

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

%matplotlib inline
```

1. Logistic Regression

In this part of the exercise, you will build a logistic regression model to predict whether a student gets admitted into a university.

Suppose that you are the administrator of a university department and you want to determine each applicant's chance of admission based on their results on two exams. You have historical data from previous applicants that you can use as a training set for logistic regression. For each training example, you have the applicant's scores on two exams and the admissions decision.

Your task is to build a classification model that estimates an applicant's probability of admission based the scores from those two exams.

In [2]:

```
data = pd.read_csv('./ex2data1.txt', names=['Exam_1', 'Exam_2', 'Admitted'])
data.head()
```

Out[2]:

	Exam_1	Exam_2	Admitted
0	34.623660	78.024693	0
1	30.286711	43.894998	0
2	35.847409	72.902198	0
3	60.182599	86.308552	1
4	79.032736	75.344376	1

In [3]:

```
def loadData(df):
    ones = pd.DataFrame({'ones': np.ones(len(df))})
    df = pd.concat([ones, df], axis=1)
    X = df.iloc[:, :-1].values
    y = df.iloc[:, -1].values.reshape(-1, 1)
    return np.matrix(X), np.matrix(y)
```

加载数据，并查看X, y的维度是否正确。

In [4]:

```
X, y = loadData(data)
X.shape, y.shape
```

Out[4]:

```
((100, 3), (100, 1))
```

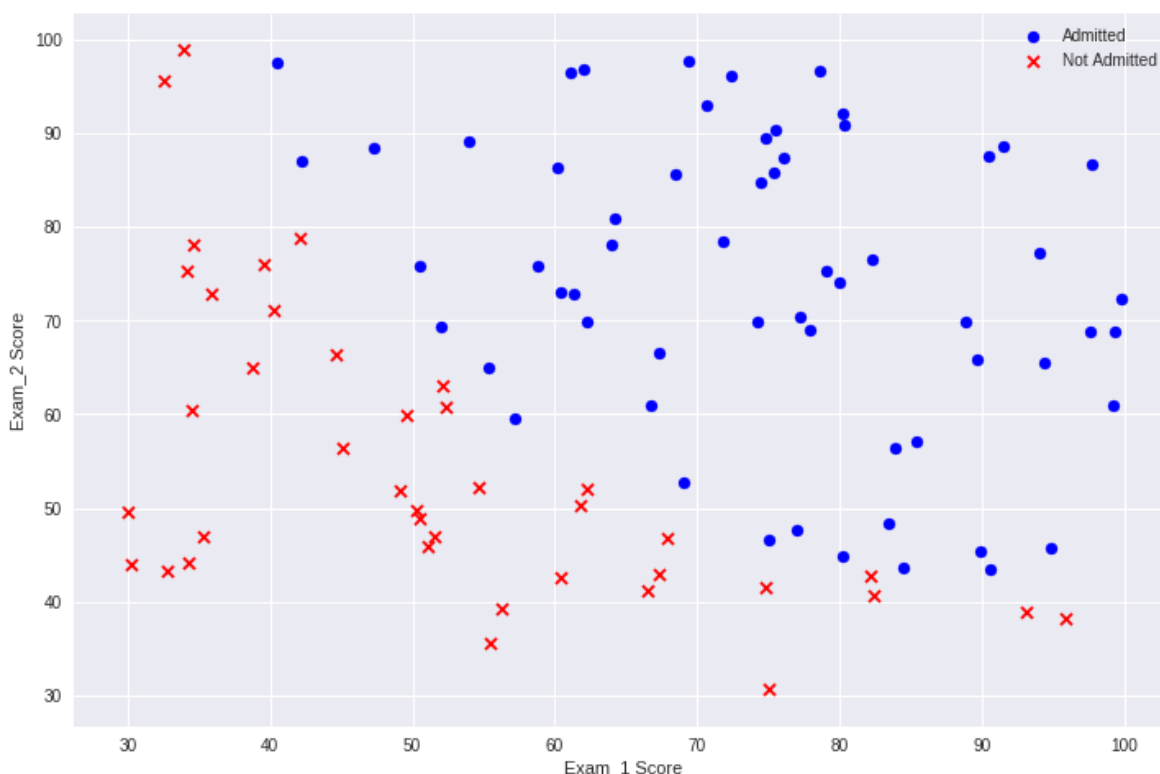
1.1 Visualizing the data

Before starting to implement any learning algorithm, it is always good to visualize the data if possible.

In [5]:

```
positive = data[data['Admitted'] == 1]
negative = data[data['Admitted'] == 0]

plt.figure(figsize=(12, 8))
plt.scatter(positive['Exam_1'], positive['Exam_2'], color='blue', marker='o', label='Admitted')
plt.scatter(negative['Exam_1'], negative['Exam_2'], color='red', marker='x', label='Not Admitted')
plt.legend()
plt.xlabel('Exam_1 Score')
plt.ylabel('Exam_2 Score')
plt.show()
```



1.2 Implementation

1.2.1 Sigmoid function

Before you start with the actual cost function, recall that the logistic regression hypothesis is defined as:

$$h_{\theta}(x) = g(\theta^T x)$$

where function g is the sigmoid function. The sigmoid function is defined as:

$$g(z) = \frac{1}{1 + e^{-z}}$$

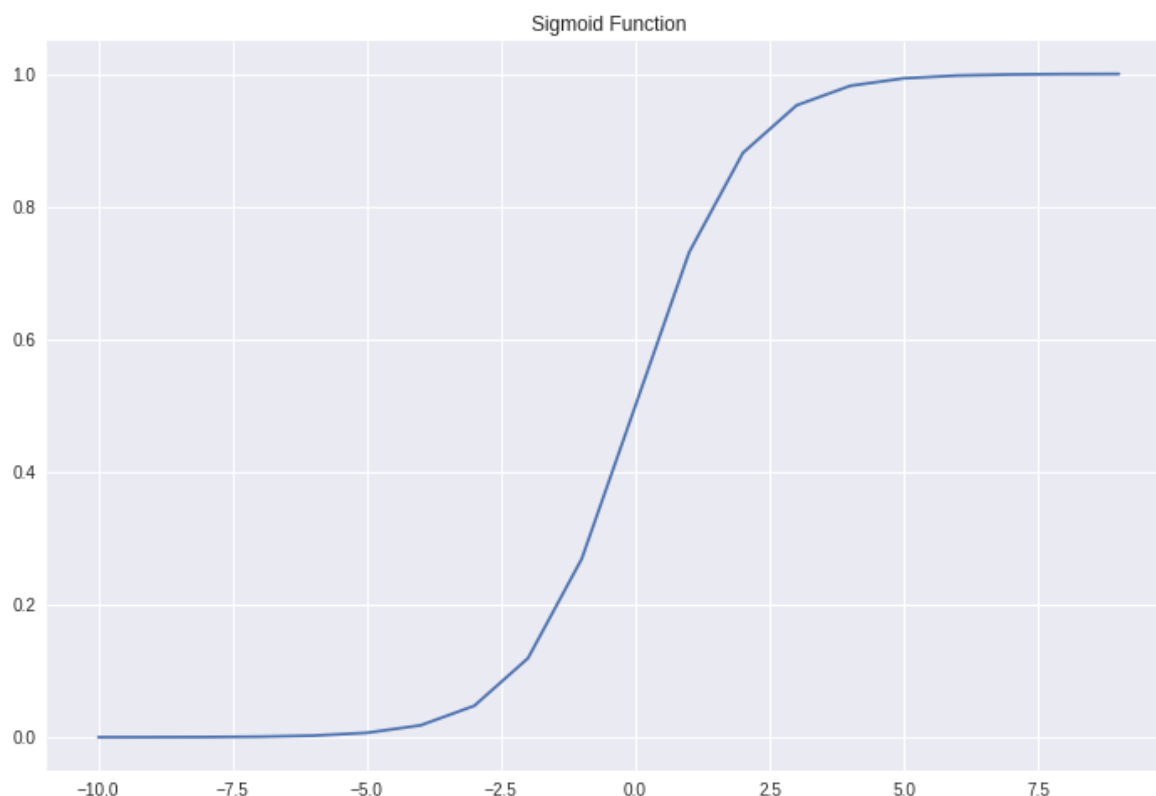
In [6]:

```
def sigmoid(Z):  
    return 1 / (1 + np.exp(-Z))
```

让我们做一个快速检查，确保它可以工作。

In [7]:

```
nums = np.arange(-10, 10, step=1)  
  
plt.figure(figsize=(12, 8))  
plt.plot(nums, sigmoid(nums))  
plt.title('Sigmoid Function')  
plt.show()
```



1.2.3 Cost function and gradient

Now you will implement the cost function and gradient for logistic regression. Recall that the cost function in logistic regression is

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m [-y^{(i)} \log(h_{\theta}(x^{(i)})) - (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))]$$

and the gradient of the cost is a vector of the same length as θ where the j^{th} element (for $j = 0, 1, \dots, n$) is defined as follows:

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

Note that while this gradient looks identical to the linear regression gradient, the formula is actually different because linear and logistic have different definitions of $h_{\theta}(x)$.

In [8]:

```
# 初始化theta为零
def initializeWithZeros(dim):
    return np.zeros((dim, 1))
```

In [9]:

```
def computeCost(X, y, theta):
    first = np.multiply(-y, np.log(sigmoid(X @ theta)))
    second = np.multiply(1 - y, np.log(1-sigmoid(X @ theta)))
    return np.mean(first - second)
```

In [10]:

```
# 正确答案为0.693
theta = initializeWithZeros(X.shape[1])
cost = computeCost(X, y, theta)
cost
```

Out[10]:

```
0.69314718055994529
```

1.2.4 使用梯度下降法寻找最优参数

In [11]:

```
def batchGradientDecent(X, y, theta, alpha, iters):  
  
    m = X.shape[0]  
    cost = np.zeros(iters)  
  
    for i in range(iters):  
        gradient = np.dot(X.T, sigmoid(X @ theta) - y) / m  
        theta = theta - alpha * gradient  
  
        cost[i] = computeCost(X, y, theta)  
  
        if i % 50000 == 0:  
            print('第{iters}轮:{cost}'.format(iters=i, cost=cost[i]))  
  
    return theta, cost
```

In [12]:

```
X, y = loadData(data)  
alpha = 0.000001  
iters = 500000  
theta = initializeWithZeros(X.shape[1])  
theta1, cost1 = batchGradientDecent(X, y, theta, alpha, iters)
```

```
第0轮:0.6928764038075428  
第50000轮:0.6295780590780752  
第100000轮:0.6293338830821339  
第150000轮:0.6290905746767632  
第200000轮:0.6288474761373234  
第250000轮:0.6286045870699931  
第300000轮:0.6283619073006185  
第350000轮:0.628119436655143  
第400000轮:0.6278771749595334  
第450000轮:0.6276351220397811
```

In [13]:

```
# 对数据进行标准化处理  
X, y = loadData(data)  
X[:,1:3] -= X[:,1:3].mean(axis=0)  
X[:,1:3] /= X[:,1:3].std(axis=0)
```

In [14]:

```
alpha = 0.01  
iters = 200000  
theta = initializeWithZeros(X.shape[1])  
theta2, cost2 = batchGradientDecent(X, y, theta, alpha, iters)
```

```
第0轮:0.6916282011267486  
第50000轮:0.20352911584883715  
第100000轮:0.2034977922166995  
第150000轮:0.2034977018657916
```

在没有对数据进行标准化的情况下，进行梯度下降的效果是非常差的，损失函数的收敛速度十分的慢。同时，学习速率 α 的值要取得非常小，否则，损失函数的波动十分明显，迭代次数也因此非常大。最后，损失函数的值为0.62。

在对数据进行标准化之后，损失函数的收敛速度十分快。最终，损失函数的值为0.20，改善效果十分明显。因此，在使用梯度下降法求解损失函数最小值时，对数据进行标准化十分重要。

1.2.5 牛顿法

事实上，也可以通过牛顿法求解逻辑回归的参数。使用牛顿法求解逻辑回归的参数往往要比使用梯度下降法快，通常只需要十几次迭代就能够收敛。但是，使用牛顿法需要计算Hessian矩阵的逆，这也就导致牛顿法的计算复杂度比较高。并且，牛顿法对目标函数有较为严格的要求，要求目标函数具有连续的一阶、二阶连续偏导数，Hessian矩阵必须正定。

关于牛顿法的更多介绍，可以参考之后的学习笔记，以及该网址
<https://blog.csdn.net/itplus/article/details/21896453>
(<https://blog.csdn.net/itplus/article/details/21896453>)

2. Regularized logistic regression

In this part of the exercise, you will implement regularized logistic regression to predict whether microchips from a fabrication plant passes quality assurance (QA). During QA, each microchip goes through various tests to ensure it is functioning correctly.

Suppose you are the product manager of the factory and you have the test results for some microchips on two different tests. From these two tests, you would like to determine whether the microchips should be accepted or rejected. To help you make decision, you have a dataset of test results on past microchips, from which you can build a logistic regression model.

2.1 Visualizing the data

首先，对数据进行可视化。

In [15]:

```
data = pd.read_csv('ex2data2.txt', names=['Test_1', 'Test_2', 'Accepted'])
data.head()
```

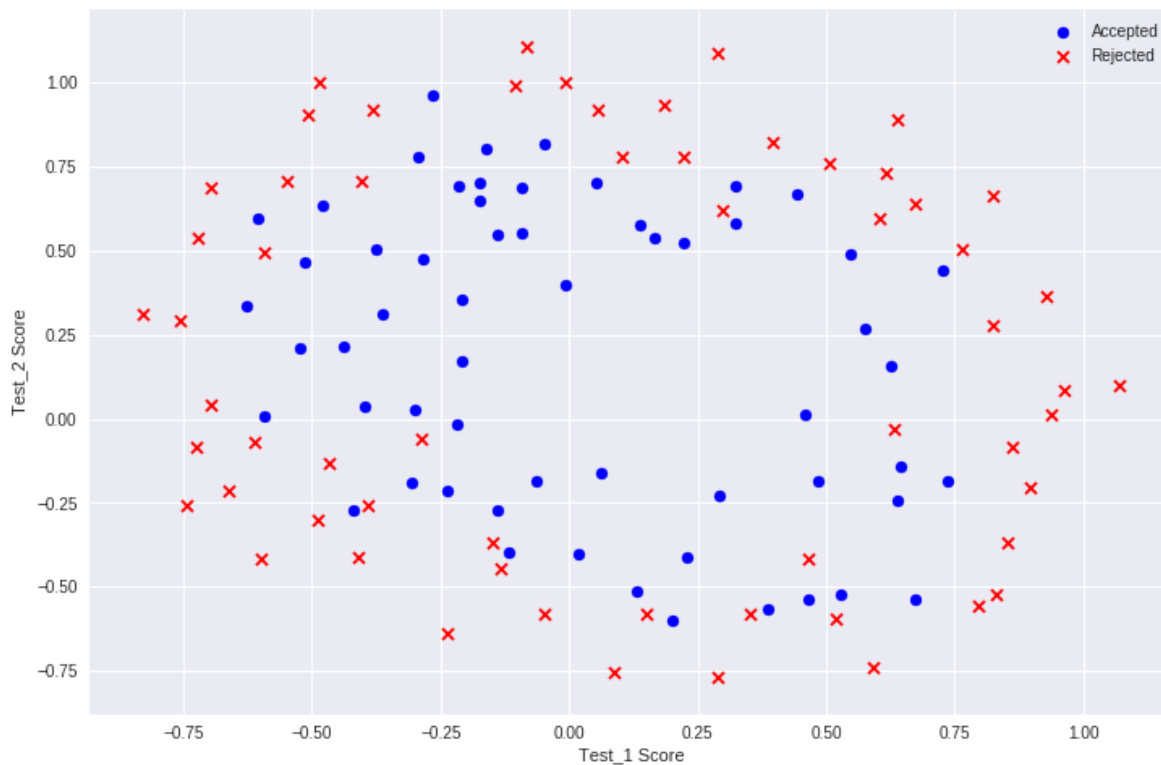
Out[15]:

	Test_1	Test_2	Accepted
0	0.051267	0.69956	1
1	-0.092742	0.68494	1
2	-0.213710	0.69225	1
3	-0.375000	0.50219	1
4	-0.513250	0.46564	1

In [16]:

```
positive = data[data['Accepted'] == 1]
negative = data[data['Accepted'] == 0]

plt.figure(figsize=(12, 8))
plt.scatter(positive['Test_1'], positive['Test_2'], color='blue', marker='o', label='Accepted')
plt.scatter(negative['Test_1'], negative['Test_2'], color='red', marker='x', label='Rejected')
plt.legend()
plt.xlabel('Test_1 Score')
plt.ylabel('Test_2 Score')
plt.show()
```



This figure shows that our dataset cannot be separated into positive and negative examples by a straight-line through the plot. Therefore, a straight-forward application of logistic regression will not perform well on this dataset since logistic regression will only be able to find a linear decision boundary.

2.2 Feature mapping

One way to fit the data better is to create more features from each data point. Now, we will map the features into all polynomial terms of x_1 and x_2 up to the sixth power.

$$\text{mapFeature}(x) = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_1^2 \\ x_1x_2 \\ x_2^2 \\ x_1^3 \\ \dots \\ x_1x_2^5 \\ x_2^6 \end{bmatrix}$$

In [17]:

```
def featureMapping(x1, x2, power, as_ndarray=False):
    data = {"{}{}".format(i - p, p): np.power(x1, i - p) * np.power(x2, p)
            for i in range(power + 1)
            for p in range(i + 1)
            }

    if as_ndarray:
        return pd.DataFrame(data).as_matrix()
    else:
        return pd.DataFrame(data)
```

In [18]:

```
x1 = np.array(data.Test_1)
x2 = np.array(data.Test_2)
```


In [19]:

```
X = featureMapping(x1, x2, 6)
X.head()
```

Out[19]:

	00	01	02	03	04	05	06	10	11
0	1.0	0.69956	0.489384	0.342354	0.239497	0.167542	0.117206	0.051267	0.035864
1	1.0	0.68494	0.469143	0.321335	0.220095	0.150752	0.103256	-0.092742	-0.063523
2	1.0	0.69225	0.479210	0.331733	0.229642	0.158970	0.110047	-0.213710	-0.147941
3	1.0	0.50219	0.252195	0.126650	0.063602	0.031940	0.016040	-0.375000	-0.188321
4	1.0	0.46564	0.216821	0.100960	0.047011	0.021890	0.010193	-0.513250	-0.238990

5 rows × 28 columns

As a result of this mapping, our vector of two features (the scores on two QA tests) has been transformed into 28-dimensional vector. A logistic regression classifier trained on this higher-dimension feature vector will have a more complex decision boundary and will appear nonlinear when drawn in our 2-dimensional plot.

While the feature mapping allows us to build a more expressive classifier, it also more susceptible to overfitting. In the next parts of the exercise, you will implement regularized logistic regression to fit the data and also see for yourself how regularization can help combat the overfitting problem.

2.3 Cost function and gradient

Now you will implement code to compute the cost function and gradient for regularized logistic regression.

Recall that the regularized cost function in logistic regression is

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \left[-y^{(i)} \log(h_{\theta}(x^{(i)})) - (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$$

Note that you should not regularize the parameter θ_0 .

In [20]:

```
def computeRegularizedCost(theta, X, y, l):  
  
    m = X.shape[0]  
    theta_j1_to_n = theta[1:, :]  
  
    first = np.multiply(-y, np.log(sigmoid(X @ theta)))  
    second = np.multiply(1-y, np.log(1-sigmoid(X @ theta)))  
    regularizedTerm = (l / (2*m)) * np.power(theta_j1_to_n, 2).sum()  
  
    return np.mean(first - second) + regularizedTerm
```

In [21]:

```
# 参数初始化以及数据预处理  
theta = initializeWithZeros(X.shape[1])  
X = featureMapping(x1, x2, 6, as_ndarray=True)  
y = data.iloc[:, -1].as_matrix().reshape(-1, 1)
```

In [22]:

```
# 正确答案为 0.693  
cost = computeRegularizedCost(theta, X, y, 1)  
cost
```

Out[22]:

0.6931471805599454

The gradient of the cost function is a vector where the j^{th} element is defined as follows:

$$\frac{\partial J(\theta)}{\partial \theta_0} = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} \quad for j = 0$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \left(\frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} \right) + \frac{\lambda}{m} \theta_j \quad for j \geq 1$$

In [23]:

```
def regularizedGradient(theta, X, y, l):  
  
    m = X.shape[0]  
    theta_j1_to_n = theta[1:, :]  
  
    grad = np.dot(X.T, sigmoid(X @ theta) - y) / m  
    regularizedTheta = (l / m) * theta_j1_to_n  
    regularizedTerm = np.concatenate([np.matrix([0]), regularizedTheta])  
  
    return grad + regularizedTerm
```

In [24]:

```
def batchGradientDescent(X, y, theta, alpha, iters, l, printFlag=True):  
  
    cost = np.zeros(iters)  
  
    for i in range(iters):  
        theta = theta - alpha * regularizedGradient(theta, X, y, l)  
        cost[i] = computeRegularizedCost(theta, X, y, l)  
        if printFlag and i % 500 == 0:  
            print('第{iters}轮:{cost}'.format(iters=i, cost=cost[i]))  
  
    return theta, cost
```

In [25]:

```
alpha= 0.3  
iters = 5000  
l=1  
theta = initializeWithZeros(X.shape[1])  
theta, cost = batchGradientDescent(X, y, theta, alpha, iters, l)
```

```
第0轮:0.689392100814767  
第500轮:0.5303383613168465  
第1000轮:0.5290349118070177  
第1500轮:0.5290039786047559  
第2000轮:0.5290027945476369  
第2500轮:0.5290027336043155  
第3000轮:0.5290027299629998  
第3500轮:0.5290027297293425  
第4000轮:0.5290027297137846  
第4500轮:0.5290027297127253
```

2.4 Plotting the decision boundary

决策边界的方程为 $\theta^T X = 0$ 的超平面。因此，需要找到所有满足该方程的x。
Instead of solving polynomial equation, just create a coridate x, y grid that is dense enough, and find all those $\theta^T X$ that is close enough to 0, then plot them.

In [26]:

```
def findDecisionBoundary(density, power, theta, threshold):  
  
    t1 = np.linspace(-1, 1.5, density)  
    t2 = np.linspace(-1, 1.5, density)  
  
    coordinates = [(x, y) for x in t1 for y in t2]  
    x_cord, y_cord = zip(*coordinates)  
    mappedCord = featureMapping(x_cord, y_cord, power)  
  
    innerProduct = mappedCord.as_matrix() @ theta  
  
    decision = mappedCord[np.abs(innerProduct) < threshold]  
  
    return decision['10'], decision['01']
```

In [27]:

```
def featureMappedLogisticRegression(power, l, alpha, iters):

    data = pd.read_csv('ex2data2.txt', names=['Test_1', 'Test_2', 'Accepted'])
    x1 = np.array(data.Test_1)
    x2 = np.array(data.Test_2)
    X = featureMapping(x1, x2, power, as_ndarray=True)
    y = data.iloc[:, -1].as_matrix().reshape(-1, 1)

    theta = initializeWithZeros(X.shape[1])

    theta, cost = batchGradientDescent(X, y, theta, alpha, iters, l, printFlag=False)

    print('The cost is {}'.format(computeRegularizedCost(theta, X, y, l)))

    return theta
```

In [28]:

```
def drawDecisionBoundary(power, l, alpha=0.3, iters=5000):

    density = 1000
    threshold = 2 * 10 ** -3

    theta = featureMappedLogisticRegression(power, l, alpha, iters)
    x, y = findDecisionBoundary(density, power, theta, threshold)

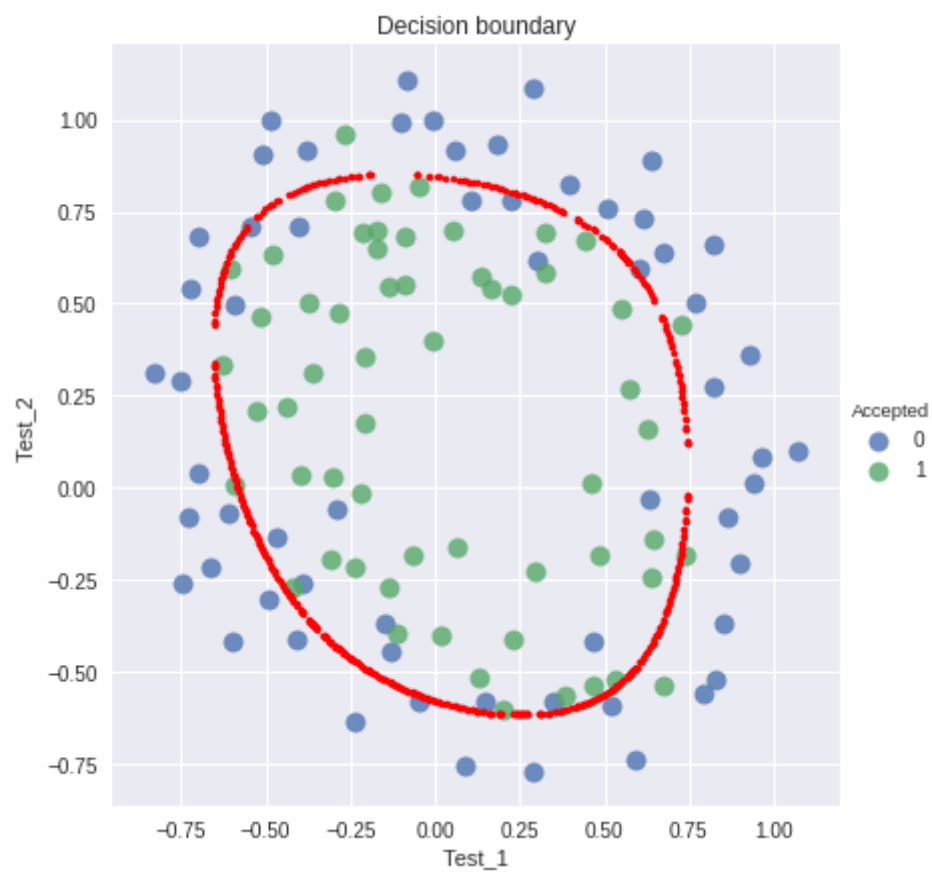
    data = pd.read_csv('ex2data2.txt', names=['Test_1', 'Test_2', 'Accepted'])
    plt.figure(figsize=(12, 8))
    sns.lmplot('Test_1', 'Test_2', hue='Accepted', data=data, size=6, fit_reg=False)
    plt.scatter(x, y, c='R', s=10)
    plt.title('Decision boundary')
    plt.show()
```

In [29]:

```
drawDecisionBoundary(power=6, l=1)
```

The cost is 0.529002729712652:

<matplotlib.figure.Figure at 0x7fad8e6df978>



In [30]:

```
# 这里感觉有点不合理，我为了展示过拟合的效果，提高了power, alpha以及iters  
# no regularization, overfitting  
# lambda=0, 没有正则化  
drawDecisionBoundary(power=10, l=0, alpha=15, iters=100000)
```

The cost is 0.25380047604665046:

<matplotlib.figure.Figure at 0x7fad8e4d8a20>

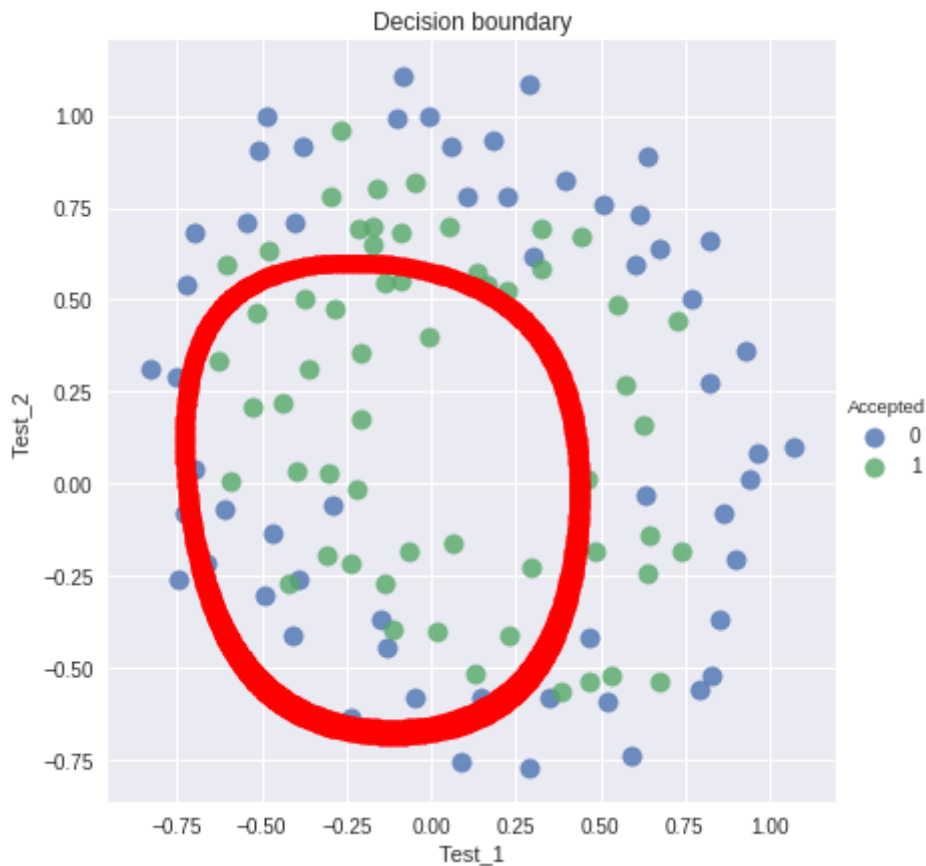


In [31]:

```
# too much regularization, underfitting
# lambda=100, 欠拟合
drawDecisionBoundary(power=6, l=100)
```

The cost is 0.6864838338726167:

<matplotlib.figure.Figure at 0x7fad8e4b9e80>



Notice the changes in the decision boundary as you vary λ . With a small λ , you will find that the classifier gets almost every training example correct, but draws a very complicated boundary, thus overfitting the data.

With a larger λ , you will see a plot that shows a simpler decision boundary which still separates the positive and negatives fairly well. However, if λ is set too high a value, you will not get a good fit and the decision boundary will not follow the data so well, thus underfitting the data.

In []: