§10

1. $|\psi\rangle = \frac{5}{13}|u\rangle - \frac{12}{13}|d\rangle$

(1) ① $\langle\psi|\hat{\sigma}_x|\psi\rangle$

$$= \begin{bmatrix} \frac{5}{13} & \frac{12}{13}i \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{5}{13} \\ -\frac{12}{13}i \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{13} & \frac{12}{13}i \end{bmatrix} \begin{bmatrix} -\frac{12}{13}i \\ \frac{5}{13} \end{bmatrix}$$

$$= 0$$

② $\langle\psi|\hat{\sigma}_y|\psi\rangle$

$$= \begin{bmatrix} \frac{5}{13} & \frac{12}{13}i \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \frac{5}{13} \\ -\frac{12}{13}i \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{13} & \frac{12}{13}i \end{bmatrix} \begin{bmatrix} -\frac{12}{13} \\ \frac{5}{13}i \end{bmatrix}$$

$$= -\frac{60}{169} - \frac{60}{169} = -\frac{120}{169}$$

(2) ① $\Delta\hat{\sigma}_x^2 = \langle\psi|\hat{\sigma}_x^2|\psi\rangle - \hat{\sigma}_x^2$

$$\langle\psi|\hat{\sigma}_x^2|\psi\rangle = \begin{bmatrix} \frac{5}{13} & \frac{12}{13}i \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{5}{13} \\ -\frac{12}{13}i \end{bmatrix}$$

$$= \langle\psi|\hat{I}|\psi\rangle = 1$$

$\therefore \Delta\hat{\sigma}_x^2 = 1 - 0 = 1$

② $\Delta\hat{\sigma}_y^2 = \langle\psi|\hat{\sigma}_y^2|\psi\rangle - \hat{\sigma}_y^2$

$\langle\psi|\hat{\sigma}_y^2|\psi\rangle = \langle\psi|\hat{I}|\psi\rangle = 1$

$\therefore \Delta\hat{\sigma}_y^2 = 1 - \left(\frac{120}{169}\right)^2 = \frac{14161}{28561}$

③ $\Delta\hat{\sigma}_x^2 + \hat{\sigma}_y^2 \geq 1$

2. (a) $U|\Phi_0\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{5} \\ -\frac{4i}{5} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} \\ 0 \\ 0 \\ -\frac{4i}{5} \end{pmatrix} = |\Phi_1\rangle$

(b) $A|\Phi_1\rangle = \begin{pmatrix} \frac{5}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{5} \\ 0 \\ 0 \\ -\frac{4}{5} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |\Phi_2\rangle$

(c) $UU^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = I$

$AA^\dagger = \begin{pmatrix} \frac{5}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{5}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{25}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \neq I$

$\therefore$ U 是幺正矩阵，而 A 不是

3. a. 波包坍缩理论

作用后：$a|\psi_1\rangle \otimes |\Phi^0_{DNA}\rangle + b|\psi_2\rangle \otimes |\Phi^m_{DNA}\rangle$

观测时，波包坍缩.

① $|a|^2$ 几率，跳变为 $|\psi_1\rangle \otimes |\Phi^0_{DNA}\rangle$ $\Rightarrow$ 粒子未有与DNA相互作用，动物未长角

② $|b|^2$ 几率，跳变为 $|\psi_2\rangle \otimes |\Phi^m_{DNA}\rangle$ $\Rightarrow$ 粒子与DNA相互作用，动物长角

b. 多世界理论.

作用后：$a|\psi_1\rangle \otimes |\Phi^0_{DNA}\rangle + b|\psi_2\rangle \otimes |\Phi^m_{DNA}\rangle$

观测时，产生纠缠，每个分支都是一个真实的平行世界

① $|\psi_1\rangle \otimes |\Phi^0_{DNA}\rangle$ $\Rightarrow$ 粒子未有与DNA相互作用，动物未长角

② $|\psi_2\rangle \otimes |\Phi^m_{DNA}\rangle$ $\Rightarrow$ 粒子与DNA相互作用，动物长角