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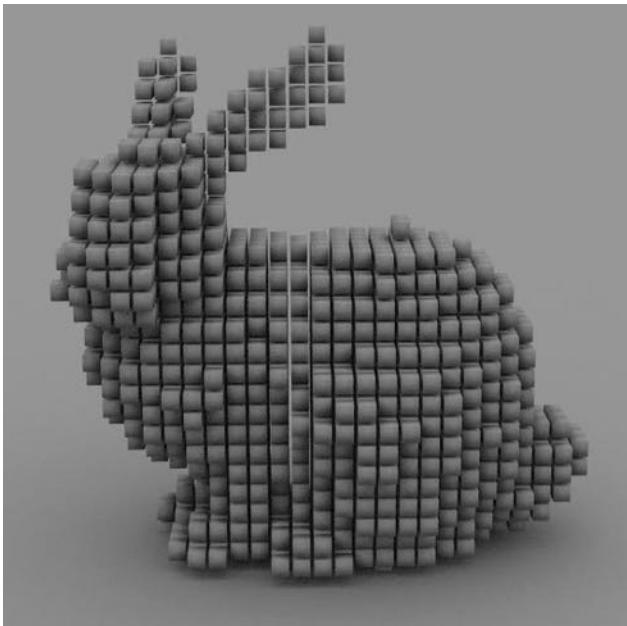
Learning Graph Structure for Point Cloud Processing

Zhuohan Li

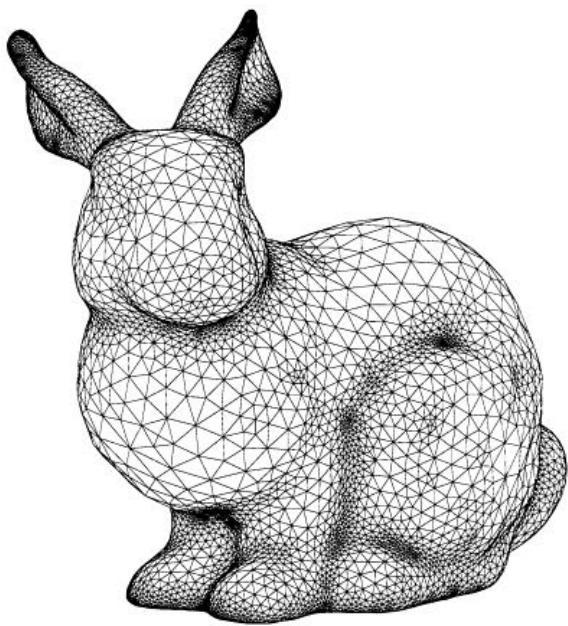
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2018/09/21

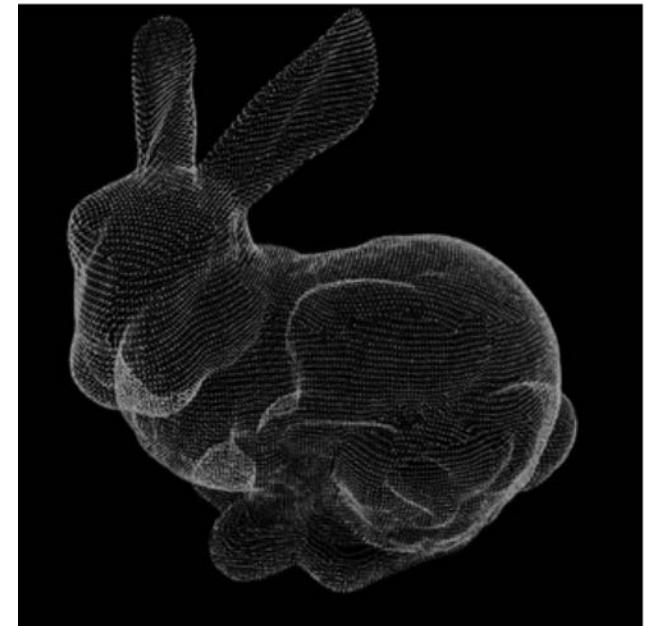
3D Data Representation



Volumetric



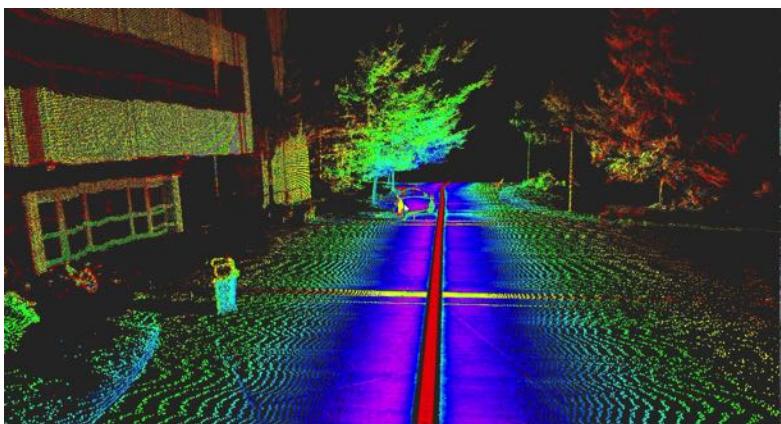
Mesh



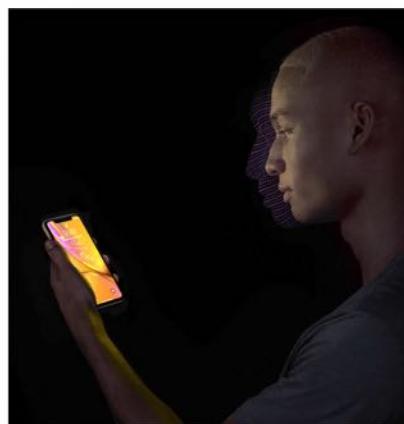
Point cloud

Point Cloud

- Unordered set $\{(x_i, y_i, z_i) \mid i = 1, \dots, N\}$
- With underlying distance metric



LiDAR

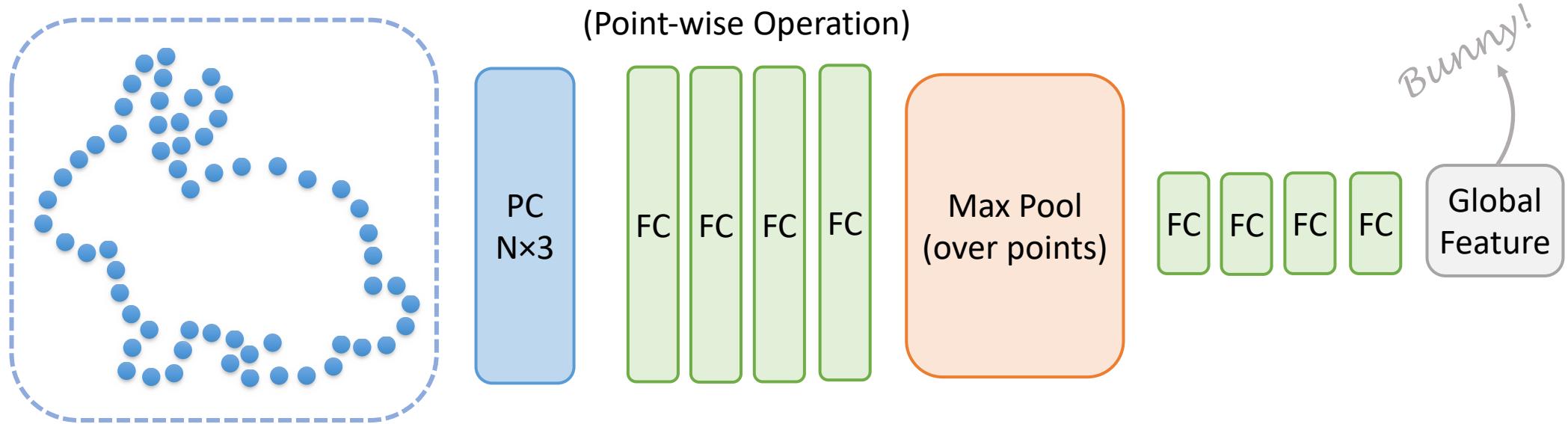


Face ID

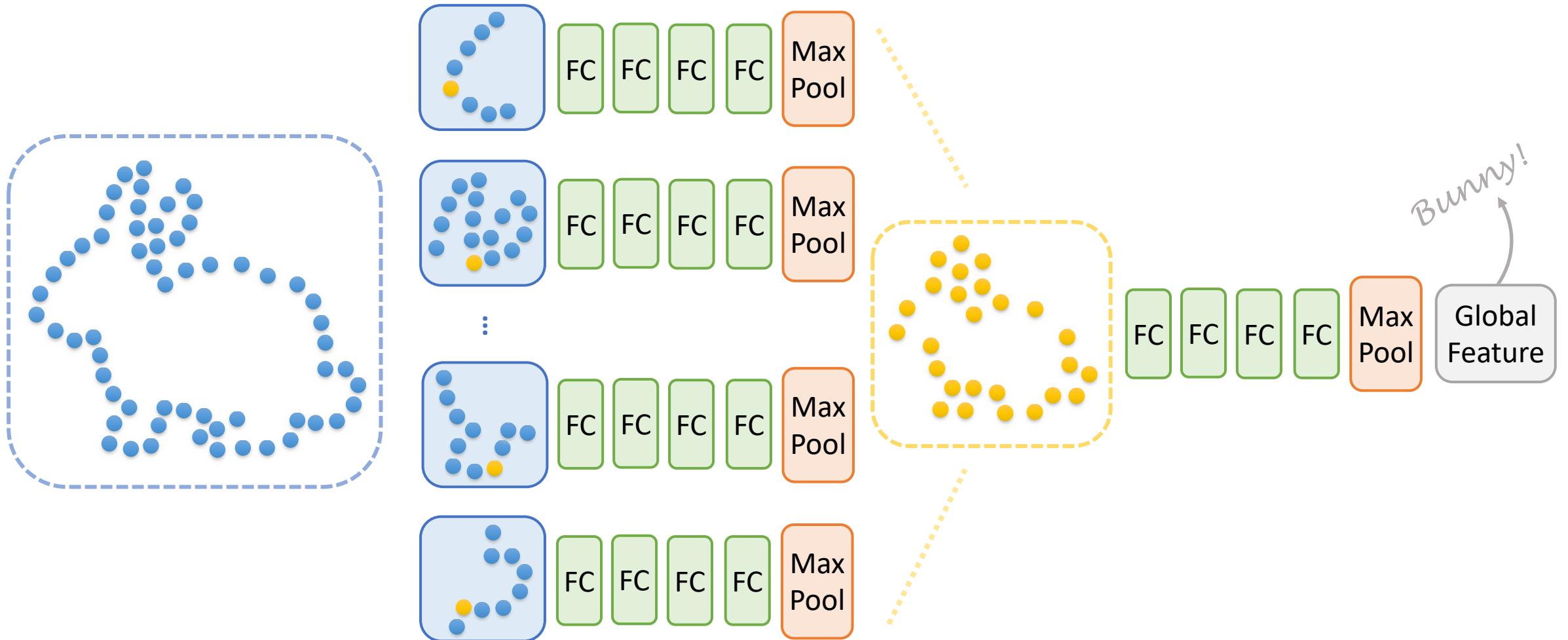


Kinect

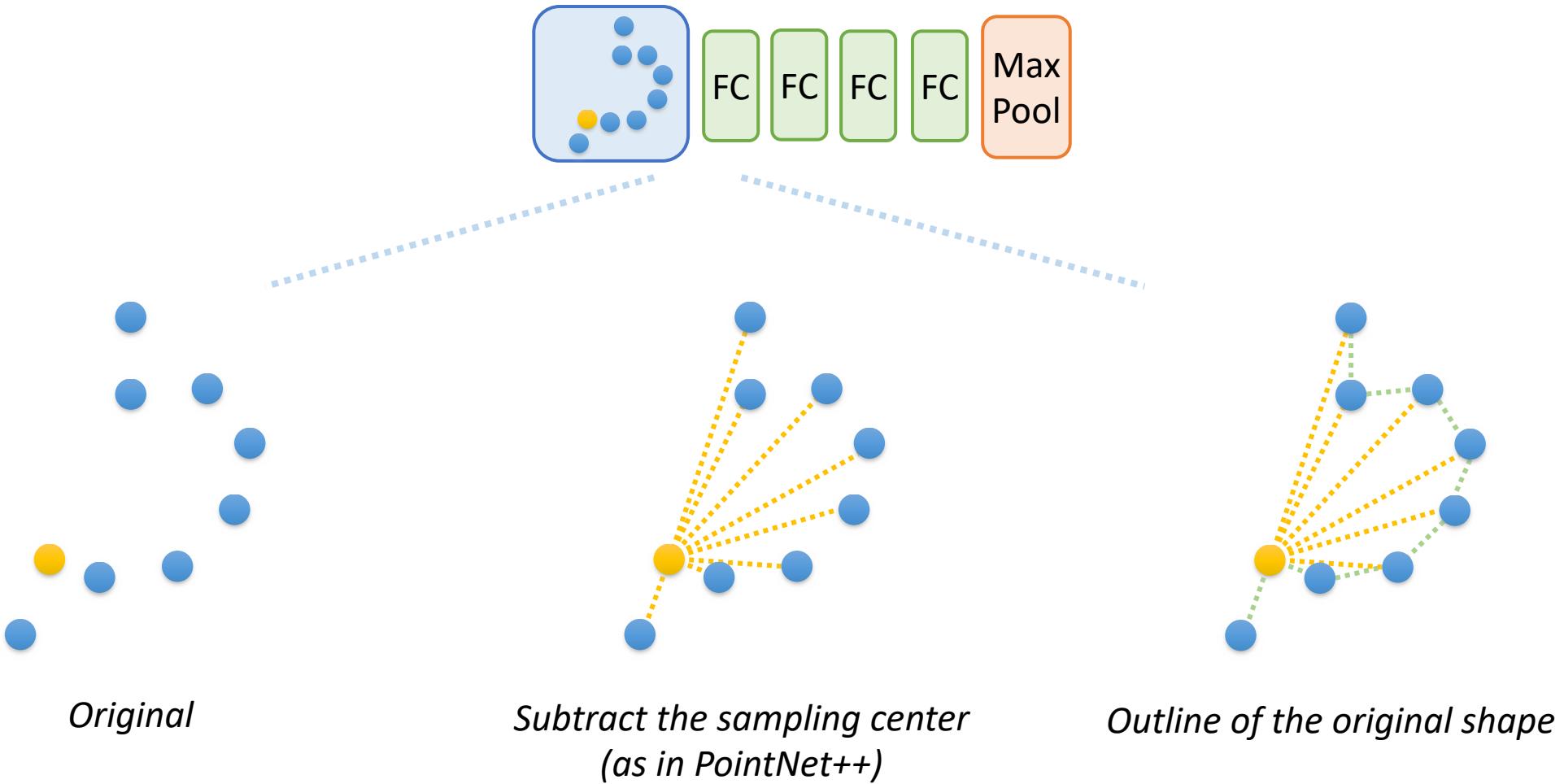
PointNet



PointNet++ (2 Layers)



Graph Structure in PointNet++



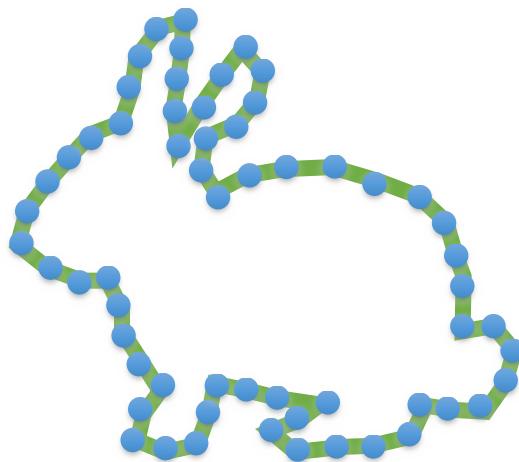
Points and Edges

- For a point cloud with $O(n)$ points, there are $O(n^2)$ edges in total
- Most of the edges are *unimportant* for the understanding of the point cloud

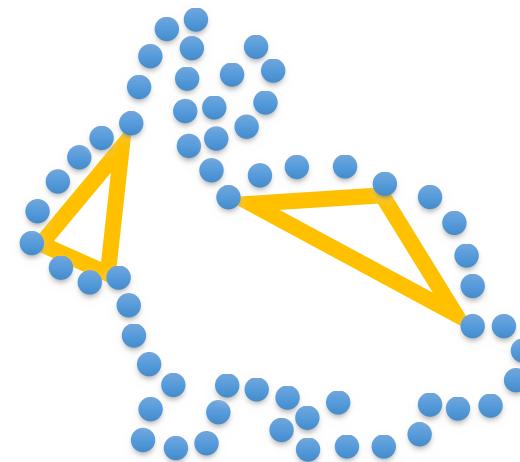


Task Dependent Graph Structure

- How to *decide* the importance of different edges?
- Different tasks need different graph structures



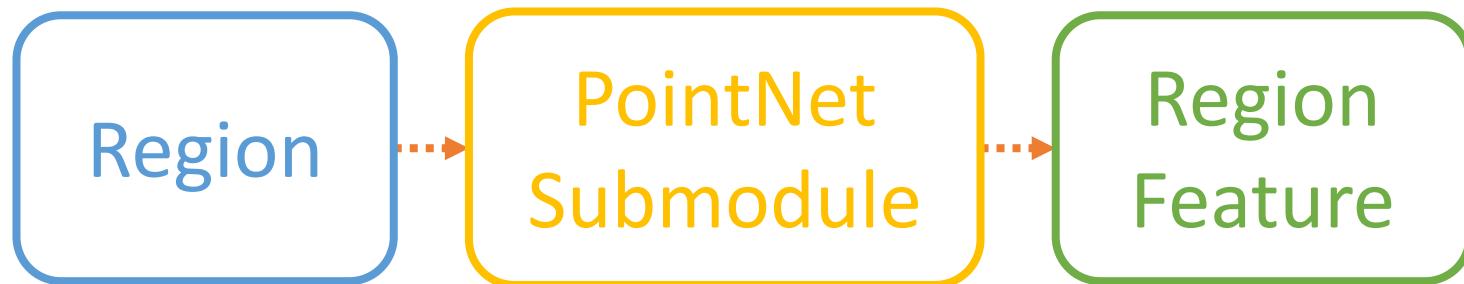
Perimeter of the bunny



Area of the bunny

Learning Task Dependent Graph Structure

- Embed a learnable graph structure into neural networks
- Let the network to choose the best graph for the tasks by *gradient-based optimization*

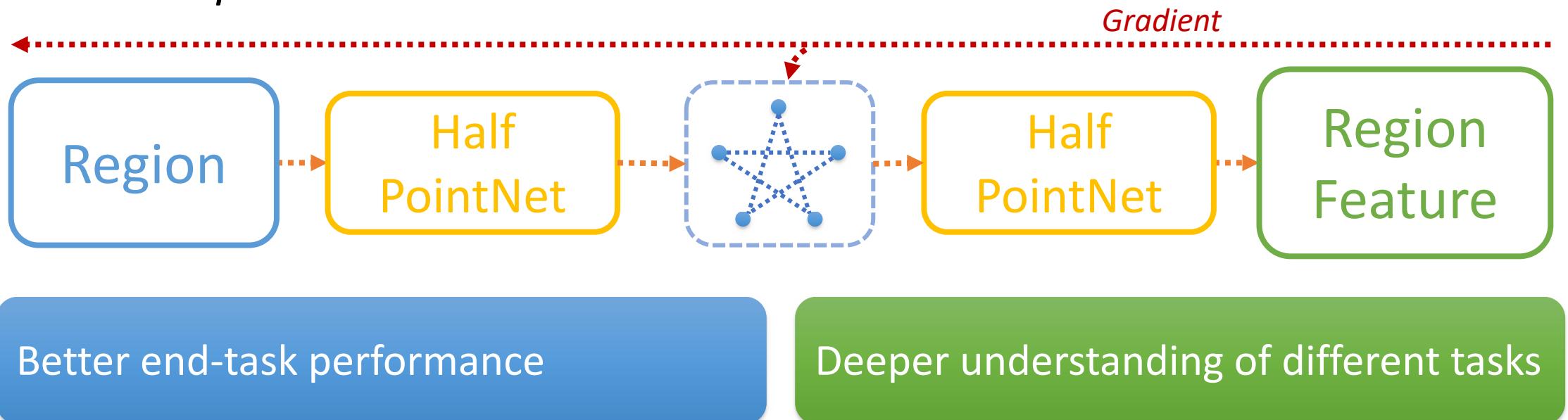


Better end-task performance

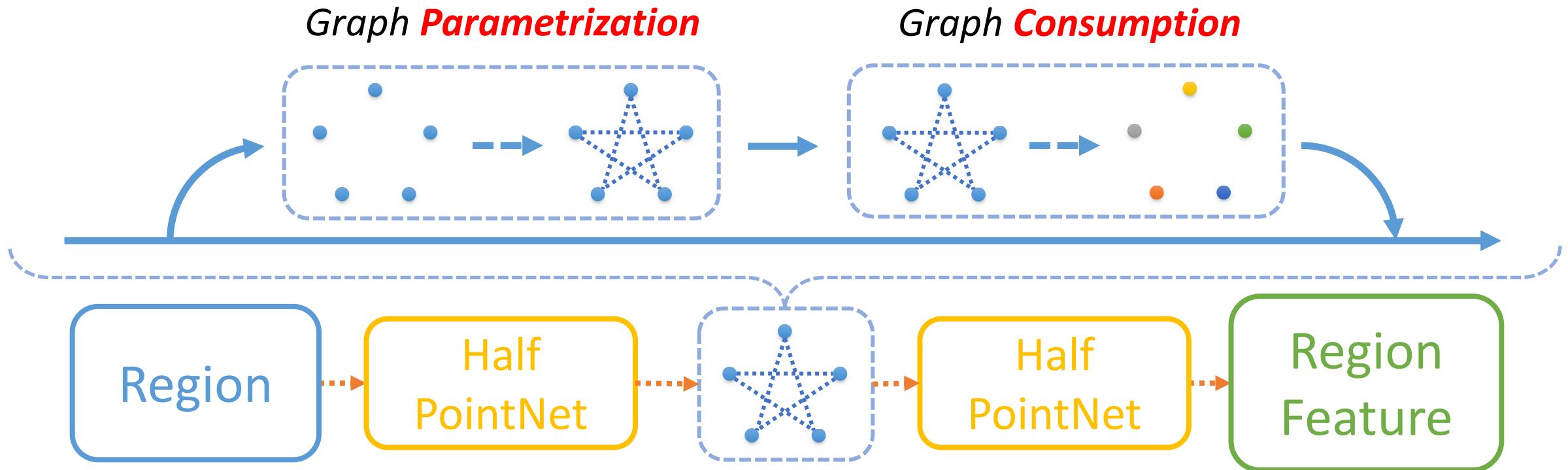
Deeper understanding of different tasks

Learning Task Dependent Graph Structure

- Embed a learnable graph structure into neural networks
- Let the network to choose the best graph for the tasks by *gradient-based optimization*

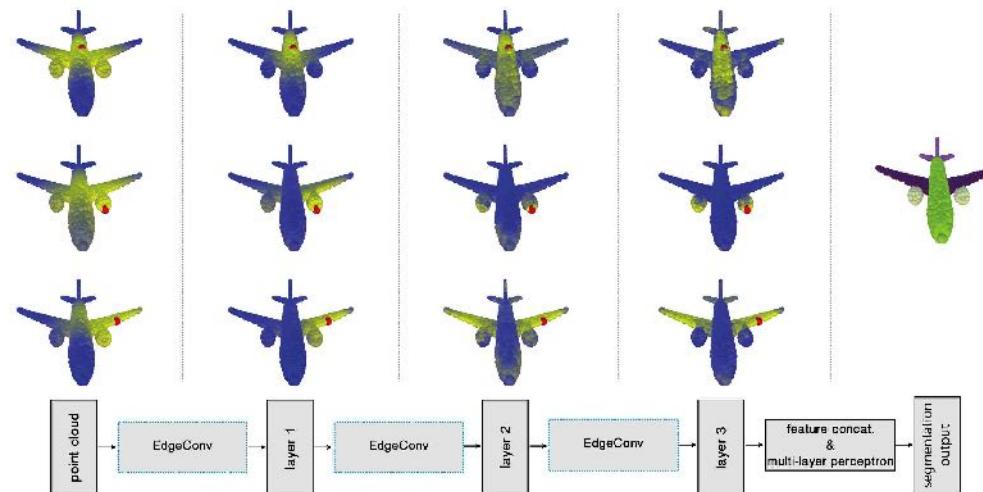


Learning Task Dependent Graph Structure



Related Works (Graph Parametrization)

- Dynamic Graph CNN for Learning on Point Clouds (Edgeconv, DGCNN)
 - Nearest neighbors in the feature space
 - Edgeconv operation to consume the graph
 - **No explicit gradient signal for graph editing**



Related Works (Graph Consumption)

- There are many works on graph-structured data processing
- Most of the works assume graph is given

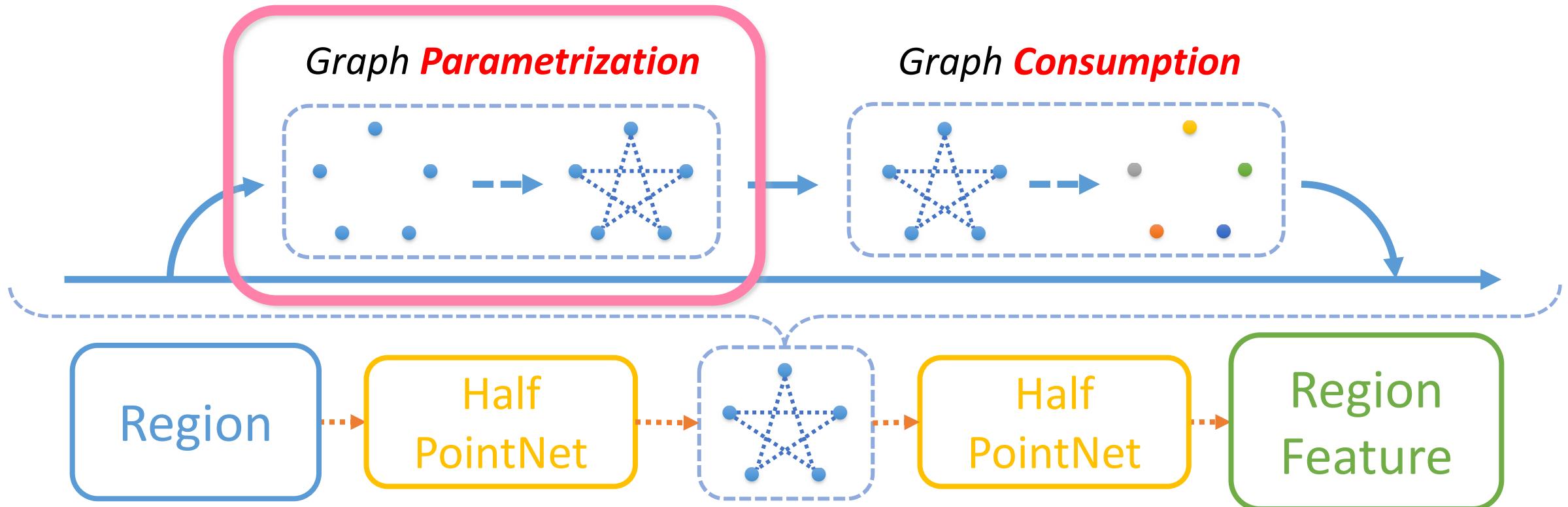


Spectral domain convolution



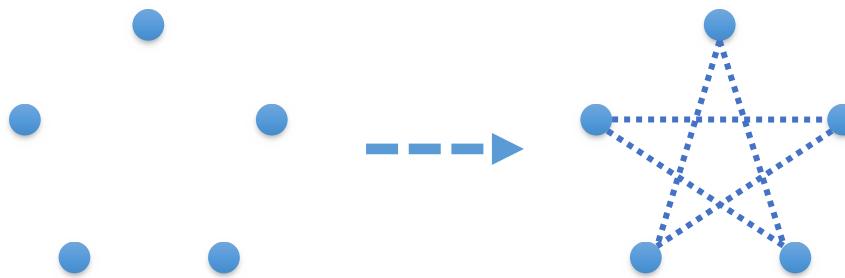
Spatial domain convolution

Learning Task Dependent Graph Structure



Graph Parametrization

- A point set $\{x_1, \dots, x_n\}$ in a sampled neighborhood of PointNet++
- Point features $\{h_1, \dots, h_n\}$
- Build a graph with edges $E = \{e_{ij}\}$ based on the point features



Attention Mechanism

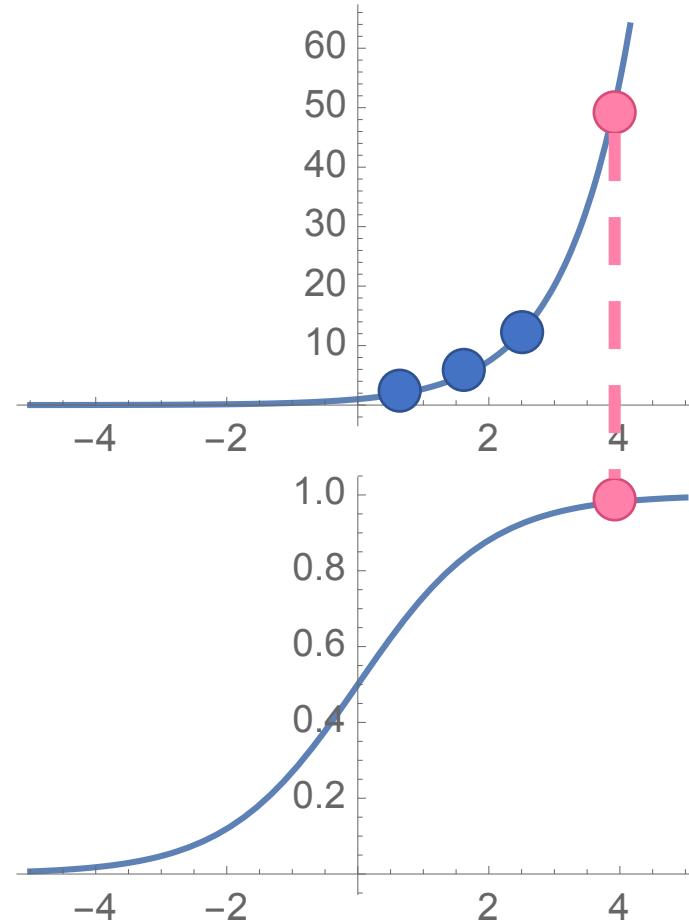
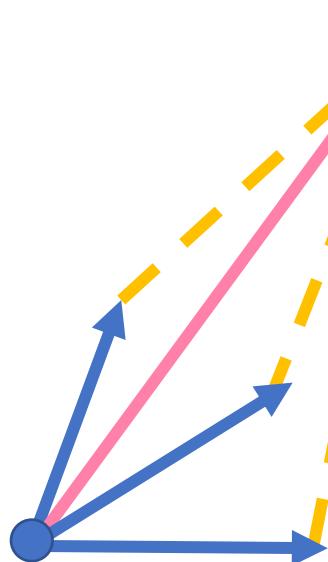
- One straightforward idea: *Attention Mechanism*
- Widely used in NLP domain to build relationship among the words
- Specifically, edge weights are defined as

$$e_{ij} = \frac{\exp((Wh_i)^T(Wh_j))}{\sum_k \exp((Wh_i)^T(Wh_k))},$$

where W is a learned weight matrix

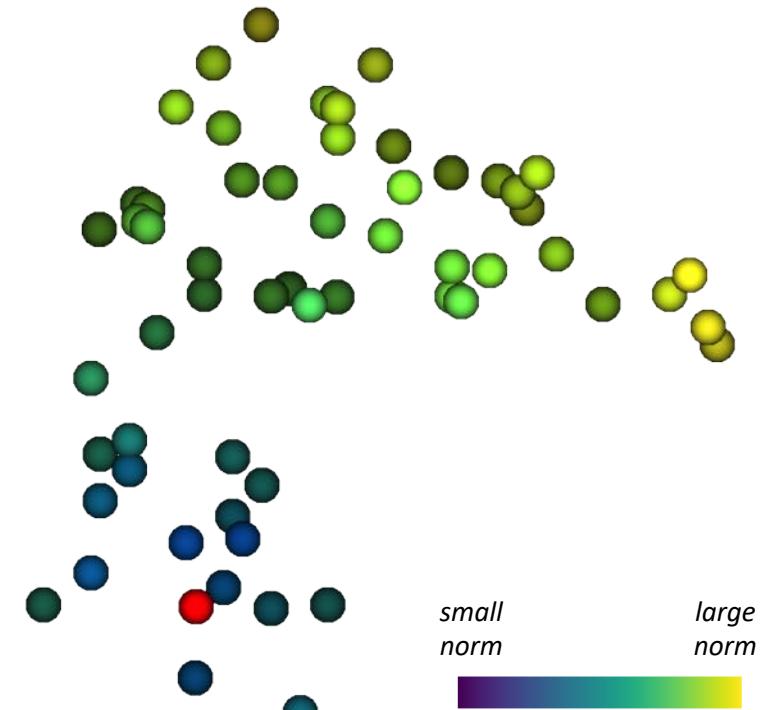
Why Attention not Work?

$$e_{ij} = \frac{\exp((Wh_i)^T(Wh_j))}{\sum_k \exp((Wh_i)^T(Wh_k))}$$



Point Clouds and Local Regions

- Points mostly lies on the **surface of an object**
- Extremely **nonuniform** sampled neighborhood
- **Farthest point sampling** samples a lot of corner points

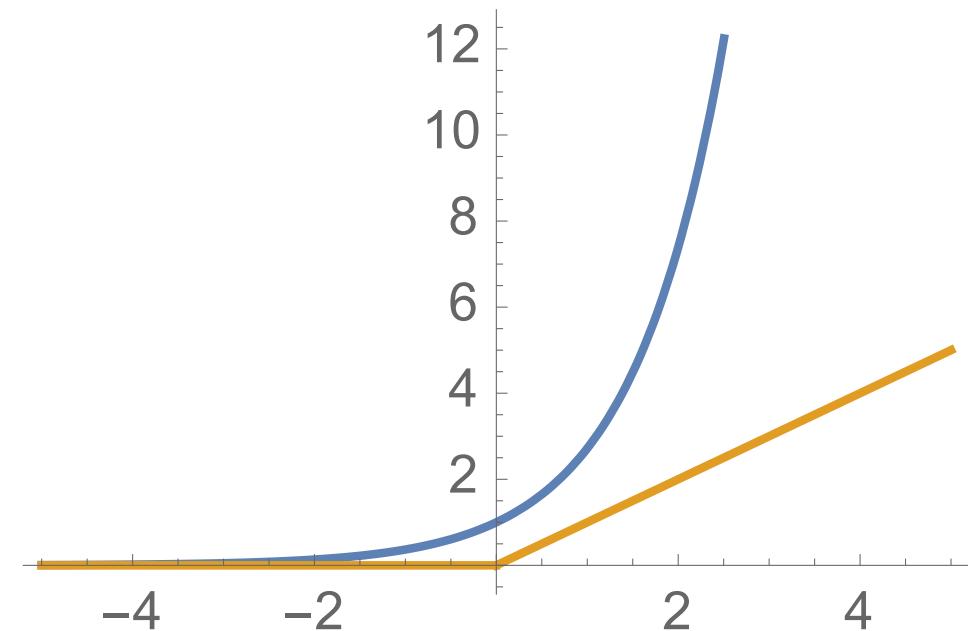


A sample neighborhood from the first layer of PointNet++

ReLU Dot-Product Graph Parametrization

- ReLU as **First-order approximation** of the SoftMax function as the graph parametrization:

$$e_{ij} = \frac{\text{ReLU}\left((Wh_i)^T(Wh_j)\right)}{\sum_k \text{ReLU}\left((Wh_i)^T(Wh_k)\right)}$$

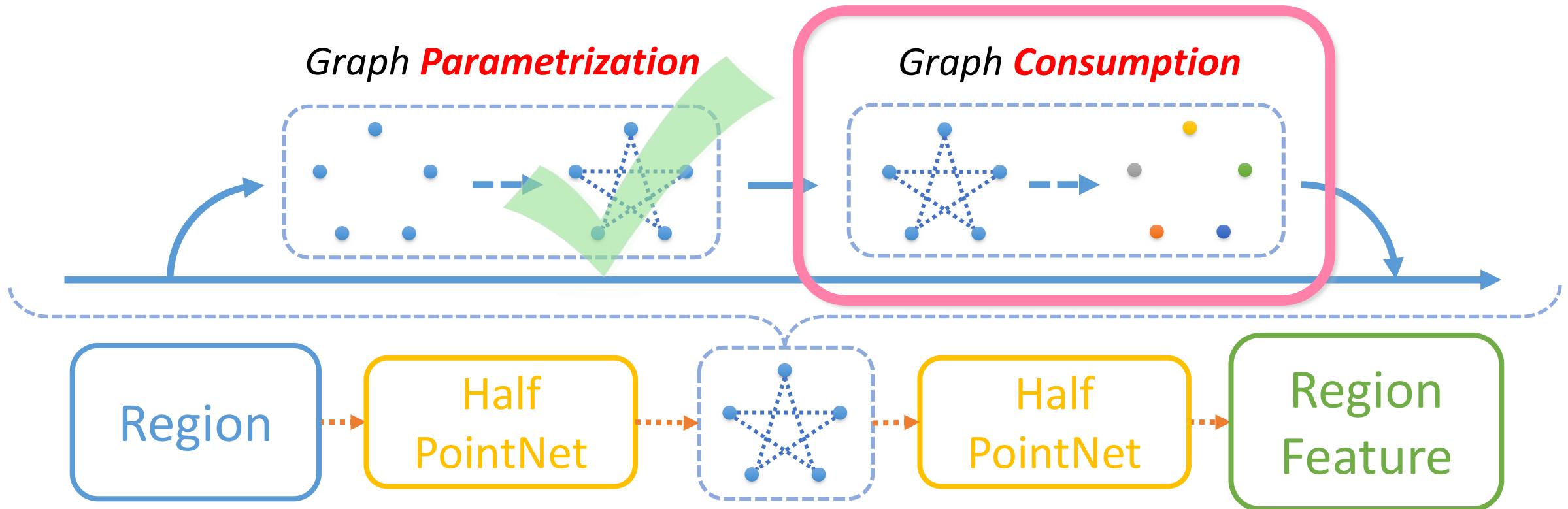


ReLU Dot-Product Graph Parametrization

$$e_{ij} = \frac{\text{ReLU}\left((Wh_i)^T(Wh_j)\right)}{\sum_k \text{ReLU}\left((Wh_i)^T(Wh_k)\right)}$$

- Denominator > 0 , numerator ≥ 0
- Connection between points **with similar point feature**
- **Smoother** weights than attention mechanism

Learning Task Dependent Graph Structure



Diffusion Process

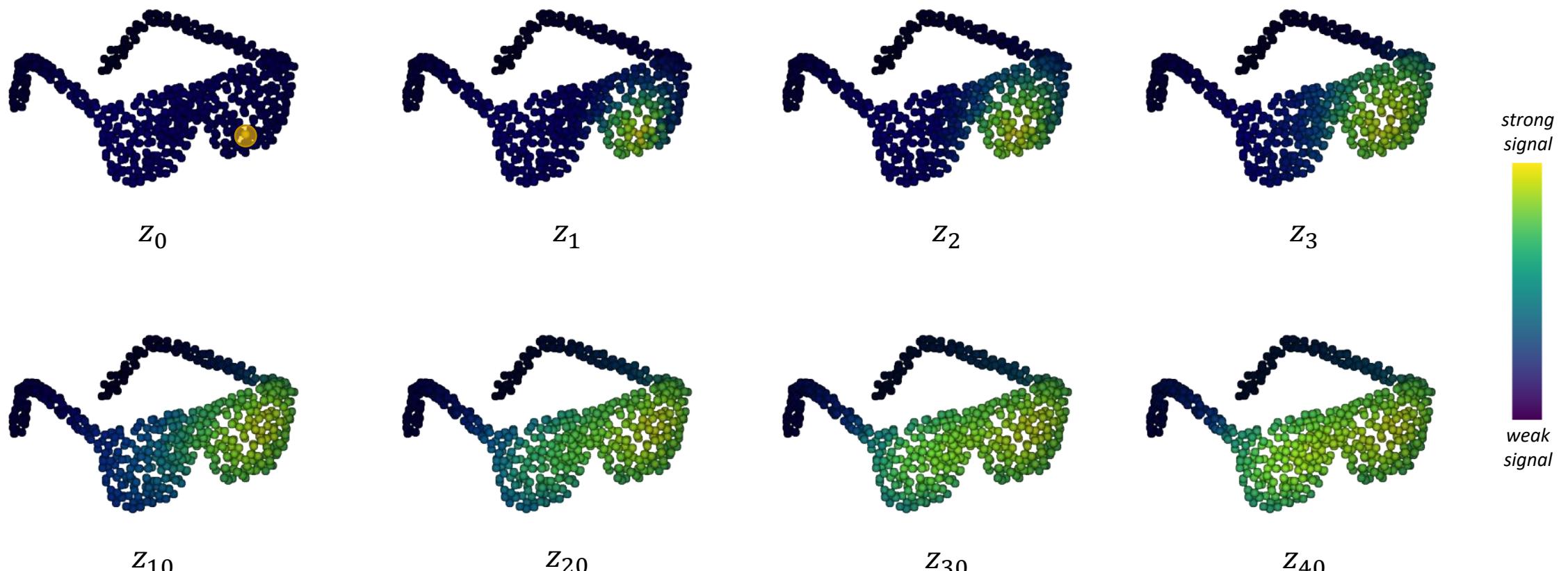
- Normalized adjacency matrix $A \in \mathbb{R}^{n \times n}$ of a graph,
- An initial signal vector $z_0 \in \mathbb{R}^n$
- The diffused signals would be

$$z_1 = Az_0, z_2 = Az_1, \dots, z_n = Az_{n-1}$$

- We can exactly recover adjacency matrix A by solving the linear system

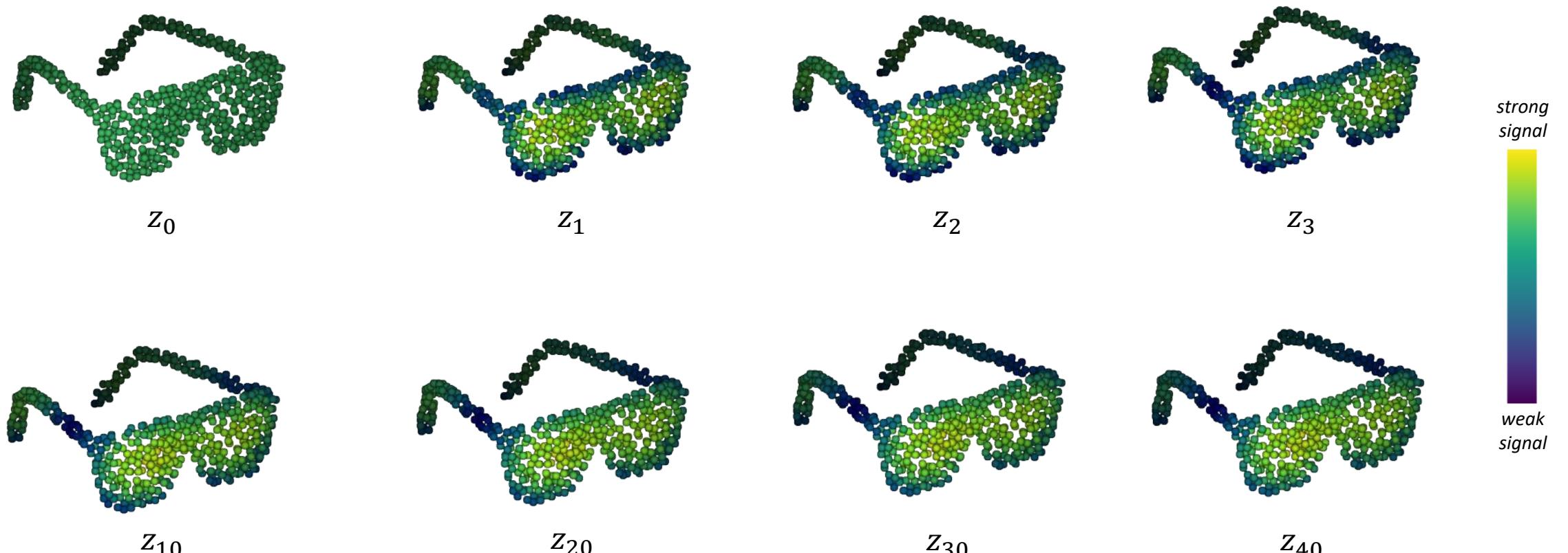
$$[z_1 \ z_2 \ \dots z_n] = A[z_0 \ z_1 \ \dots z_{n-1}]$$

Multiscale Resolution



*Diffusion process example starting from a single point signal,
using Gaussian Kernel as graph weights*

Structure Emerging from Uniform Signal



*Diffusion process example starting from uniform signal,
using Gaussian Kernel as graph weights*

Diffusing Multiple Signals

- With multiple initial signals $z_0^{(1)}, z_0^{(2)}, \dots, z_0^{(m)} \in \mathbb{R}^n$, the diffusion steps can be reduced to n/m to recover the matrix A

$$\begin{bmatrix} z_1^{(0)} & z_2^{(0)} & \dots & z_{n/m}^{(0)} \end{bmatrix} = A \begin{bmatrix} z_0^{(0)} & z_1^{(0)} & \dots & z_{n/m-1}^{(0)} \end{bmatrix}$$

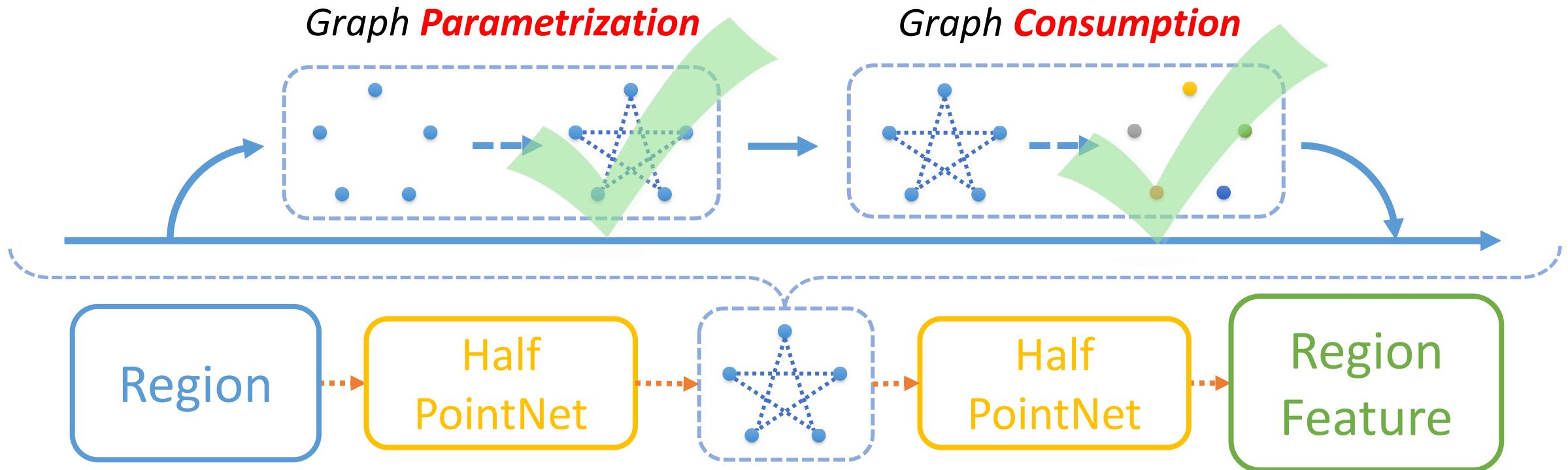
$$\begin{bmatrix} z_1^{(1)} & z_2^{(1)} & \dots & z_{n/m}^{(1)} \end{bmatrix} = A \begin{bmatrix} z_0^{(1)} & z_1^{(1)} & \dots & z_{n/m-1}^{(1)} \end{bmatrix}$$

...

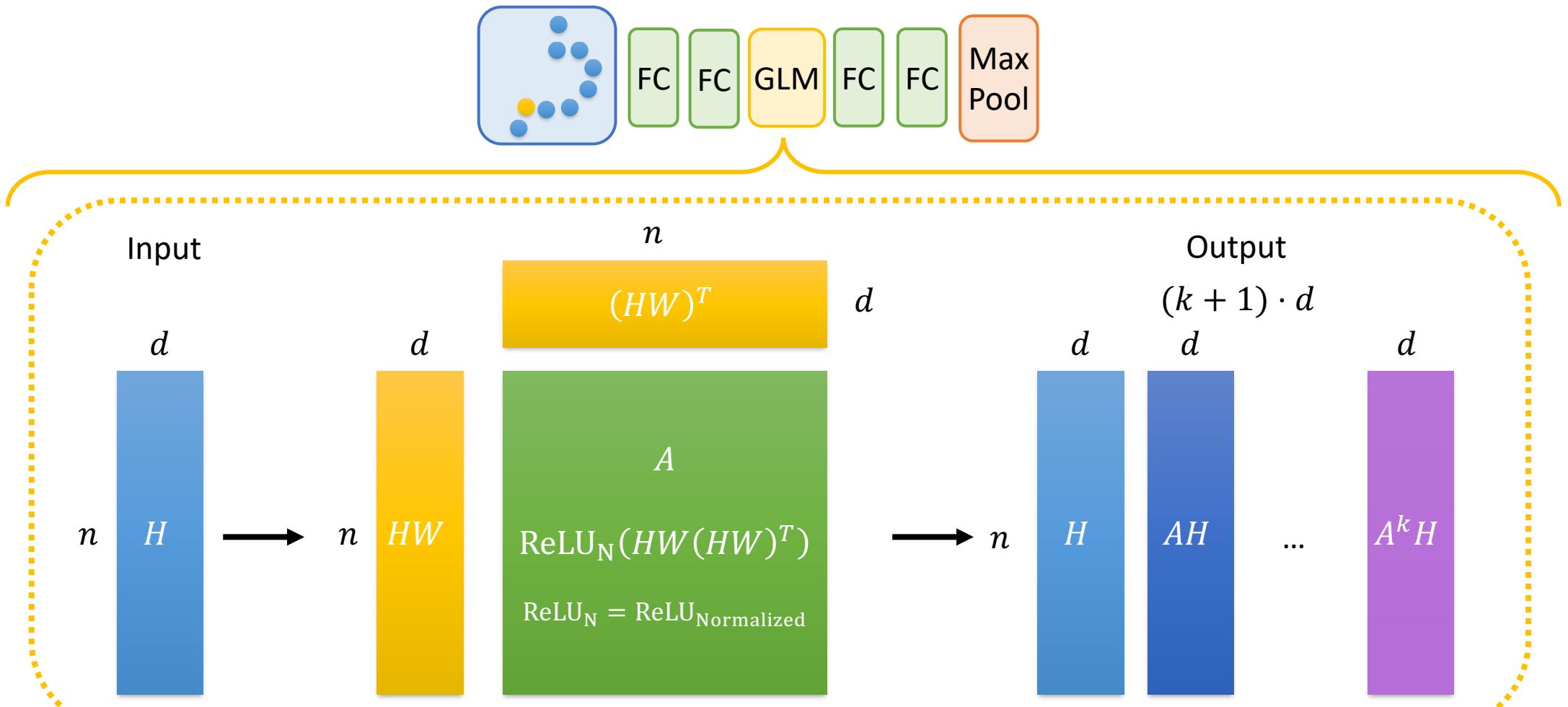
$$\begin{bmatrix} z_1^{(m)} & z_2^{(m)} & \dots & z_{n/m}^{(m)} \end{bmatrix} = A \begin{bmatrix} z_0^{(m)} & z_1^{(m)} & \dots & z_{n/m-1}^{(m)} \end{bmatrix}$$

- We use feature vectors ($m = d$) as initial signals for convenience

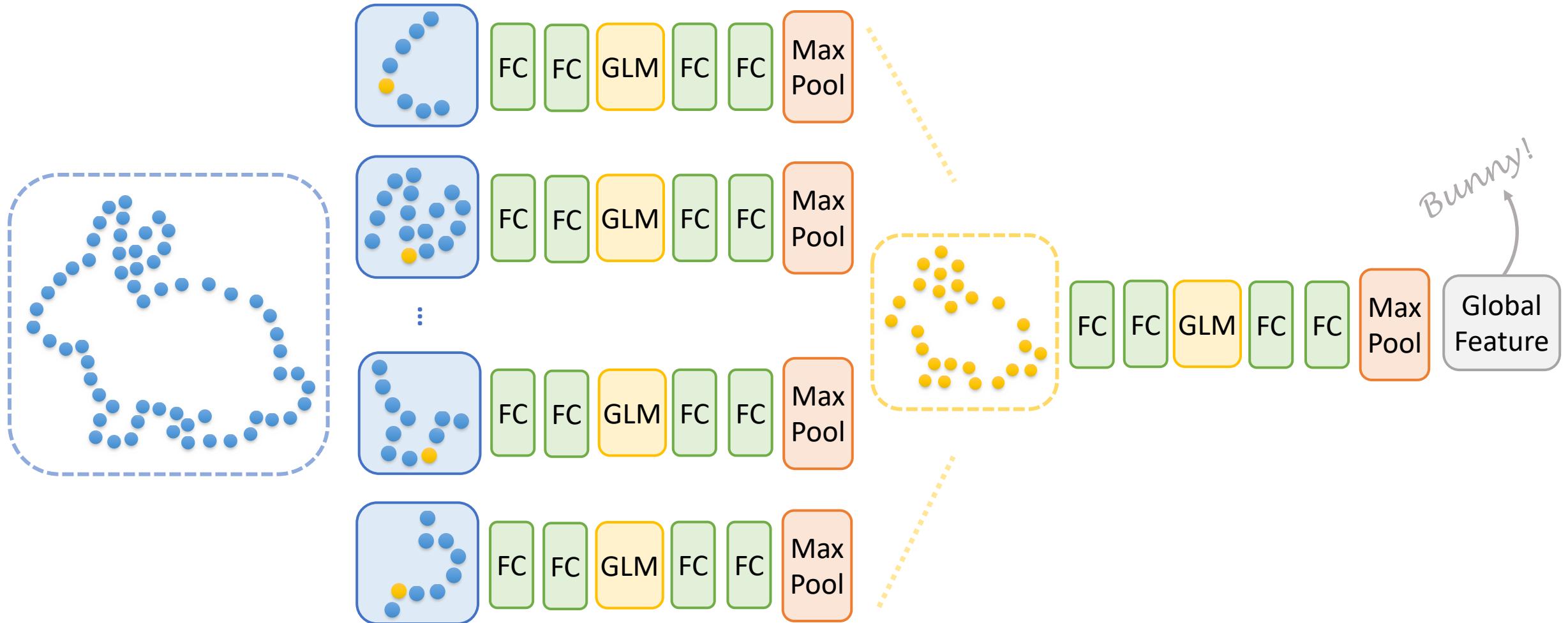
Learning Task Dependent Graph Structure



Graph Learning Module (GLM) Structure



PointNet++ with GLM (2 Layers)



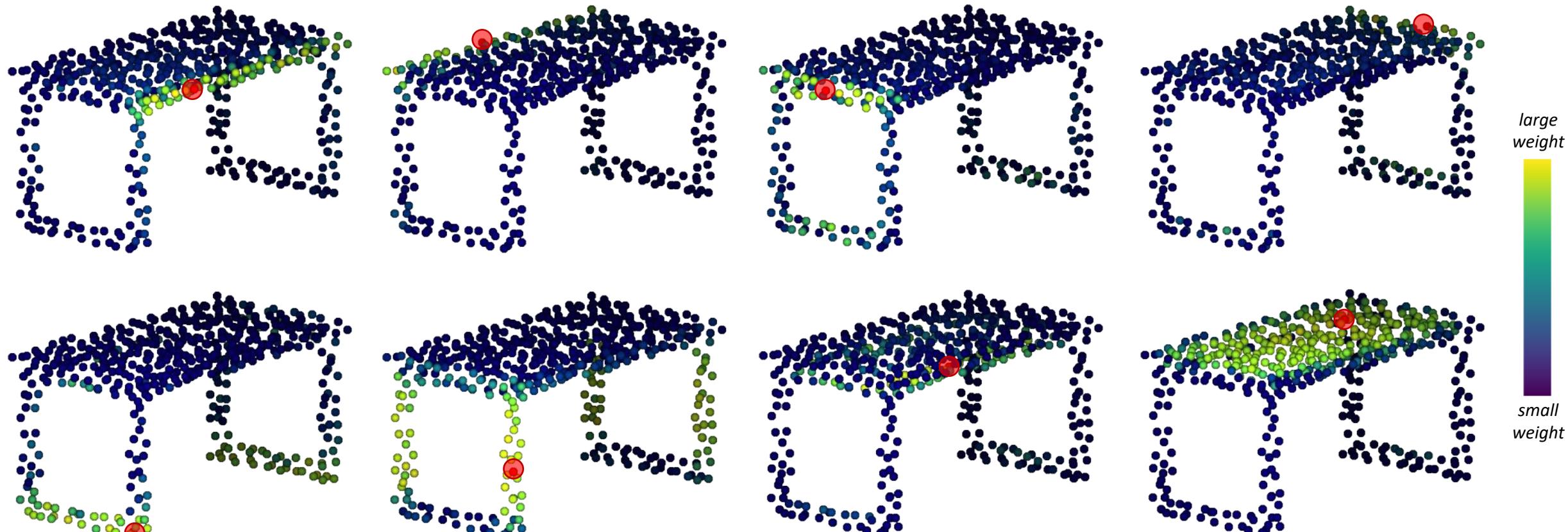
ModelNet40 Classification



Task	ModelNet40 Classification (Accuracy)
2-layer PointNet++ without GLM	90.3%
2-layer PointNet++ with GLM	90.9%

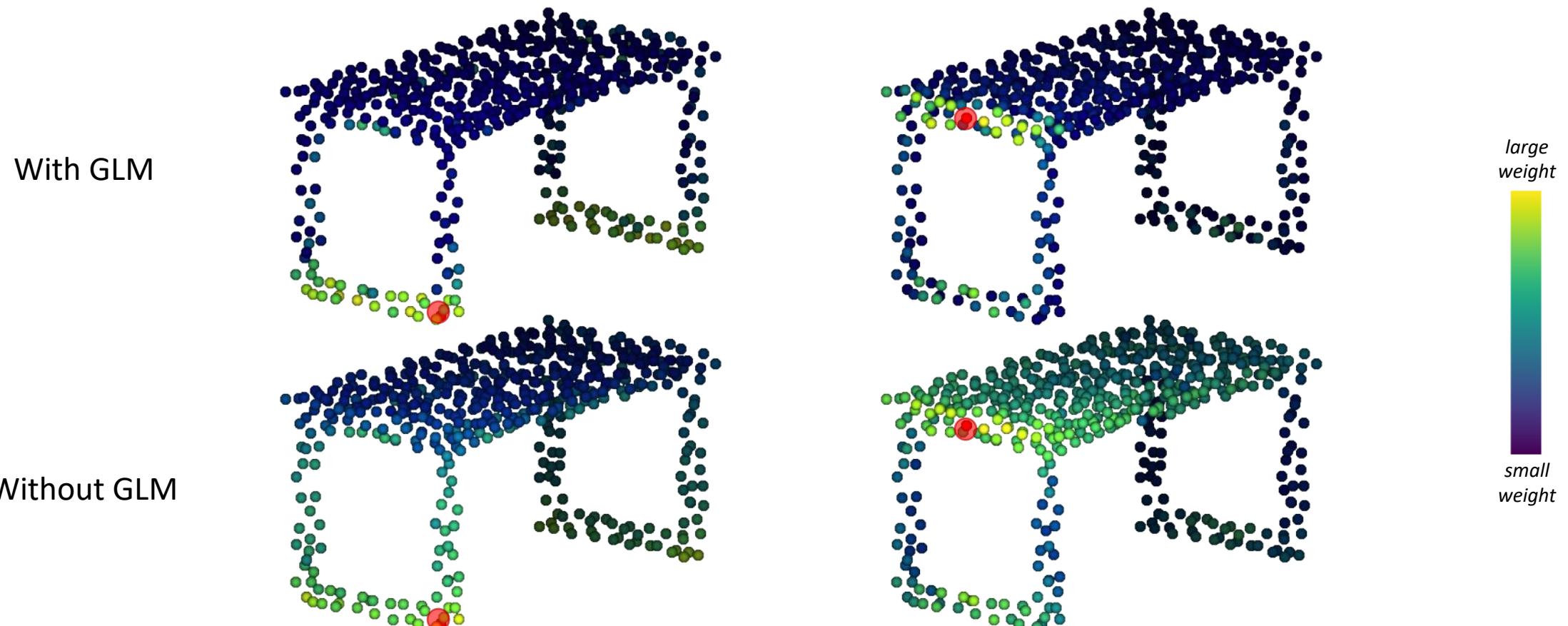
*Two models share all the hyperparameters
except for the GLM module*

Learned Graph from Classification Task (Table)



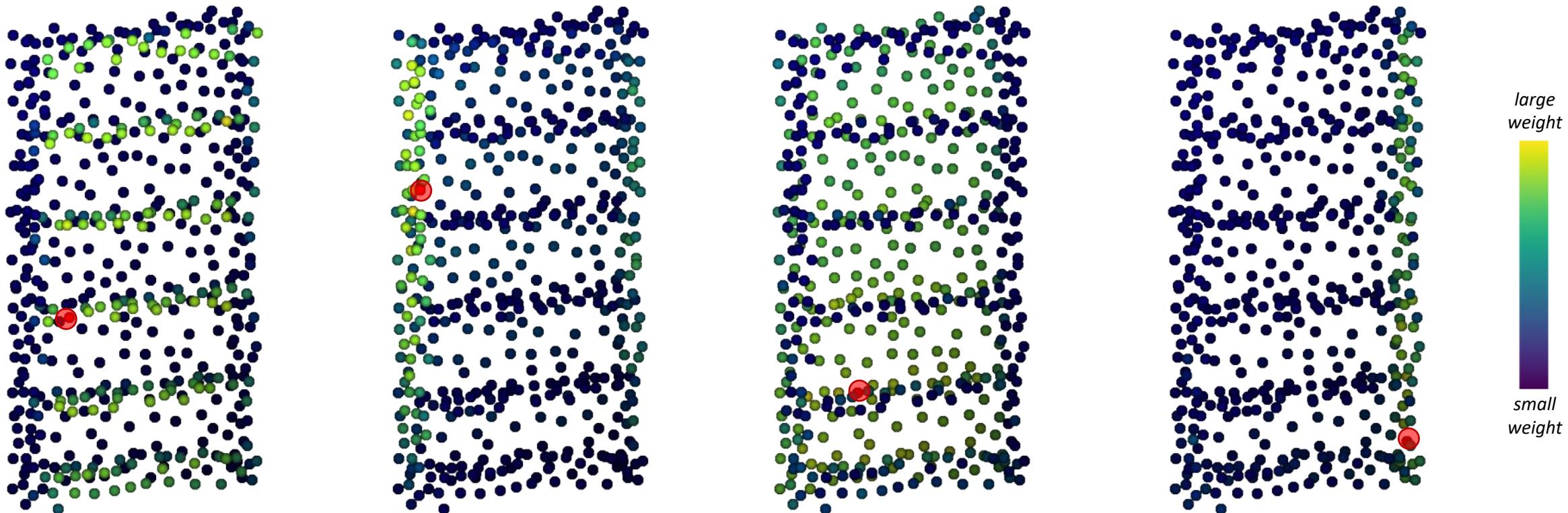
*Edge weight distribution of the red point,
learned from ModelNet40 classification task*

Graph Structure Comparison



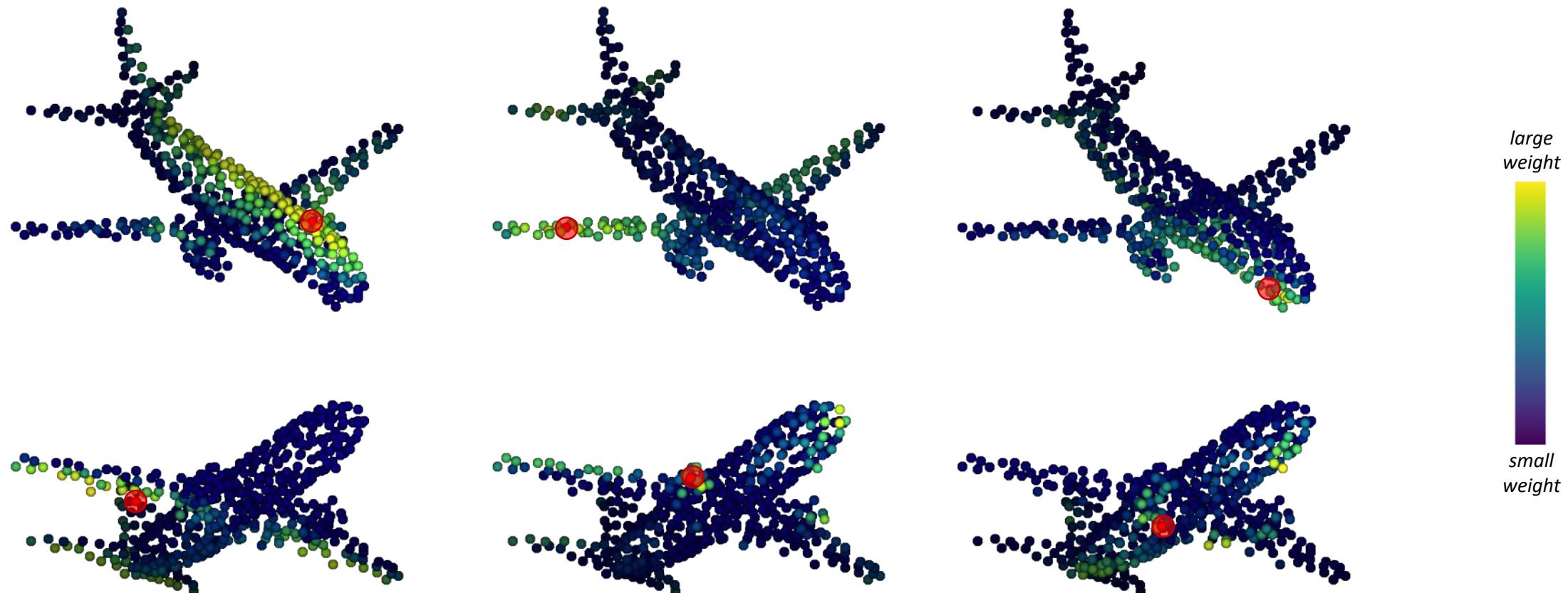
*For PointNet without GLM, we use the inner-product of point features
as the graph weights to show the function of GLM module*

Learned Graph from Classification Task (Shelf)



*Edge weight distribution of the red point,
learned from ModelNet40 classification task*

Learned Graph from Classification Task (Plane)



*Edge weight distribution of the red point,
learned from ModelNet40 classification task*

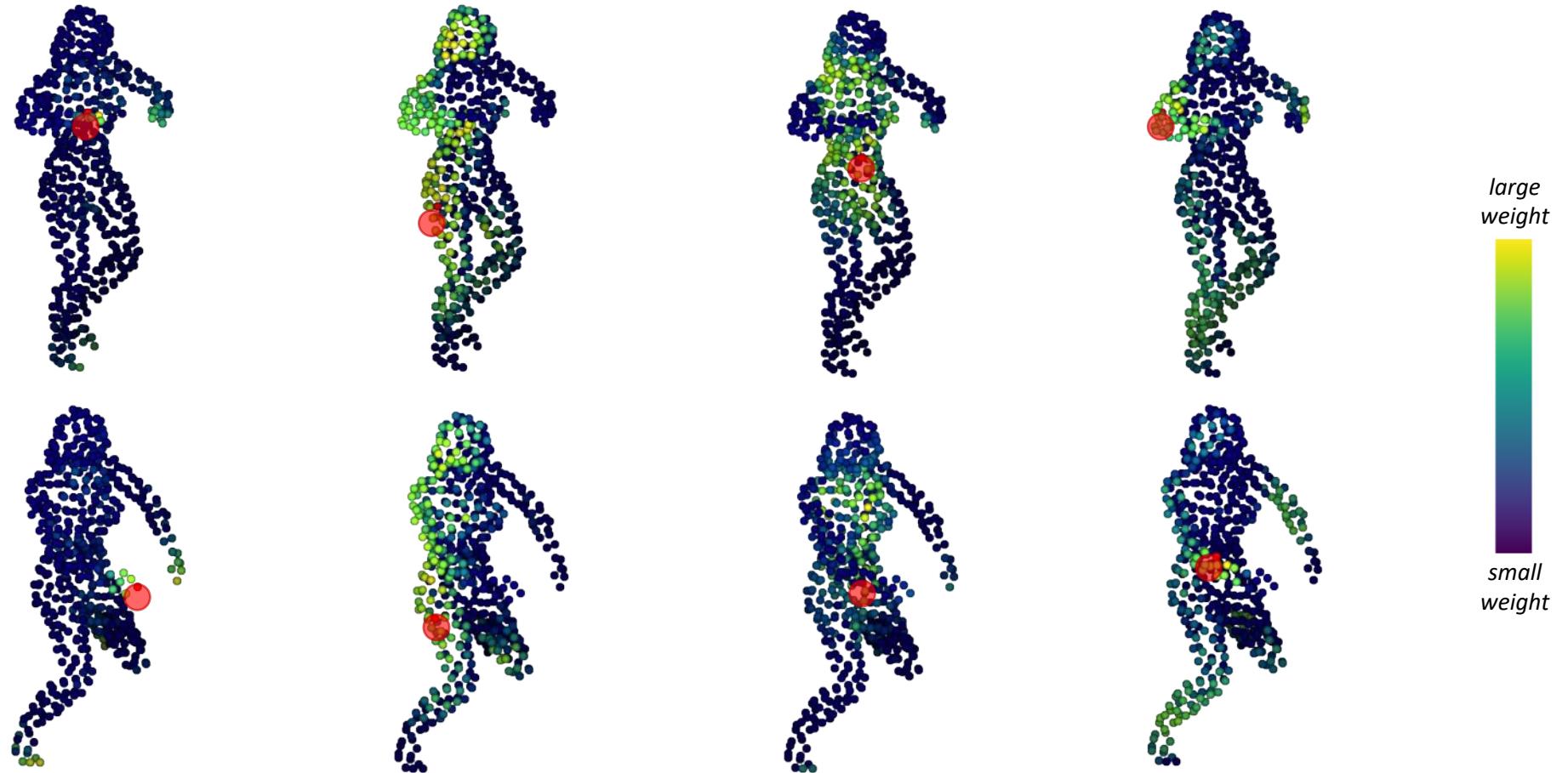
SHREC15 Non-Rigid Shape Classification



Task	SHREC15 Non-Rigid Shape Classification (Accuracy)
2-layer PointNet++ without GLM	96.8%
2-layer PointNet++ with GLM	99.0%

*Two models share all the hyperparameters
except for the GLM module*

Non-rigid Graph Structure



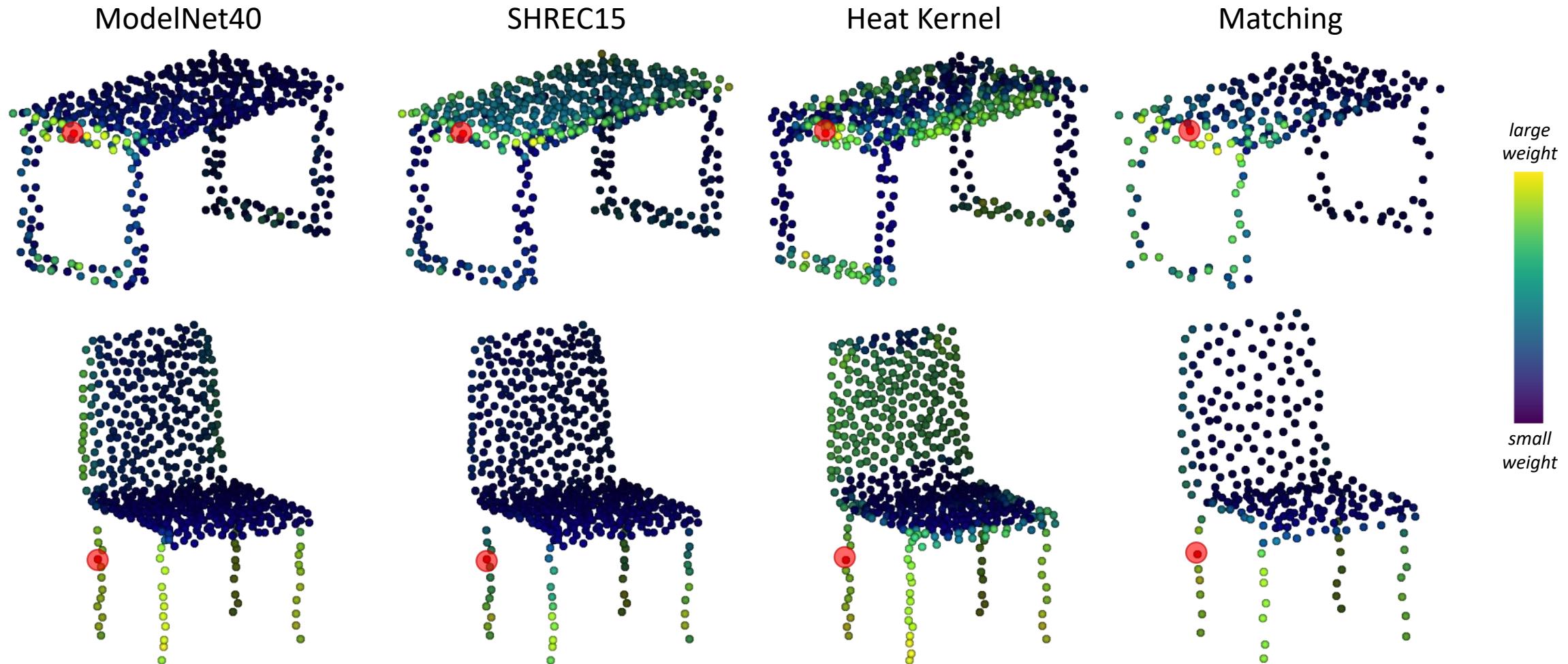
*Edge weight distribution of the red point,
learned from SHREC15 classification task*

Experimental Results

Task	ModelNet40 Classification (Accuracy)	SHREC15 Non- Rigid Shape Classification (Accuracy)	ShapeNet Part Segmentation (Mean IoU)	Heat Kernel Signature Regression (L2 Loss)	Matching (Contrastive Loss)
2-layer PointNet++ without GLM	90.3%	96.8%	84.8%	0.03744	0.1087
2-layer PointNet++ with GLM	90.9%	99.0%	85.0%	0.02425	0.1044

*Two models share all the hyperparameters
except for the GLM module*

Graph Structure among Tasks



Ablation Study on Diffusion Steps

Task	Heat Kernel Signature Regression (L2 Loss)
2-layer PointNet++ without GLM	0.03744
2-layer PointNet++ with GLM (1 step)	0.03044
2-layer PointNet++ with GLM (3 steps)	0.02425

*All models share all the hyperparameters
except for the GLM module*

Thanks!

Zhuohan Li