'(s+s')^{-1/2}(||\alpha - \alpha_s||_1 + ||\beta - \beta_{s'}||_1), \tag{22}$$

where $C_0' = 2\frac{1-(1-\sqrt{2})\delta_{2(s+s')}}{1-(1+\sqrt{2})\delta_{2(s+s')}}$. In particular, if α is s-sparse and β is s'-sparse, the recovery is exact.

Proof: From definition in Def. II.2, it can be deduced that the solution γ^* to problem shown in Equ. (12) obeys

$$||\gamma^* - \gamma||_1 \leq C_0'||\gamma - \gamma_{(s+s')}||_1 \tag{23}$$

and

$$||\gamma^* - \gamma||_2 \leq C_0'(s+s')^{-1/2}||\gamma - \gamma_{(s+s')}||_1, \tag{24}$$

where $C_0' = 2\frac{1-(1-\sqrt{2})\delta_{2(s+s')}}{1-(1+\sqrt{2})\delta_{2(s+s')}}$. If γ is $(s+s')$-sparse, the recovery is exact.

$$||\gamma^* - \gamma||_1 = ||\alpha^* - \alpha||_1 + ||\beta^* - \beta||_1 \tag{25}$$

Moreover,

$$||\gamma - \gamma_{(s+s')}||_1 \leq ||\alpha - \alpha_s||_1 + ||\beta - \beta_{s'}||_1 \tag{26}$$

Therefore

$$||\alpha^* - \alpha||_1 + ||\beta^* - \beta||_1 \leq C_0'||\alpha - \alpha_s||_1 + C_0'||\beta - \beta_{s'}||_1. \tag{27}$$

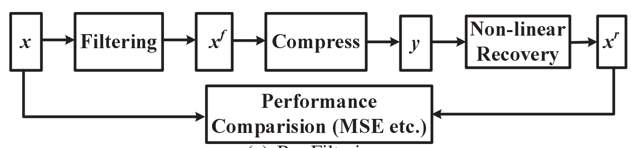
By eliminating the second part of the left equation, we can produce that

$$||\alpha^* - \alpha||_1 \leq C_0'||\alpha - \alpha_s||_1 + C_0'||\beta - \beta_{s'}||_1, \tag{28}$$

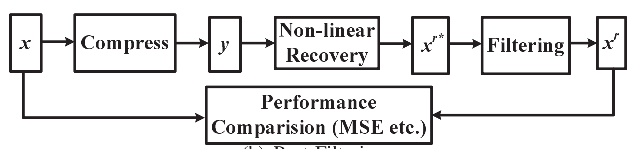
which shows a loose upper bound for the solution of the revised optimization problem, which proves the results shown in Theorem III.3. ■

B. Compression and recovery with filtering

The general strategy for erroneous data compression is done by adding the front-filtering technologies, which can only work after obtaining all required data set. Consequently, this characteristic will increase the calculating complexity and processing time for the front sampling device, which is inconsistent with the asymmetric processing ability between OBUs and road side units (RSUs). Under this structure, the errors that exist in the inherit data structure are eliminated before or after the compression operation, which can be shown as the block diagram as shown in Fig. 2(a) and 2(b). After the data filtering process, we can apply the traditional data compression technologies to accomplish the total target.



(a) Pre-Filtering



(b) Post-Filtering

Fig. 2. Compression and Recovery Block Diagram with Filtering Technology

In this paper, we place the filtering process behind the CS process. This is because that the performance gain obtained by CS may be omitted by the pre-executed filtering process. The CS technologies promise the ability of recover both the sparse target data and the errors, which guarantee the integral system performance. In this paper, the vehicle tracking problem can be inspired by the derivation in [15].

IV. Performance Evaluation

In this section, the real trajectory datasets cited from [16] are utilized to illustrate the performance of the proposed compression and recovery algorithms. Since the target of this paper is on the erroneous data, we add some errors manually on some selected error-free trajectory datasets. The process for the longitude data can be represented as

$$x_i = \begin{cases} x_i^f + e_i \ , & \text{for randomly } n_e \text{ positions from } N \\ x_i^f , & \text{otherwise} \end{cases} , \tag{29}$$

where n_e means the position where errors happen, and x_i^f means the input error-free trajectory data in the i-th place, and the data in the latitude domain can be represented similarly. Moreover, the error e_i happens in the i-th position depends on the scale parameter c. To evaluate the performance of the compression and recovery process, we adopt the mean square error, which can be denoted as

$$MSE(p_x^r, p_y^r, p_x^f, p_y^f) = ||p_x^r - p_x^f||_2^2 + ||p_y^r - p_y^f||_2^2 \tag{30}$$

where p_x^f, p_y^f denote as the original error-free coordinate datasets, and p_x^r, p_y^r represent the equally-long sequence recovered from the compressed data.

Moreover, we evaluate the performance of our approaches through extensive simulations. For comparison, two main principles are carried out, including the uniform sampling algorithm (USA) and the Dauglas-Paucker algorithm (DPA). Instantaneously, the data filtering technologies are applied to enhance the performance of the original DP algorithm and CS-based algorithm (CSA). The major parameters for the simulation are shown in TABLE I.

TABLE I
Reference Values for Main Parameters in Simulation

Parameter	Description	Value
τ	The trajectory data observation interval	5 s
σ	The standard variance of the coordinates	1-10 m
σ_s	The standard variance of the moving speed	0.1–1 m/s
N	The length of the processed sequence	1000
n	The number of Monte Carlo simulations	100
n_e	The number of errors in the original data	1–10
c	The scale of the error in the original data	0.1–1

In Fig. 5, the comparison is carried out among the trajectory of the original error-free data, the data with n_e errors, the data compressed and recovered with pre-Kalman filtering, and data processed with post-Kalman filtering. It can be deduced that the post-Kalman filter processing can show a more closed trajectory results with the error-free data than that dealt with pre-Kalman filtering. Based on the results from this figure, we can obtain one proof for the superiority of the post-Kalman filter processing to pre-processing.

In Fig. 6, a comparison is carried out among the MSE performance with different compression and recovery algorithms, including the DP algorithm, the CS based algorithm proposed in section III-A, the DP algorithm with Kalman filtering, and the CS based algorithm with pre- and post-Kalman filtering.

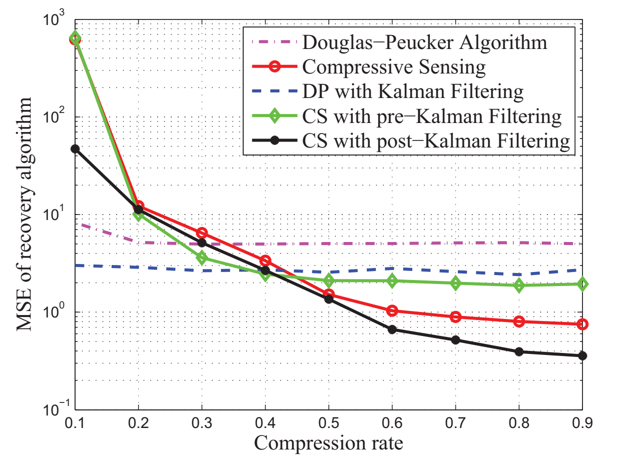


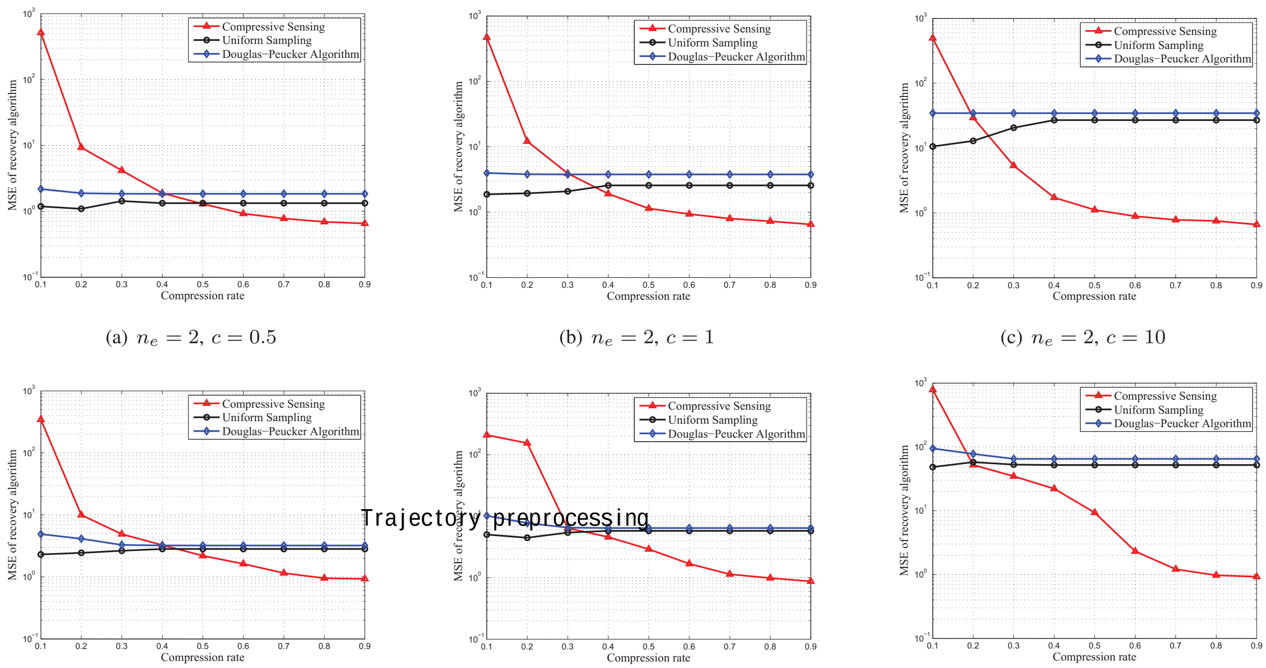
Fig. 4. MSE performance comparision with five different compression and recovery algorithms

Comparing the results from original DP algorithm and DP algorithm with Kalman filtering, it can be seen that kalman filtering technology can improve the MSE performance. This reveals the theory presented in section III-B that the Kalman filter can smooth some great fluctuations. As for performance from CS based algorithms, additions with pre- and post-Kalman filtering show the opposite results. Post-Kalman filter processing helps enhance the recovery performance slightly, while results from CS plus pre-Kalman filtering elucidate a mild decrease. This happens because that the original data structure may be destroyed by the pre-Kalman filtering, while the CS technology can only provide little influence on the data structure.

V. Conclusion and Future Proposal

In this paper, we analyze and design a compressive sensing based erroneous trajectory compression and recovery algorithm for vehicle tracking scenario. Moreover, some theoretical bounds for the proposed recovery optimization problem are analyzed and proved. The CS-based method proposed in this paper can not only achieve a fairly high compression rate and recovery accuracy, but fit the bandwidth mismatch between the RSU and OBU. In another aspect, the Kalman filtering technology is applied for further optimizing the system performance. Extensive simulations show that the CS-based compression and recovery algorithm can enhance the system performance in deal with erroneous vehicular trajectory datasets especially when the compression rate is relatively high, compared with traditional algorithms. The higher of the value of the number and scale of errors, the improved performance is even more significant. Moreover, the additional Kalman filtering, especially the controlled KF processing can increase the total performance.

For future targets, possible joint compression and recovery principles may maximize the information contained in the original data structures. The performance enhanced by adding



(a) $n_e = 2$, $c = 0.5$ (b) $n_e = 2$, $c = 1$ (c) $n_e = 2$, $c = 10$

(d) $n_e = 10$, $c = 0.5$ (e) $n_e = 10$, $c = 1$ (f) $n_e = 10$, $c = 10$

Fig. 3. MSE performance for erroneous vehicular trajectory data recovery

filtering concept may be highlighted when merging it into the CS-based compression algorithm. Moreover, the work presented in this paper is based on one single vehicular trajectory processing. The study on multi vehicular trajectory compression may be our follow-on work in the near future.

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