Adaptive Sorting-Based Evolutionary Algorithm for Many-Objective Optimization

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***Abstract*—Evolutionary algorithms have shown their promise in coping with many-objective optimization problems. However, the strategies of balancing convergence and diversity and the effectiveness of handling problems with irregular Pareto fronts (PFs) are still far from perfect. To address these issues, this paper proposes an adaptive sorting-based evolutionary algorithm based on the idea of decomposition. First, we propose an adaptive sorting-based environmental selection strategy. Solutions in each subpopulation (partitioned by reference vectors) are sorted based on their convergence. Those with better convergence are further sorted based on their diversity, then being selected according to their sorting levels. Second, we provide an adaptive promising subpopulation sorting-based environmental selection strategy for problems which may have irregular PFs. This strategy provides additional sorting-based selection effort on promising subpopulations after the general environmental selection process. Third, we extend the algorithm to handle constraints. Finally, we conduct an extensive experimental study on the proposed algorithm by comparing with start-of-the-state algorithms. Results demonstrate the superiority of the proposed algorithm.**

***Index Terms*—Decomposition, evolutionary algorithm, irregular Pareto front (PF), many-objective optimization, reference vector, sorting.**

I. Introduction

MANY real-world applications can be modeled as multi-objective optimization problems (MOPs) which require optimizing several conflicting objectives simultaneously [1]

$$\begin{aligned} \min \mathbf{f}(\mathbf{x}) &= (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})) \\ \text{s.t.} \quad & g_u(\mathbf{x}) \le 0, u = 1, 2, \dots, q \\ & h_v(\mathbf{x}) = 0, v = 1, 2, \dots, p \\ & \mathbf{x} \in \Omega \end{aligned} \tag{1}$$

where $\Omega \subseteq \mathbb{R}^n$ is the decision space, $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is a candidate solution, $F : \Omega \rightarrow \mathbb{R}^m$ constitutes m objective functions to be minimized, and $\mathbb{R}^m$ is the objective space. $g_u(\mathbf{x})$ and $h_v(\mathbf{x})$ are inequality and equality constraints, respectively, that solutions should satisfy.

Multiobjective evolutionary algorithms (MOEAs) have been widely used on MOPs [2], [3]. However, there are many problems having more than three objectives. These problems are known as many-objective optimization problems (MaOPs) [4], [5]. MaOPs pose great difficulties to conventional MOEAs especially those relying on principles of nondominance.

The main difficulty is that Pareto dominance-based selection strategies are inefficient for MaOPs. The reason is that solutions are prone to become nondominant with respect to each other and the numbers of such solutions increase with the number of objectives, resulting in weak selection pressure [6], [7]. The second is the difficulty of diversity maintenance. In the objective space of MaOPs, solutions tend to be sparse [8], making it difficult to estimate the similarity by existing techniques, such as crowing distance [9] and kth nearest distance [10]. Third, variation operators may lose efficiency on MaOPs, as offspring is likely to be far away from their parents in high-dimensional objective spaces [8].

Approaches for addressing these difficulties can be roughly divided into three categories. The first category modifies the traditional Pareto dominance relationship to enhance the ability to distinguish solutions. Examples include fuzzy dominance [11], L-optimal [12], and ε-dominance [13]. In particular, study [14] proposed a grid-based algorithm, in which the objective space was divided by grids, and the dominance information of solutions were decided by the grids that the solutions belonged.

The second category makes use of indicator-based selection strategies, such as hypervolume [15], S-metric [16], and I_ε indicator [17]. Due to the strictly monotonic relation with Pareto dominance [18], hypervolume has been widely used as a secondary selection criterion after Pareto

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This paper has supplementary downloadable material available at http://ieeexplore.ieee.org, provided by the author. This material provides a detailed description of experimental settings, IGD results, and the ASEA version on constrains handling. This material is 752 KB in size.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

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dominance [19]. In [20], hypervolume was first used as the only criterion in both mating and environmental selection processes. Reference [21] further improved the performance of hypervolume-based selection on MaOPs. In a recent study [22], the whole objective space was divided into small grids, the hypervolume value of a solution was calculated only considering the information of the grid the solution belonged. This strategy reduced the computational cost of hypervolume computation.

The third category is decomposition-based approaches. These methods decompose an MOP into a number of single-objective problems [23] or sub-MOPs [24] and conduct a collaborative evolution of these subproblems through aggregation functions. One of the representative algorithms is MOEA/D [23], in which solutions associated with a certain subproblem was selected according to the information from their neighboring subproblems. NSGAIII [8], [25] extended the idea of decomposition to the environmental selection of Pareto dominance-based methods. It used a set of reference vectors to decompose the objective space to maintain diversity of solutions. Following the idea of NSGAIII, studies [26]–[28], and [29] improved the fitness evaluation methods of decomposition approaches on MaOPs; Li *et al.* [30] and Yuan *et al.* [31] proposed new environmental selection strategies: [30] combined dominance and decomposition for environmental selection, while [31] maintained the diversity by exploiting the perpendicular distance from solutions to reference vectors in the objective space.

There are also some algorithms that cannot be categorized into the above classes, such as preference-based algorithms [32], [33], objective reduction methods [34]–[37], the knee point-driven approach [38], two-archive algorithms [39], [40] (i.e., solutions were divided into two archives, in which they were evolved with aims of convergence and diversity, respectively), and the recent angle-based environmental selection strategies [41], [42] (i.e., the diversity of solutions was measured by the angles between solutions in the objective space, the solution with the best diversity is selected recursively).

Among the above classes of methods, decomposition-based algorithms have presented the greatest promise for MaOPs. The guidance of predefined reference vectors is particularly attractive as they help to maintain a good diversity of solutions while at the same time provide a high selection pressure toward Pareto fronts (PFs). However, there are still some open challenges.

First, the environmental selection strategies of decomposition-based algorithms have not been well investigated in the context of MaOPs. The weighted sum method is not suitable for nonconvex problems [43]. Recently, a localized weighted sum approach [29] was proposed to solve such classes of problems but it still faces problems with maintenance of diversity [29]. The Chebyshev method has been reported to be equivalent with Pareto dominance methods [44], thus it is likely that the Chebyshev method is not effective on MaOPs [29], [44], [45]. The penalty-based boundary intersection (PBI) approach needs a careful configuration of the penalty parameter θ [46], [47]. Study [28] suggested an angle-penalized distance scalarizing function, which adaptively implemented diversity-based penalization in fitness evaluation. However, these scalarizing functions may face difficulty in balancing convergence and diversity, as intuitively they cannot always capture the appropriate proportion of each solution's convergence and diversity information for various PF types. Furthermore, existing scalarizing functions restrict the effective use of reference vectors. Since scalarizing functions only consider the distribution of solutions with respect to its own associated vectors, it is the best to select only one solution from each reference vector. Thus, the only way of obtaining more solutions is to generate more reference vectors, which increases the computational effort especially in high-dimensional objective spaces. In addition, the predefined parameters in these functions may reduce the robustness of algorithms.

Second, the performance of decomposition-based algorithms degrades if the distribution of reference vectors is not consistent with PF shapes [48]. To handle this problem, one theoretical way is to adjust the distribution of reference vectors during the evolutionary process [25], [28], [49]–[52]. The MOEA/D-AWA [49] introduced an adaptive adjustment strategy to delete overcrowded reference vectors and add new vectors into sparse regions periodically. The A-NSGAIII [25] generated new reference vectors around existing reference vectors, then deleted the less useful ones among the new generated vectors. This "generate-then-delete" procedure may reduce its computational efficiency [28]. The RVEA* [28] replaced invalid reference vectors by the new randomly generated ones among valid reference vectors. This strategy has higher computational efficiency, but the diversity of solutions can be compromised [28]. However, similar with the adaptive-reference-vector strategies for MOPs [53]–[55], the strategies of [25], [28], and [49]–[52] may adversely affect convergence [56]. The reason is intuitive: the changing reference vectors change the set of subproblems during the process of evolution and changing subproblems too often slows down convergence [56]. It is clear from the above discussion that it is desirable to develop efficient schemes to better deal with MaOPs and especially ones with irregular PFs.[1]

To tackle the above issues, in this paper we propose an adaptive sorting-based evolutionary algorithm (ASEA). Our major contributions are summarized as follows.

1) We propose an adaptive sorting-based environmental selection strategy. Different from existing methods fusing solutions' convergence and diversity information into single scalarizing functions, this strategy proposes a two-stage sorting scheme: first, solutions are sorted based on their convergence, then the solutions with better convergence are sorted based on their diversity. This strategy does not need to consider the relative proportion of convergence and diversity information of solutions. Furthermore, the number of solutions participating in diversity-based sorting is adaptive during the evolution

[1] Irregular PF means the shape of the PF is not a regular geometrical structure, i.e., not smooth, continuous, or well spread.

process, and there are no predefined parameters in this strategy, which guarantees its robustness.

2) We provide an adaptive promising subpopulation sorting-based environmental selection strategy for MaOPs which may have irregular PFs. This strategy provides a further selection effort on promising subpopulations (subpopulations which used to have selected nondominant solutions) after the general environmental selection procedure. It improves the number and the diversity of final solutions without adding reference vectors, and can be used for any decomposition-based algorithms.
3) We further extend ASEA to solve constrained MaOPs. Three test problems with different constraint types are used to verify its effectiveness.
4) We conduct an extensive experimental study on DTLZ [57], WFG [58] and their minus versions DTLZ^{-1} and WFG^{-1} [48] test suites. Our proposed algorithm wins the best or is comparable with the best on 82 out of the 128 test instances, showing its competitiveness.

The remainder of this paper is as follows. Section II describes background technologies. Section III details components of the proposed ASEA. Experimental studies are drawn in Section IV. Finally, Section V concludes this paper.

II. Preliminaries

A. Reference Vectors

Various reference vector generation methods have been proposed in decomposition-based MOEAs, such as the simplex-lattice design [59], the uniform random sampling [60], the uniform design [61] and the generalized decomposition method [62]. This paper employs the widely used simplex-lattice design method [59] for reference vector generalization. For an m-dimensional (0, 1) normalized objective space, this method uniformly divides the $m-1$-dimensional hyperplane into p parts, and generates H reference points

$$H = \binom{m-1+p}{p}. \tag{2}$$

Specifically, two-layered reference vectors [8] can be used to limit the number reference vectors for problems with larger number of objectives.

B. Objective Space Normalization

Since the reference vectors start from the coordinate origin, the objective values should be normalized [8]

$$f'_k(\mathbf{x}) = f_k(\mathbf{x}) - z_k^{\text{ideal}}, k = 1, 2, \ldots, m \tag{3}$$

where $f_k(\mathbf{x})$ is the kth objective value of solution $\mathbf{x}$, z_k^{ideal} is the kth dimension of the ideal point which is determined by the minimum value of each objective function in $\cup_{t=0}^{t} G_t$.

For MaOPs whose objective values are disparately scaled, $f'_k(\mathbf{x})$ should be further normalized

$$f_k^n(\mathbf{x}) = \frac{f'_k(\mathbf{x})}{z_k^{\text{nadir}} - z_k^{\text{ideal}}}, k = 1, 2, \ldots, m \tag{4}$$

Algorithm 1 Framework of ASEA

Require: reference vector initialization parameter H, population size N, number of solutions needing to be selected in the promising subpopulation sorting-based environmental selection procedure N^*

```
P_0 ← population initialization
G = 1, P_G ← P_0, Q_G ← ∅
Λ ← reference vector initialization(H)
while G ⩽ G_max do
    Q_G ← crossover and mutation(P_G)
    P_G = P_G ∪ Q_G
    objective space normalization(P_G)
    subpopulation partition(P_G, Λ)
    P_{G+1} ← adaptive sorting-based selection(P_G, Λ, N)
    P_{G+1} ← adaptive promising subpopulation sorting-based
        selection(P_G, P_{G+1}, N*)
    G = G + 1
end while
```

where z_k^{nadir} is the kth dimension of the nadir point. In this paper, the nadir point is calculated by the method of [27]. While we adopt the above normalization scheme in this paper, we acknowledge that dominant resistant solutions can affect such a scaling. Use of maximum value of the objectives in the population, use of Nadir computed using the archive or use solution with the closest Euclidean distance from the ideal from the subpopulations corresponding to the axial reference directions have been suggested as alternatives.

C. Subpopulation Partition

To guide the evolution process by reference vectors, the whole population is divided into a number of subpopulations in the normalized objective space [26]–[29]. For solution $\mathbf{x}_i \in P_G$ (G is the generation index), the acute angle between $\mathbf{x}_i$, and each reference vector λ_j, $j \in \{1, 2, \ldots, N\}$ is

$$\theta_{\mathbf{x}_i, \lambda_j} = \arccos\left(\frac{f(\mathbf{x}_i) \cdot \lambda_j}{\|f(\mathbf{x}_i)\|}\right). \tag{5}$$

Solution $\mathbf{x}_i$ is associated with reference vector λ_j if and only if the angle between $\mathbf{x}_i$ and λ_j is the smallest. The solutions associated with λ_j constitute subpopulation P_G^j

$$P_G^j = \left\{\mathbf{x}_i | j = \underset{j \in \{1,\ldots,N\}}{\operatorname{argmax}} \cos\theta_{\mathbf{x}_i, \lambda_j}\right\}, i = 1, 2, \ldots, |P_G|. \tag{6}$$

III. Proposed Algorithm

A. Framework

The framework of our proposed ASEA is presented in Algorithm 1. ASEA starts with randomly generating an initial population P_0 of size N from a uniform distribution, where there are N reference vectors generated (refer to Section II-A). In each generation G, step 1, the crossover and mutation operators are implemented in P_G to generate an offspring set Q_G. We employ the simulated binary crossover [63] and the polynomial mutation [64] without explicit mating selection strategies, due to their good performance found in references [8], [27], and [28]. Step 2, objective space normalization and subpopulation partition are conducted (refer to Section II-B and II-C, respectively). Step 3, the adaptive sorting-based environmental selection is executed to select N solutions for the next generation. Step 4,

Algorithm 2 Adaptive Sorting-Based Selection

Require: P_G, Λ, N

1: **for** $i = 1{:}|P_G|$ **do**
2: $\quad C_{\mathbf{x}_i}, D_{\mathbf{x}_i} \leftarrow$ convergence and diversity of $\mathbf{x}_i$ //refer to (7) (8)
3: **end for**
4: **while** $|P_{G+1}| < N$ **do**
5: $\quad$ **for** $j = 1 : N$ **do**
6: $\quad\quad P_G^j \leftarrow$ solutions in P_G^j sorted in ascending order based on their convergence C
7: $\quad\quad E_G^j \leftarrow$ solutions ranked in top r_j in P_G^j //refer to (9)
8: $\quad\quad E_G^j \leftarrow$ solutions in E_G^j sorted in ascending order based on their diversity D
9: $\quad\quad$ **for** $k = 1 : |E_G^j|$ **do**
10: $\quad\quad\quad \{level\, k\} = \{level\, k\} \cup$ the solution ranked k in E_G^j
11: $\quad\quad$ **end for**
12: $\quad$ **end for**
13: $\quad$ **for** $k = 1 : l$ **do**
14: $\quad\quad$ **if** $|\{level\, k\}| > N - |P_{G+1}|$ **then**
15: $\quad\quad\quad \{level\, k\} \leftarrow$ randomly select $N - |P_{G+1}|$ solutions from $\{level\, k\}$
16: $\quad\quad$ **end if**
17: $\quad\quad P_{G+1} = P_{G+1} \cup \{level\, k\}$
18: $\quad$ **end for**
19: $\quad P_G = P_G \setminus P_{G+1}$
20: **end while**

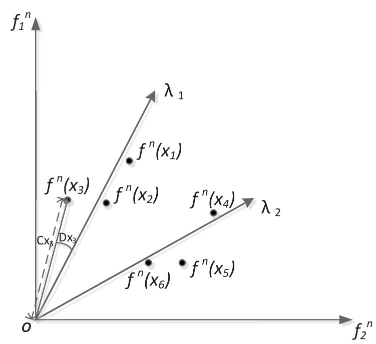

Fig. 1. Example for understanding the convergence and diversity measurement and the adaptive sorting-based selection.

the adaptive promising subpopulation sorting-based environmental selection is conducted when the true PF of the problem may be irregular. For clarity, we refer ASEA as the algorithm without line 10 and the algorithm including line 10 is referred as ASEA$^+$ hereafter.

B. Adaptive Sorting-Based Selection

The pseudocode of the adaptive sorting-based selection is given in Algorithm 2.

1) Convergence and Diversity Measurement: For solution $\mathbf{x}_i \in P_G^j$, $j \in \{1, 2, \ldots, N\}$, we measure its convergence $C_{\mathbf{x}_i}$ by the distance from the ideal point to $f^n(\mathbf{x}_i)$ in the normalized objective space

$$C_{\mathbf{x}_i} = \|f^n(\mathbf{x}_i)\|. \tag{7}$$

We measure the diversity of $\mathbf{x}_i$, $D_{\mathbf{x}_i}$ by the acute angle between $f^n(\mathbf{x}_i)$ and its associated reference vector λ_j [j is identified by (6)]

$$D_{\mathbf{x}_i} = \theta_{\mathbf{x}_i, \lambda_j}. \tag{8}$$

An example is given in Fig. 1, where $C_{\mathbf{x}_3}$ and $D_{\mathbf{x}_3}$ measure the convergence and diversity of $\mathbf{x}_3$, respectively.

2) Sorting: First, we sort solutions in each subpopulation P_G^j, $j \in \{1, 2, \ldots, N\}$, respectively, in ascending order based on their convergence C. The solutions ranked in top r_j are copied to E_G^j, where r_j is calculated by

$$r_j = \left\lceil e^{1 - \frac{G_{\max}}{G}} \cdot |P_G^j| \right\rceil. \tag{9}$$

Then, we sort the solutions in each $E_G^j, j \in \{1, 2, \ldots, N\}$, respectively, in ascending order based on their diversity. We move the solutions ranked first to $\{Level\, 1\}$, and the like. Finally, we obtain $\{Level\, k, k = 1, 2, \ldots, l\}$, where $l = \max\{r_1, r_2, \ldots, r_N\}$. This procedure is shown in lines 5–12 of Algorithm 2.

3) Selection: Starting from $\{Level\, 1\}$, we move solutions to P_{G+1} until $|P_{G+1}| = N$. Specifically, solutions in the last accepted level are randomly selected, if not all solutions in this level can be selected. Steps 2 and 3 will iterate, if $|P_{G+1}| < N$.

We depict Fig. 1 to better understand the selection procedure. In this figure, $P_G^1 = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$, $P_G^2 = \{\mathbf{x}_4, \mathbf{x}_5, \mathbf{x}_6\}$, let $N = 2$, $G = 50$ and $G_{\max} = 100$. First, we sort solutions in P_G^1, and P_G^2, respectively, in ascending order based on their convergence, i.e., $P_G^1 = \{\mathbf{x}_3, \mathbf{x}_2, \mathbf{x}_1\}$ and $P_G^2 = \{\mathbf{x}_6, \mathbf{x}_5, \mathbf{x}_4\}$. According to (9), $r_1 = 2$, $r_2 = 2$, hence we achieve $E_G^1 = \{\mathbf{x}_3, \mathbf{x}_2\}$ and $E_G^2 = \{\mathbf{x}_6, \mathbf{x}_5\}$. Then, we sort solutions in E_G^1 and E_G^2, respectively, in ascending order based on their diversity, i.e., $E_G^1 = \{\mathbf{x}_2, \mathbf{x}_3\}$ and $E_G^2 = \{\mathbf{x}_6, \mathbf{x}_5\}$. Finally, we move the solutions ranked first in E_G^1 and E_G^2, i.e., $\mathbf{x}_2$ and $\mathbf{x}_6$, to P_{G+1}.

We propose the adaptive sorting-based environmental selection strategy based on empirical observations as follows.

First, existing methods which fuse the convergence and diversity information into single scalarizing functions [27]–[29] may face difficulty in balancing convergence and diversity and restrict the effective use of the reference vectors. To address these issues, the proposed strategy sorts solutions based on their convergence and diversity, respectively. It then selects solutions according to their sorting levels. It does not need to consider the relative proportion of the convergence and diversity information of the solutions. Furthermore, due to the two-stage sorting scheme (i.e., convergence-then-diversity-based sorting), this strategy enables to select more than one solutions from the associated solutions of each reference vector. This improves the effective use of reference vectors and makes it easy to be used in MaOPs which may have irregular PFs without adding reference vectors (this will be introduced in Section III-C).

Second, use of a constant selection behavior is inappropriate for the sparse solutions in high-dimensional objective spaces. Theoretically, the convergence should be emphasized at the early stage of evolution to push the population toward PFs. When the population is close to PFs, diversity should be considered to deliver well distributed set of solutions [28]. Therefore, we provide the adaptive sorting scheme, i.e., (9). At the early stage, a small number of solutions with better convergence are selected for diversity-based sorting, which guarantees convergence. As the evolution goes by, more solutions are selected for diversity-based sorting, which makes the diversity being emphasized gradually to maintain well distributed set of solutions.

Algorithm 3 Adaptive Promising Subpopulation Sorting-Based Selection

Require: P_G, P_{G+1}, N^*

$\Lambda^* \leftarrow$ reference vectors associating with nondominant solutions in P_{G+1}
if $\sum_{j=1}^{|\Lambda^*|} |P_G^{j^*}| > N^*$ **then**
  while $N^* > 0$ **do**
    update Λ^* by the reference vectors in Λ^* which have associated solutions of P_G
    if $N^* < \Lambda^*$ **then**
      update Λ^* by N^* eference vectors randomly selected from Λ^*
    end if
    for $j = 1 : |\Lambda^*|$ **do**
      $P_G^{j^*} \leftarrow$ solutions in P_G which associate with λ_j
      for $i = 1 : |P_G^{j^*}|$ **do**
        $D^*_{\mathbf{x}_i} \leftarrow$ the diversity of $\mathbf{x}_i$ //refer to (10)
      end for
      $P_G^{j^*} \leftarrow$ solutions in $P_G^{j^*}$ sorted in ascending order based on their convergence C
      $E_G^{j^*} \leftarrow$ solutions ranked in top r_j in $P_G^{j^*}$ //refer to (9)
      $P_{G+1} = P_{G+1} \cup$ the solution with the maximum D^* value in $E_G^{j^*}$
      $P_G^j = P_G^j \setminus$ the solution with the maximum D^* value in $E_G^{j^*}$
      $N^* = N^* - 1$
    end for
  end while
else
  $P_{G+1} = P_{G+1} \cup \bigcup_{j=1}^{|\Lambda^*|} \{P_G^{j^*}\}$
end if

C. Adaptive Promising Subpopulation Sorting-Based Selection

To handle MaOPs which may have irregular PFs, we provide an adaptive promising subpopulation sorting-based selection strategy. This strategy provides a further sorting-based selection effort on promising subpopulations after the general environmental selection procedure, which improves the number and the diversity of final solutions without adding reference vectors. The pseudocode is presented in Algorithm 3.

1) Promising Subpopulation Identification: The Pareto nondominant solutions of P_{G+1} can be regarded as promising solutions in the current generation, therefore we denote their associated reference vectors as promising reference vectors and collect them in Λ^*. Solutions in P_G which are associated with $\lambda_j \in \Lambda^*$ constitute promising subpopulation $P_G^{j^*}$.

2) Convergence and Diversity Measurement:* For solution $\mathbf{x}_i \in P_G^{j^*}$, its convergence is same as that measured by (7), i.e., $C_{\mathbf{x}_i}$. However, different from (8), here we measure the diversity of $\mathbf{x}_i$, $D^*_{\mathbf{x}_i}$ by

$$D^*_{\mathbf{x}_i} = \min\left(\theta_{\mathbf{x}_i,\mathbf{x}_k}\right) \tag{10}$$

where $\mathbf{x}_i \in P_G^{j^*}$, $k \in K$, the set K consists of the indexes of solutions in P_{G+1} which are associated with λ_j or with the adjacent promising reference vectors of λ_j .

Taking Fig. 2 as an example, $\Lambda^* = \{\lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_6^*\}$ are promising reference vectors, $P_{G+1} = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5\}$, $P_G^{3^*} = \{\mathbf{x}_6, \mathbf{x}_7\}$, and $P_G^{6^*} = \{\mathbf{x}_8\}$. The adjacent promising reference vectors of λ_3^* are λ_2^* and λ_4^* , while λ_6^* does not have adjacent promising reference vectors. According to (10), $D^*_{\mathbf{x}_6} = \theta_{\mathbf{x}_2,\mathbf{x}_6}$, $D^*_{\mathbf{x}_7} = \theta_{\mathbf{x}_3,\mathbf{x}_7}$, and $D^*_{\mathbf{x}_8} = \theta_{\mathbf{x}_5,\mathbf{x}_8}$.

We design (10) due to the following empirical observations. For the solution whose associated reference vector has adjacent

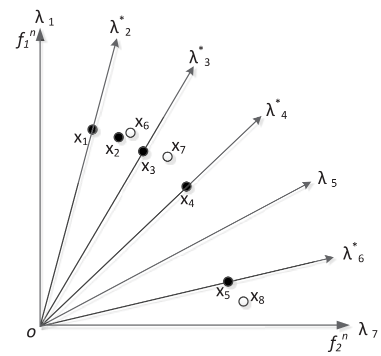

Fig. 2. Example for understanding the diversity measurement in promising subpopulations.

promising reference vectors, its diversity is not only depended on solutions around its own associated reference vector, but also influenced by the solutions around its adjacent promising reference vectors. For example, the diversity of $\mathbf{x}_6$ in Fig. 2 is not only decided by $\mathbf{x}_3$, but also $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$. Therefore, it is feasible to measure the diversity by (10), as it reflects the maximum similarity between $\mathbf{x}_6$ and the solutions having been selected, i.e., $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$. The solution with a larger D^* has smaller similarity with respect to the solutions which have been selected. In Fig. 2, $\mathbf{x}_7$ is better than $\mathbf{x}_6$ in terms of diversity, as $D^*_{\mathbf{x}_7} > D^*_{\mathbf{x}_6}$.

3) Sorting: First, we sort solutions in each promising subpopulation $P_G^{j^*}$, $j \in \Lambda^*$, respectively, in ascending order based on their convergence C. The solutions ranked in top r_j are copied to $E_G^{j^*}$, where r_j is calculated by (9).

4) Selection: We move the solutions with the maximum D^* values in each $E_G^{j^*}$, $j \in \Lambda^*$ to P_{G+1}. Steps 2–4 will iterate until N^* solutions have been selected.

D. Computational Complexity Analysis

In each generation of ASEA, the computational complexity is dominated by the environmental selection procedure: the complexity of objective space normalization is $O(m^2N)$ [28]. The subpopulation partition requires $O(mN^2)$ computationtions [29]. The calculation of C and D are included in subpopulation partition. The calculation of D^* requires $O(N^2)$ in the worst case. Both the sorting and selection processes require $O(N)$ computations in the worst case. Therefore, the overall computational complexity is $O(mN^2)$, which shows the computational efficiency of ASEA.

IV. Experimental Study

First, we investigate the performance of ASEA on general MaOPs by comparing with four state-of-the-art MOEAs: NSGAIII [8], MOEA/DD [30], θ-DEA [27], and RVEA [28] on DTLZ1-7 [57] and WFG1-9 [58] problems. Similar to ASEA, all four algorithms compared in this paper used the concept of reference vectors. However, NSGAIII used reference vectors only for selecting solutions in the last accepted level which was divided by Pareto dominance; MOEA/DD employed a "dominance-first and PBI aggregation function-second" strategy; θ-DEA and RVEA fused the convergence and diversity into scalarizing fitness evaluation functions;

while our ASEA deals with the convergence and diversity aspects separately without predefined parameters, and makes a more effective use of reference vectors, which are quite different from the above approaches.

Second, we examine ASEA$^+$ on MaOPs with irregular PFs through comparison with the original ASEA and the adaptive reference vector versions of NSGAIII and RVEA (i.e., A-NSGAIII and RVEA*) on DTLZ1-7^{-1} and WFG1-9^{-1} [48]. We also further study the environmental selection strategies of ASEA and ASEA$^+$. Finally, we extend ASEA to handle constrained MaOPs.

We employ the inverse generational distance (IGD) [65] to estimate the performance of algorithms. The IGD configuration and experimental parameter settings are provided in Sections S-I-B and S-I-A in the supplementary materials, respectively.

A. Performance on DTLZ and WFG Problems

Experimental results are summarized in Table I. ASEA wins 168 out of the 256 comparisons, demonstrating its efficiency in handling general MaOPs. Essential experimental findings are described below. A detailed analysis can be found in Section S-II-A in the supplementary material.

From Table I, ASEA obtains the best performance in 7 out of the 8 instances of DTLZ1 and DTLZ3, indicating its superiority in performance. Similar results are achieved on DTLZ2 and DTLZ4, which shows its promise in handling convex problems and maintaining diversity. For DTLZ7 which has a disconnected PF, ASEA is also competitive on high-dimensional cases. MOEA/DD shows its efficiency on DTLZ5 and DTLZ6 with 2-D curved degenerate PFs, while ASEA, θ-DEA, and RVEA, fail to achieve high-quality performance since they all have far few reference vectors that intersect the true irregular PFs.

In the WFG suit, ASEA is in leading position on 6 out of the 8 cases of WFG1 and WFG3, demonstrating its effectiveness on handling flat bias and mixed PF structures. However, for the disconnected PFs of WFG2, ASEA, θ-DEA, and RVEA are outperformed by NSGAIII and MOEA/DD on most cases. These results indicate the performance degradation when the distribution of reference vectors is not consistent with the PF shapes [48]. On WFG4-7, ASEA shows its outperformance on most cases, verifying its validity in escaping local optima and coping with various difficulties in decision spaces. On WFG8-9, ASEA wins one best result, and reaches acceptable values on the other scenarios. These results illustrate its ability on handling nonseparable problems.

B. Performance on DTLZ^{-1} and WFG^{-1} Problems

A-NSGAIII and RVEA* are decomposition-based algorithms tailoring for MaOPs with irregular PFs. Their population sizes are specially designed.[2] To make a fair comparison as much as possible, we set the parameter N^* in ASEA$^+$ equals to $N - |\textit{promising reference vectors}|$. Following this, the population size of ASEA$^+$ will be similar with that of RVEA* and will be slightly smaller than that of A-NSGAIII. In addition, in each generation of the original ASEA, we also select N^* additional solutions after line 9 of Algorithm 1, but with random selection.

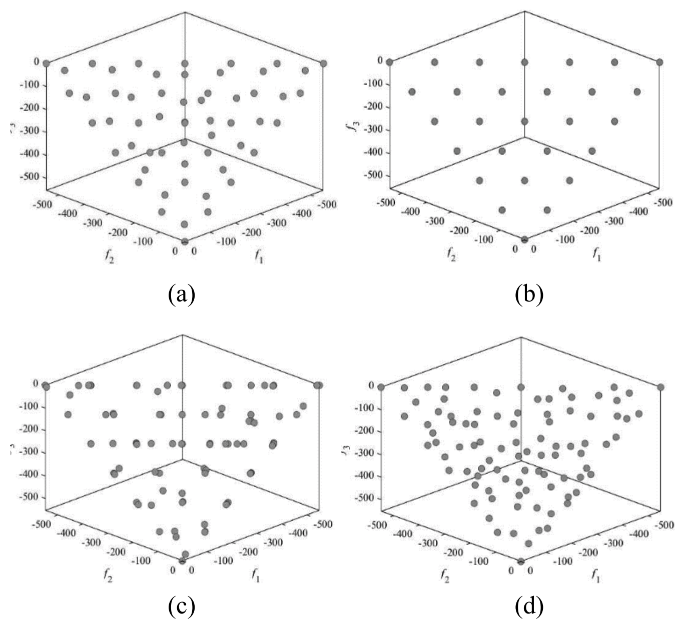

Fig. 3. Final Pareto nondominant solutions obtained by (a) ASEA$^+$, (b) ASEA, (c) A-NSGAIII, and (d) RVEA* on DTLZ1^{-1} with three objectives.

To visually demonstrate the performance of the promising subpopulation sorting-based selection strategy of ASEA$^+$, Fig. 3 is depicted. We can see from the figure that ASEA$^+$ obtains more solutions than the original ASEA due to the additional selection effort on promising subpopulations. Furthermore, the diversity of solutions of ASEA$^+$ is much better than that of A-NSGAIII and RVEA*. These observations illustrate the efficiency of the promising subpopulation sorting-based selection strategy.

To make a comprehensive investigation, we further compare ASEA$^+$ with the original ASEA, A-NSGAIII and RVEA* on DTLZ1-7^{-1} and WFG1-9^{-1} problems. The performance ranks of the algorithms are presented in Fig. 4. Detailed IGD values can be found in Table IV in the supplementary material. According to Fig. 4, the performance of ASEA$^+$ is much better than that of ASEA on all instances except for DTLZ7^{-1} with ten objectives. However, the result of ASEA$^+$ on this case is competitive compared with other algorithms. These results illustrate the superiority of the promising subpopulation sorting-based selection with respect to random selection. Furthermore, ASEA$^+$ obtains better or comparable performance on 51 out of the 64 instances compared with A-NSGAIII, and is significantly better than RVEA* on all cases. This performance demonstrates the effectiveness of ASEA$^+$ on MaOPs with irregular PFs.

C. Further Investigations of ASEA

1) Investigation of the Adaptive Sorting Scheme: In the adaptive sorting-based selection strategy of ASEA, the parameter r_j in (9) plays a key role to balance convergence

[2]The numbers of additional reference vectors in A-NSGAIII and RVEA* equal to the number of nonuseful vectors and the number of invalid vectors, respectively. So the population sizes in the next generation are $N + |\textit{nonuseful vectors}|$ and $N + |\textit{invalid vectors}|$, respectively. Note that *nonuseful* in A-NSGAIII means no associated solutions, while *invalid* in RVEA* means no associated nondominant solutions.

TABLE I
MEAN AND STANDARD DEVIATION OF IGD VALUES ON DTLZ1-7 AND WFG1-9 PROBLEMS. BEST RESULTS ARE SHADED

Problem	m	ASEA	NSGAIII	MOEA/DD	θ-DEA	RVEA
DTLZ1	4	4.1204e-2 (9.95e-6)	4.1214e-2 (2.16e-5) >	4.1205e-2 (5.37e-6) ≈	4.1208e-2 (1.93e-5) ≈	4.1205e-2 (1.20e-5) ≈
	6	8.0960e-2 (4.40e-4)	8.1182e-2 (1.00e-4) >	8.1091e-2 (1.15e-5) >	8.1140e-2 (9.93e-5) >	8.1095e-2 (2.74e-5) >
	8	9.6259e-2 (1.14e-2)	9.7320e-2 (6.77e-2) >	9.7112e-2 (2.59e-4) >	9.7727e-2 (9.68e-4) >	9.6883e-2 (4.16e-4) >
	10	1.0636e-1 (2.51e-2)	1.1528e-1 (1.34e-2) >	1.0923e-1 (5.38e-5) >	1.0767e-1 (3.66e-4) >	1.0830e-1 (5.37e-4) >
DTLZ2	4	1.2118e-1 (3.56e-6)	1.2119e-1 (1.19e-6) ≈	1.2119e-1 (6.51e-7) ≈	1.2118e-1 (7.61e-7) ≈	1.2119e-1 (1.08e-6) ≈
	6	2.5563e-1 (4.90e-5)	2.5579e-1 (1.20e-4) >	2.5568e-1 (5.51e-6) ≈	2.5569e-1 (2.39e-5) ≈	2.5569e-1 (1.64e-5) ≈
	8	3.1495e-1 (1.12e-4)	3.1492e-1 (6.77e-3) ≈	3.1488e-1 (1.08e-5)	3.1488e-1 (1.62e-5)	3.1489e-1 (1.14e-5)
	10	4.2595e-1 (1.02e-2)	4.4725e-1 (5.46e-2) >	4.2193e-1 (9.55e-5)	4.2070e-1 (2.08e-4)	4.2120e-1 (3.03e-4)
DTLZ3	4	1.2123e-1 (1.46e-4)	1.2156e-1 (1.84e-4) >	1.2161e-1 (4.97e-4) >	1.2154e-1 (5.16e-4) >	1.2132e-1 (1.17e-4) ≈
	6	2.5532e-1 (3.73e-2)	2.5825e-1 (1.31e-3) >	2.5631e-1 (5.37e-4) >	2.5648e-1 (3.40e-4) >	2.5611e-1 (3.30e-4) >
	8	5.3858e-1 (2.10e-1)	9.3856e-1 (6.77e-2) >	5.7519e-1 (3.56e-1) >	8.8017e-1 (1.23e+0) >	7.8886e-1 (4.28e-1) >
	10	4.5207e-1 (5.10e-2)	5.8568e-1 (3.17e-1) >	4.2137e-1 (5.09e-4)	4.2113e-1 (1.10e-3)	4.1991e-1 (7.32e-4)
DTLZ4	4	1.2118e-1 (4.61e-6)	2.2157e-1 (1.62e-1) >	1.2119e-1 (3.01e-7) ≈	2.2197e-1 (1.62e-1) >	1.2119e-1 (2.19e-6) ≈
	6	2.5685e-1 (4.96e-4)	2.5589e-1 (1.87e-4)	2.8947e-1 (7.54e-2) >	2.5592e-1 (2.37e-4)	2.5584e-1 (3.11e-4)
	8	3.1666e-1 (8.29e-4)	3.1489e-1 (6.77e-2)	3.3853e-1 (5.29e-2) >	3.1489e-1 (4.12e-5)	3.3736e-1 (5.02e-2) >
	10	4.1953e-1 (1.90e-3)	4.5783e-1 (7.91e-2) >	4.2142e-1 (2.94e-4) >	4.2008e-1 (5.75e-4) >	4.2058e-1 (4.49e-4) >
DTLZ5	4	1.4032e-1 (4.94e-2)	5.1846e-2 (7.22e-3)	7.9168e-2 (2.78e-3)	1.4673e-1 (1.02e-2) >	1.9750e-1 (5.47e-3) >
	6	1.9097e-1 (3.44e-2)	2.9082e-1 (1.97e-2) >	1.1817e-1 (1.05e-2)	2.6019e-1 (1.04e-1) >	3.9221e-1 (2.16e-1) >
	8	2.4052e-1 (7.53e-2)	2.9643e-1 (8.22e-2) >	1.5319e-1 (1.88e-2)	2.1158e-1 (2.54e-2)	2.6835e-1 (7.12e-2) >
	10	3.4123e-1 (4.27e-2)	6.3318e-1 (1.50e-1) >	1.4274e-1 (9.91e-3)	1.3820e-1 (4.35e-2)	2.4530e-1 (5.19e-2)
DTLZ6	4	2.1033e-1 (3.70e-2)	8.1725e-2 (1.30e-2)	7.0160e-2 (6.04e-3)	2.1493e-1 (9.50e-2) >	2.1317e-1 (3.25e-2) >
	6	2.4052e-1 (2.95e-2)	3.9366e-1 (1.20e-1) >	1.2235e-1 (9.40e-3)	2.6294e-1 (7.01e-2) >	2.3011e-1 (1.95e-2)
	8	2.3017e-1 (1.39e-2)	1.1817e+0 (6.00e-1)	1.7249e-1 (3.49e-2)	2.8295e-1 (7.42e-2) >	2.4803e-1 (1.43e-1) >
	10	2.9719e-1 (1.15e-1)	1.2329e+0 (1.24e+0) >	1.4142e-1 (1.84e-2)	2.9076e-1 (5.63e-2)	2.9560e-1 (6.29e-2)
DTLZ7	4	2.8936e-1 (1.15e-2)	2.2157e-1 (8.28e-2)	1.0421e+0 (7.15e-1) >	2.6011e-1 (2.63e-3)	2.7652e-1 (2.97e-3)
	6	5.3626e-1 (7.92e-2)	5.7347e-1 (2.88e-2) >	1.4411e+0 (5.49e-1) >	4.9725e-1 (2.76e-2)	7.8439e-1 (1.24e-2) >
	8	7.4445e-1 (2.41e-1)	8.0157e-1 (2.01e-2) >	1.6761e+0 (3.71e-1) >	8.6815e-1 (1.12e-1) >	1.1605e+0 (1.06e-1) >
	10	9.4718e-1 (1.55e-1)	9.8918e-1 (6.73e-2) >	2.3504e+0 (2.55e-1) >	1.1654e+0 (1.81e-1) >	2.2154e+0 (5.75e-1) >

Continued on the next page.
>: The result of ASEA is significantly better than the outcome according to Wilcoxon Sign Test [66] at a 0.05 significance level, hereafter.
≈: The result of ASEA is comparable with the outcome according to Wilcoxon Sign Test at a 0.05 significance level, hereafter.

and diversity. To verify the benefits of this adaptive sorting scheme, we build four representative schemes with different features for comparison

$$r_j = 1 \tag{11}$$

$$r_j = |P_G^j| \tag{12}$$

$$r_j = \left\lceil \frac{G}{G_{\max}} \cdot |P_G^j| \right\rceil \tag{13}$$

$$r_j = \left\lceil \left\{ \ln\left[\left(\frac{G}{G_{\max}} \right)^{\frac{1}{2}} \right] \cdot (e-1) + 1 \right\} \cdot |P_G^j| \right\rceil. \tag{14}$$

Following (11), the environmental selection is based on convergence only, while according to (12), the environmental selection is totally based on diversity. Equations (13) and (14) are adaptive schemes: from Fig. 5 we can see that compared with (13), the original scheme (9) lies more emphasis on convergence; on the contrary, (14) makes more effort on diversity.

We compare ASEA with the ASEA versions adopting the above r_j schemes on DTLZ1-4 and WFG1-9 problems and compare ASEA$^+$ with the ASEA$^+$ versions employing these schemes on DTLZ1-4^{-1} and WFG1-9^{-1} problems. The population size of ASEA$^+$ versions is listed in Section IV-B. IGD results are shown in Tables V and VI in the supplementary material. According to the results, ASEA and ASEA$^+$ are the best or comparable (best on 84 out of the 104 test cases). Noticeably, for DTLZ2, ASEA is slightly worse than the version with (12) and (14) on 6- and 10-objective cases, because it is easy to converge to the true PFs for such general convex problems. Similar results are obtained for DTLZ2^{-1}. For several high-dimensional WFG^{-1} instances, ASEA$^+$ is slightly worse than the versions with (13) and (14) which lay more emphasis on diversity. The reason may be that solutions are very sparse in such high-dimensional objective spaces, having a focus on convergence at the early stage of evolution may limit the diversity of solutions. However, the performance of ASEA$^+$ with (9) is still better than the results of the state-of-the-art algorithms as can be seen from Section IV-B (IGD results of these algorithms can be found in Table IV in the supplementary material). To summarize, we can conclude that the adaptive schemes of ASEA and ASEA$^+$ are overall effective and robust in different optimization scenarios.

2) Study of the Promising Subpopulation Sorting-Based Selection on General DTLZ and WFG Problems: We have demonstrated the efficiency of the promising subpopulation sorting-based selection strategy on DTLZ^{-1} and WFG^{-1} problems in Section IV-B. Here, we compare ASEA$^+$ with ASEA on general DTLZ and WFG problems. This further helps us

TABLE I
CONTINUED

Problem	m	ASEA	NSGAIII	MOEA/DD	θ-DEA	RVEA
WFG1	4	2.8522e-1 (7.68e-3)	2.9252e-1 (6.57e-3) >	1.6919e+0 (3.43e-3) >	2.9920e-1 (1.05e-2) >	2.9521e-1 (7.95e-3) >
	6	6.3373e-1 (5.21e-1)	6.5988e-1 (3.52e-2) >	4.9534e+0 (3.85e-2) >	5.9987e-1 (6.56e-2)	6.5946e-1 (1.10e-1) >
	8	8.4672e-1 (4.45e-2)	8.5427e-1 (4.43e-2) >	1.1235e+0 (1.10e-1) >	1.0429e+0 (4.48e-2) >	1.0333e+0 (1.41e-1) >
	10	9.7797e-1 (2.01e-2)	1.0189e+0(3.95e-2) >	1.5234e+1 (3.04e-2) >	1.1404e+0 (3.74e-2) >	1.2150e+0 (6.13e-2) >
WFG2	4	4.1312e-1 (4.23e-2)	4.1821e-1 (1.43e-2) >	3.9991e-1 (5.80e-3)	5.5118e-1 (3.02e-3) >	3.9710e-1 (1.28e-2) >
	6	9.9057e-1 (2.67e-1)	8.6060e-1 (1.68e-1)	1.0631e+0 (4.40e-2) >	1.3891e+0 (1.50e-2) >	1.8088e+0 (2.80e-1) >
	8	3.2290e+0 (6.89e-1)	5.5041e+0 (1.98e+0) >	7.4521e+0 (8.98e-2) >	3.5070e+0 (1.42e+0) >	4.7988e+0 (7.87e-1) >
	10	3.4477e+0 (1.19e+0)	5.2180e+0(1.97e+0) >	2.6025e+0 (6.67e-2)	2.6898e+0 (2.10e-1)	6.8267e+0 (2.38e+0) >
WFG3	4	2.9741e-1 (5.23e-2)	2.6213e-1 (3.13e-2)	6.6048e-1 (2.42e-3) >	2.9034e-1 (1.43e-2)	3.0919e-1 (3.14e-2) >
	6	6.1321e-1 (3.15e-1)	8.7051e-1 (8.55e-2) >	1.9166e+0 (2.43e-2) >	8.1573e-1 (1.32e-1) >	9.3018e-1 (7.70e-2) >
	8	1.3040e+0 (1.45e-1)	2.7898e+0 (1.98e+0) >	1.9514e+0 (3.02e-2) >	1.8905e+0 (3.76e-1) >	3.9115e+0 (1.06e+0) >
	10	9.2775e-1 (1.40e-1)	2.2155e+0 (6.34e-1) >	6.3967e+0 (1.22e-1) >	1.2202e+0 (2.77e-1) >	3.1099e+0 (2.02e-1) >
WFG4	4	6.0759e-1 (4.54e-4)	6.0764e-1 (1.03e-4) ≈	6.4286e-1 (2.14e-3) >	6.0781e-1 (2.86e-4) >	6.0850e-1 (8.53e-4) >
	6	1.7466e+0 (3.33e-3)	1.7482e+0 (1.32e-3) ≈	1.8909e+0 (3.05e-2) >	1.7480e+0 (1.85e-3) ≈	1.7491e+0 (3.09e-3) >
	8	2.9629e+0 (6.71e-2)	3.0380e+0 (1.88e-1) >	4.5226e+0 (1.42e-1) >	2.9742e+0 (1.33e-3) >	3.0028e+0 (3.95e-2) >
	10	4.4410e+0 (1.82e-2)	4.5429e+0 (4.39e-3) >	6.5032e+0 (1.36e-1) >	4.5432e+0 (8.34e-3) >	4.4321e+0 (1.90e-2)
WFG5	4	6.0289e-1 (1.04e-4)	6.0308e-1 (8.71e-5) >	6.5393e-1 (2.22e-3) >	6.0302e-1 (1.07e-4) ≈	6.0379e-1 (2.94e-4) >
	6	1.7313e+0 (2.45e-3)	1.7314e+0 (5.46e-4) ≈	1.9085e+0 (2.47e-2) >	1.7335e+0 (3.20e-4) >	1.7326e+0 (8.17e-4) ≈
	8	2.9596e+0 (9.10e-3)	3.0380e+0 (1.88e-1) >	3.7720e+0 (1.02e-1) >	2.9406e+0 (4.07e-4)	2.9632e+0 (1.62e-2) >
	10	4.5546e+0 (3.60e-2)	4.5294e+0 (2.93e-3)	6.5872e+0 (8.29e-2) >	4.5280e+0 (1.80e-3)	4.4155e+0 (2.00e-2)
WFG6	4	6.0439e-1 (1.36e-3)	6.0532e-1 (1.56e-3) >	6.6040e-1 (2.98e-3) >	6.0542e-1 (2.85e-3) >	6.0592e-1 (1.40e-3) >
	6	1.7271e+0 (1.83e-2)	1.7321e+0 (5.13e-3) >	1.9022e+0 (1.30e-2) >	1.7397e+0 (2.79e-3) >	1.7388e+0 (9.36e-3) >
	8	2.9371e+0 (3.68e-1)	2.9393e+0 (2.62e-3) >	3.2253e+0 (2.17e-1) >	2.9382e+0 (1.46e-3) ≈	3.0907e+0 (1.95e-1) >
	10	4.5516e+0 (3.81e-2)	4.5483e+0 (9.99e-3)	6.1131e+0 (1.17e-1) >	4.5367e+0 (4.32e-3)	4.4100e+0 (2.85e-2)
WFG7	4	6.0789e-1 (2.57e-4)	6.0840e-1 (3.46e-4) >	6.8598e-1 (7.47e-3) >	6.0817e-1 (2.02e-4) >	6.1046e-1 (5.20e-4) >
	6	1.7473e+0 (3.14e-3)	1.7476e+0 (1.56e-3) ≈	1.8866e+0 (1.81e-2) >	1.7513e+0 (4.71e-4) >	1.7496e+0 (3.96e-3) >
	8	2.9628e+0 (8.42e-3)	2.9653e+0 (5.04e-3) >	3.2543e+0 (1.95e-1) >	2.9633e+0 (7.25e-3) ≈	3.0018e+0 (3.49e-2) >
	10	4.5605e+0 (2.25e-2)	4.5704e+0 (7.63e-2) >	6.2083e+0 (2.66e-1) >	4.5450e+0 (5.96e-3)	4.4547e+0 (2.31e-2)
WFG8	4	6.3688e-1 (8.94e-3)	6.3359e-1 (7.41e-3)	6.4146e-1 (6.50e-3) >	6.3331e-1 (7.55e-3)	6.2984e-1 (1.70e-3)
	6	1.7342e+0 (2.83e-3)	1.7252e+0 (9.95e-3)	1.8663e+0 (3.07e-2) >	1.7155e+0 (2.00e-3)	1.7438e+0 (1.09e-2) >
	8	3.2639e+0 (4.15e-2)	3.2087e+0 (4.10e-2)	3.9171e+0 (2.54e-1) >	3.0847e+0 (5.56e-2)	3.0979e+0 (6.55e-2)
	10	4.3946e+0 (2.35e-1)	4.7604e+0 (3.88e-1) >	6.3350e+0 (2.47e-1) >	4.3462e+0 (5.82e-2)	4.4019e+0 (5.72e-2) ≈
WFG9	4	5.9382e-1 (1.42e-3)	5.9440e-1 (2.71e-3) >	1.6919e+0 (3.43e-3) >	6.0233e-1 (2.79e-3) >	5.9629e-1 (1.70e-3) >
	6	1.6986e+0 (6.90e-3)	1.6898e+0 (4.56e-3)	4.9534e+0 (3.85e-2) >	1.6902e+0 (5.68e-3)	1.7113e+0 (4.60e-3) >
	8	2.9419e+0 (1.08e-2)	2.9441e+0 (3.56e-2) >	3.9129e+0 (1.62e-1) >	2.9279e+0 (6.41e-3)	2.9912e+0 (2.61e-2) >
	10	4.4232e+0 (4.40e-2)	4.4415e+0 (2.91e-2) >	1.5234e+1 (3.04e-2) >	4.4352e+0 (1.58e-2) >	4.3130e+0 (3.90e-2)

in understanding the performance of the promising subpopulation sorting-based selection on problems with regular PFs (i.e., DTLZ1-4 and WFG1-9) and with irregular PFs (i.e., DTLZ5-7). For fair comparison, in each generation of ASEA$^+$, we select at most one solution in each subpopulation in line 9 of Algorithm 1 and select $N^* = N - |P_{G+1}|$ solutions in line 10 of Algorithm 1, such that the population size of ASEA$^+$ will be N, which equals to that of ASEA.

IGD results of the experiment are summarized in Table VII in the supplementary material. According to the results, ASEA$^+$ obtains better or comparable performance with respect to ASEA on 30 out of the 52 cases of DTLZ1-4 and WFG1-9 problems. These results illustrate that the promising subpopulation sorting-based selection strategy is an effective secondary selection criterion after the general selection strategy (i.e., after line 9 of Algorithm 1). This supports the view that the promising subpopulation sorting-based selection can maintain a much better partial diversity in each subpopulation than the random selection in the last accepted level (i.e., lines 14–16 of Algorithm 2) during early stages of evolution. For DTLZ5-7 with irregular PFs, ASEA$^+$ is constantly better than ASEA, and is also better than most of the state-of-the-art algorithms as presented in Section IV-A (refer to Table I). This further demonstrates the performance of the promising subpopulation sorting-based selection on irregular PFs.

D. ASEA on Constraints Handling

In this section, we extend ASEA to solve constrained MaOPs by employing the constraint violation [67], [68]. We examine the performance of ASEA by comparing it with the constraints handling versions of NSGAIII and RVEA on C1-DTLZ1, C2-DTLZ2, and C3-DTLZ4 [25] problems. The procedure of constraint handling employed in ASEA and the experimental results on the above listed problems are presented in Section S-II-C in the supplementary material.

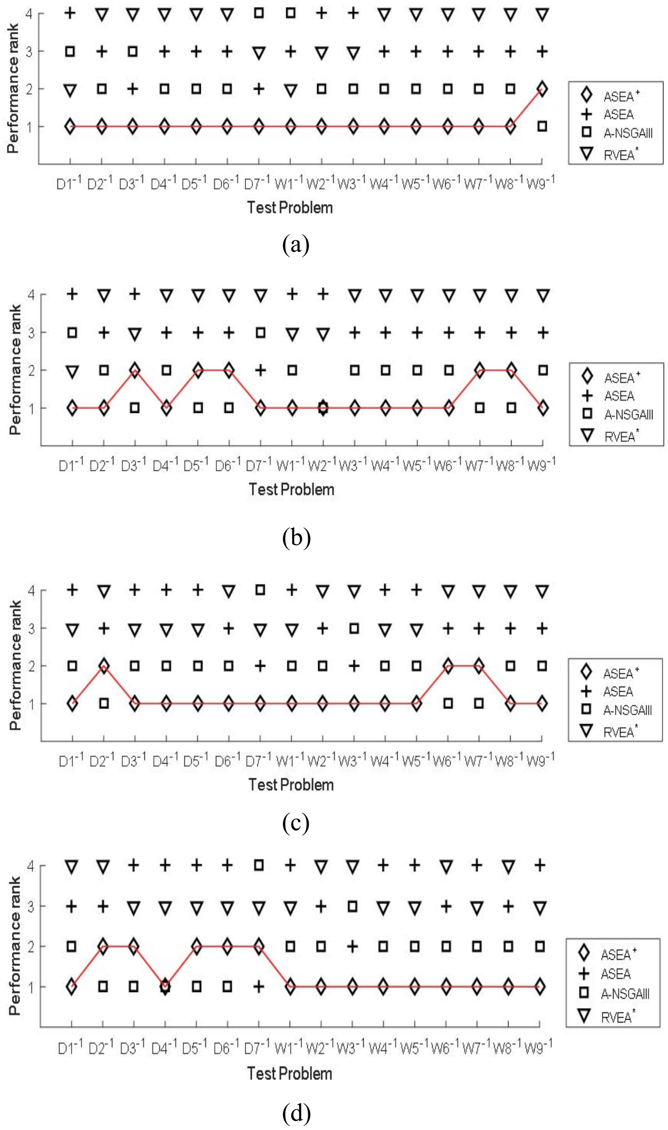

Fig. 4. Average IGD performance ranks on DTLZ^{-1} (Dx^{-1}) and WFG^{-1} (Wx^{-1}) problems. The smaller the rank, the better performance the algorithm has. (a) $m = 4$. (b) $m = 6$. (c) $m = 8$. (d) $m = 10$.

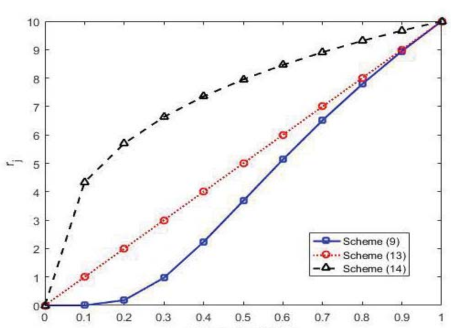

Fig. 5. Three r_j adaptive schemes versus generations in case of $|P_G^j| = 10$.

V. CONCLUSION

In this paper, an ASEA is proposed for MaOPs. The proposed ASEA includes two major innovations: first, an adaptive sorting-based selection strategy is developed for selecting solutions in each subpopulation partitioned by uniformly distributed reference vectors. Solutions in each subpopulation are sorted according to their convergence and diversity information, respectively, where ones in higher sorting levels are selected preferentially. This strategy tackles the convergence and diversity of solutions separately rather than fusing these features into single scalarizing functions, which does not need to consider the relative proportion of convergence and diversity information of solutions. At the same time, it is adaptive during the evolution process without predefined parameters, which guarantees its effectiveness and robustness. Second, to solve MaOPs which may have irregular PFs, an adaptive promising subpopulation sorting-based strategy is proposed. It allocates additional sorting-based selection effort on promising subpopulations without adding reference vectors. This strategy significantly improves the number and diversity of solutions. In addition, a constraint handling technique is also incorporated into ASEA to extend its ability to deal with constrained MaOPs.

The performance of the proposed algorithm is investigated by comparing its performance with four other state-of-the-art algorithms on DTLZ, DTLZ^{-1}, WFG and WFG^{-1} test problems with 4–10 objectives. Results of ASEA and ASEA^{+} on unconstrained and constrained MaOPs with general and irregular PFs clearly highlight the benefits of the algorithms. Therefore, it can be concluded that the proposed algorithm is very competitive on MaOPs.

While this research focused on environmental selection, there are still challenges in developing effective recombination strategies, management of dominance resistant solutions, and accounting for the scalability in the variable space.

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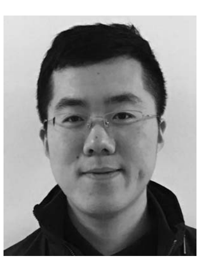

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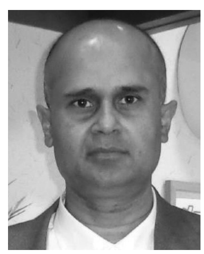

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