§4

1. $X=\begin{pmatrix}0&1\\1&0\end{pmatrix}$

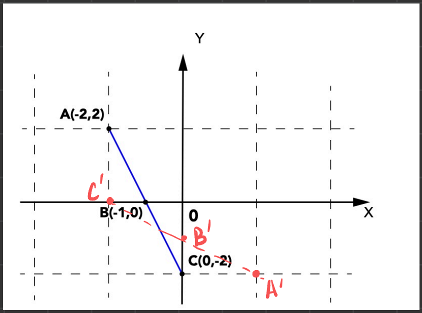
$A=\begin{pmatrix}-2\\2\end{pmatrix}$, $A'=XA=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}-2\\2\end{pmatrix}=\begin{pmatrix}2\\-2\end{pmatrix}$

$B=\begin{pmatrix}-1\\0\end{pmatrix}$, $B'=XB=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}-1\\0\end{pmatrix}=\begin{pmatrix}0\\-1\end{pmatrix}$

$C=\begin{pmatrix}0\\-2\end{pmatrix}$, $C'=XC=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0\\-2\end{pmatrix}=\begin{pmatrix}-2\\0\end{pmatrix}$

注意到 $\overrightarrow{C'A'}=\begin{pmatrix}4\\-2\end{pmatrix}$, $\overrightarrow{C'B'}=\begin{pmatrix}2\\-1\end{pmatrix}$, $\overrightarrow{C'A'}=2\overrightarrow{C'B'}$

$\therefore$ A', B', C' 仍共线



2. $M_1=\begin{pmatrix}2&2+i\\2-i&1+i\end{pmatrix}$ $\qquad$ $M_2=\begin{pmatrix}2&1+2i\\1-2i&3\end{pmatrix}$

(1) $M_1M_2=\begin{pmatrix}2&2+i\\2-i&1+i\end{pmatrix}\begin{pmatrix}2&1+2i\\1-2i&3\end{pmatrix}=\begin{pmatrix}8-3i&8+7i\\7-3i&7+6i\end{pmatrix}$

$M_2M_1=\begin{pmatrix}2&1+2i\\1-2i&3\end{pmatrix}\begin{pmatrix}2&2+i\\2-i&1+i\end{pmatrix}=\begin{pmatrix}8+3i&3+5i\\8-7i&7\end{pmatrix}$

$$M_1^T=\begin{pmatrix}2 & 2-i\\ 2+i & 1+i\end{pmatrix}\qquad M_2^T=\begin{pmatrix}2 & 1-2i\\ 1+2i & 3\end{pmatrix}$$

$$M_1^\dagger=\begin{pmatrix}2 & 2+i\\ 2-i & 1-i\end{pmatrix}\qquad M_2^\dagger=\begin{pmatrix}2 & 1+2i\\ 1-2i & 3\end{pmatrix}$$

(2) $M_1M_2\neq M_2M_1$

(3) $M_1\neq M_1^\dagger$，M_1 不是

$M_2=M_2^\dagger$，M_2 是

3. $X=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\qquad |\psi\rangle=\begin{pmatrix}1\\ -1\end{pmatrix}$

(1) $X|\psi\rangle=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix}=\begin{pmatrix}-1\\ 1\end{pmatrix}=-|\psi\rangle$

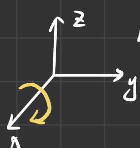
$\therefore |\psi\rangle$ 是 X 的本征态

(2) 本征值为 -1.

(3) $\because \langle\psi|\psi\rangle=(1\ \ -1)\begin{pmatrix}1\\ -1\end{pmatrix}=2$

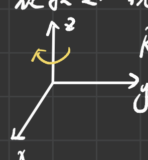
$\therefore$ 归一化的向量为 $\frac{\sqrt{2}}{2}|\psi\rangle=\begin{pmatrix}\frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2}\end{pmatrix}$

4. 旋转1：绕 x 轴顺时针旋转90°



$$R_1=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

旋转2：绕 z 轴顺时针旋转90°



$$R_2=\begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$R_1R_2=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & 0 & 0\end{pmatrix}\qquad R_2R_1=\begin{pmatrix}0 & 0 & 1\\ -1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$$

由 $R_1R_2\neq R_2R_1$，知命题成立

即旋转的结果依赖两次的顺序