§8

1. 未通电时，波函数为

$\frac{1}{\sqrt{2}}(|\psi_1\rangle+|\psi_2\rangle)$

达探测器时，$\frac{1}{\sqrt{2}}(|\psi_1\rangle+|\psi_2\rangle) \to \frac{1}{\sqrt{2}}\sum_{j=1}^{9}(|\psi_1\rangle+|\psi_2\rangle)\,|d_j\rangle$

于 d_5，其探测到电子的概率为 $\frac{1}{2}|a_5+b_5|^2$

由对称性，$a_5=b_5$

$\therefore p=2|a_5|^2=\frac{150}{3000}$

$\therefore |a_5|^2=\frac{75}{3000}=0.025$

$\therefore$ 通电时，$\frac{1}{2}(|\psi_1\rangle+e^{\frac{i\pi}{3}}|\psi_2\rangle) \to \frac{1}{2}\sum_{j=1}^{9}(a_j+e^{\frac{i\pi}{3}}b_j)\,|d_j\rangle$

于 d_5，其探测到电子的概率为 $\frac{1}{2}|a_5+e^{\frac{i\pi}{3}}b_5|^2=\frac{3}{2}|a_5|^2=0.0375$

$\therefore$ d_5 上探测了约 $3000\times0.0375\approx113$ 个电子.

2. 设 $|\psi\rangle=a_1|u\rangle+b_1|d\rangle$，$|\phi\rangle=a_2|u\rangle+b_2|d\rangle$

则 $|\Phi_{1,2}\rangle=|\psi\rangle\otimes|\phi\rangle$

$=a_1a_2|uu\rangle+a_1b_2|ud\rangle+b_1a_2|du\rangle+b_1b_2|dd\rangle$

若 $|\Phi\rangle$ 亦为直积态

$\because |\Phi\rangle=\frac{1}{2}|ud\rangle-\frac{i}{2}|du\rangle+\frac{1}{\sqrt{2}}|dd\rangle$

$\therefore \begin{cases} a_1a_2=0 & ① \\ a_1b_2=\frac{1}{2} & ② \\ b_1a_2=-\frac{i}{2} & ③ \\ b_1b_2=\frac{1}{\sqrt{2}} & ④ \end{cases}$

①④ $\Rightarrow a_1a_2b_1b_2=0$

②③ $\Rightarrow a_1a_2b_1b_2=-\frac{i}{4}$

$\therefore$ 矛盾.

$\therefore |\Phi\rangle$ 是纠缠态.

3. (1) $\because \hat{\sigma}_x |u\rangle = |d\rangle,\ \hat{\sigma}_x |d\rangle = |u\rangle$

$\therefore \hat{\sigma}_x |\Phi\rangle = \frac{1}{2}(\hat{\sigma}_x |u\rangle) \otimes |d\rangle - \frac{i}{2}(\hat{\sigma}_x |d\rangle) \otimes |u\rangle + \frac{1}{2}(\hat{\sigma}_x |d\rangle) \otimes |d\rangle$

$= \frac{1}{2}|dd\rangle - \frac{i}{2}|uu\rangle + \frac{1}{2}|ud\rangle$

$\therefore |uu\rangle\ |ud\rangle\ |du\rangle\ |dd\rangle$ 是一组正交归一基.

$\therefore \langle \Phi | \hat{\sigma}_x | \Phi \rangle = (\frac{1}{2}\langle ud| + \frac{i}{2}\langle du| + \frac{1}{2}\langle dd|)(\frac{1}{2}|dd\rangle - \frac{i}{2}|uu\rangle + \frac{1}{2}|ud\rangle)$

$= \frac{\sqrt{2}}{2}$

(2) $\because \hat{\sigma}_z |u\rangle = |u\rangle,\ \hat{\sigma}_z |d\rangle = |d\rangle,\ \hat{\tau}_x |u\rangle = |d\rangle,\ \hat{\tau}_x |d\rangle = |u\rangle$

$\therefore \hat{\sigma}_z \otimes \hat{\tau}_x |\Phi\rangle = \frac{1}{2}(\hat{\sigma}_z |u\rangle) \otimes (\hat{\tau}_x |d\rangle) - \frac{i}{2}(\hat{\sigma}_z |d\rangle) \otimes (\hat{\tau}_x |u\rangle) + \frac{1}{\sqrt{2}}(\hat{\sigma}_z |d\rangle) \otimes (\hat{\tau}_x |d\rangle)$

$= \frac{1}{2}|uu\rangle - \frac{i}{2}|dd\rangle + \frac{1}{\sqrt{2}}|du\rangle$

$\therefore \langle \Phi | \hat{\sigma}_z \otimes \hat{\tau}_x | \Phi \rangle = (\frac{1}{2}\langle ud| + \frac{i}{2}\langle du| + \frac{1}{\sqrt{2}}\langle dd|)(\frac{1}{2}|uu\rangle - \frac{i}{2}|dd\rangle + \frac{1}{\sqrt{2}}|du\rangle)$

$= 0$

4. $\because |n_+\rangle = \cos\frac{\theta}{2}|u\rangle + e^{i\varphi}\sin\frac{\theta}{2}|d\rangle \qquad |n_-\rangle = \sin\frac{\theta}{2}|u\rangle - e^{i\varphi}\cos\frac{\theta}{2}|d\rangle$

$\therefore |n_+ n_-\rangle = |n_+\rangle \otimes |n_-\rangle$

$= (\cos\frac{\theta}{2}|u\rangle + e^{i\varphi}\sin\frac{\theta}{2}|d\rangle) \otimes (\sin\frac{\theta}{2}|u\rangle - e^{i\varphi}\cos\frac{\theta}{2}|d\rangle)$

$= \sin\frac{\theta}{2}\cos\frac{\theta}{2}|uu\rangle - e^{i\varphi}\cos^2\frac{\theta}{2}|ud\rangle + e^{i\varphi}\sin^2\frac{\theta}{2}|du\rangle - (e^{i\varphi})^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}|dd\rangle$

$\therefore |n_- n_+\rangle = \sin\frac{\theta}{2}\cos\frac{\theta}{2}|uu\rangle + e^{i\varphi}\cos^2\frac{\theta}{2}|ud\rangle - e^{i\varphi}\sin^2\frac{\theta}{2}|du\rangle - (e^{i\varphi})^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}|dd\rangle$

$\therefore -\frac{e^{-i\varphi}}{\sqrt{2}}(|n_+ n_-\rangle - |n_- n_+\rangle) = \frac{-e^{-i\varphi}}{2}\left(e^{i\varphi}(-\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2})|ud\rangle + e^{i\varphi}(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2})|du\rangle\right)$

$= \frac{-e^{-i\varphi}}{2} \cdot e^{i\varphi}(|ud\rangle - |du\rangle)$

$= \frac{1}{2}(|ud\rangle - |du\rangle)$

$= |S\rangle$