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Efficient Aerodynamic Optimization of Propeller using Hierarchical Kriging Models

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Abstract. The number of aerodynamic analysis in the design optimization of propeller has been reduced significantly by using the surrogate model, such as kriging. In this paper, a more efficient aerodynamic design optimization method of propeller is proposed by using the HK (hierarchical kriging) model. The high-fidelity model is defined as a RANS (Reynolds-Averaged Navier-Stokes) simulation for propeller and the low-fidelity model is defined as the results by blade-element/vortex theory. Initial samples are selected for two levels of fidelity via LHS (Latin Hypercube Sampling). The aerodynamic performance of varying fidelity is used to build variable-fidelity surrogate models for functions of objective (e.g. thrust) and constraint (e.g. shaft power). Infill sampling criteria, including MSP (minimizing of surrogate prediction), EI (expected improvement), PI (probability of improvement), LCB (lower-confidence bounding) and MSE (mean-squared error) are used to obtain new samples, and the surrogate models are repetitively updated until a global optimum is found. High-altitude propellers are optimized by the kriging model and hierarchical kriging model, respectively. Compared to kriging model, the number of RANS solving by the hierarchical kriging model is reduced 37.5%, and the optimization time is reduced 24.3%. The results have shown that the proposed design method for propeller can significantly improve the optimization efficiency

1. Introduction

Due to its high efficiency, propeller is widely used on transport aircrafts, UAVs (Unmanned Aerial Vehicles) and high-altitude airships. Many methods to calculate aerodynamic performance of propeller have been proposed, such as blade-element theory[1], vortex theory[2], RANS (Reynolds-Averaged Navier-Stokes) method[3] and hybrid RANS/LES (Large Eddy Simulation)[4] method. Blade-element theory and vortex theory can give the acceptable results in seconds, thus they are still widely used in aerodynamic analysis and design of propeller today. RANS method has been widely used in engineering design, while hybrid RANS/LES method has not been used due to the very time consuming.

The time spent in a single RANS calculation is acceptable. However, when RANS is used to design optimization, many RANS calculations are necessary, making the design process very time consuming. For example, GA (Genetic Algorithm)[5], a global optimization method, usually needs thousands of RANS calculations. Even if each step of the optimization iteration performs the RANS calculations of many samples in parallel, this problem cannot be solved well due to that optimization iteration cannot be paralleled. Gradient optimization method can significantly reduce the number of RANS

calculations, but it is easy to fall into local optimization, and also optimization iteration cannot be paralleled. The SBO (surrogate-based optimization) method[6], which has received widely attention in recent years, inherits the global characteristics of the genetic algorithm and greatly reduces the number of RANS calculations. The prediction accuracy of the constructed surrogate model even directly determines the number of RANS calculations. More accurate surrogate model is expected using the limited RANS data, then a better optimal solution is obtained.

The objective of this study is to propose a more efficient design method of Propeller using hierarchical kriging models. The remainder of this paper is organized as follows. The aerodynamic analysis methods of propellers are introduced, including ROTNS, the in-house RANS solver for propeller and QPROP using the blade-element/vortex method. Then, the aerodynamic optimization design method using kriging and hierarchical kriging models is presented, and also the parameterization of propeller. In section of results and discussion, the blade shape of high-altitude propellers are optimized by kriging model and hierarchical kriging model, respectively. The last section is for the conclusions.

2. Aerodynamic analysis methods of propellers

In this section, two different aerodynamic analysis methods of propellers are used. High-fidelity data are obtained by RANS solver for propeller and low-fidelity data are obtained by QPROP based on blade-element/vortex theory.

2.1. RANS solver for propeller

The integral form of Reynolds-averaged Navier-Stokes (RANS) equations expressed in the blade-attached rotational frame can be written as follows:

$$\frac{\partial}{\partial t}\iiint_{\Omega} \mathbf{Q} dV + \iint_{\partial\Omega} \mathbf{F} \bullet \mathbf{n} dS - \iint_{\partial\Omega} \mathbf{F}_v \bullet \mathbf{n} dS + \iiint_{\Omega} \mathbf{G} dV = 0 \tag{1}$$

where, t is time, Ω and $\partial\Omega$ stand for control volume and the corresponding boundary, respectively. dS and dV are elemental area and volume of the control volume, respectively. $\mathbf{n}$ is the outward unit normal. $\mathbf{Q}$ is the conservation fluid variables. $\mathbf{F}$ and $\mathbf{F}_v$ are the inviscid and viscous flux term, respectively. $\mathbf{G}$ is the Coriolis force term.

The cell-centred finite-volume method proposed by Jameson[7] is used to solve the above governing equations on the chimera grid (Figure 1 and Figure 2), and an in-house RANS solver, named ROTNS is developed. An improved Newton-like LU-SGS method[8] is utilized for time stepping, and a high-efficient FAS multi-grid method[8] on Chimera grid is developed to improve the computational efficiency. Jameson's central scheme (JST)[7] is adopted for the spatial discretization scheme. Spalart-Allmaras (S-A)[9] one-equation model is used for turbulence enclosure.
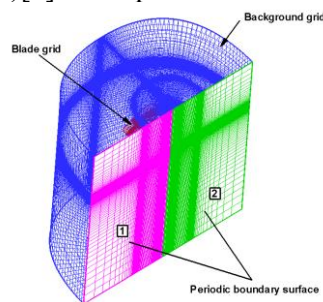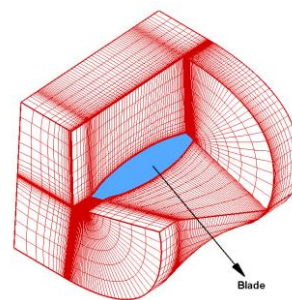

Figure 1. Chimera grid system.   **Figure 2.** Cutaway view of the blade grid.

2.2. QPROP

QPROP is an analysis program for predicting the performance of propeller-motor combinations developed by Prof. Drela[10]. It has a relatively sophisticated and accurate prop aerodynamic model. The propeller is modelled with an advanced blade-element/vortex method, which is essentially a considerably enhanced version of the analysis method of Larrabee[11][12].

2.3. Validation of aerodynamic analysis methods

Designed by Wichita State University and tested in their 3ft × 4ft Low Speed Wind Tunnel[13], the 2-bladed propeller with a diameter of 6 inches is modelled to validate the two methods. The Reynolds number based on chord at 75% of the blade radius is within the range of 90,000 to 120,000 when the rotational speed is fixed at 6000rpm. Figure 3 shows the propeller shape with a blade pitch angle of 15°. Figure 4 demonstrates that results of the thrust coefficient, power coefficient and efficiency against the advance ratio by both ROTNS and QPROP are in good agreement with the experimental results.
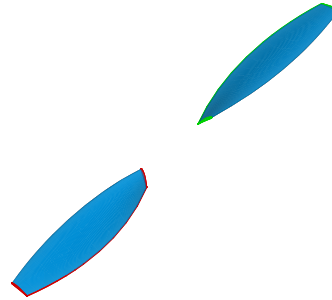

Figure 3. The 2-bladed propeller with a diameter of 6 inches and a blade pitch angle of 15°.

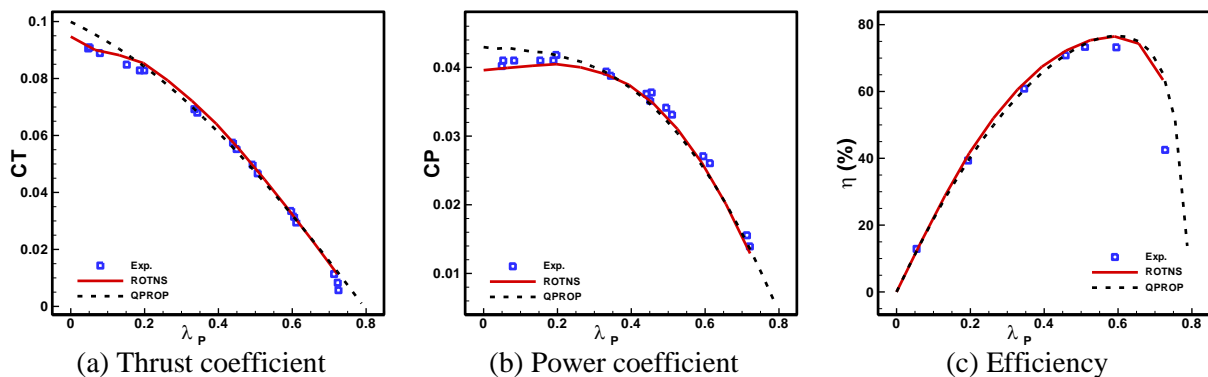| (a) Thrust coefficient | (b) Power coefficient | (c) Efficiency |

Figure 4. Results of the thrust coefficient, power coefficient, and efficiency against the advance ratio by ROTNS and QPROP are in good agreement with the experimental results[13]

3. Aerodynamic optimization design method of propellers

3.1. Optimization based on kriging model and hierarchical kriging model

Based on an in-house surrogate-based optimizer (as shown in Figure 5), "SurroOpt"[14][15], the aerodynamic design platform for the propeller is established. Kinds of surrogate models are developed in this optimizer for different problems. These surrogate models include response surface model (RSM), kriging model[16], hierarchical kriging (HK) model[17][18], gradient-enhanced kriging (GEK) model[19], weighted gradient-enhanced kriging (WGEK) model[20], etc.

For design optimization based on kriging model, ROTNS is used to obtain the aerodynamic data, and infill sampling criteria, including MSP (minimizing of surrogate prediction), EI (expected improvement), PI (probability of improvement), LCB (lower-confidence bounding) and MSE (mean-squared error), are used to obtain new samples, and the surrogate models are repetitively updated until a global optimum is found.

For design optimization based on hierarchical kriging model, QPROP is used as low-fidelity (low-fi) analysis and ROTNS is used as high-fidelity (hi-fi) analysis. Low-fidelity kriging models for thrust and shaft power are built based low-fidelity data. Then, hierarchical kriging models are built based on low-fidelity kriging models and high-fidelity data. MSP, EI, PI, LCB and MSE are also used as the

infill sampling criteria to obtain new high-fidelity samples, and the surrogate models are repetitively updated until a global optimum is found. It is noted that QPROP is significantly more efficient than ROTNS, thus the number of samples by QPROP can be much more than that of ROTNS. Figure 6 shows the flowchart of optimization based on kriging model and hierarchical kriging model.
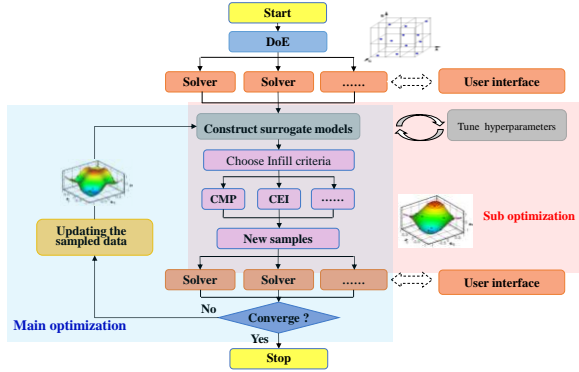

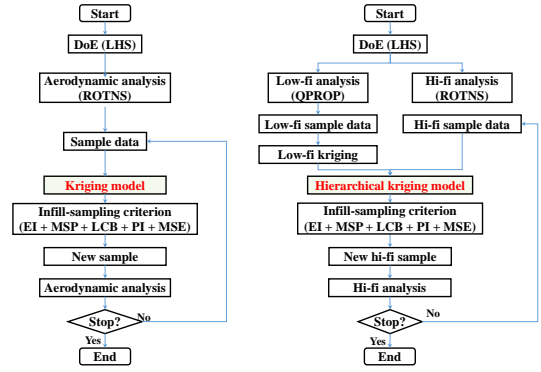**Figure 5.** Framework of the efficient surrogate-based optimizer: SurroOpt.

Figure 6. Optimization based on kriging model and hierarchical kriging model.

3.2. Parameterization of propeller

In present work, the sectional airfoils of propeller are fixed. The design variables are distributions of chord and twist, as shown in Figure 7 (a). These distributions are parameterized by an airfoil parameterization method, named CST (Class-Shape function Transformation)[21][22]. The distribution functions should be normalized, as shown in Figure 7 (b).
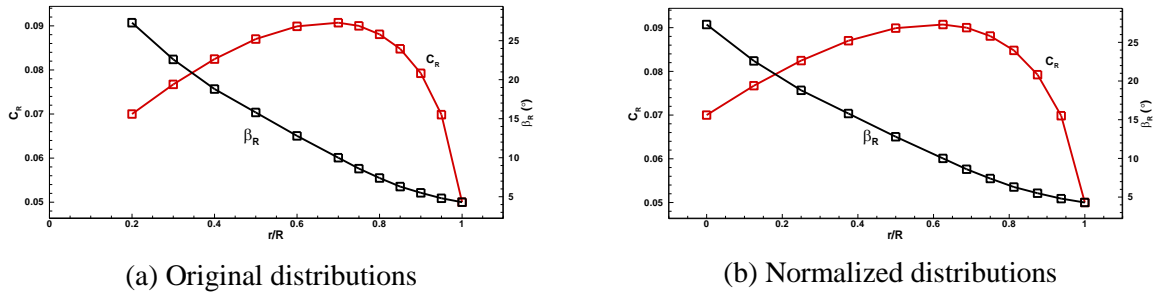

(a) Original distributions

(b) Normalized distributions

Figure 7. Parameterization of propeller.

The functions of distributions are written as:

$$y = C(x) \bullet S(x) + x \bullet C(1) + (1-x) \bullet C(0) \tag{2}$$

where, y is chord or twist, and

$$C(x) = x^{N1}(1-x)^{N2}$$

$$S(x) = \sum_{i=0}^{N} A_i \bullet S_i(x) \tag{3}$$

$$S_i(x) = \frac{N!}{i!(N-i)!} x^i (1-x)^{N-i}$$

where, C(x) is called class function and S(x) is called shape function. Si(x) is Bernstein polynomial with the order of N. Thus, the design variables are: C(1), C(0), N1, N2 and Ai (i=0~N). C(1) denotes chord or twist at blade tip, C(0) denotes chord or twist at blade root. The number of design variables

for chord distribution and twist distribution are the same each other. Thus, the total number of design variables is: (N+5)×2. In this study, N = 7, therefore the total number of design variables is 24.

4. Results and discussions
The 2-bladed propeller with a diameter of 7m are optimized at the air speed of 10 m/s, and the rotational speed of 370 rpm. The shaft power is constrained by 10 kW. The optimization model is described as

$$\begin{aligned} Maximize: \quad & Thrust \\ s.t. \quad & Shaft\ power \leq 10\ kW \end{aligned} \tag{4}$$

The blade shape of the high-altitude propellers are optimized by kriging model and hierarchical kriging model, respectively. The initial number of high-fidelity (ROTNS) sample is 48, and increases to a maximum value of 150 by infill sampling criteria. It is noted that the number of low-fidelity (QPROP) sample is fixed at 200 in the process of optimization design.

It is obvious that optimization using hierarchical kriging (HK) model has a faster convergence than that of kriging model, as shown in Figure 8. In addition, the optimized maximum thrust is almost the same. It is also shown in Figure 9 and Figure 10 that the optimized distribution of chord and twist are almost the same except the inboard of the blade, resulting to a similar geometry of the optimized blade. Table 1 shows a comparison of design efficiency using two surrogate models, where the thrust is optimized to 553.9 N. The construction of hierarchical kriging model consumes much more time than that of kriging model. However, the number of ROTNS calling is significantly reduced 37.5%, using hierarchical kriging model. The time consuming to call ROTNS is much more than that of surrogate model construction and QPROP. Thus, the total time of design optimization is reduced 17.4%.
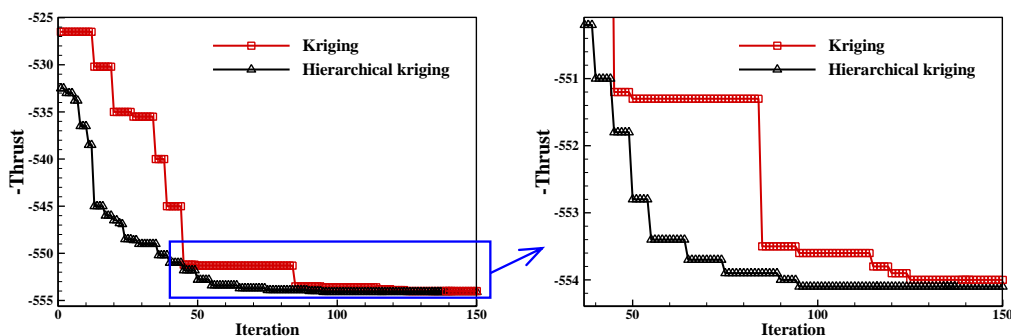

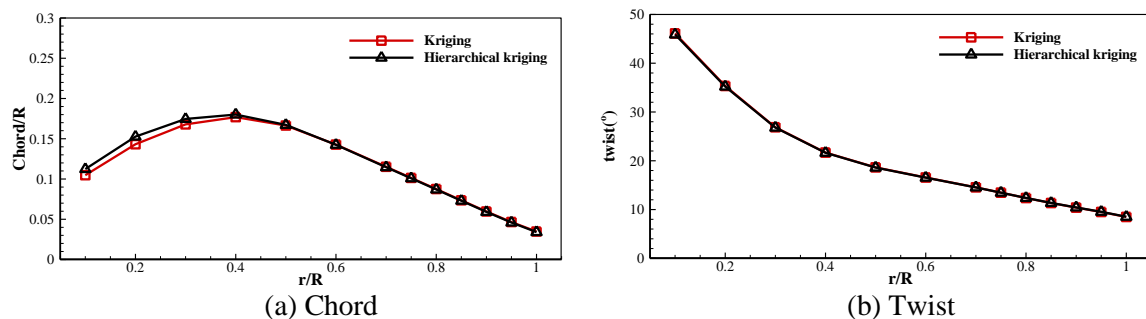**Figure 8.** History of design optimization using kriging model and hierarchical kriging model.

(a) Chord   (b) Twist
Figure 9. Comparison of the optimized distributions of chord and twist

5. Summary
Kriging model and hierarchical kriging model are applied to optimize aerodynamic shape of propeller. The design optimization efficiency is significantly improved using hierarchical kriging model. The

optimized shape of propeller are almost the same using the two surrogate models. Although the construction of hierarchical kriging model consumes much more time than that of kriging model, the design optimization using hierarchical kriging model behaves much more efficient.

Table 1. Comparison of design efficiency using kriging model and hierarchical kriging model.

	Kriging model	HK model	Δ
Thrust (N)	553.9	553.9	0
Time of surrogate model construction (h)	0.53	4.59	+4.06
No. of ROTNS	120	75	-45 (37.5%)
Time of total optimization (h)	144	109	-35 (-24.3%)
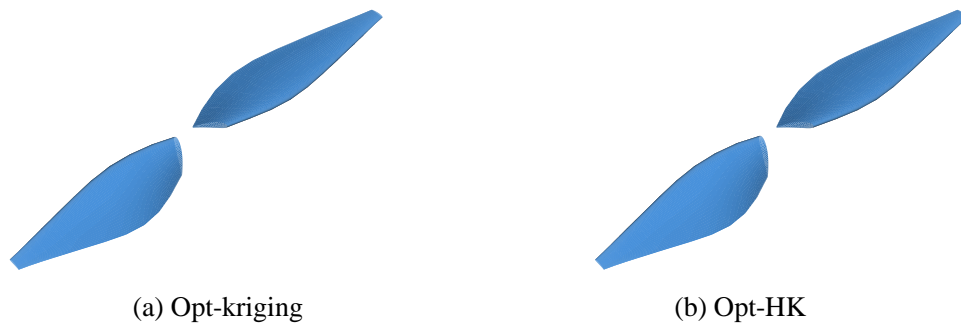
(a) Opt-kriging   (b) Opt-HK
Figure 10. Comparison of the optimized shape

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