$I_{\mathrm{SDE}}{}^{+}$—An Indicator for Multi and Many-Objective Optimization

Trinadh Pamulapati, Rammohan Mallipeddi, and Ponnuthurai Nagaratnam Suganthan, *Fellow, IEEE*

***Abstract*—In this letter, an efficient indicator for multi and many-objective optimization is proposed. The proposed indicator ($I_{\mathrm{SDE}}{}^{+}$) is a combination of sum of objectives and shift-based density estimation and benefits from their ability to promote convergence and diversity, respectively. An evolutionary multiobjective optimization framework based on the proposed indicator is shown to perform comparably or better than the state-of-the-art on a variety of scalable benchmark problems.**

***Index Terms*—Indicator-based, many-objective, multiobjective, shift-based density estimation (SDE), weighted sum (WS) of objectives.**

I. Introduction

In evolutionary multiobjective optimization (EMO), the aim is to minimize the distance to the optimal Pareto front (i.e., convergence) while maximizing the distribution of solutions over the Pareto front (i.e., diversity). Depending on techniques employed to achieve them, EMO algorithms can be classified as—Pareto-based, aggregation-based, and indicator-based. The ability of EMO algorithms in achieving the appropriate balance between convergence and diversity during the evolution deteriorates as the number of objectives increases in many-objective optimization (MaOP) problems, where the number of objectives is greater than three [1].

In Pareto-based methods [2], dominance relationship between solutions and density estimation are employed as primary and secondary criteria, to promote converge and diversity, respectively. In aggregation-based EMO algorithms, scalarizing functions are employed to map the objective values of a multiobjective problem into a single scalar value that drives the convergence, while diversity is achieved through a set of well-distributed reference points [3]. However, in indicator-based algorithms [4], the individual solutions in the population are compared using a single scalar value referred to as indicator that accounts for both convergence and distribution of solutions.

In literature, indicators based on hypervolume (HV) were employed [4], [5]. The binary additive ε indicator (Iε+) [4] was the first to be proposed. However, due to its computational complexity, an alternative HV indicator based on Monte Carlo simulations was proposed [5]. Recently, R_2 [6] and additive approximation (α) [7] indicators were proposed to enhance the computational efficiency of indicator-based algorithms on MaOPs. However, the evaluation of R_2 indicator requires a set of utility functions to map the different objectives to a single value and reference points. In addition, the available utility functions do not scale properly for MaOPs and the diversity is sensitive to the choice of reference points. To evaluate α indicator at a given generation, an archive consisting of nondominated vectors found so far is needed. Hence, the quality of the indicator depends on the size and distribution of the solutions in the archive. Recently, a diversity indicator based on shift-based density estimation (I_{SDE}) was proposed [8].

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T. Pamulapati and R. Mallipeddi are with the School of Electronics Engineering, Kyungpook National University, Daegu 702 701, South Korea (e-mail: trinadhpamulapati@gmail.com; mallipeddi.ram@gmail.com).

P. N. Suganthan is with the Department of Electrical and Computer Engineering, Nanyang Technological University, Singapore 639798 (e-mail: epnsugan@ntu.edu.sg).

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Among the various indicators, Iε+ promotes convergence and I_{SDE} promotes diversity [8]. Therefore, the use of single indicators might bias the search to a subregion of the Pareto front. In other words, different indicators have bias to different sections of the Pareto front which might complement each other. To benefit from the complementary nature of multiple indicators (Iε+ and I_{SDE}), an indicator-based EMO algorithm was proposed in [8] where the trade-off between convergence and diversity is achieved by a stochastic ranking procedure.

In this letter, we propose an indicator that can effectively balance convergence and diversity. The proposed indicator is a combination of sum-of-objectives and the shift-based density estimation (SDE). The performance of EMO algorithm using the proposed indicator seems to be consistent even when the number of objectives increases making it suitable for multiobjective optimization and MaOP.

The remainder of this letter is organized as follows. Section II presents the background and motivation. Section III presents the proposed indicator and the EMO algorithm framework using the proposed indicator. Section IV presents experimental setup, results, and discussions. Section V concludes this letter.

II. Background and Motivation

A. Shift-Based Density Estimation

In general, to estimate density of individuals in a population, density estimation algorithms consider only the distribution of individuals. Hence, the density of an individual p in a population P of N individuals can be expressed as [9]

$$\mathrm{Density}(p, P) = \mathrm{SF}\{\mathrm{dist}(p, q_1), \mathrm{dist}(p, q_2), \ldots, \mathrm{dist}\big(p, q_{N-1}\big)\} \quad (1)$$

where $q_i \in P$ and $q_i \neq p$. dist() is the Euclidean distance that represents the degree of similarity between individuals. The function SF{ } measures the degree of similarity between p and the other individuals in P.

In Pareto-based MOEAs, density estimation is generally used as secondary criterion when the primary dominance-based criterion fails to distinguish solutions when solving MaOPs. To enhance their performance, SDE that considers both the distribution and convergence information of population members was proposed [9]. The convergence characteristics of an individual p in population P is obtained by shifting the positions of the remaining individuals of P in the objective space. When solving a multiobjective problem involving minimization of m objectives, to estimate the density of p, a solution q is shifted to a new position q' as

$$q'(j) = \begin{cases} p(j) & \text{if } q(j) < p(j) \\ q(j) & \text{otherwise} \end{cases} \quad j \in (1,\ 2, \ldots, m) \quad (2)$$

where $p(j)$, $q(j)$, and $q'(j)$ denote the jth objective value of individuals p, q, and q', respectively. Therefore, in SDE, the positions of the individuals in the objective space are adjusted to reflect their convergence with respect to p in P. Therefore, shift-based density of individual p in P can be expressed as

$$\mathrm{SDE}(p, P) = \mathrm{SF}\{\mathrm{dist}(p, q'_1), \mathrm{dist}(p, q'_2), \ldots, \mathrm{dist}(p, q'_{N-1})\}. \quad (3)$$

From (2) and (3), it is evident that SDE pushes the individuals with poor convergence into crowded regions and assigns them high density values. Hence, in Pareto-based EMO algorithms, the chance of eliminating such solutions increases. In other words, when SDE is employed as a secondary criterion, two nondominated solutions would be distinguished taking into account both convergence and diversity. The incorporation of SDE significantly improved the performance of Pareto-based EMO algorithms [9] due to its ability to promote convergence in addition to being a diversity measure.

Based on SDE, a computationally efficient indicator referred to as I_{SDE} was proposed [8]. I_{SDE} indicator value of an individual depends on position and convergence of all the individuals in the population. Therefore, to evaluate I_{SDE} of an individual, all the other individuals in the population are shifted. For an individual p in P, the indicator is evaluated by

$$I_{\mathrm{SDE}}(p) = \min_{p \in P,\ p \neq q}\{\mathrm{dist}(p, q'_1), \mathrm{dist}(p, q'_2), \ldots,\ \mathrm{dist}(p, q'_{N-1})\} \quad (4)$$

where SF{ } is taken to be the min. Therefore, I_{SDE} indicator is suitable to promote diversity among the solutions in the population during the evolution. Even though it considers the convergence information of solutions, it alone cannot impart the necessary selection pressure to guide the search to the optimal Pareto front. Hence, it is employed in conjunction with other convergence indicators [8].

B. Sum of Objectives

When solving a multiobjective optimization problem (MOP), to enable direct comparison between individuals using a scalar value, fitness assignment [10] is required to map the multiobjective space into a single dimension. In literature, the different fitness assignment processes available are: weighted sum (WS) [11], average ranking, maximum ranking, favor relation, preference order ranking, global detriment, profit, and distance to best known solution [10]. In a comparison study, it was concluded that simple WS exhibits better convergence properties compared to the others mentioned above and Pareto dominance, the performance of which deteriorates as the number of objectives increases. Sum of objectives (SB) is a special case of WS with all weights set to 1. Normalization of objective values is necessary to make SB range independent. In this letter, the minimum and maximum objective values of the current population are used. In SB, the quality of the solution x is expressed as

$$\mathrm{SB}(x) = \sum_{i=1}^{m} f_i(x). \quad (5)$$

III. Proposed I_{SDE}+ Indicator and EMO Algorithm

In this section, we propose a new indicator ($I_{\mathrm{SDE}}{}^{+}$) by fusing the sum of objectives into SDE and an EMO framework employing $I_{\mathrm{SDE}}{}^{+}$ indicator is presented.

A. $I_{SDE}{}^{+}$ Indicator

In EMO algorithms, given a set of $2N$ individuals where N is the population size, the aim of the selection process is to select N solutions that are better in convergence and diversity. Therefore, evaluating the diversity of an individual by considering all the $2N$ individuals does not provide the correct information. In other words, the selection of an individual that is diverse with respect to the solutions that have highest probability of getting selected would be appropriate. Hence, a good indicator needs to give priority to solutions that exhibit better convergence and the diversity of the solutions should be evaluated with respect to the solutions that are better than it in terms of convergence. Since the primary goal of an indicator is to provide convergence followed by diversity.

Algorithm 1 General EMO Framework With I_{SDE}+

Input: N (population size)
1: $P \leftarrow$ ***Initialize*** (N)
2: I_{SDE}+ $\leftarrow$ ***Evaluate indicator*** (P)
3: **while** termination criteria not met **do**
4: $P' \leftarrow$ ***Mating selection*** (P, N, I_{SDE}+)
5: $Q \leftarrow$ P $\cup$ ***Variation*** (P', N)
6: I_{SDE}+ $\leftarrow$ ***Evaluate indicator*** (Q)
7: [P, I_{SDE}+] $\leftarrow$ ***Environmental selection*** (Q, N, I_{SDE}+)
8: **end while**
Output: P

When solving an MOP with m objectives to be minimized, to evaluate the $I_{\mathrm{SDE}}{}^{+}$ indicator, the solutions in the population are sorted in the ascending order of sum of objectives. The solution with the least sum of objectives is assigned the highest possible indicator value of one. Then, to evaluate the $I_{\mathrm{SDE}}{}^{+}$ of a given solution, only the solutions that are better in convergence with least sum of objectives compared to it are shifted. The $I_{\mathrm{SDE}}{}^{+}$ of an individual p in P is evaluated as

$$I_{\mathrm{SDE}^{+}}(p) = \min_{q \in P_{\mathrm{SB}(p)}, p \neq q}\Big\{\mathrm{dist}(p, q'_1), \mathrm{dist}(p, q'_2), \ldots \mathrm{dist}\Big(p, q'_{N_{\mathrm{SB}(p)}-1}\Big)\Big\} \quad (6)$$

where $P_{\mathrm{SB}(p)} \in P$ and $q \in P_{\mathrm{SB}(p)}$ such that $\mathrm{SB}(q) < \mathrm{SB}(p)$. $N_{\mathrm{SB}(p)}$ is the size of $P_{\mathrm{SB}(p)}$. The solutions with highest $I_{\mathrm{SDE}}{}^{+}$ values are considered to be better.

$I_{\mathrm{SDE}}{}^{+}$is a combination of sum of objectives and SDE; and benefits from their convergence and density estimation abilities, respectively. The incorporation of sum of objectives into I_{SDE} is expected to improve the selection pressure of the proposed indicator toward the optimal Pareto front while maintaining the diversity among the population members. In addition, the diversity estimation considering only the solutions with better convergence reflects the actual contribution of solution in terms of diversity.

B. Basic Structure of Proposed Algorithm

The general framework of an indicator-based EMO algorithm using the proposed indicator is shown in Algorithm 1. Given N, a uniform random initialization followed by objective and indicator evaluation corresponding to each individual is done. Then operations such as mating selection, variation, objective and indicator evaluation followed by environmental selection are repeated until a predefined stopping criterion is met.

During mating selection, some promising solutions from the current population are selected depending on the I_{SDE}+ indicator values of individuals. For mating selection, binary tournament selection strategy shown in Algorithm 2 is employed. Among two randomly selected solutions x and y, the solution with highest I_{SDE}+ value is preferred (lines 4–7).

Then solutions selected during mating selection are used to produce new solutions using the variation operators such as simulated

Algorithm 2 Mating Selection Using I_{SDE^+}

Input: P (population), N (population size), I_{SDE^+}

1: $P' \leftarrow \emptyset$
2: **while** $|P'| < N$ **do**
3: randomly select two individuals x, y from P
4: **if** $I_{\mathrm{SDE}^+}(x) > I_{\mathrm{SDE}^+}(y)$ **then** $P' \leftarrow P' \cup (x)$
5: **else** $P' \leftarrow P' \cup (y)$
6: **end if**
7: **end while**

Output: P'

Algorithm 3 Environmental Selection Based on I_{SDE^+}

Input: Q (combined population), N (population size), I_{SDE^+}

1. Sort individuals in Q in descending order of I_{SDE^+}
2. $[P, I_{\mathrm{SDE}^+}] \leftarrow$ select first N solutions and indicator values

Output: P, I_{SDE^+}

TABLE I
PARAMETER SETTINGS FOR DTLZ AND WFG TEST SUITES

Parameter	DTLZ1	DTLZ2-DTLZ6	DTLZ7	WFG1-9				
k	5	10	20	-				
m	2,4,6,8,10			2	4	6	8	10
K	--			4	6	10	7	9
L	--			10				
D	m-1 + k			$(K+L)$				

k – DTLZ Problem specific parameter [13], m – Number of objectives, K - Position vector, L - Distance vector and D - Number of variables.

binary crossover and polynomial mutation [12]. Finally, environmental selection based on the proposed indicator is performed to pick N individuals with the highest I_{SDE^+} values from the union of the current population and their offspring. The procedure is described in Algorithm 3.

IV. EXPERIMENTAL SETUP, RESULTS, AND DISCUSSION

Experiments were conducted on 16 scalable test problems from two widely used test suites, DTLZ [13] and WFG [14], comprising of 7 and 9 problems, respectively. For each test problem, 2, 4, 6, 8, and 10-objectives are considered. The parameter values employed are presented in Table I [12]. Thirty independent runs were performed for each algorithm on each test instance on a PC with a 3.30 GHz Intel Core i5-6600CPU and Windows 10 Pro 64-bit operating system with 16 GB RAM. As a stopping criterion, the maximum number of generations for DTLZ1 and WFG2 is set to 700 and is set to 1000 for DTLZ3 and WFG1. For the other problems (DTLZ2, DTLZ4–DTLZ7, and WFG3–WFG9) it is set to 250 [12].

In this letter, all algorithms employ population size (N) of 100, 120, 132, 156, and 275 for 2, 4, 6, 8, and 10-objectives, respectively [12]. Simulated binary crossover and polynomial mutation with distribution indices and probabilities set to $n_m = 20$, $n_c = 20$, $p_c = 1.0$, and $p_m = 1/D$, respectively, are employed. Sample size in HypE is set to 10 000 and parameters of peer algorithms are taken from respective publications.

To compare different algorithms, we employ quantitative indicators such as HV, generational distance (GD), and spread (Δ) [15], [16]. HV can evaluate both the convergence ability and the diversity of the solutions provided by different EMO algorithms. GD assesses convergence ability while spread (Δ) accounts for diversity of the solutions. To evaluate quality indicators, we employed the procedure described in [17]. A larger HV and smaller values of GD and Δ indicate the superiority of the algorithm. Computational complexity of algorithms is reported in terms of average CPU time (t) in seconds for a single run taken by the algorithm.

To show the effectiveness of the proposed algorithm we consider state-of-the-art EMO algorithms like KnEA [12], BiGE [18], GrEA [19], SPEA2+SDE [9], NSGA-III [20], SRA [8] HypE [5], and IBEA [4]. The experimental results (mean and standard deviation values of normalized HV) on benchmark suites are presented in Table II. In addition, we performed a statistical significance test with a confidence level of 0.05 to compare the performance of the proposed algorithm with the state-of-the-art algorithms. The "+," "=," and "−" signs against the HV values indicate that the proposed algorithm is statistically "better," "comparable," or "worse" with the corresponding algorithm. The last row of Table II summarizes the overall performance of the proposed algorithm in terms of number of instances it is better, comparable and worst with respect to the corresponding state-of-the-art algorithm.

The proposed algorithm is better than and/or comparable to KnEA, BiGE, GrEA, NSGA-III, SPEA2+SDE, SRA, HypE, and IBEA in 91.25%, 87.50%, 93.75%, 98.75%, 82.50%, 85.00%, 86.25%, and 80.00% of cases, respectively, of the total 80 instances. The proposed method significantly outperforms Pareto-based KnEA, BiGE, GrEA, and NSGA-III. The overall performance of SPEA2+SDE, HypE and SRA is similar. Among the state-of-the-art algorithms, IBEA exhibits competitive performance compared to the proposed algorithm.

The proposed algorithm performs better than and/or comparable in 96.87%, 89.84%, 85.15%, 85.93%, and 82.81% of cases on 2, 4, 6, 8, and 10-objective versions of the test problems, respectively. The performance degrades slightly as the number of objectives increases but significantly better than the state-of-the-art even at higher objectives. Therefore, the proposed method is suitable for both multiobjective optimization/MaOP problems. The parallel coordinates of the solution sets corresponding to the best run (large HV) on 4 and 8-objective instances of DTLZ1 are presented in Figs. 1 and 2, respectively. Corresponding Δ, GD, and t values are presented in Table III. KnEA, BiGE, GrEA, HypE, and IBEA converge to the subregions of the Pareto front in different forms and it becomes more evident as the number of objectives increases. In other words, the search bias in these algorithms increases as the number of objectives increases. Among all the algorithms, the spread provided by NSGA-III is the best on lower objectives. However, as the number of objectives increases the diversity loss in NSGA-III becomes obvious from the Δ values in Table III. The consistent performance of SPEA2+SDE, SRA, and proposed algorithm on DTLZ1 can be observed even when the number of objectives increases as shown by parallel plots, Δ and GD values. However, the run time (t) taken by the proposed algorithm is significantly less.

SPEA2+SDE and SRA algorithms perform consistently better than the proposed algorithm on higher objective instances of unimodal separable problems WFG1 and WFG7. In addition, they perform better in 4 and 6-objective instances of DTLZ7 but perform significantly worse in 2, 8, and 10-objective instances. However, unlike SPEA2+SDE, SRA performs better than proposed algorithm on higher objective instances of multimodal DTLZ1, which generally tests the convergence capability of EMO algorithms. The superior performance of SRA can be attributed to the complementary nature of the two indicators employed. However, excluding unimodal separable problems (WFG1 and WFG7), the performance of SRA deteriorates significantly on all other problems (mainly at higher objectives) of WFG test suite.

Indicator-based algorithms, HypE and IBEA, perform consistently better than the proposed method on WFG3 and WFG4. In addition, IBEA performs consistently better in higher objective instances of DTLZ7. In a few problem instances, HypE and

TABLE II
COMPARISON OF HV AND STATISTICAL RESULTS ON DTLZ AND WFG TEST PROBLEMS (W—WIN, T—TIE, L— LOSS)

#	m	KnEA	BiGE	GrEA	NSGA-III	SPEA2+SDE	SRA	HypE	IBEA	PROPOSED
DTLZ1	2	0.4926 (0.0137) =	0.4695 (0.0525) =	0.4224 (0.0993) +	0.4968 (0.0175) =	0.4937 (0.0156) =	0.4929 (0.0176) =	0.4921 (0.0163) =	0.2913 (0.0279) +	0.4931 (0.0163)
	4	0.6417 (0.1286) +	0.8102 (0.0785) +	0.7088 (0.1660) +	0.9113 (0.0094) -	0.8956 (0.0113) =	0.9045 (0.0099) =	0.5723 (0.1290) +	0.3756 (0.1183) +	0.8982 (0.0110)
	6	0.5192 (0.1084) +	0.7769 (0.1741) +	0.5884 (0.2748) +	0.9790 (0.0090) +	0.9606 (0.0064) +	0.9745 (0.0054) -	0.5229 (0.1263) +	0.4843 (0.1010) +	0.9690 (0.0071)
	8	0.3255 (0.1057) +	0.6719 (0.2043) +	0.4222 (0.2203) +	0.9729 (0.1052) +	0.9819 (0.0046) +	0.9911 (0.0033) -	0.5302 (0.1086) +	0.6196 (0.0999) +	0.9868 (0.0054)
	10	0.5147 (0.2183) +	0.1871 (0.2327) +	0.4682 (0.1876) +	0.9044 (0.2103) +	0.9938 (0.0028) +	0.9988 (0.0010) +	0.5720 (0.1187) +	0.8402 (0.0412) +	0.9965 (0.0017)
DTLZ2	2	0.1697 (0.0174) +	0.2026 (0.0136) +	0.2095 (0.0118) =	0.2086 (0.0131) =	0.2091 (0.0127) =	0.2088 (0.0128) =	0.2114 (0.0130) =	0.2072 (0.0157) =	0.2113 (0.0141)
	4	0.5606 (0.0171) +	0.5415 (0.0166) +	0.5870 (0.0157) =	0.5805 (0.0127) =	0.5842 (0.0158) =	0.5736 (0.0184) +	0.5131 (0.0177) +	0.5746 (0.0159) +	0.5849 (0.0176)
	6	0.8100 (0.0144) +	0.7994 (0.0175) +	0.8412 (0.0109) =	0.8258 (0.0433) +	0.8432 (0.0122) =	0.8196 (0.0115) +	0.7198 (0.0433) +	0.8436 (0.0130) =	0.8429 (0.0094)
	8	0.9626 (0.0064) +	0.9594 (0.0066) +	0.9592 (0.0075) +	0.9580 (0.0254) +	0.9718 (0.0043) =	0.9552 (0.0075) +	0.9129 (0.0133) +	0.9748 (0.0042) =	0.9729 (0.0047)
	10	0.9764 (0.0186) +	0.9987 (0.0014) +	0.9987 (0.0012) +	0.9984 (0.0018) +	0.9989 (0.0012) =	0.9971 (0.0017) +	0.9941 (0.0021) +	0.9991 (0.0008) =	0.9994 (0.0007)
DTLZ3	2	0.0580 (0.0176) +	0.2009 (0.0094) +	0.2077 (0.0158) =	0.2088 (0.0108) =	0.2100 (0.0138) =	0.2084 (0.0108) =	0.2124 (0.0134) =	0.0000 (0.0002) +	0.2079 (0.0135)
	4	0.4223 (0.0736) +	0.3774 (0.1394) +	0.1484 (0.1655) +	0.5810 (0.0201) =	0.5881 (0.0135) =	0.5743 (0.0185) =	0.1468 (0.0436) +	0.0486 (0.0077) +	0.5821 (0.0175)
	6	0.9783 (0.0117) +	0.9806 (0.0146) +	0.9887 (0.0072) +	0.9989 (0.0047) +	0.9999 (0.0004) =	0.9989 (0.0010) +	0.9984 (0.0049) +	0.9989 (0.0014) +	1.0000 (0.0002)
	8	0.6596 (0.4545) +	0.9998 (0.0005) +	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000) =	0.9999 (0.0004) =	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000)
	10	0.9997 (0.0011) +	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000) =	1.0000 (0.0000)
DTLZ4	2	0.1627 (0.0753) +	0.1638 (0.0840) +	0.1791 (0.0722) =	0.1002 (0.1082) +	0.1589 (0.0898) +	0.1884 (0.0648) =	0.1514 (0.0943) =	0.1467 (0.0980) +	0.2095 (0.0105)
	4	0.5547 (0.0146) +	0.5324 (0.0166) +	0.5268 (0.0795) =	0.4613 (0.1111) +	0.5439 (0.0629) =	0.5550 (0.0719) =	0.4092 (0.1219) +	0.5597 (0.0612) =	0.5671 (0.0354)
	6	0.8303 (0.0159) +	0.8260 (0.0113) +	0.8440 (0.0122) =	0.7524 (0.0784) +	0.8536 (0.0159) -	0.8341 (0.0181) =	0.6458 (0.1390) +	0.8475 (0.0260) =	0.8419 (0.0176)
	8	0.9762 (0.0049) +	0.9760 (0.0053) +	0.9762 (0.0045) +	0.9622 (0.0260) +	0.9799 (0.0073) =	0.9743 (0.0047) +	0.9024 (0.0432) +	0.9809 (0.0089) =	0.9820 (0.0059)
	10	0.9999 (0.0003) =	1.0000 (0.0002) =	1.0000 (0.0002) =	0.9999 (0.0003) =	1.0000 (0.0000) =	1.0000 (0.0000) =	0.9991 (0.0010) +	1.0000 (0.0000) =	1.0000 (0.0000)
DTLZ5	2	0.1727 (0.0204) +	0.2018 (0.0118) +	0.2086 (0.0122) =	0.2132 (0.0147) =	0.2107 (0.0123) =	0.2058 (0.0139) =	0.2132 (0.0120) =	0.2087 (0.0141) =	0.2094 (0.0134)
	4	0.7526 (0.0160) +	0.7723 (0.0111) =	0.7646 (0.0127) +	0.7613 (0.0123) +	0.7728 (0.0147) =	0.7753 (0.0135) =	0.7651 (0.0155) +	0.7736 (0.0145) =	0.7721 (0.0142)
	6	0.8285 (0.0132) +	0.8420 (0.0118) =	0.8397 (0.0106) =	0.7990 (0.0125) +	0.8429 (0.0110) -	0.8415 (0.0128) -	0.8389 (0.0143) =	0.8509 (0.0110) -	0.8371 (0.0103)
	8	0.8411 (0.0135) +	0.8584 (0.0103) =	0.8446 (0.0121) +	0.8151 (0.0177) +	0.8616 (0.0106) =	0.8564 (0.0168) =	0.8676 (0.0127) -	0.8688 (0.0109) -	0.8611 (0.0107)
	10	0.8624 (0.0110) +	0.8774 (0.0137) =	0.8523 (0.0151) +	0.8627 (0.0149) +	0.8852 (0.0092) -	0.8773 (0.0142) =	0.8895 (0.0084) -	0.8889 (0.0094) -	0.8783 (0.0100)
DTLZ6	2	0.1188 (0.0214) +	0.0461 (0.0200) +	0.2118 (0.0115) =	0.1875 (0.0188) +	0.2123 (0.0132) =	0.2079 (0.0111) =	0.2113 (0.0155) =	0.2084 (0.0127) =	0.2132 (0.0134)
	4	0.9282 (0.0103) +	0.9058 (0.0099) +	0.9330 (0.0063) +	0.9334 (0.0073) =	0.9341 (0.0063) =	0.9376 (0.0082) =	0.9235 (0.0076) +	0.9347 (0.0065) =	0.9363 (0.0067)
	6	0.9843 (0.0046) =	0.9109 (0.0097) +	0.9843 (0.0077) =	0.9824 (0.0061) +	0.9843 (0.0037) =	0.9796 (0.0115) +	0.9766 (0.0040) +	0.9875 (0.0035) =	0.9857 (0.0044)
	8	0.9881 (0.0034) +	0.9112 (0.0073) +	0.9820 (0.0071) +	0.9854 (0.0061) +	0.9909 (0.0029) =	0.9822 (0.0118) +	0.9860 (0.0044) +	0.9903 (0.0037) =	0.9900 (0.0031)
	10	0.9897 (0.0034) =	0.9081 (0.0082) +	0.9749 (0.0139) +	0.9859 (0.0058) +	0.9913 (0.0028) =	0.9839 (0.0100) +	0.9904 (0.0026) =	0.9903 (0.0030) =	0.9894 (0.0060)
DTLZ7	2	0.1400 (0.0139) =	0.1359 (0.0176) =	0.1332 (0.0101) =	0.1378 (0.0118) =	0.1424 (0.0126) =	0.1202 (0.0288) =	0.1022 (0.0229) +	0.1377 (0.0181) =	0.1385 (0.0120)
	4	0.1896 (0.0115) -	0.1742 (0.0138) +	0.1835 (0.0151) =	0.1802 (0.0116) =	0.1950 (0.0149) -	0.1954 (0.0126) -	0.1168 (0.0091) +	0.1916 (0.0171) -	0.1821 (0.0104)
	6	0.1634 (0.0174) =	0.1733 (0.0150) =	0.1740 (0.0124) =	0.1336 (0.0142) +	0.1781 (0.0139) -	0.1727 (0.0106) =	0.0939 (0.0112) +	0.1786 (0.0160) -	0.1700 (0.0153)
	8	0.0676 (0.0236) +	0.1383 (0.0133) =	0.1258 (0.0108) +	0.0989 (0.0197) +	0.0894 (0.0253) +	0.0767 (0.0193) +	0.0728 (0.0119) +	0.1557 (0.0124) -	0.1401 (0.0172)
	10	0.0322 (0.0114) +	0.1056 (0.0155) +	0.0640 (0.0211) +	0.1002 (0.0219) +	0.0347 (0.0092) +	0.0715 (0.0127) +	0.0684 (0.0091) +	0.1428 (0.0116) -	0.1227 (0.0203)
WFG 1	2	0.5747 (0.0190) +	0.6080 (0.0125) +	0.6157 (0.0167) =	0.6171 (0.0121) =	0.6187 (0.0142) =	0.6150 (0.0117) =	0.1394 (0.0222) +	0.6120 (0.0148) =	0.6177 (0.0161)
	4	0.9708 (0.0078) -	0.9650 (0.0065) -	0.9322 (0.0096) +	0.9573 (0.0558) +	0.9726 (0.0050) -	0.9700 (0.0069) -	0.5367 (0.0223) +	0.9579 (0.0109) =	0.9600 (0.0068)
	6	0.9881 (0.0046) -	0.9800 (0.0058) -	0.9651 (0.0064) +	0.9227 (0.0754) =	0.9858 (0.0047) -	0.9862 (0.0056) -	0.6736 (0.0207) +	0.9671 (0.0065) +	0.9750 (0.0051)
	8	0.9886 (0.0046) -	0.9885 (0.0044) -	0.9667 (0.0072) +	0.8803 (0.0862) +	0.9904 (0.0031) -	0.9911 (0.0039) -	0.8301 (0.0201) +	0.9822 (0.0058) =	0.9839 (0.0050)
	10	0.9923 (0.0038) -	0.9936 (0.0029) -	0.9705 (0.0071) +	0.8532 (0.1102) +	0.9942 (0.0026) -	0.9958 (0.0018) -	0.9004 (0.0151) +	0.9842 (0.0033) +	0.9874 (0.0051)
WFG 2	2	0.4491 (0.0144) =	0.4432 (0.0109) +	0.4454 (0.0157) =	0.4299 (0.0168) +	0.4497 (0.0132) =	0.4489 (0.0164) =	0.4520 (0.0169) =	0.4565 (0.0157) =	0.4527 (0.0151)
	4	0.5218 (0.0290) +	0.5709 (0.0246) +	0.5826 (0.0193) =	0.5432 (0.0633) +	0.5035 (0.0301) +	0.3904 (0.0412) +	0.5580 (0.0302) +	0.5812 (0.0311) =	0.5887 (0.0265)
	6	0.4370 (0.0502) +	0.5665 (0.0259) +	0.5747 (0.0233) =	0.4995 (0.0642) +	0.5085 (0.0278) +	0.2713 (0.0879) +	0.5624 (0.0297) +	0.5792 (0.0330) =	0.5816 (0.0339)
	8	0.4364 (0.0462) +	0.6430 (0.0136) +	0.6525 (0.0168) +	0.6190 (0.0208) +	0.6052 (0.0174) +	0.3731 (0.0700) +	0.6676 (0.0197) =	0.6748 (0.0196) =	0.6756 (0.0140)
	10	0.5664 (0.0688) +	0.6648 (0.0163) +	0.6787 (0.0157) +	0.6189 (0.0281) +	0.6469 (0.0231) +	0.4489 (0.0446) +	0.6846 (0.0125) +	0.6956 (0.0156) =	0.6986 (0.0165)
WFG 3	2	0.4954 (0.0132) =	0.4873 (0.0151) =	0.4899 (0.0166) =	0.4669 (0.0232) +	0.4909 (0.0157) =	0.4879 (0.0121) =	0.4977 (0.0136) =	0.4991 (0.0142) =	0.4921 (0.0205)
	4	0.2565 (0.0151) =	0.2525 (0.0174) +	0.2597 (0.0142) =	0.1913 (0.0582) +	0.2594 (0.0136) =	0.2512 (0.0154) +	0.2706 (0.0161) -	0.2714 (0.0121) -	0.2616 (0.0153)
	6	0.1431 (0.0184) +	0.1637 (0.0145) +	0.1766 (0.0125) =	0.0673 (0.0285) +	0.1742 (0.0125) =	0.1578 (0.0097) +	0.1912 (0.0105) -	0.1919 (0.0129) -	0.1780 (0.0157)
	8	0.0862 (0.0145) +	0.1497 (0.0151) =	0.1590 (0.0120) -	0.0318 (0.0186) +	0.1448 (0.0108) =	0.1080 (0.0129) +	0.1763 (0.0123) -	0.1723 (0.0114) -	0.1499 (0.0143)
	10	0.0706 (0.0147) +	0.1455 (0.0126) -	0.1658 (0.0111) -	0.0016 (0.0026) +	0.1366 (0.0093) =	0.0925 (0.0163) +	0.1687 (0.0149) -	0.1675 (0.0133) -	0.1366 (0.0156)

(continued)

TABLE II
CONTINUED

WFG 4	2	0.2579 (0.0157) =	0.2452 (0.0150) =	0.2489 (0.0124) =	0.2518 (0.0133) =	0.2533 (0.0151) =	0.2520 (0.0137) =	0.2554 (0.0146) =	0.2514 (0.0125) =	0.2521 (0.0140)
	4	0.3737 (0.0171) =	0.3575 (0.0142) +	0.3788 (0.0203) =	0.3330 (0.0464) +	0.3639 (0.0137) +	0.3658 (0.0134) =	0.3882 (0.0147) -	0.3632 (0.0177) +	0.3753 (0.0182)
	6	0.3448 (0.0239) +	0.3638 (0.0139) =	0.3707 (0.0125) =	0.2266 (0.0808) +	0.3579 (0.0145) +	0.3472 (0.0135) +	0.3917 (0.0220) -	0.3495 (0.0158) +	0.3719 (0.0181)
	8	0.4135 (0.0246) +	0.5395 (0.0204) =	0.5320 (0.0158) +	0.4427 (0.0506) +	0.4892 (0.0183) +	0.4456 (0.0224) +	0.5492 (0.0398) =	0.5679 (0.0165) -	0.5494 (0.0173)
	10	0.4356 (0.0355) +	0.5796 (0.0158) -	0.5915 (0.0165) -	0.3794 (0.1369) +	0.4629 (0.0175) +	0.4506 (0.0311) +	0.5737 (0.0315) =	0.6029 (0.0202) -	0.5586 (0.0157)
WFG 5	2	0.2722 (0.0163) +	0.2995 (0.0119) +	0.3023 (0.0161) +	0.3039 (0.0131) =	0.3093 (0.0141) =	0.2986 (0.0159) +	0.3079 (0.0111) =	0.3031 (0.0135) =	0.3074 (0.0085)
	4	0.2651 (0.0142) +	0.2685 (0.0155) +	0.2521 (0.0141) +	0.2618 (0.0153) +	0.2779 (0.0151) =	0.2631 (0.0146) +	0.2835 (0.0115) =	0.2681 (0.0122) +	0.2767 (0.0164)
	6	0.1529 (0.0231) +	0.2570 (0.0129) +	0.2402 (0.0167) +	0.2423 (0.0182) +	0.2732 (0.0151) =	0.2283 (0.0192) +	0.2811 (0.0125) =	0.2624 (0.0142) +	0.2770 (0.0147)
	8	0.1851 (0.0225) +	0.3060 (0.0124) +	0.2910 (0.0129) +	0.2648 (0.0364) +	0.3357 (0.0143) -	0.2457 (0.0208) +	0.3440 (0.0135) -	0.3092 (0.0137) +	0.3247 (0.0161)
	10	0.1668 (0.0240) +	0.3244 (0.0124) +	0.3200 (0.0152) +	0.2755 (0.0254) +	0.3427 (0.0138) =	0.2051 (0.0244) +	0.3545 (0.0184) -	0.3231 (0.0151) +	0.3397 (0.0133)
WFG 6	2	0.3067 (0.0146) =	0.2979 (0.0164) +	0.3034 (0.0126) +	0.3030 (0.0156) +	0.3083 (0.0154) =	0.3002 (0.0155) +	0.3051 (0.0137) =	0.3082 (0.0129) =	0.3119 (0.0154)
	4	0.2157 (0.0243) +	0.2710 (0.0168) +	0.2534 (0.0171) +	0.2672 (0.0318) +	0.2867 (0.0188) =	0.2672 (0.0176) +	0.2853 (0.0192) =	0.2881 (0.0214) =	0.2888 (0.0149)
	6	0.0352 (0.0271) +	0.1953 (0.0413) +	0.1837 (0.0429) +	0.1742 (0.0370) +	0.2093 (0.0456) =	0.1199 (0.0329) +	0.2105 (0.0420) =	0.2080 (0.0444) =	0.2283 (0.0412)
	8	0.0584 (0.0221) +	0.2106 (0.0275) +	0.2074 (0.0368) +	0.1862 (0.0640) +	0.2230 (0.0349) =	0.1290 (0.0227) +	0.2337 (0.0404) =	0.2316 (0.0314) =	0.2403 (0.0450)
	10	0.0368 (0.0156) +	0.1986 (0.0526) =	0.1949 (0.0409) =	0.1518 (0.0452) +	0.2102 (0.0415) =	0.0999 (0.0231) +	0.1974 (0.0404) =	0.2119 (0.0519) =	0.2170 (0.0405)
WFG 7	2	0.2777 (0.0146) +	0.2806 (0.0120) +	0.2871 (0.0150) =	0.2131 (0.0592) +	0.2823 (0.0131) =	0.2851 (0.0139) =	0.1806 (0.0177) +	0.2857 (0.0131) =	0.2885 (0.0132)
	4	0.4935 (0.0135) =	0.4815 (0.0169) +	0.4925 (0.0178) =	0.4286 (0.0743) +	0.5012 (0.0181) =	0.4940 (0.0159) =	0.2703 (0.0303) +	0.4834 (0.0170) +	0.4951 (0.0166)
	6	0.5452 (0.0155) -	0.5376 (0.0191) =	0.5299 (0.0143) =	0.4356 (0.0845) +	0.5571 (0.0123) -	0.5420 (0.0195) -	0.1786 (0.0231) +	0.5066 (0.0193) +	0.5303 (0.0218)
	8	0.5829 (0.0230) -	0.5990 (0.0159) -	0.5322 (0.0158) +	0.2905 (0.1732) +	0.5985 (0.0158) -	0.5786 (0.0187) -	0.4928 (0.0393) +	0.5344 (0.0198) +	0.5534 (0.0204)
	10	0.5509 (0.0376) =	0.6458 (0.0162) -	0.6002 (0.0166) -	0.3264 (0.1097) +	0.6397 (0.0178) -	0.6139 (0.0162) -	0.5435 (0.0393) +	0.5567 (0.0208) =	0.5541 (0.0207)
WFG 8	2	0.4621 (0.0208) =	0.4555 (0.0255) =	0.4765 (0.0210) -	0.3554 (0.0213) +	0.4783 (0.0187) -	0.4770 (0.0171) -	0.4429 (0.0238) +	0.4746 (0.0203) -	0.4583 (0.0217)
	4	0.0256 (0.0203) +	0.1803 (0.0206) +	0.1991 (0.0297) +	0.1875 (0.0238) +	0.2128 (0.0174) +	0.1941 (0.0298) +	0.2380 (0.0183) =	0.2598 (0.0174) -	0.2340 (0.0197)
	6	0.0076 (0.0066) +	0.1663 (0.0226) +	0.1788 (0.0196) +	0.1104 (0.0212) +	0.1925 (0.0166) =	0.1447 (0.0234) +	0.2030 (0.0183) =	0.2291 (0.0186) -	0.2034 (0.0235)
	8	0.0256 (0.0086) +	0.1874 (0.0182) +	0.1854 (0.0182) +	0.1470 (0.0188) +	0.1998 (0.0143) +	0.0975 (0.0152) +	0.2567 (0.0145) =	0.2413 (0.0141) =	0.2506 (0.0201)
	10	0.0129 (0.0076) +	0.1833 (0.0176) +	0.1715 (0.0164) +	0.1107 (0.0236) +	0.1764 (0.0164) +	0.0760 (0.0136) +	0.2605 (0.0136) -	0.2225 (0.0183) =	0.2313 (0.0183)
WFG 9	2	0.2168 (0.0137) =	0.2177 (0.0120) =	0.2260 (0.0132) =	0.2225 (0.0126) =	0.2263 (0.0160) =	0.2270 (0.0150) =	0.2214 (0.0164) =	0.2239 (0.0094) =	0.2211 (0.0138)
	4	0.5067 (0.0309) =	0.4925 (0.0239) +	0.5021 (0.0313) =	0.4677 (0.0316) +	0.5103 (0.0222) =	0.5003 (0.0155) +	0.3060 (0.0391) +	0.5082 (0.0338) =	0.5153 (0.0262)
	6	0.5741 (0.0337) =	0.5941 (0.0273) -	0.5551 (0.0306) =	0.5150 (0.0406) +	0.5706 (0.0332) =	0.5663 (0.0328) =	0.2675 (0.0383) +	0.5263 (0.0453) +	0.5538 (0.0580)
	8	0.6301 (0.0642) +	0.7247 (0.0455) =	0.6992 (0.0350) +	0.6650 (0.1262) +	0.6860 (0.0317) +	0.6618 (0.0304) +	0.2972 (0.0338) +	0.7170 (0.0736) =	0.7190 (0.0679)
	10	0.6323 (0.0897) +	0.8031 (0.0252) -	0.7492 (0.0294) =	0.6932 (0.0740) +	0.7466 (0.0289) =	0.7195 (0.0264) +	0.3179 (0.0314) +	0.7721 (0.0666) =	0.7563 (0.0695)
W – T – L		55-18-7	50-20-10	40-35-5	62-17-1	18-48-14	40-28-12	41-28-11	22-42-16	

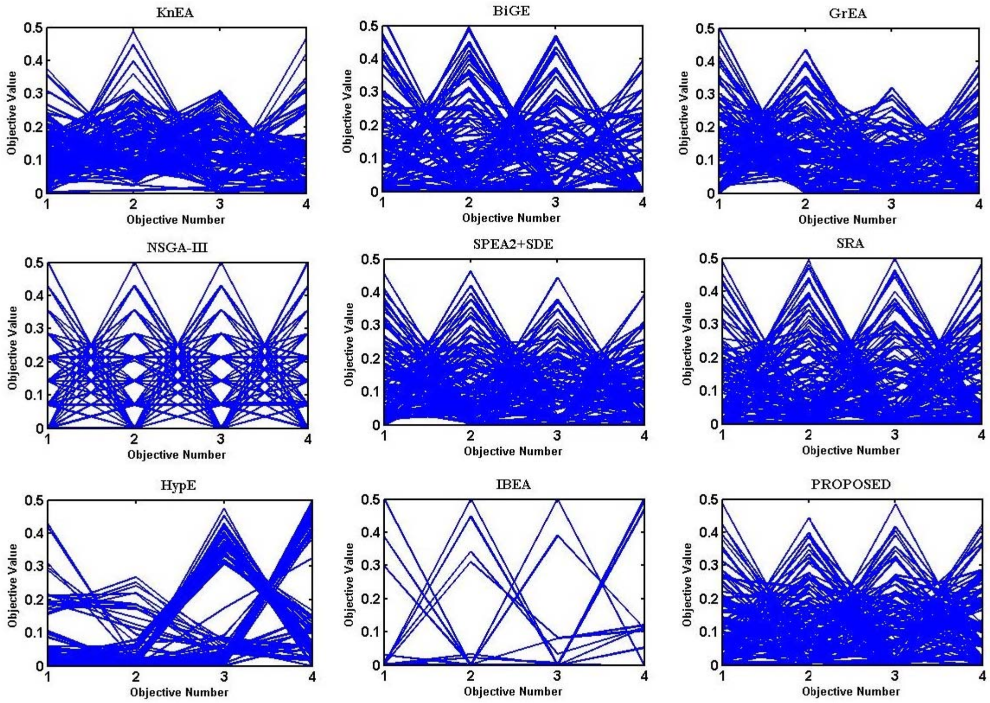

Fig. 1. Parallel coordinates of the best solution set of different algorithms on 4-objective instance of DTLZ1.

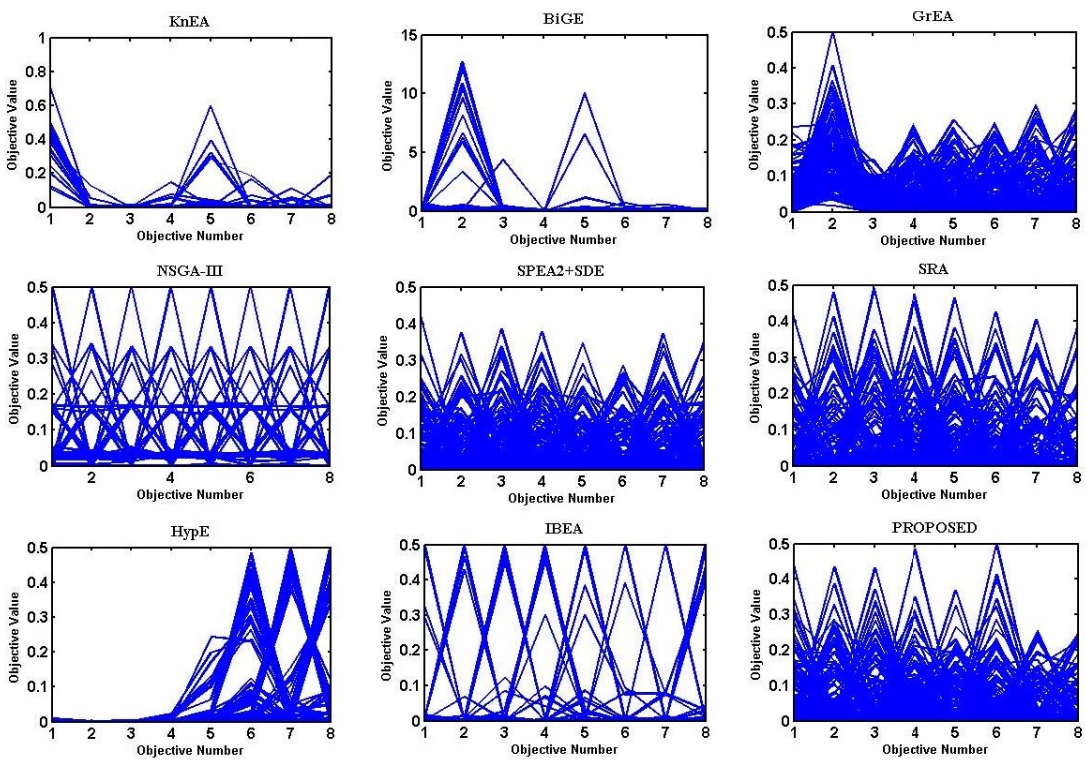

Fig. 2. Parallel coordinates of the best solution set of different algorithms on 8-objective instance of DTLZ1.

IBEA perform better than the proposed method but do not exhibit consistent performance. The better performance of HypE and IBEA is due to the computationally expensive HV indicator as shown in Table III.

From the results, it can be observed that HypE performs better than IBEA on lower objectives (2 and 4) of DTLZ1 and DTLZ3 while IBEA performs better on higher objectives (8 and 10). This can be confirmed from the parallel plots in Figs. 1 and 2. On DTLZ7, HypE outperforms IBEA. In other words, the two indicators exhibit complementary behavior. However, the performance of the proposed algorithm is consistent and better as the number of objectives increases (DTLZ1 and DTLZ3) and as the characteristics of the problems changes (DTLZ7 with disconnected Pareto front).

TABLE III
COMPARISON OF SPREAD (Δ), GD, CPU TIME (t IN SECONDS) ON 4 AND 8-OBJECTIVES OF DTLZ1 TEST PROBLEM

m		**KnEA**	**BiGE**	**GrEA**	**NSGA-III**	**SPEA2+SDE**	**SRA**	**HypE**	**IBEA**	**PROPOSED**
4	Δ	0.9554 (0.6090) +	1.4307 (0.5076) +	0.9589 (0.4737) +	0.0085 (0.0129) **-**	0.1603 (0.0116) **-**	0.3600 (0.4117) +	1.2764 (0.4027) +	1.5607 (0.4562) +	0.1828 (0.0200)
	GD	0.1229 (0.2212) +	0.3443 (0.4701) +	0.1145 (0.3008) =	0.0059 (0.0000) =	0.0059 (0.0002) =	0.0439 (0.1585) +	0.1178 (0.2428) +	0.0059 (0.0039) =	0.0058 (0.0002)
	t	11.9343	3.7750	47.4343	14.4343	194.2562	188.2843	4206.2000	42.0156	3.1781
8	Δ	1.7831 (0.2838) +	1.7084 (0.1790) +	1.6000 (0.4483) +	0.2706 (0.3701) =	0.1751 (0.0113) **-**	0.2095 (0.0667) =	1.2137 (0.3751) +	1.5331 (0.2403) +	0.2012 (0.0192)
	GD	0.9788 (0.9188) +	1.0692 (0.8357) +	1.1216 (0.7954) +	0.0329 (0.1168) =	0.0109 (0.0002) =	0.0093 (0.0005) **-**	0.0864 (0.1983) +	0.0462 (0.2336) +	0.0109 (0.0003)
	t	14.5875	8.2187	294.1600	29.5250	497.0600	353.9600	13174.2000	70.4093	4.9531

V. CONCLUSION

In this letter, we propose a computationally effective indicator by combining the sum of objectives and SDE better known for their ability to promote convergence and diversity, respectively. An EMO framework using the proposed indicator demonstrates consistent performance even when the number of objectives increases indicating its scalability. The performance of the proposed method is compared with various state-of-the-art methods on diverse scalable benchmark test problems.

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