Many-Objective Evolutionary Algorithms: A Survey

BINGDONG LI, JINLONG LI, and KE TANG, University of Science and Technology of China
XIN YAO, University of Birmingham

Multiobjective evolutionary algorithms (MOEAs) have been widely used in real-world applications. However, most MOEAs based on Pareto-dominance handle many-objective problems (MaOPs) poorly due to a high proportion of incomparable and thus mutually nondominated solutions. Recently, a number of many-objective evolutionary algorithms (MaOEAs) have been proposed to deal with this scalability issue. In this article, a survey of MaOEAs is reported. According to the key ideas used, MaOEAs are categorized into seven classes: relaxed dominance based, diversity-based, aggregation-based, indicator-based, reference set based, preference-based, and dimensionality reduction approaches. Several future research directions in this field are also discussed.

CCS Concepts: • **Computing methodologies** → **Randomized search**; • **Mathematics of computing** → *Bio-inspired Optimization*; Evolutionary Algorithms;

Additional Key Words and Phrases: Many-objective optimization, evolutionary algorithm, scalability

ACM Reference Format:
Bingdong Li, Jinlong Li, Ke Tang, and Xin Yao. 2015. Many-objective evolutionary algorithms: A survey. ACM Comput. Surv. 48, 1, Article 13 (September 2015), 35 pages.
DOI: http://dx.doi.org/10.1145/2792984

1. INTRODUCTION

In the real world, we are often faced with problems with multiple objectives, which are called multiobjective problems (MOPs).[1] MOPs with at least four objectives are informally known as many-objective problems (MaOPs) [Farina and Amato 2002], although some papers specified problems with three or more objectives as MaOPs [Wang et al. 2014]. As an important class of MOPs, MaOPs appear widely in many real-world applications, such as engineering design [Fleming et al. 2005], air traffic control

[1]Without loss of generality, we consider minimization problems where all objectives are to be minimized.

This work was supported in part by the 973 Program of China under grant 2011CB707006, the National Natural Science Foundation of China under grants 61175065 and 61329302, the Program for New Century Excellent Talents in University under grant NCET-12-0512, the Science and Technological Fund of Anhui Province for Outstanding Youth under grant 1108085J16, EPSRC grant EP/J017515/1, and the European Union Seventh Framework Programme under grant 247619. Xin Yao was supported by a Royal Society Wolfson Research Merit Award.
Authors' addresses: B. Li, J. Li (corresponding author), and K. Tang, USTC-Birmingham Joint Research Institute in Intelligent Computation and Its Applications (UBRI), School of Computer Science and Technology, University of Science and Technology of China (USTC), Hefei, Anhui 230027, China; emails: lbingd17@mail.ustc.edu.cn; {jlli, ketang}@ustc.edu.cn; X. Yao, Center of Excellence for Research in Computational Intelligence and Applications (CERCIA), School of Computer Science, University of Birmingham, Birmingham B15 2TT, UK; email: x.yao@cs.bham.ac.uk.

ACM 0360-0300/2015/09-ART13 $15.00
DOI: http://dx.doi.org/10.1145/2792984

Table I. Number of Papers Tackling Problems with at Least Four Objectives and Corresponding Citations (in Parentheses) in Major Conferences and Journals in Recent Years

Year	2007	2008	2009	2010	2011	2012	2013
CEC	17(666)	11(435)	13(276)	14(148)	9(94)	15(102)	16(37)
GECCO	3(358)	3(140)	8(146)	6(115)	8(66)	4(46)	19(58)
EMO	9(970)	—	8(183)	—	9(136)	—	13(102)
PPSN	—	6(118)	—	7(57)	—	2(8)	—
TEC	3(1306)	2(615)	4(239)	4(312)	3(187)	3(185)	4(146)
ECJ	2(335)	1(31)	5(281)	1(28)	1(263)	2(81)	1(63)
AIJ	0	0	0	0	1(12)	0	2(14)
Total	34(3,635)	23(1,339)	38(1,125)	32(660)	31(758)	26(422)	54(420)

Note: We conducted a comprehensive literature survey using the following databases: Google Scholar, IEEE Xplore Digital Library, ACM Digital Library, and SpringerLink, using "many-objective" as the search keyword. References that address many-objective optimization but do not explicitly use the term *many-objective* are also included. The citation number is counted until April 5th, 2015.

[Herrero et al. 2009], nurse rostering [Sülflow et al. 2007], car controller optimization [Narukawa and Rodemann 2012], water supply portfolio planning [Kasprzyk et al. 2012], with up to hundreds of objectives.[2] Recently, MaOPs have drawn steady attention (238 related papers with 8,359 citations) in the evolutionary multiobjective optimization (EMO) community, as can be seen from Table I. It appears that in 2013 there was a recent jump in the interest in this topic, including two papers in generic artificial intelligence journals rather than just papers in specialist evolutionary computation journals/conferences.

A multiobjective optimization problem can be stated as follows [Miettinen 1999]:

$$\begin{aligned} &\mathit{minimize}\ \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}), \ldots,\ f_m(\mathbf{x}))^T \\ &\mathit{subject\ to}\ \mathbf{x} \in \Omega, \end{aligned} \tag{1}$$

where the decision vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ belongs to the (nonempty) *decision space* Ω, the objective function vector $\mathbf{F}: \Omega \rightarrow \Lambda$ consists of m $(m \geq 2)$ objectives, and the *objective space* Λ usually equals to $S \subset R^m$. For MaOPs, the number of objectives m is larger than 3. Since different solutions may have an advantage over each other on different objectives, the concept of Pareto dominance is critical when comparing solutions of a MOP. The goal of multiobjective optimization is to approximate the Pareto front (PF) in the objective space so that no further improvement on any objective can be achieved without harming the rest of objectives.

It should be noted that many-objective optimization is closely related to multiple criteria decision making (MCDM). When dealing with MaOPs, researchers from MCDM usually use different preference models to compare nondominated solutions [Figueira et al. 2005]. The preference models used by the MCDM methods can be divided into three categories [Løken 2007]: value measurement models that assign a numerical score to each solution,[3] goal, aspiration, and reference level models that prefer solutions closest to a determined goal or aspiration level, and outranking models that use pairwise comparisons to select solutions.[4]

[2]For more information about real-world applications and synthetic problems with at least four objectives, please refer to the online appendix of this article, which can be found on the author's homepage (http://home.ustc.edu.cn/~lbingd17/files/OnlineAppendixToSurvey.pdf).

[3]According to Velasquez and Hester [2013], the MCDM methods based on value measurement models, such as multiattribute utility theory (MAUT) and analytic network process (ANP), are very popular recently.

[4]Extensively reviewing these methods is outside of the scope of this article. For more information, please refer to Løken [2007], Velasquez and Hester [2013], and Purshouse et al. [2014].

Evolutionary Algorithms (EAs) are population-based, black-box search/optimization methods and do not need particular assumptions like continuity or differentiability. They are very suitable for dealing with MOPs [Yang et al. 2013]. In the past few decades, researchers have proposed plenty of multiobjective evolutionary algorithms (MOEAs), which can be categorized into four classes: Pareto-based approach, aggregation-based approach, indicator-based approach, and preference-based approach [Zhou et al. 2011]. However, when dealing with MaOPs, traditional MOEAs are more likely to fail to converge to the PF since the problem of deterioration becomes more prevalent with an increasing number of objectives [Wang et al. 2014]. Thus, researchers have designed several algorithms to overcome the obstacles.

Although numerous studies on MaOPs have been conducted, few surveys on MaOEAs are available. Some of recent studies focus on certain subareas of multiobjective optimization: multiobjective algorithms for data mining [Mukhopadhyay et al. 2014a, 2014b] and decomposition-based multiobjective algorithms [Santiago et al. 2014], whereas some are more closely related to many-objective optimization. Ishibuchi et al. [2008] demonstrated the difficulties of MaOPs through experiments and reviewed a number of techniques for improving scalability of MOEAs. Hughes [2008] reviewed different fitness assignment methods for many-objective optimization. Purshouse et al. [2014] focused on hybrid methods that combine MCDM and multiobjective optimization. Von Lücken et al. [2014] analyzed different methods, main findings, and open questions in many-objective optimization. Compared to it, our article has carried out a more detailed investigation on the following aspects:

—*Completeness*: Von Lücken et al. [2014] conducted a good review over 112 references. Our article provides a comprehensive survey based on more than 200 related papers.
—*Taxonomy*: We provide a different and more comprehensive categorization of different algorithms and techniques for MaOPs. First, our categorization analyzes key techniques in algorithms to describe the algorithms in more detail. Second, the reference set based methods are included in our categorization but not in previous surveys. Third, for each category and subcategory, we include important details, such as methods and test problems, in our survey, supported by summary tables.
—*Test problems*: To understand what algorithms were used to solve what problems, it is important to include problems in a comprehensive survey. That is what we have done in our survey, including the real-world problems listed in the appendix.

Based on the key idea used, we categorize MaOEAs into seven classes: the relaxed dominance based, diversity-based, aggregation-based, indicator-based, reference set based, preference-based, and dimensionality reduction approaches, summarized in Figure 1.[5] In this article, we conduct an extensive survey on MaOEAs and discuss the characteristics of each class of approaches.

The rest of the article is organized as follows. Section 2 introduces the background of many-objective optimization. Sections 3 and 4 reviews the relaxed dominance based and the diversity-based approaches, respectively. The aggregation-based approach is presented in Section 5. Section 6 is devoted to the indicator-based approach. Section 7 presents the reference set based approach. The preference-based and dimensionality reduction approaches are explained in Sections 8 and 9, respectively. The last section summarizes the development of MaOEAs and future research directions.

[5]The first and second classes here are related to the Pareto-based methods in Zhou et al. [2011]. The aggregation-based, indicator-based, and preference-based methods are similar to those in Zhou et al. [2011]. The remaining two classes are not covered by Zhou et al. [2011]. It should be noted that Zhou et al. focused on introducing algorithms for MOPs instead of MaOPs [Zhou et al. 2011].

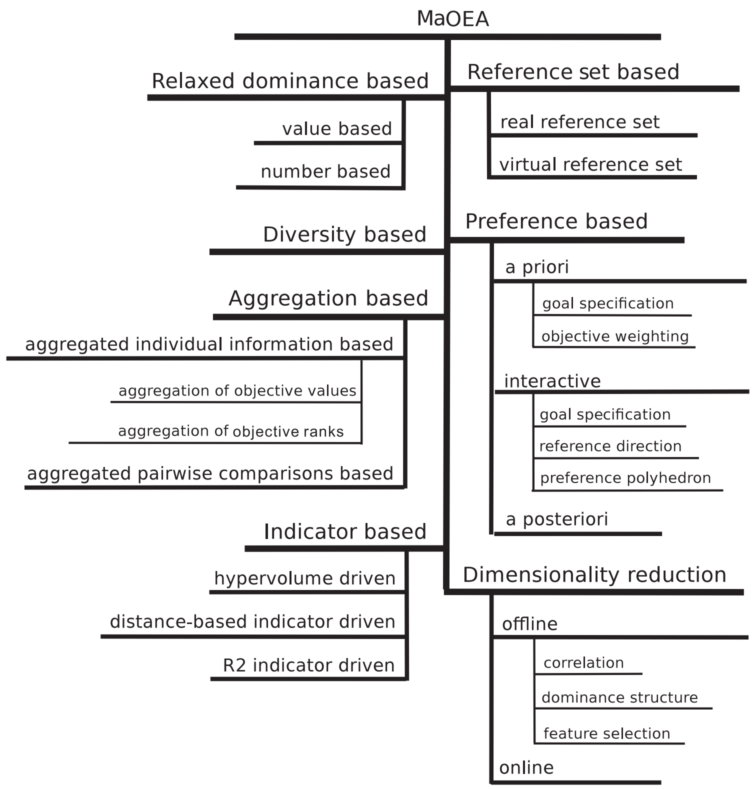

Fig. 1. Classification of MaOEAs.

2. BACKGROUND OF MANY-OBJECTIVE OPTIMIZATION

2.1. Problem Definition

There is no generally agreed-on definition of MaOPs. The primary motivation behind MaOPs is to highlight the challenges posed by a large number of objectives to existing MOEAs. We therefore define a MaOP as a MOP with $m \geq 4$. Although we use four objectives as the dividing line between MOPs and MaOPs now, the number would increase as stronger MaOEAs are developed in the future. Hence, the definition presented here is an evolving one and serves a more practical than theoretical purpose.

The Pareto dominance relation is widely used to compare solutions of a MaOP. Based on the Pareto dominance relation, Pareto optimal solution, Pareto set (PS), and PF are further defined. These terms are defined as follows:

Definition 2.1 (*Pareto dominance* [*Yu 1974*]). Given two solutions $\mathbf{x}, \mathbf{y} \in \Omega_f$ and their corresponding objective vectors $\mathbf{F}(\mathbf{x}), \mathbf{F}(\mathbf{y}) \in R^m$, $\mathbf{x}$ *dominates* $\mathbf{y}$ (denoted as $\mathbf{x} \prec \mathbf{y}$) if and only if $\forall i \in \{1, 2, \ldots, m\}$, $f_i(\mathbf{x}) \leq f_i(\mathbf{y})$ and $\exists j \in \{1, 2, \ldots, m\}$, $f_j(\mathbf{x}) < f_j(\mathbf{y})$.

Definition 2.2 (*Pareto optimal solution*). A solution $\mathbf{x}^* \in \Omega_f$ is *Pareto optimal* if there does not exist another solution $\mathbf{x} \in \Omega_f$ that dominates it.

Definition 2.3 (*Pareto set*). The union of all Pareto optimal solutions are called *Pareto set* (PS): $PS = \{\mathbf{x} \in \Omega_f | \nexists \mathbf{y} \in \Omega_f, \mathbf{y} \prec \mathbf{x}\}$.

Definition 2.4 (*Pareto front*). The corresponding objective vector set of the PS is called the *Pareto front* (PF).

2.2. Quality Measurements of Many-Objective Optimization Algorithms

The goal of optimizing a MaOP, descriptively defined as follows, is the same as that of optimizing a general MOP [Li et al. 2014]:

Definition 2.5 (*Goal of optimizing an MaOP*). The goal of optimizing an MaOP is to obtain an approximation set A to the PF, including the following two subgoals:
(1) All solutions in A are as close as possible to the PF,
(2) All solutions in A are as diverse as possible in the objective space,

where an approximation set is defined as follows [Zitzler et al. 2003].

Definition 2.6 (*Approximation set*). Let A $\subset \Lambda$ be a set of objective vectors, denoted as $A = \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_{|A|}\}$. A is called an *approximation set* if any element of A does not dominate or is not equal to any other objective vector in A.

Corresponding to the optimization goal, the quality of an approximation set is measured in terms of two aspects: (1) convergence, the closeness to the real PF in the objective space, and (2) diversity, whether the solutions are well distributed over the PF. Quality indicators are quantified quality measurements [Zitzler et al. 2003].

Definition 2.7 (*Quality Indicator*). An k-ary quality indicator I is a function I: $\Gamma^k \to R$, which assigns each vector $(A_1, A_2, \ldots, A_k)$ of k approximation sets a real value I$(A_1, A_2, \ldots, A_k)$.

The hypervolume indicator (I_{HV}, aka S metric) was described as the Lebesgue measure L of the union of the hypercubes c_i corresponding to each solution $\mathbf{a}_i \in A$ [Zitzler and Thiele 1998; Emmerich et al. 2005]: $I_{HV}(\mathbf{z}^{\dagger}, A) = L(\{\bigcup_i c_i | \mathbf{a}_i \in A\}) = L(\{\bigcup_{\mathbf{a}\in A}\{\mathbf{b}|\mathbf{a} \prec \mathbf{b} \prec \mathbf{z}^{\dagger}\})$, where $\mathbf{z}^{\dagger}$ is the worst point in the objective space. It measures both convergence and diversity of an approximation set.

Generational distance (GD) [Van Veldhuizen and Lamont 1998] and inverted generational distance (IGD) [Bosman and Thierens 2003] are defined as follows: $I_{GD} = \frac{1}{|A|}(\sum_{i=1}^{|A|} dis(\mathbf{a}_i, PF')^p)^{\frac{1}{p}}$, $I_{IGD} = \frac{1}{|PF'|}(\sum_{i=1}^{|PF'|} dis(\mathbf{p}_i, A)^p)^{\frac{1}{p}}$, where $A = \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_{|A|}\}$ is the approximation set, $PF' = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_{|PF'|}\}$ is a subset of the PF, $dis(\mathbf{a}_i, PF')$ is the distance between $\mathbf{a}_i$ and PF', and $dis(\mathbf{p}_i, A)$ is the distance between $\mathbf{p}_i$ and A.[6] GD measures the convergence quality, whereas IGD measures both convergence and diversity of an approximation set. Similar distance-based indicators, such as the additive approximation metric[7] [Bringmann et al. 2011], and the Δ_p indicator [Schütze et al. 2010] are proposed for performance measurement.

The generalized spread (I_{GS}) [Zhou et al. 2006], which measures the diversity of an approximation set, is defined as follows:

$$I_{GS} = \frac{\sum_{i=1}^{s} d(\mathbf{e}_i, A) + \sum_{\mathbf{p}\in PF'} |d(\mathbf{p}, A) - \bar{d}|}{\sum_{i=1}^{s} d(\mathbf{e}_i, A) + |PF'|\bar{d}}, \tag{2}$$

where $\{\mathbf{e}_1, \ldots, \mathbf{e}_s\}$ are s extreme solutions in PF', $d(\mathbf{p}, A)$ is the Euclidean distance between $\mathbf{p}$ and A, and $\bar{d}$ is the mean of $d(\mathbf{p}, A)$ for all $\mathbf{p} \in PF'$.

The R2 indicator is a class of utility function based indicators. Given a reference set R, the $R2$ indicator can be defined as $R2(A, U) = -\frac{1}{|U|}\sum_{u\in U}(\max_{\mathbf{a}\in A}\{u(\mathbf{a})\})$, where

[6] In practice, p is usually set to 2. The distance between a solution and a solution set is the distance between the solution and the member in the solution set that is closest to the solution.
[7] It is also known as the α indicator.

U is a utility function set [Hansen and Jaszkiewicz 1998; Brockhoff et al. 2012]. If one sets U to a series of weighted Tchebycheff functions with a set of k weighting vectors V and selects the utopian point z^* into R, then the R2 indicator is defined as $R2(z^*, A, V) = \frac{1}{|V|}\sum_{\lambda\in V}\min_{\mathbf{a}\in A}\{\max_{j\in\{1,\ldots,m\}}\{\lambda_j|z_j^* - \mathbf{a}_j|\}\}$. The R2 indicator can measure both convergence and diversity of an approximation set [Brockhoff et al. 2012].

According to Zitzler et al. [2003], there exists no unary quality measure that can indicate whether an approximation A is better than another approximation B. Instead, most quality measures indicate that A is better B, which actually means that A is not worse than B. Binary indicators, such as the binary ϵ-indicator of Zitzler et al. [2003], might be able to overcome this limitation.

2.3. Key Challenges of Many-Objective Optimization

When the number of objectives increases, one has to deal with the following issues:

—In the dominance resistance (DR) phenomenon [Fonseca and Fleming 1998; Purshouse and Fleming 2007; Knowles and Corne 2007], the incomparability of solutions caused by the enormously increasing proportion of nondominated solutions.
—In the limited solution set size, under nondegenerated scenarios the PF of an m-objective problem is an (m-1)-dimensional manifold [Ishibuchi et al. 2015; Jin and Sendhoff 2003]. To describe such a front, we have to increase the number of solutions exponentially.
—The visualization of solution set in the objective space needs special techniques, such as projection to a lower dimension space and parallel coordinates [Walker et al. 2013].

It has been shown that the Pareto-based approaches, such as nondominated sorting genetic algorithm II (NSGA-II) [Deb et al. 2002] and the improved strength pareto evolutionary algorithm (SPEA2) [Zitzler et al. 2001], may deteriorate their performance significantly on MaOPs [Knowles and Corne 2007]. This is mainly due to the DR phenomenon and the active diversity promotion (ADP) mechanisms [Purshouse and Fleming 2007]. ADP refers to that when the dominance-based primary criterion fails to discriminate solutions, the density-based secondary criterion is activated to determine which solutions are able to survive the environmental selection. As a result of DR and ADP, the final solution set might not even converge to PF but stagnate far away from it [Wagner et al. 2007].

Intuitively, there are two avenues to improve the scalability of Pareto-based algorithms: the relaxed dominance based approach and the diversity-based approach, related to DR and ADP, respectively. On the one hand, to mitigate the adverse impact of DR, researchers have proposed several variants of the Pareto dominance to enhance the selection pressure toward the PF [Laumanns et al. 2002; Sato et al. 2007]. Compared to the classical Pareto dominance, these methods are more capable of discriminating solutions of MaOPs. On the other hand, several advanced diversity management strategies are proposed to deal with the ADP problem [Adra and Fleming 2011]. In general, these methods tend to pay more attention to the convergence.

Non-Pareto-based MOEAs, such as indicator-based and aggregation-based approaches, do not suffer from the selection pressure problem, as they do not rely on the Pareto dominance to push the population toward the PF. However, they still suffer from the "curse of dimensionality"—that is, to search simultaneously in an exponentially increasing number of directions. For the aggregation-based approach, the key point is the setting of weighting vectors, as it plays a vital role in maintaining the distribution of the population [Giagkiozis et al. 2013]. For the hypervolume-based MOEAs, an important class of indicator-based MOEAs, the high computational cost of the indicator is a limitation [Wagner and Neumann 2013]. In addition to the approaches

Table II. Value-Based Dominance

Technique	Algorithm	Paper	Test Problems (m)
ϵ-Dom	ϵ-MOEA	[Hadka et al. 2012; Hadka and Reed 2012]	UF(5) DTLZ(8)
		[Yang et al. 2013]	DTLZ(10) DTLZ5(I,M)(9) MOTSP(10)
	ϵ-NSGA-II	[Kollat and Reed 2006]	LTM test case(4)
		[Hadka et al. 2012; Hadka and Reed 2012]	UF(5) DTLZ(8)
CDAS	CDAS	[Sato et al. 2007]	0/1 MKP(5)
	PPD-MOEA	[Sato et al. 2010a]	0/1 MKP(10)
	S-CDAS	[Sato et al. 2010b]	0/1 MKP(10)
	CCG-S-CDAS	[Sato et al. 2011b]	0/1 MKP(10)
	Adaptive CCG	[Sato et al. 2012]	0/1 MKP(10)
α-Dom	—	—	—
Grid-dom	GrEA	[Yang et al. 2013]	DTLZ(10) DTLZ5(I,M)(9) MOTSP(10)
Cone ϵ-dom	Cone ϵ-MOEA	—	—
Subspace ϵ-dom	ϵR-EMO	[Aguirre and Tanaka 2009a, 2009b, 2010b]	MNK-landscape
	μFAϵR^E	[Aguirre and Tanaka 2010a]	MNK-landscape
	[Jaimes et al. 2011a]	[Jaimes et al. 2011a]	DTLZ5(I,M) (15)
			0/1 knapsack(15)
	A$\epsilon_s\epsilon_h$EMyO	[Aguirre et al. 2013b]	DTLZ(6)
Volume-dom	SEAMO2′ SPEA2′ NSGA-II′	[Le et al. 2009]	MKP(4)
Multilevel grid	Random search	[Laumanns and Zenklusen 2011]	-

mentioned previously, the reference set based approach that uses a reference set to evaluate and select solutions provides a new alternative to solving MaOPs [Deb and Jain 2014]. Different from other approaches, the dimensionality reduction approach aims at reducing the number of objectives by analyzing the relationship among the objectives or using feature selection techniques. To incorporate user preferences into the search process, researchers designed the preference-based approach to focus on a subregion of the PF. The dimensionality reduction approach also attempts to eliminate the less important objectives to reduce the difficulty of the original problem [Brockhoff and Zitzler 2009].

3. RELAXED DOMINANCE BASED APPROACH

To differentiate nondominated solutions and enhance the selection pressure toward the PF, many variants of dominance have been proposed. We categorize them into the following two classes: value-based and number-based dominance.

3.1. Value-Based Dominance

As its name indicates, these methods modify the Pareto dominance by changing the objective values of the solutions when comparing them. In general, these modifications aim at enlarging the dominating area of the nondominated solutions so that some of them are more likely to be dominated by others. The summary of these kind of modifications is shown in Table II. The first column is the names of relaxed forms of dominance. The second column shows the MaOEAs where the techniques are integrated. The last two columns indicate the papers and the testing problems used in the experiments.

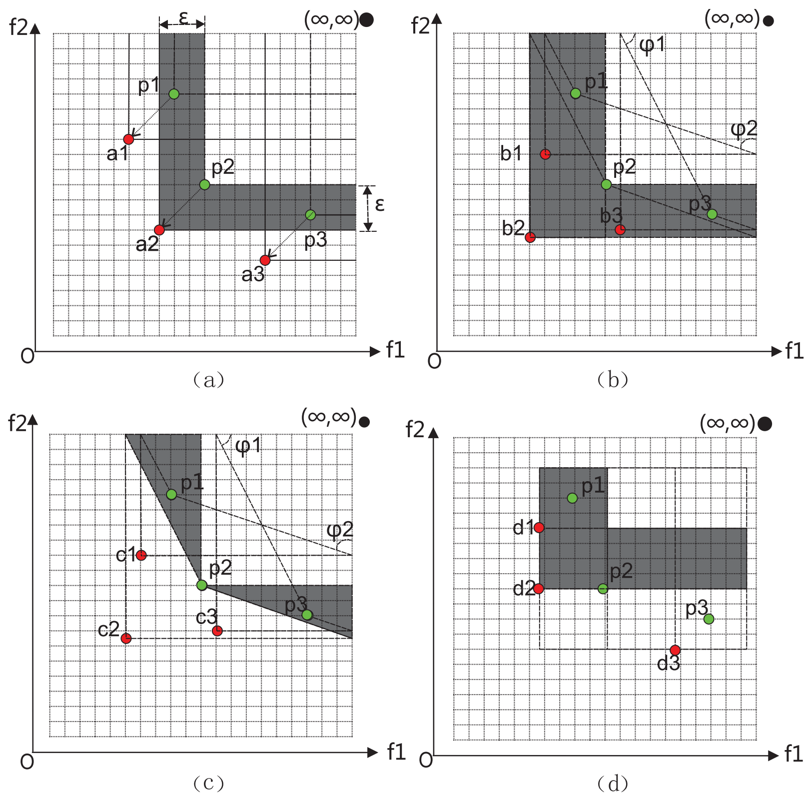

Fig. 2. Relaxed forms of dominance. The area on the upper right of Xi is considered relaxed-dominated by pi, $i = 1, 2, 3$ in subfigure (X), X = {a,b,c,d}. (a) An example relationship of ϵ-dominance among points in the objective space. The grey area is the increased part of objective space where solutions are considered ϵ-dominated by p2, similarly hereinafter. (b) The relationship of CDAS, the shape of grey area of which is quite similar to ϵ-dominance. The angles $\varphi 1$ and $\varphi 2$ are used to control the dominance area, playing a similar role to ϵ in ϵ-dominance. (c) An example of α-dominance. $\varphi 1$ and $\varphi 2$ are used to tune the trading ratio between objectives. (d) An example of grid dominance. A 3*3 grid is created by the current population {p1,p2,p3}. The shape of the grey area is also similar to ϵ-dominance.

In the remaining part of this section, we denote the newly generated objective vector corresponding to the $\mathbf{F}(\mathbf{x})$ as $\mathbf{G}(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_m(\mathbf{x}))^T$.

Laumanns et al. [2002] proposed the concept of ϵ-dominance. Given two solutions $\mathbf{x}, \mathbf{y} \in \Omega$ and $\epsilon > 0$, $\mathbf{x}$ is said to ϵ-dominate $\mathbf{y}$, if and only if $\forall i \in \{1, \ldots, m\}, (1 - \epsilon) f_i(\mathbf{x}) \leq f_i(\mathbf{y})$. In this case, $g_i(\mathbf{x}) = (1 - \epsilon) f_i(\mathbf{x}) \leq f_i(\mathbf{x}),\ \forall i \in \{1, \ldots, m\}$. An example is shown as Figure 2(a). The grey area is the increased part of objective space where individuals are considered ϵ-dominated by p2 while not dominated by p2. It is obvious that the dominating space of p2 is enlarged. Based on ϵ-dominance, Deb et al. [2005] proposed a steady-state algorithm named ϵ-MOEA. In ϵ-MOEA, the objective space is divided into hyperboxes, each of which is assigned at most one solution. Solutions in different hyperboxes are compared based on ϵ-dominance. Otherwise, traditional Pareto dominance is used to compare two solutions. The performance of ϵ-MOEA on MaOPs is tested [Hadka et al. 2012; Hadka and Reed 2012]. Laumanns and Zenklusen [2011] combine the multilevel grid archiving with random search and obtains similar convergence properties as hypervolume archiving. It is computationally much cheaper than hypervolume-based algorithms.

By deriving the Sine theorem, Sato et al. [2007] proposed a method to control the dominance area of solutions (CDAS). It is defined as follows:

$$g_i(\mathbf{x}) = \frac{r \cdot sin(\omega_i + S_i \cdot \pi)}{sin(S_i \cdot \pi)}, \tag{3}$$

where r is the norm of $\mathrm{F}(\mathbf{x})$, ω_i is the declination angle between $\mathrm{F}(\mathbf{x})$ and $f_i(\mathbf{y})$, S_i is a user defined parameter to control the degree of expansion or contraction, and $\phi_i = S_i \cdot \pi$. As the right part of Figure 2(b) shows, the grey area is the increased part of objective space where individuals are considered CDAS dominated by p2 while not Pareto dominated by p2. From Figure 2, we can see that ϵ-dominance and CDAS are quite similar, despite their difference in controlling mechanisms of $g_i(x)$s. Empirical results on 4 and 5 objective 0/1 knapsack problems have shown that by changing the degree of expansion or contraction of the dominance area, this method can improve or reduce the selection pressure, thus emphasizing convergence or diversity of results of the algorithm. However, the setting of the angle can be a challenging task because the optimal setting differs depending on the problem at hand [Narukawa 2013]. Based on the studies on parameter settings [Sato et al. 2010a, 2010b], Narukawa [2013] proposed CDAS-D, which controls the angle dynamically during optimization. Specifically, the angle is replaced by a random value once the hypervolume is decreased. Experimental results on DTLZ functions with up to nine objectives showed that CDAS-D is comparable to CDAS with the optimal ratio of the dominating space.

Ikeda et al. [2001] proposed another variant of dominance named α-domination. When comparing two solutions on some objective, not only the value of this objective but all other objective values are taken into account. The α-domination uses linear trade-off functions to define the tolerance of dominance. It allows a solution to dominate another if it is slightly inferior to the other in one objective but largely superior in other objectives. An example of α-domination is presented in Figure 2(c). An identical idea is used in the guided multiobjective evolutionary algorithm (G-MOEA), although explained from the perspective of integrating the user's preference [Branke et al. 2001]. However, the experiments of G-MOEA are carried out on bi-objective problems, and the α-domination has not been included into any MaOEAs to the best of our knowledge.

Yang et al. [2013] proposed a grid-based evolutionary algorithm (GrEA) for optimizing MaOPs. In GrEA, Yang et al. adopted the adaptive construction of grids, borrowing ideas from Knowles and Corne [2003b]. Based on the concept of grids, the selection pressure is increased by the grid dominance and grid difference. An example of grid dominance is depicted in Figure 2(d). Compared to the traditional box-centered calculation of grids, individually centered calculation of grids in Yang et al. [2013] and Knowles and Corne [2003b] can depict locations of points in the population that is changing during the evolution. The grid-based mechanism, together with a fitness adjustment strategy to avoid local overcrowding, is shown to be highly effective on 36 out of 52 test instances from three benchmark function families (i.e., 4 to 10 objective DTLZ, DTLZ5(I,M), and MOTSP). However, it should be noted that individually centered calculation of grids cannot guarantee convergence and thus will suffer from potential deterioration, which is a serious drawback in the context of many-objective optimization [Knowles and Corne 2003b].

The adaptive ϵ-ranking can be seen as an extension of ϵ-dominance to some subspaces of the objective space [Aguirre and Tanaka 2009b; Batista et al. 2011b]. The ϵ-ranking multiobjective optimizer (ϵR-EMO) [Aguirre and Tanaka 2009b] applies a space partitioning strategy and the adaptive ϵ-ranking procedure. Based on the partitioning strategy, the subspace sampling schedule enables different emphasis on different subspaces at each generation. Besides, the selection pressure is enhanced using the adaptive ϵ-ranking procedure. The experiment revealed that it provided a

Table III. Number-Based Dominance

Technique	Algorithm	Paper	Test Problems (m)
Favour	DDB[a]	[Drechsler et al. 2001]	5 benchmarks(7)
$(1-k)$-Dom	—	[Farina and Amato 2002]	test case(12)
$(1-k)_F$-Dominance	—	[Farina and Amato 2002]	—
$(1-\alpha)_F$-Dominance	—	[Farina and Amato 2002]	—
	MDMOEA	[Zou et al. 2008]	DTLZ(9)
L-dominance (LD)	LD	[Garza-Fabre et al. 2010a]	DTLZ(20)

[a]The name of algorithm was not explicitly mentioned in Dreschler et al. [2001]; we name the algorithm with the initials of the authors' names.

remarkable improvement in the convergence and diversity of the results on MNK-landscapes [Aguirre and Tanaka 2007] with up to 10 objectives. Some extensions, such as adaptive space partitioning [Aguirre and Tanaka 2010b] and ϵ-hood mating [Aguirre et al. 2013b] further improve the performance.

In addition to adaptive ϵ-ranking, another variant of ϵ-dominance, namely cone ϵ-dominance, is also proposed [Batista et al. 2011b]. The cone ϵ-dominance can be seen as a hybridization of ϵ-dominance and α-dominance. Batista et al. [2011a] analyzed three relaxed dominance criteria: α-dominance, ϵ-dominance, and cone ϵ-dominance. The results on quadratic and DTLZ test problems with from 2 to 50 objectives demonstrated that all three relaxed forms of dominance showed better scalability to the number of objectives than Pareto dominance.

3.2. Number-Based Dominance

The number-based methods try to compare a solution to another by counting the number of objectives where it is better than, the same as, or worse than the other. Only three methods are categorized into this class, as shown in Table III.

The $(1-k)$-dominance proposed in Farina and Amato [2002] is quite simple. For two solutions $\mathbf{x}, \mathbf{y} \in \Omega$, n_b, n_e, n_w are the numbers of objectives where $\mathbf{x}$ is better than, equal to,or worse than $\mathbf{y}$. $\mathbf{x}$ is said to $(1-k)$-dominate $\mathbf{y}$ if and only if

$$\begin{cases} n_e & < \ m \\ n_b & \geq \ \frac{m-n_e}{k+1} \end{cases}, \tag{4}$$

where $0 \leq k \leq 1$. It is obvious that k = 0 corresponds to the classical definition of Pareto dominance. The larger k is, the more relaxed the new dominance is. In addition, two extensions are proposed by incorporating fuzzy arithmetic techniques. In Farina and Amato [2002], no specific MaOEAs were proposed. Instead, they only tested the effects of modified dominance on the comparability of many-objective solutions. Favor relation [Drechsler et al. 2001] seems to be simpler than $(1-k)$-dominance. A solutions is considered to be superior than another in terms of favor relation if $n_b > n_e$.

The L-dominance (LD) [Zou et al. 2008] can be seen as an extension of the favor relation [Drechsler et al. 2001]. We say that $\mathbf{x}$ L-dominates $\mathbf{y}$ if and only if

$$n_b - n_w = L > 0 \wedge \|\mathbf{F}(\mathbf{x})\|_p < \|\mathbf{F}(\mathbf{y})\|_p \quad (\text{for certain} p), \tag{5}$$

where $\|\mathbf{F}(\mathbf{x})\|_p$ is the p-norm of $\mathbf{F}(\mathbf{x})$. A solution is said to be L-optimal if there is no other solutions which can L-dominate it. L-optimal set and L-optimal front are the sets of L-optimal solutions in the decision space and objective space, respectively. From Theorems 1 and 2 in Zou et al. [2008], we can show that Pareto dominance implies LD, but the other way around is not true. An L-optimal set is a subset of the PS. An L-optimal front is a subset of the PF. A dynamical MOEA based on LD, namely MDMOEA [Zou et al. 2008], can converge to the true L-optimal front and maintain a well distributed population on DTLZ problems with up to 9 objectives. In

Table IV. Diversity-Based Ranking Methods

Technique	Algorithm	Paper	Test Problems (m)
Grid + adaptive neighborhood	Layering selection	[Li et al. 2010]	DTLZ(15)
Grid crowding distance, Adaptive neighborhood, Fitness adjustment	GrEA	[Yang et al. 2013]	DTLZ(10) DTLZ5(I,M)(9) MOTSP(10)
DM1	NSGA-II/DM1	[Adra and Fleming 2011]	DTLZ(20)
DM2	NSGA-II/DM2		
SDE	SPEA2+SDE NSGA-II+SDE PESA-II+SDE	[Li et al. 2013]	DLTZ(10) TSP(10)

addition, experiments on 20-objective DTLZ functions showed that LD can accelerate the convergence [Garza-Fabre et al. 2010a].

One obvious advantage of the number-based dominance is that the normalization of objectives is done automatically by comparing elements with each other. This may result in significant speed-ups and simplify MOEAs [Drechsler et al. 2001].

4. DIVERSITY-BASED APPROACH

Although most of recent work focuses on enhancing the selection pressure toward the PF, another avenue to adjusting Pareto-based algorithms for MaOPs is to apply a customized diversity-based approach. In general, these methods (as shown in Table IV) intend to improve the performance by reducing the adverse impact of diversity maintaining.

Adra and Fleming [2011] introduced a diversity management mechanism named DM1. DM1 determines whether or not to activate diversity promotion according to the spread of the population. The diversity promotion mechanism in the selection is deactivated once the population is excessively diverse. On a set of test functions with up to 20 objectives, NSGA-II/DM1 repeatedly outperformed NSGA-II in terms of convergence and distribution.

In addition to using the grid dominance to enhance the selection pressure, three grid-based criteria—grid ranking, grid crowding distance, and grid coordinate point distance—are introduced into GrEA [Yang et al. 2013] to help maintain the distribution among solutions, as it is important to balance convergence and diversity [Črepinšek et al. 2013]. To avoid local overcrowding, they used an adaptive neighborhood to estimate the density of individuals, which is similar to Li et al. [2010]. The fitness of individuals is adjusted in the selection process considering both neighborhood and grid dominance relations. The experimental results showed GrEA can achieve a better coverage of the PF than the other six state-of-the-art algorithms on 36 out of 52 test instances. Similar grid-based diversity management is also used in Li et al. [2010] for the average ranking (AR) algorithm and has been shown to have a good balance among convergence, uniformity, and spread.

Recently, a shift-based density estimation (SDE) strategy is proposed in Li et al. [2013]. Specifically, when measuring the density of a solutions neighborhood, SDE shifts the position of other solutions according to their convergence comparison with the current solution, thus both the distribution and the convergence information[8] of the solutions are taken into account. As a result, it can reduce the detrimental impact of

[8]Convergence information of a solution means the relative proximity of the solution to the PF. Given two solutions, SDE estimates the density of one solution's neighborhood by considering only distance of the better objective dimensions compared to the other one. By ignoring the distance of the worse dimensions of the first solution, its convergence information is reflected during the density estimation.

Table V. Aggregation of Objective Values

Algorithm	Variant	Paper	Test Problems (m)
MSOPS	MSOPS	[Hughes 2003]	Hypersphere(20) [Hughes 2011]
	MSPOS-II	[Hughes 2007a]	Test functions(5) [Hughes 2007a]
	MODELS	[Hughes 2011]	Hypersphere(20) [Hughes 2011]
MOEA/D	MOEA/D-PBI	[Zhang and Li 2007]	DTLZ(3)
	W-MOEA/D	[Zhang and Li 2007]	DTLZ(3)
	T-MOEA/D	[Zhang and Li 2007]	DTLZ(3)
	MOEA/D-DRA	[Zhang et al. 2009]	CEC09(5) [Zhang et al. 2008]
	UMOEA/D	[Tan et al. 2013]	DTLZ(5) F1(5),F2(5) [Tan et al. 2013] MKP(4) [Zitzler and Thiele 1999]
	MOEA/D-b	[Wang et al. 2013]	WFG(7)
DBEA-Eps	DBEA-Eps	[Asafuddoula et al. 2013]	DTLZ(15) CSIP(3) [Jain and Deb 2014] WRMP(5) [Ray et al. 2001] GAADP(10) [Hadka et al. 2012]
DQGA	DQGA	[Ray et al. 2013]	DTLZ(8) WROP(5) [Ray et al. 2001]
WS	WS	[Garza-Fabre et al. 2009]	DTLZ(50)
MDFA	MDFA	[Garza-Fabre et al. 2010b]	DTLZ(50)
PGA	PGA	[Garza-Fabre et al. 2011]	DTLZ(50)

dominance resistant solutions on the convergence of Pareto-based MOEAs. Another advantage of SDE is that it can be easily adopted by traditional Pareto-based algorithms such as NSGA-II [Deb et al. 2002] and SPEA2 [Zitzler et al. 2001]. Its effectiveness in balancing convergence and diversity is demonstrated by experimental results on DTLZ and TSP test functions with up to 10 objectives.

Pasia et al. [2011] examined multiobjective random one-bit climbers (moRBCs) with tabu moves and adaptive ϵ-ranking on MNK-landscapes [Aguirre and Tanaka 2009b]. In tabu moves, each solution is associated with a tabu list of moves to avoid revisiting previous solutions. Experimental results indicated that tabu moves can greatly improve the spread quality of moRBCs, whereas the adaptive ϵ-ranking contributed mostly to the convergence improvement. Tabu moves and adaptive ϵ-ranking complement each other and led to improvement in terms of both convergence and diversity performance of moRBCs on MNK-landscapes.

5. AGGREGATION-BASED METHODS

Using aggregation functions is another way to differentiate many-objective solutions. According to the information aggregated, the methods are categorized into two classes: aggregation of individual information and aggregation of pairwise comparisons.

5.1. Aggregated Individual Information Based Methods

These kinds of methods usually use the aggregated individual information to compare solutions. The methods can be divided into two subclasses: aggregation of objective values and aggregation of objective ranks, as shown in Tables V and VI, respectively.

5.1.1. Aggregation of Objective Values. Weighted sum, weighted Tchebycheff (a.k.a. weighted min-max methods), vector angle distance scaling, and the boundary intersection methods are commonly used aggregation functions [Hughes 2011; Zhang and Li 2007]. Garza-Fabre et al. [2009] tested the weighted sum as a ranking method for MaOPs, where the weighting coefficients are used to specify importance of objectives.

Table VI. Aggregation of Objective Ranks

Algorithm	Variant	Paper	Test Problems (m)
MR	MR	[Garza-Fabre et al. 2009]	DTLZ(50)
AR	AR	[Garza-Fabre et al. 2009]	DTLZ(50)
RD	GDE3-R_{sum}	[Kukkonen and Lampinen 2007]	DTLZ(50)
	GDE3-R_{min}	[Kukkonen and Lampinen 2007]	DTLZ(50)

By employing a series of weighting vectors, the researchers are able to decompose a MaOP into many single-objective subproblems, each of which corresponds to a weighting vector [Ray et al. 2013].[9] Since these MOEAs do not rely on the Pareto dominance when conducting the selection, they can be applied directly to MaOPs without any additional change. Three aspects are critical to these algorithms: what aggregation functions are used, how to set the weighting vectors for subproblems, and how to update the best-known solution for each subproblem.

In multiple single objective Pareto sampling (MSOPS), population members are ranked according to both the vector angle distance scaling and weighted Tchebycheff methods [Hughes 2003]. MSOPS enables users to analyze a MaOP at hand, especially in terms of bounds and discontinuities of the PF.[10] However, this feature of MSOPS is only demonstrated by examples with two and three objectives. When dealing with MaOPs, it is highly possible that the large angle is due to a limited solution set size (a sample size on the PF) compared to the high PF dimensionality. The many-objective directed evolutionary line search (MODELS) algorithm [Hughes 2011] combines the aggregation-based method from MSOPS and the directed line search based on approximated local gradient. Both exploration and exploitation within the search landscape are taken into account in MODELS.

Zhang and Li [2007] proposed a multiobjective evolutionary algorithm based on decomposition (MOEA/D). Unlike MSOPS where an individual is evaluated with a number of weighting vectors, MOEA/D specifies an solution with only one vector [Yang et al. 2013]. In MOEA/D, each subproblem has a neighborhood defined by their weighting vectors and local information is shared by neighboring subproblems. This scheme has two benefits: lowering the computational complexity and improving results using local neighborhood information sharing. Ishibuchi et al. [2013] studied the relation between the neighborhood size for local mating and local replacement and the performance of MOEA/D on MaOPs.

Asafuddoula et al. [2013] introduced a decomposition-based evolutionary algorithm named DBEA-Eps with systematic sampling [Deb and Jain 2012a] for generating reference directions and adaptive epsilon control to balance convergence and diversity. The experimental results on DTLZ functions (with up to 15 objectives) and several engineering problems demonstrated that it outperformed M-NSGA-II [Deb and Jain 2012a] and MOEA/D-PBI when tackling unconstrained and constrained MaOPs [Deb and Jain 2014].

To deal with the diversity loss due to the high selection pressure, Garza-Fabre et al. [2011] proposed the parallel genetic algorithm (PGA), which combines effective ranking with speciation. Different from the approaches mentioned previously, PGA

[9]The aggregation-based approach (a.k.a. compromise programming) can be dated back to the 1970s [Miettinen 1999].
[10]In MSOPS, users know the investigated area of the PF by setting the weighting vectors. They can compute the angle between the weighting vector and the solution whose direction is closest to the weighting vector. If the angle corresponding to a certain weighting vector is extremely large, then one might say that discontinuity exists in the neighboring area of the weighting vector direction.

uses a subpopulation for each subproblem derived by aggregation functions, guided by the belief that isolated subpopulation leads to speciation and might help improve the performance of the algorithm. The experimental results demonstrated that using multiple subpopulations outperformed the conventional method in terms of convergence and isolation improves the diversity in the final approximation set.

How to set the weighting vectors is a key issue that determines the searching direction of these aggregation-based algorithms. Yet the selection of weighting vectors is still an open question, and ad hoc methods are widely used [Giagkiozis et al. 2013].

Some methods are designed to generate a restricted number of weighting vectors. This kind of generating methods are based on the normal-boundary insertion method [Das and Dennis 1998]), and the number of weighting vectors (H) for an m-objective MaOP satisfies the following equation: $H = \binom{m+p-1}{p}$, where p is the number of divisions along each objective. However, this may cause problems when the objective number increases. For example, to obtain at least one intermediate reference point for a eight-objective problem, $p = 8$ is the least number of divisions that we can set. This results in a total of 5,040 reference points. To avoid this situation, Deb and Jain [2014] proposed a two-layer generation method where the division numbers of the boundary and inside layer are set to (3,2) or (2,1) so that the total number of reference points is acceptable.

Several methods are also proposed to generate an arbitrary number of weighting vectors. Hughes [2007a] used an on-the-fly weighting vector generating method where the weighting vectors are updated using current population members in each generation. Thus, they relieved the algorithm users of the burden of setting the target vector a priori and enabled a more general algorithm for handling MaOPs. Zhang et al. [2009] tested MOEA/D on CEC09 unconstrained MOP test instances. They suggested an iterative way for setting weighting vectors with random sampling combined with iteratively selection. Tan et al. [2013] adopted a uniform design method to set the aggregation coefficient vectors, and thus the distribution of the coefficient vectors is more uniform over the objective space. The number of the coefficient vectors is unrestricted. Wang et al. [2013] argued that a uniformly distributed searching direction, instead of weighting vectors, could produce a uniformly distributed population. They set the weight values to the normalized reciprocal of the corresponding value in the original vector (a small positive is added to avoid division by zero). The method is evaluated by experimental results on DTLZ and WFG test suites with two and seven objectives. Giagkiozis et al. [2013] proposed the generalized decomposition (gD), which can achieve the optimal distribution of solutions once the geometry of the front is known a priori. The generated distribution can be adapted with the intervention of user's preferences. If no a priori knowledge is given, setting the vectors with a linear PF assumption is suggested. Experimental results demonstrated that it performed best in comparison to evenly distributed weighting vectors from Das and Dennis [1998] and Zhang and Li [2007] and the method suggested by Jaszkiewicz [2002] to generate a set of weighting vectors uniformly distributed on the probability simplex.

5.1.2. Aggregation of Objective Ranks. Instead of objective values, ranking dominance (RD) [Kukkonen and Lampinen 2007] aggregates the ranks of each objective values of a nondominated solution. A solution is said to ranking dominate another if its aggregation value is smaller than the other one. In Kukkonen and Lampinen [2007], two aggregation functions—sum of elements and minimum of elements—are introduced to RD. Experimental results demonstrated that RD showed a better coverage of PF than Pareto dominance. However, in some cases, it might mislead the search since it allows objective deterioration. Similar methods are also examined in Garza-Fabre et al. [2009].

Table VII. Aggregation of Pairwise Comparisons

Technique	Algorithm	Paper	Test Problems (m)
Maxmin fitness	MCDE	[Mendez and Coello 2012]	DTLZ(5)
	MC-MOEA	[Mendez and Coello 2013]	DTLZ(8)
SV-DOM	SV-DOM	[Köppen and Yoshida 2007]	DTLZ(15)
$-\epsilon$-DOM	$-\epsilon$-DOM		
FPD	FPD		
SOD-CNT	SV-DOM		

5.2. Aggregated Pairwise Comparisons Based Methods

In addition to the individual information, pairwise comparison results with other solutions in the population can be used in aggregation-based methods to evaluate many-objective solutions. A summary can be seen in Table VII. For a solution $\mathbf{x}$, its fitness value is the aggregation of the comparisons of $\mathbf{x}$ and all remaining solutions.

A general description of these methods is as follows: $APC(\mathbf{x}) = agg(comp(\mathbf{x}, \mathbf{y}))$, where $agg()$ is an aggregation function. For global detriment (GD) [Garza-Fabre et al. 2009], $comp(\mathbf{x}, \mathbf{y})$ is defined as $\sum_{k=1}^{m} \max(f_k(\mathbf{x}) - f_k(\mathbf{y}), 0)$. A similar mechanism is used in Profit (PF) [Garza-Fabre et al. 2009], except that the quality of a solution is expressed according to their profit other than detriment. The results on DTLZ functions with up to 50 objectives showed that these approaches behaved better than six state-of-the-art fitness assignment methods [Bentley and Wakefield 1998] in terms of convergence quality.

For the maxmin fitness function [Balling and Wilson 2001], agg is the maximum function, whereas $comp(\mathbf{x}, \mathbf{y})$ is defined as $\min_k(f_k(\mathbf{x}) - f_k(\mathbf{y}))$. Thus, a solution is dominated (nondominated, weakly dominated) if its maxmin fitness value is greater than (less than, equal to) zero. In Mendez and Coello [2013], the properties of maxmin fitness function are analyzed, and the maximin-clustering multiobjective evolutionary algorithm (MC-MOEA), which encompasses three selection operators, is proposed to overcome the disadvantages of maxmin fitness function. The experimental results showed that MC-MOEA is a good choice to solve MOPs having both low dimensionality (two or three) and high dimensionality (more than three) in the objective space.

As shown in Mendez and Coello [2013], these methods have a great advantage: good results in less time when compared to the hypervolume-based approach, which we will discuss later. Jaimes and Coello [2009] presented a quantitative analysis of different preference relations for MaOPs. The study revealed that average ranking and preference order relation stress the solutions far from the knee region, which might limit the applicability of them, as it is commonly assumed that the user prefers solutions in the knee region in general.

5.3. Discussion

Although aggregation-based methods do not rely on Pareto dominance to push the population toward the PF, these methods face their own challenges, which point to potential future research directions:

—*To configure the weighting vectors*: For aggregated individual information based methods, the weighting vectors dramatically affect the diversity performance. How to deal with the dilemma between limited computational resources and the exponentially increasing amount of weighting vectors is still an open question.

—*To choose the aggregation functions*: In Yang et al. [2013], the experimental studies showed that the solution set of MOEA/D is close to the real PF for some test cases, whereas the coverage of the PF is not as good. As indicated by Yang et al. [2013], Deb and Jain [2014], and Ishibuchi et al. [2010], it might do harm to the diversity

Table VIII. Hypervolume-Driven Methods

Indicator	Algorithm	Paper	Test Problems (m)
Hypervolume	SMS-EMOA	[Wagner and Neumann 2013]	DTLZ(20) WFG, LZ
	HypE	[Bader and Zitzler 2011]	DTLZ(50) MKP(50) WFG(50)
	HypE$_{uni}$ HypE$_p$	[Auger et al. 2009]	DTLZ(25)

maintenance when the Tchebycheff function is chosen as the aggregation method. It is possible that some weighting vectors may correspond to only one Pareto-optimal point by the Tchebycheff function [Deb and Jain 2014; Ishibuchi et al. 2010]. As for the weighted sum aggregation methods, their incapability of dealing with problems with nonconvex PFs was identified [Zhang and Li 2007] and needs to be improved.

6. INDICATOR-BASED APPROACH

Using indicator values to guide the search process seems to be a direct way to solve MaOPs, as the approximation set is evaluated according to the indicator. According to the indicator applied, we categorize these methods into three classes: hypervolume-driven, distance-based indicator driven, and R2 indicator driven algorithms.

6.1. Hypervolume-Driven Algorithms

The hypervolume indicator is the only metric known to be strictly monotonic to the Pareto dominance [Bader and Zitzler 2011]. This property gives rise to a series of hypervolume-based algorithms, as shown in Table VIII.

To maximize the hypervolume value of the solution set, Emmerich et al. [2005] proposed the S-metric selection evolutionary multiobjective algorithm (SMS-EMOA) [Emmerich et al. 2005]. SMS-EMOA is a $(\mu + 1)$ evolutionary algorithm,[11] guided by the gradient of the hypervolume metric [Bosman 2012], where each solution $\mathbf{a} \in A$ is evaluated by its hypervolume contribution: $Con_{HV}(\mathbf{a}, A) = I_{HV}(A) - I_{HV}(A \backslash \{\mathbf{a}\})$. The convergence and diversity of the resulting population is quite promising on low-dimensional MOPs. However, SMS-EMOA suffers from the exponentially increasing computational load in terms of the dimension of the objective space. As described by Beume and Rudolph [2006], the runtime of a generation of SMS-EMOA is O $(\mu^{m/2+1})$, where μ is the population size and m is the number of objectives.

To deal with the high computational time of calculating hypervolume values, Bader and Zitzler [2011] proposed a hypervolume estimation algorithm (HypE). They used Monte Carlo sampling to approximate the exact hypervolume values, allowing a trade-off between accuracy and time. In HypE, the nondominated solutions are compared according to their hypervolume-based fitness values. Specifically, the fitness value of a solution $\mathbf{a}$, denoted as $I_h^k(\mathbf{a}, A)$, is defined as the expected hypervolume loss that can be attributed to $\mathbf{a}$ when it is removed from the population A along with $k - 1$ other uniformly randomly chosen solutions. During selection, k equals to the number of solutions to be removed from the nondominated solution set Q, which is the latest front of the combined population to be added to the next generation. The experimental results showed that HypE achieved competitive performance in terms of the average hypervolume on the DTLZ, WFG, and MKP problem suites with up to 50 objectives.

[11] In each generation, the current population of a $(\mu + 1)$ evolutionary algorithm (a.k.a. a steady-state evolutionary algorithm) includes μ individuals while the number of offspring is 1.

Table IX. Distance-Based Indicator Driven Algorithms

Indicator	Algorithm	Paper	Test Problems (m)
α Indicator	AGE	[Bringmann et al. 2011]	DTLZ(20)
	AGE-II	[Wagner and Neumann 2013]	DTLZ(20)
IGD	MyO-DEMR	[Denysiuk et al. 2013]	DTLZ(20)
Δ_p Indicator	DDE	[Villalobos and Coello 2012]	DTLZ(10)
IBEA	IBEA	[Zitzler and Künzli 2004]	EXPO(4)

Table X. R2 Indicator Based Approaches

Indicator	Algorithm	Paper	Test Problems (m)
$R2$ indicator	MOMBI	[Gómez and Coello 2013]	DTLZ(8) WFG(8)
	R2-MOGA	[Manriquez et al. 2013]	DTLZ(10) WFG(3)
	R2-MODE		

6.2. Distance-Based Indicator Driven Algorithms

Since computing the hypervolume indicator is too time consuming, several distance-based indicator driven algorithms are proposed (Table IX). Among them, the indicator-based evolutionary algorithm (IBEA) uses a binary additive ϵ-indicator I_{ϵ^+}, the Δ_p differential evolution (DDE) incorporates the Δ_p indicator into the selection mechanism, and the approximation-guided evolutionary (AGE) algorithm aims at minimizing the α indicator, whereas many-objective differential evolution with mutation restriction (MyO-DEMR) takes advantage of the IGD values [Villalobos and Coello 2012; Bringmann et al. 2011]. The performance of these algorithms is quite competitive according to experimental studies. For example, the experimental results on DTLZ functions with up to 20 objectives indicated that AGE could achieve better performance in terms of the α and hypervolume indicator when compared to IBEA, NSGA-II, SMS-EMOA, and SPEA2 [Bringmann et al. 2011].

6.3. R2 Indicator Driven Algorithms

R2-MOGA and R2MODE integrate the $R2$ indicator into a modified version of Goldberg's non-dominated sorting method [Manriquez et al. 2013]. The empirical results on DTLZ with up to 10 objectives indicate that these algorithms can outperform SMS-EMOA by using much less computational time. Another algorithm based on the $R2$ indicator, namely many-objective metaheuristic based on $R2$ indicator (MOMBI), achieved similar performance [Gómez and Coello 2013] (Table X).

6.4. Discussion

When it comes to the indicator-based algorithms, their difficulties are different from Pareto-based approaches, as they do not rely on Pareto dominance to push the solutions toward the PF. However, the indicator-based MaOEAs do have their own issues.

First, the computational cost of the hypervolume value is high. As the experimental results indicated [Bringmann et al. 2011], the running time of SMS-EMOA has severely limited its application to MaOPs with more than eight objectives. Although other indicators cost much less time, they are not strictly monotonic with the Pareto dominance and might lead to performance degration. There has been some recent work on faster computation of the hypervolume indicator, such as hypervolume by slicing objectives (HSO) with worst-case complexity being $O(N^{M-1})$ [While et al. 2006], the

Table XI. Reference Set Based Approach

Algorithm	Variants	Paper	Test Problems (m)
TAA	TAA	[Praditwong and Yao 2006]	DTLZ(8)
	TAA	[Praditwong et al. 2011]	SMCP(5)
	Two_Arch2	[Wang et al. 2014]	DTLZ(20),WFG(20)
TC-SEA	TC-SEA	[Moen et al. 2013]	DTLZ(20)
NSGA-III	NSGA-III	[Deb and Jain 2012a]	DTLZ(10)
	*A*NSGA-III	[Jain and Deb 2013]	DTLZ(8)
	A^2NSGA-III	[Jain and Deb 2013]	DTLZ(8)

WFG algorithm with worst-case complexity being $O(2^{N-1})$ [While et al. 2012], and so forth.[12]

Second, the hypervolume might not be appropriate when the decision maker aims to find a uniform spread optimal set. For SMS-EMOA, which aims at obtaining the maximal hypervolume value, the algorithm prefers knee points and returns a uniformly distributed population only when the PF is linear [Emmerich et al. 2005]. Although these observations are based on the results on two-objective problems, it can be inferred that SMS-EMOA also has difficulties in generating a uniformly spreading solution set when applied to MaOPs. Additional archiving strategies or niching methods might be needed to improve the distribution quality.

Third, to compute the indicator values, a reference point(s) is needed by the hypervolume and R2 indicator. However, for other indicators, such as IGD, GD, and Δ_p, a subset of the real PF is needed. Yet to get such an appropriate subset *a priori* is a challenging task.

7. REFERENCE SET BASED APPROACH

In recent years, several reference set based MaOEAs have been proposed. These methods use a set of reference solutions to measure the quality of solutions. Thus, the search process is guided by the solutions in the reference solution set. There are two key points about the reference set based approach: (1) how to construct the reference set and (2) how to measure the quality of the populations members using the reference set. As shown in Table XI, there are three reference set based MaOEAs: the two archive algorithms (TAAs) [Praditwong and Yao 2006; Wang et al. 2014], many-objective NSGA-II (NSGA-III) [Deb and Jain 2014; Jain and Deb 2014], and the taxi-cab surface evolutionary algorithm (TC-SEA) [Moen et al. 2013]. TAA selects solutions from historical or current populations to construct the reference set. NSGA-III and TC-SEA, in contrast, just use a set of virtual solutions to form an ideal solution front as the search goal. For virtual solutions, we only know the objective values without having any idea about their corresponding decision vectors.

7.1. Real Reference Set Based Approach

Praditwong and Yao [2006] and Wang et al. [2014] proposed a novel two archive algorithm (TAA) and its improved version (Two_Arch2), which separate nondominated solutions of each generation into two archives, namely convergence archive (CA) and diversity archive (DA). CA can be seen as an online-updated real reference set and contains only nondominated solutions that once dominated some existing archive members. When the total solutions in the union of CA and DA overflow, the solution in DA with the shortest distance to CA is removed iteratively until the archives meet the capacity constraint. Experimental results [Wang et al. 2014] demonstrate that TAA

[12]Here, N is the population size and M is the number of objectives.

and its latest variant (Two_Arch2) outperform other state-of-the-art MaOEAs significantly in terms of convergence with comparable diversity quality. This approach was also applied to real-world problems in software engineering [Praditwong et al. 2011] and in sensornet protocol optimization [Tate et al. 2012].

7.2. Virtual Reference Set Based Approach

Different from TAA, NSGA-III and TC-SEA use virtual points in the objective space to constitute the reference set. The two algorithms use similar methods to construct the reference set,[13] but there are differences between their selection mechanisms.

To reduce the deterioration of NSGA-II on MaOPs, NSGA-III replaces the crowding distance based diversity-preservation operator in NSGA-II by a reference set based niche-preservation strategy [Deb and Jain 2014]. After determining the reference solutions on a hyperplane, the population members are normalized and then associated to a reference solution according to the perpendicular distances to the reference lines defined by joining the reference solution with the origin. Solutions associated with the reference solution, which is associated with less population members, are preferred. Thus, the search power of all reference directions is balanced. NSGA-III achieves desirable performance on a number of problems with up to 10 objectives [Deb and Jain 2014]. Some adaptive variants of NSGA-III are also proposed and obtain better distributions of solutions [Jain and Deb 2013, 2014].

The TC-SEA [Moen et al. 2013] differs from NSGA-III in three aspects. First, TC-SEA uses an online constructed reference set, whereas NSGA-III uses a predefined structured reference set. Second, TC-SEA associates solutions to reference solutions according to the TC value instead of the orthogonal distance as in NSGA-III. Third, TC-SEA uses the rank and the value of TC metric as two sorting criteria for selection. The algorithm is tested on the DTLZ benchmark problems with up to 20 objectives.

7.3. Discussion

Although TAA was originally proposed several years ago, the reference set based approach is still in its infancy, and there are many topics that deserve more investigation. First, whether the real reference set is better than the virtual one for guiding the search process is still an open question. The real reference set seems to contains more information of the evolution process, whereas the virtual reference set is better organized. Second, TAA, NSGA-III, and TC-SEA use Euclidean distance, orthogonal distance, and TC value to associate solutions with reference solutions. One might wonder which one is the most appropriate for the association. Third, both NSGA-III and TC-SEA seem to follow a convergence first, divergence second pattern, whereas TAA manages two archives focusing on convergence and diversity simultaneously. Like other evolutionary algorithms, the balance between convergence and diversity is an important research topic in the reference set based approach.

8. PREFERENCE-BASED APPROACH

To approximate the entire PF using MaOEAs, we need to increase the population size exponentially in terms of the objective space dimension [Ishibuchi et al. 2008]. However, in most real-world MaOPs, the population size is often too limited compared to the large objective space to produce any results in a meaningful way [Rachmawati and Srinivasan 2006]. Thus, it seems to be a good idea to focus on a subset of the PF according to the users' preference. This kind of approach has been discussed in the literature [Ishibuchi et al. 2008].

[13]Both of them incorporate the normal-boundary insertion method from Das and Dennis [1998].

Table XII. A Priori Preference-Based Algorithms

Algorithm	Paper	Test Problems (m)	Preference Model
ϵ-Constraint+CDE	[Becerra et al. 2013]	WFG(10)	Goal constraint
2p-NSGA-II	[Qiu et al. 2012]	DTLZ(15)	Distances to goal points
PBEA	[Thiele et al. 2009]	LPMS(5)	Goal point
MQEA-PS	[Kim et al. 2012]	DTLZ(7)	Objective weighting
MQEA-PS2	[Ryu et al. 2012]	DTLZ(7)	Objective weighting
SBGA	[Gong et al. 2013b]	DTLZ(20)	Preferred region
Reference point (RP) Guided dominance PROMETHEE II (P2)	[Eppe et al. 2011]	mTSP(4)	Distances to one point Objective weighting Objective weighting
MOPSO-PS	[Lee and Kim 2011]	DTLZ(7)	Objective weighting
HypE$_{uni}$	[Auger et al. 2009]	DTLZ(25)	Weight distribution functions

There are two key points in the preference-based approach: what preference models are chosen and when to integrate the preference information. There are a wide variety of preference models, such as goal specification, trade-off between objectives, objective rankings, and so on [Rachmawati and Srinivasan 2006]. According to the timing of integrating the preference information into the optimizing process, three classes of algorithms can be defined [Evans 1984; Jaimes et al. 2011a]:

—*A priori algorithms (selection before search)*: The preference information is set up before the search, and it will guide the population to converge to a subset of the PF [Auger et al. 2009; Qiu et al. 2012; Kim et al. 2012; Becerra et al. 2013].
—*Interactive algorithms (selection during search)*: The optimization process asks the decision makers (DMs) for the preference information interactively to direct the search to the region of interest, which is a subset of the PF [Deb and Chaudhuri 2005; Deb et al. 2006; Deb and Kumar 2007; Thiele et al. 2009; Jaimes et al. 2011b; Gong et al. 2013a].
—*A posteriori algorithms (selection after search)*: The preference information is introduced after running MaOEAs and obtaining a solution set that approximates the real PF [Purshouse et al. 2011; Wang et al. 2012, 2013a, 2013c].

8.1. A Priori Preference-Based Algorithms

Although sharing the same timing of integrating preferences, these algorithms use quite different methods for modeling the preference information. As shown in Table XII, there are eight preference models, which can be categorized into two classes: goal specification and objective weighting.

8.1.1. Goal Specification. Goal specification allows a user to provide one solution, a set of solutions, or a subregion of the solution space that they are most interested in as preference information. The preference-based evolutionary algorithm (PBEA) [Thiele et al. 2009] takes into account the preference information by integrating an achievement function that relies on the reference point. It is evaluated experimentally on a five-objective problem from Miettinen [2003]. In Becerra et al. [2013], a goal vector is provided as the preference information and the goal-constraint method is used to embed it into many-objective optimization. In Qiu et al. [2012], a bipolar preference dominance is proposed, where two points is required from the DM: the ideal point and the nadir point. The bipolar preference pushes the population toward the ideal point and away from the nadir point. Gong et al. [2013b] used preferred regions to describe the DM's preferences. Solutions are compared according to their ranks and distances from the DM's preferred regions.

Table XIII. Interactive Preference-Based Algorithms

Algorithm	Paper	Test Problems (m)	Preference Model
P-MOET	[Kalboussi et al. 2013]	MOAATP(7)	Goal point
ITuCPR	[Jaimes et al. 2011b]	ADP(6)	Goal point
R-NSGA-II	[Deb et al. 2006]	DTLZ(10)	Goal points
RD-NSGA-II	[Deb and Kumar 2007]	DTLZ(10)	Reference direction
IEA with preference polyhedron	[Gong et al. 2013a]	DTLZ(5)	Preference polyhedron

8.1.2. Objective Weighting. In objective weighting models, a user will assign each objective some sort of weighting to aggregate them. Kim et al. [2012] used objective weighting to represent a user's preference in MQEA-PS. λ-Fuzzy measures are used to integrate the objective weighting and the objectives of the solutions and to measure the quality of the solutions in terms of user's preference. The results on 7-objective DTLZ problems and the 3-objective fuzzy path planner optimization problem demonstrated that MQEA-PS improves the performance for both problems compared to MQEA with dominance-based selection and other MOEAs like NSGA-II and MOPBIL. An improved version, MQEA-PS2, which uses multiple populations, is further developed in Ryu et al. [2012]. Guided dominance [Branke et al. 2001] and PROMETHEE II (P2) [Brans and Mareschal 2005] are two similar methods, except guided dominance incorporates linear trade-off functions, whereas PROMETHEE II uses aggregated preference functions. Auger et al. [2009] used weight distribution functions in HypE$_{uni}$ to emphasize specific solutions or subregions in the objective space. The efficiency of HypE$_{uni}$ was tested on test problems with up to 25 objectives.

8.2. Interactive Preference-Based Algorithms

In interactive algorithms (selection during search, shown in Table XIII), the DMs need to provide preference information to the algorithms interactively. These algorithms can reduce the computational load online and introduce the preference information progressively [Fonseca and Fleming 1998]. There are several different preference models, such as goal point(s), reference direction, and preference polyhedron.

8.2.1. Goal Specification. Reference point based NSGA-II (R-NSGA-II) introduced a preference operator that prefers solutions closer to the goal points and de-emphasizes solutions within a ϵ-neighborhood of a goal point [Deb et al. 2006]. In Kalboussi et al. [2013], R-NSGA-II was integrated into a preference-based many-objective evolutionary testing (P-MOET) method to solve a seven-objective software engineering problem.

Jaimes et al. [2011b, 2013] used the Tchebycheff preference relation to compare solutions. The region of interest (ROI) is defined as a neighborhood region near the goal point. For two solutions in the ROI, the Tchebycheff preference becomes classical Pareto dominance. Otherwise, the one nearer to the goal point is preferred. The experimental results showed that the Tchebycheff relation improves the convergence of NSGA-II without sacrificing the distribution [Jaimes et al. 2013].

8.2.2. Reference Direction. In reference direction based NSGA-II (RD-NSGA-II) [Deb and Kumar 2007], the DMs are supposed to supply a reference direction in the objective space in each iteration. Then the solutions are ranked with the aid of an achievement scalarizing function and the crowding distance value. The algorithm is tested on DTLZ functions with up to 10 objectives.

8.2.3. Preference Polyhedron. Gong et al. [2013a] proposed an interactive evolutionary algorithm (IEA) based on the preference polyhedron theory. During the search process, the DM periodically obtains a nondominated solution set and chooses the most preferred one.

Table XIV. A Posteriori Preference-Based Algorithms

Algorithm	Paper	Test Problems (m)	Keywords
a-PICEA-g	[Wang et al. 2013a]	WFG(7)	Preference solutions
PICEA-w	[Wang et al. 2013b]	WFG(7)	
PICEA-g	[Wang et al. 2013c]	WFG(10)	
	[Purshouse et al. 2011]	WFG(13)	
LPICEA-g	[Wang et al. 2012]	WFG(4)	
MUF-EA	[Li and Liu 2012]	DTLZ(10)	Marginal utility function

8.3. A Posteriori Preference-Based Algorithms

The a posteriori preference-based algorithms (selection after search) try to avoid the intervention of the DMs before or during the optimization process. Users do not need to participate until the solution set is obtained. As shown in Table XIV, there are two main algorithms: PICEA-g and MUF-EA.

In preference-inspired coevolutionary algorithms (PICEAs), preferences are modeled as a set of solutions that coevolve along with the population [Wang et al. 2013a, 2013b]. It should be noted that these algorithms aim at approximating the whole PF, different from the previously mentioned preference-based algorithms. By systemically comparing a variant of PICEA-g [Purshouse et al. 2011] with representative methods of other classes, Wang et al. [2013c] explored the potential of PICEAs. The empirical results on the WFG test problems with up to 10 objectives indicated that PICEA-g was always the first or second ranked method in terms of convergence and spread when compared to several state-of-the-art methods.

Different from PICEAs, Li and Liu [2012] used a marginal utility function to describe the user's preferred range for each objective. In that way, the algorithm can supply the user with a set of solutions in the region in which he or she is interested. The algorithm is tested on DTLZ test problems with up to 10 objectives.

8.4. Discussion

Since the search direction is biased toward the ROI, the a priori and interactive algorithms can reduce the computational load during the search process and pay more attention to preferred solutions. The a posteriori preference-based algorithms are inferior to the other two classes, as they might obtain a large number of solutions in which the DM is not interested. One problem about the interactive algorithms lies in that the algorithms need to interact with the DMs frequently, who are prone to fatigue [Gong et al. 2013b]. In some occasions, the preference information provided by a tired user might be very misleading.[14]

9. DIMENSIONALITY REDUCTION APPROACH

Whereas some algorithms try to tackle the difficulties of MaOPs directly, a series of dimensionality reduction algorithms are proposed to circumvent the roadblock of MaOPs [Van der Maaten et al. 2009; Saxena et al. 2013]. The dimensionality reduction approach aims to deal with MaOPs with redundant objectives. Ishibuchi et al. [2011] showed that the performance of NSGA-II was not degraded by the increased objectives when they are highly correlated or dependent. When a high-dimensional MaOP has a similar PF to another problem with fewer objectives, we can try to optimize the lower-dimensional problem instead of the original one. According to the timing of incorporating the dimensionality reduction techniques into a MaOEA, this approach can be categorized into two classes: offline and online methods.

[14]For more information about preference incorporation in MOEA, please refer to Deb [2001], Rachmawati and Srinivasan [2006], and Bechikh [2013].

Table XV. Offline Dimensionality Reduction Methods

Algorithm	Paper	Test Problems (m)
L-PCA	[Saxena et al. 2013]	DTLZ(25), WFG(25) DTLZ5(I,M)(50)
NL-MVU-PCA		
Exact-δ-MOSS/k-EMOSS	[Brockhoff and Zitzler 2009]	-
Greedy-δ-MOSS	[Brockhoff and Zitzler 2006a, 2006b, 2009]	DTLZ(25), MKP(25)
Greedy-k-EMOSS	[Brockhoff and Zitzler 2006a, 2006b, 2009]	DTLZ(25), MKP(25)
Greedy-k-EMOSS-omission	[Brockhoff and Zitzler 2009]	-
Greedy-$\mathrm{OA}_{\mathrm{max/avg}}$	[Brockhoff and Zitzler 2010]	DTLZ(15), WFG(15) $\mathrm{DTLZ2}_{\mathrm{BZ}}$(15) $\mathrm{DTLZ3}_{\mathrm{BZ}}$(15) MKP(15), EXPO(4)
PCSEA	[Singh et al. 2011]	DTLZ2(20) DTLZ5(I,M)(100), WFG(3)
ϵ-Optimization	[Lindroth et al. 2010]	Truck model(12)
Algorithm1	[Jaimes et al. 2008]	DTLZ5(I,M)(10), MKP(20) $\mathrm{DTLZ2}_{\mathrm{BZ}}$(20)
Algorithm2		

9.1. Offline Dimensionality Reduction

For offline methods (summarized in Table XV), dimensionality reduction is carried out after obtaining a set of Pareto-optimal solutions. According to the key techniques used, they can be further divided into three subclasses [Singh et al. 2011]: correlation-based, dominance structure based, and feature selection based methods.

9.1.1. Correlation-Based Methods. One direction for dimensionality reduction is to examine the correlation among objectives. Saxena et al. [2013] proposed L-PCA based on principal component analysis and NL-MVU-PCA based on maximum variance unfolding for linear and nonlinear objective reduction, respectively [Saxena et al. 2013]. The performance of these algorithms has been studied on 30 test instances and two real-world problem instances. When minimizing the differences between the respective Pareto-optimal sets, the ϵ-optimization from Lindroth et al. [2010] is an alternative method based on the correlations between objectives.

9.1.2. Dominance Structure Based Methods. The dominance structure based methods aim to reduce the number of objectives by considering the dominance relationships among the solutions obtained by an entire run of a MaOEA. The more dominance structure the population keeps, the less side effects the information loss coming with the dimensionality reduction will cause.

By measuring the degree of conflict of objective sets based on ϵ-dominance, Brockhoff and Zitzler [2009] proposed both exact and heuristic algorithms to reduce the number of objectives while minimizing the harm to the dominance structure. They [Brockhoff and Zitzler 2010] proposed a heuristic that iteratively merges two objectives into a new one using a weighted combination method. The method is demonstrated to be highly useful for reducing the information loss.

Singh et al. [2011] proposed the Pareto corner search evolutionary algorithm (PCSEA). Instead of search for the whole PF, PCSEA searches corners of the PF to obtain individuals to identify the relevant objectives. After obtaining a solution set from PCSEA, a heuristic technique based on the change of number of nondominated solutions is used to iteratively reduce the objectives. The method is evaluated by experiments on objectives with up to 100 objectives.

Table XVI. Online Dimensionality Reduction Methods

Algorithm	Variants	Paper	Test Problems (m)
SIBEA	$\text{SIBEA}_{\text{fixed,kEMOSS}}$ $\text{SIBEA}_{\text{fixed,random}}$	[Brockhoff and Zitzler 2007]	DTLZ(9) DTLZ2_{BZ}(9)
	$\text{SIBEA}_{\text{online}}$ $\text{SIBEA}_{\text{random}}$	[Brockhoff and Zitzler 2009]	DTLZ(9) DTLZ2_{BZ}(9)
C-PCA-NSGA-II	C-PCA-NSGA-II	[Saxena and Deb 2007]	DTLZ5(I,M)(50) DLTZ2(5)
MVU-PCA-NSGA-II	MVU-PCA-NSGA-II		
PCA-NSGA-II	PCA-NSGA-II	[Brockhoff et al. 2008]	DTLZ5(I,M)(50)
REDGA	REDGA-S-1	[Jaimes et al. 2009]	MKP(20),DTLZ2_{BZ}(20)
	REDGA-S-m		
	REDGA-X-m		
MICA-NORMOEA	MICA-NORMOEA	[Guo et al. 2012]	DTLZ2(M)(5) DTLZ5(I,M)(20)
OC-ORA	OC-ORA	[Guo et al. 2013]	DTLZ(20) DTLZ5(I,M)(20) SDSSP(5)[a]

[a]SDSSP is short for Storm Drainage Systems Planning Problem.

9.1.3. Feature Selection Based Method. Jaimes et al. [2008] proposed two algorithms to reduce objective numbers based on a feature selection technique from Mitra et al. [2002]. Both methods select essential objectives according to the correlation between each pair of objectives so that the least conflicting objectives are discarded. Experimental results showed that they could reduce the computational cost compared to algorithms from Brockhoff and Zitzler [2006a] and Deb and Saxena [2006].

9.2. Online Dimensionality Reduction

By iteratively obtaining solution sets and invoking the dimensionality reduction technique, the number of objectives can be reduced gradually during the search process. As shown in Table XVI, there are seven online dimensionality reduction algorithms.

As far as we know, all pf the online algorithms follow correlation-based pattern. C-PCA-NSGA-II, MVU-PCA-NSGA-II, and PCA-NSGA-II use the same framework of iteratively obtaining solution sets and reducing the objectives using information of correlations among the objectives [Saxena and Deb 2007; Brockhoff et al. 2008]. Using the interdependence coefficient to represent relations among objectives, Guo et al. [2012] proposed an objective reduction algorithm named MICA-NORMOEA. In MICA-NORMOEA, the partitioning around medoids (PAM) [Kaufman and Rousseeuw 2009] clustering algorithm and NSGA-II are invoked iteratively to reduce the objective numbers until the stop criterion is met. Jaimes et al. [2009] considered two schemes to embed objective reduction into a MaOEA: one scheme continuously reduces the objective dimension and only uses the original objective set in the last generation, and the other uses reduced and original objective sets in turn to guide the search process. Results on 10 objective DTLZ and MKP problems showed that both schemes improve the convergence of a MaOEA.

9.3. Discussion

The dimensionality reduction approach has three main advantages. First, it can reduce the computational load of a MaOEA. Second, it can help DMs understand the MaOP better by pointing out the redundant objectives. Third, it is consistent with other approaches and is easy to be combined other approaches. For example, Sinha et al. [2013] proposed a framework named PI-EMO-VF, which combines an objective reduction based problem simplification procedure with an interaction based solution procedure. Brockhoff and Zitzler [2007] integrated the objective reduction technique into hypervolume-based algorithms to save the time of computing hypervolumes.

The dimensionality reduction approach assumes that the MaOP at hand has redundant objectives. This assumption may limit the application of the approach. When faced with MaOPs without redundant objectives, these algorithms may fail to reduce the number of objectives or return a solution set that does not cover the complete PF [Singh et al. 2011]. Moreover, the aim of saving computational time is achieved at the cost of losing information of the dominance structure of the original problem that is related to all of the objectives [Brockhoff and Zitzler 2010]. Whether the information loss might cause problems is still an open question.

10. CONCLUSIONS AND FUTURE DIRECTIONS

In this article, we have conducted a comprehensive survey of different MaOEAs. They are categorized into seven classes: relaxed dominance based, diversity-based, aggregation-based, indicator-based, reference set based, preference-based, and dimensionality reduction approaches.

The relaxed dominance based approach tries to relax the definition of dominance and increase the selection pressure toward the PF. To this end, a series of methods have been proposed, such as value-based dominance modifications (ϵ-dominance, CDAS, grid dominance, etc.), and number-based methods that evaluate a solution according to the number of objectives that it is better than, the same as, or worse than the other. However, the drive toward a more aggressive selection pressure could make diversity maintenance more difficult in a relaxed dominance based MaOEA.

The diversity-based approach tries to improve the performance of MaOEAs by reducing the adverse impact of diversity maintaining. To this end, several customized diversity maintaining strategies are proposed, such as activation/deactivation of the diversity promotion, grid-based neighborhood niching, SDE, and tabu moves, which are demonstrated to be highly effective.

The aggregation-based approach optimizes MaOPs using a series of scalarizing functions of the individual information. Using scalarizing functions to rank the solutions overcomes the incomparability of the nondominated solutions. Both the weighting vectors and the scalarizing functions could influence the performance of these algorithms. Compared to the exponentially increasing objective space, the number of scalarizing functions are usually very limited. This situation might cause difficulties for diversity maintenance of the solution set.

The indicator-based approach optimizes MaOPs under the guidance of an indicator value of the solution set, since a better indicator value indicates that the solution set is a better approximation to the true PF. The hypervolume metric has gained much attention because of its inherent consistency with the definition of Pareto dominance. However, the computation cost has hindered the application of hypervolume-based methods. Having this in mind, some researchers use other indicators with less computational costs to steer the optimizing process.

The reference set based methods use a set of reference solutions to estimate the quality of solutions and guide the search process. There are two key points about the reference set based approach: how to manage the reference set and how to use the reference set to estimate the quality of the populations members. Both of the two points are worth further investigation for developing new algorithms.

The preference-based approach was originally designed to bias the search to a subregion of the PF with the aid of user's preference. According to the timing when the user's preference is incorporated into the search process, the preference-based approach can be categorized into three classes: a priori, interactive, and a posteriori methods, which take advantage of the user's preference before, during, and after optimization.

Different from other classes of approaches, the dimensionality reduction approach tries to reduce the number of objectives and change the MaOP at hand into another one with less objectives with a similar PS. According to the dimensionality reduction techniques used, we categorize these algorithms into three classes: correlation-based, dominance structure based, and feature selection based methods. These methods can reduce the computational load but potentially lose some information as a result of the reduced objectives.

In addition to the previously mentioned approaches, there have been some other topics closely related to MaOPs, such as a large population size [Kowatari et al. 2012; Aguirre et al. 2013a], customized mutation and crossover operators [Sato et al. 2011a], nondominated sorting methods [Wang and Yao 2014], and better crowding distance measures [Wang et al. 2010; Minku and Yao 2013].

Even though a number of MaOEAs have been proposed in recent years, there are still many open problems that need to be solved. On the one hand, it is useful to improve the current approaches by developing a better understanding of their behaviors and mechanisms and overcoming their drawbacks. For example, for the relaxed dominance based approach, more adaptive, flexible, and robust selection pressure enhancement methods need to be studied. More effective diversity maintaining strategies need to be designed for the aggregation-based approach. In the indicator-based approach, it might be an interesting idea to combine computationally costly indicator (i.e., hypervolume) with computationally cheap indicators (i.e., GD, IGD) to guide the search process. For the reference set based approach, the updating strategy of reference sets is worth studying. On the other hand, it would be very interesting to combine two or more approaches together. The resulting hybrid algorithms (e.g., Aguirre and Tanaka [2009b] and Brockhoff and Zitzler [2007]) might show better performance on MaOPs. For example, the relaxed dominance based methods can be combined with the dimensionality reduction methods to further increase the selection pressure toward the PF.

ELECTRONIC APPENDIX

The electronic appendix for this article can be accessed in the ACM Digital Library.

ACKNOWLEDGMENTS

The authors would like to thank Yu Sun from UBRI, USTC for proofreading.

REFERENCES

S. F. Adra and P. J. Fleming. 2011. Diversity management in evolutionary many-objective optimization. *IEEE Transactions on Evolutionary Computation* 15, 2, 183–195.

H. E. Aguirre, A. Liefooghe, S. Verel, and K. Tanaka. 2013a. A study on population size and selection lapse in many-objective optimization. In *Proceedings of the 2013 IEEE Congress on Evolutionary Computation (CEC'13)*. IEEE, Los Alamitos, CA, 1507–1514.

H. E. Aguirre, A. Oyama, and K. Tanaka. 2013b. Adaptive ε-sampling and ε-hood for evolutionary many-objective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 322–336.

H. E. Aguirre and K. Tanaka. 2004. Insights on properties of multiobjective MNK-landscapes. In *Proceedings of the 2004 IEEE Congress on Evolutionary Computation (CEC'04)*, Vol. 1. IEEE, Los Alamitos, CA, 196–203.

H. E. Aguirre and K. Tanaka. 2007. Working principles, behavior, and performance of MOEAs on MNK-landscapes. *European Journal of Operational Research* 181, 3, 1670–1690.

H. E. Aguirre and K. Tanaka. 2009a. Many-objective optimization by space partitioning and adaptive ε-ranking on MNK-landscapes. In *Evolutionary Multi-Criterion Optimization*. Springer, 407–422.

H. E. Aguirre and K. Tanaka. 2009b. Space partitioning with adaptive ε-ranking and substitute distance assignments: A comparative study on many-objective MNK-landscapes. In *Proceedings of the 11th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 547–554.

H. E. Aguirre and K. Tanaka. 2010a. A hybrid scalarization and adaptive ε-ranking strategy for many-objective optimization. In *Parallel Problem Solving from Nature, PPSN XI*. Springer, 11–20.

H. E. Aguirre and K. Tanaka. 2010b. Space partitioning evolutionary many-objective optimization: Performance analysis on MNK-landscapes. *Information and Media Technologies* 5, 2, 636–649.

M. Asafuddoula, T. Ray, and R. Sarker. 2013. A decomposition based evolutionary algorithm for many objective optimization with systematic sampling and adaptive epsilon control. In *Evolutionary Multi-Criterion Optimization*. Springer, 413–427.

A. Auger, J. Bader, D. Brockhoff, and E. Zitzler. 2009. Articulating user preferences in many-objective problems by sampling the weighted hypervolume. In *Proceedings of the 11th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 555–562.

J. Bader and E. Zitzler. 2011. HypE: An algorithm for fast hypervolume-based many-objective optimization. *Evolutionary Computation* 19, 1, 45–76.

R. Balling and S. Wilson. 2001. The maxi-min fitness function for multi-objective evolutionary computation: Application to city planning. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'01)*. 1079–1084.

L. Batista, F. Campelo, F. Guimaraes, and J. Ramírez. 2011a. A comparison of dominance criteria in many-objective optimization problems. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 2359–2366.

L. Batista, F. Campelo, F. Guimarães, and J. Ramírez. 2011b. Pareto cone ε-dominance: Improving convergence and diversity in multiobjective evolutionary algorithms. In *Evolutionary Multi-Criterion Optimization*. Springer, 76–90.

R. L. Becerra, C. A. C. Coello, and G. T. Pulido. 2013. Goal-constraint: Incorporating preferences through an evolutionary ε-constraint based method. In *Proceedings of the 2013 IEEE Congress on Evolutionary Computation (CEC'13)*. IEEE, Los Alamitos, CA, 741–747.

S. Bechikh. 2013. *Incorporating Decision Makers Preference Information in Evolutionary Multi-Objective Optimization*. Ph.D. Dissertation. High Institute of Management of Tunis, University of Tunis, Tunisia.

P. Bentley and J. Wakefield. 1998. Finding acceptable solutions in the Pareto-optimal range using multi-objective genetic algorithms. In *Soft Computing in Engineering Design and Manufacturing*. Springer, 231–240.

N. Beume and G. Rudolph. 2006. Faster S-metric calculation by considering dominated hypervolume as Klee's measure problem. In *Proceedings of the 2nd IASTED Conference on Computational Intelligence (CI'06)*. 231–236.

P. A. N. Bosman. 2012. On gradients and hybrid evolutionary algorithms for real-valued multiobjective optimization. *IEEE Transactions on Evolutionary Computation* 16, 1, 51–69.

P. A. N. Bosman and D. Thierens. 2003. The balance between proximity and diversity in multiobjective evolutionary algorithms. *IEEE Transactions on Evolutionary Computation* 7, 2, 174–188.

J. Branke, T. Kaußler, and H. Schmeck. 2001. Guidance in evolutionary multi-objective optimization. *Advances in Engineering Software* 32, 6, 499–507.

J. P. Brans and B. Mareschal. 2005. PROMETHEE methods. In *Multiple Criteria Decision Analysis: State of the Art Surveys*. Springer, 163–186.

K. Bringmann, T. Friedrich, F. Neumann, and M. Wagner. 2011. Approximation-guided evolutionary multi-objective optimization. In *Proceedings of the 22nd International Joint Conference on Artificial Intelligence.* 1198–1203.

D. Brockhoff, D. K. Saxena, K. Deb, and E. Zitzler. 2008. On handling a large number of objectives a posteriori and during optimization. In *Multiobjective Problem Solving from Nature*. Springer, 377–403.

D. Brockhoff, T. Wagner, and H. Trautmann. 2012. On the properties of the R2 indicator. In *Proceedings of the 14th Annual Conference on Genetic and Evolutionary Computation (GECCO'12)*. ACM, New York, NY, 465–472.

D. Brockhoff and E. Zitzler. 2006a. Are all objectives necessary? On dimensionality reduction in evolutionary multiobjective optimization. In *Parallel Problem Solving from Nature, PPSN IX*. Springer, 533–542.

D. Brockhoff and E. Zitzler. 2006b. Dimensionality reduction in multiobjective optimization with (partial) dominance structure preservation: Generalized minimum objective subset problems. *TIK Report* 247.

D. Brockhoff and E. Zitzler. 2007. Improving hypervolume-based multiobjective evolutionary algorithms by using objective reduction methods. In *Proceedings of the 2007 IEEE Congress on Evolutionary Computation (CEC'07)*. IEEE, Los Alamitos, CA, 2086–2093.

D. Brockhoff and E. Zitzler. 2009. Objective reduction in evolutionary multiobjective optimization: Theory and applications. *Evolutionary Computation* 17, 2, 135–166.

D. Brockhoff and E. Zitzler. 2010. Automated aggregation and omission of objectives for tackling many-objective problems. In *New Developments in Multiple Objective and Goal Programming*. Springer, 81–102.

G. W. Characklis, B. R. Kirsch, J. Ramsey, K. E. M. Dillard, and C. T. Kelley. 2006. Developing portfolios of water supply transfers. *Water Resources Research* 42, 5, 1–14.

D. W. Corne and J. D. Knowles. 2007. Techniques for highly multiobjective optimisation: Some nondominated points are better than others. In *Proceedings of the 9th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 773–780.

W. R. Cotton, R. A. Pielke Sr, R. L. Walko, G. E. Liston, C. J. Tremback, H. Jiang, R. L. McAnelly, J. Y. Harrington, M. E. Nicholls, G. G. Carrio, and J. P. McFadden. 2003. RAMS 2001: Current status and future directions. *Meteorology and Atmospheric Physics* 82, 1–4, 5–29.

M. Črepinšek, S. H. Liu, and M. Mernik. 2013. Exploration and exploitation in evolutionary algorithms: A survey. *ACM Computing Surveys* 45, 3, 35.

I. Das and J. E. Dennis. 1998. Normal-boundary intersection: A new method for generating the Pareto surface in nonlinear multicriteria optimization problems. *SIAM Journal on Optimization* 8, 3, 631–657.

K. Deb. 2001. Multi-objective optimization. In *Multi-Objective Optimization Using Evolutionary Algorithms*. Wiley, 13–46.

K. Deb and S. Chaudhuri. 2005. I-EMO: An interactive evolutionary multi-objective optimization tool. In *Pattern Recognition and Machine Intelligence*. Springer, 690–695.

K. Deb, S. Gupta, D. Daum, J. Branke, A. K. Mall, and D. Padmanabhan. 2009. Reliability-based optimization using evolutionary algorithms. *IEEE Transactions on Evolutionary Computation* 13, 5, 1054–1074.

K. Deb and H. Jain. 2012a. Handling many-objective problems using an improved NSGA-II procedure. In *Proceedings of the 2012 IEEE Congress on Evolutionary Computation (CEC'12)*. IEEE, Los Alamitos, CA, 1–8.

K. Deb and H. Jain. 2012b. *An Improved NSGA-II Procedure for Many-Objective Optimization, part II: Handling Constraints and Extending to an Adaptive Approach*. KanGAL Report 2012010.

K. Deb and H. Jain. 2014. An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints. *IEEE Transactions on Evolutionary Computation* 18, 4, 577–601.

K. Deb and A. Kumar. 2007. Interactive evolutionary multi-objective optimization and decision-making using reference direction method. In *Proceedings of the 9th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 781–788.

K. Deb, M. Mohan, and S. Mishra. 2005. Evaluating the ε-domination based multi-objective evolutionary algorithm for a quick computation of Pareto-optimal solutions. *Evolutionary Computation* 13, 4, 501–525.

K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation* 6, 2, 182–197.

K. Deb and D. K. Saxena. 2006. Searching for Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. In *Proceedings of the World Congress on Computational Intelligence (WCCI'06)*. 3352–3360.

K. Deb, J. Sundar, N. Udaya Bhaskara Rao, and S. Chaudhuri. 2006. Reference point based multi-objective optimization using evolutionary algorithms. *International Journal of Computational Intelligence Research* 2, 3, 273–286.

K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. 2002. Scalable multi-objective optimization test problems. In *Proceedings of the 2002 IEEE Congress on Evolutionary Computation (CEC'02)*. IEEE, Los Alamitos, CA, 825–830.

R. Denysiuk, L. Costa, and I. Espírito Santo. 2013. Many-objective optimization using differential evolution with variable-wise mutation restriction. In *Proceedings of the 15th Annual Conference on Genetic and Evolutionary Computation (GECCO'13)*. ACM, New York, NY, 591–598.

K. W. DeRonne and G. Karypis. 2013. Pareto optimal pairwise sequence alignment. *IEEE/ACM Transactions on Computational Biology and Bioinformatics* 10, 2, 481–493.

N. Drechsler, R. Drechsler, and B. Becker. 2001. Multi-objective optimisation based on relation favour. In *Evolutionary Multi-Criterion Optimization*. Springer, 154–166.

M. Emmerich, N. Beume, and B. Naujoks. 2005. An EMO algorithm using the hypervolume measure as selection criterion. In *Evolutionary Multi-Criterion Optimization*. Springer, 62–76.

S. Eppe, M. López-Ibáñez, T. Stutzle, and Y. D. Smet. 2011. An experimental study of preference model integration into multi-objective optimization heuristics. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 2751–2758.

G. W. Evans. 1984. An overview of techniques for solving multiobjective mathematical programs. *Management Science* 30, 11, 1268–1282.

M. Farina and P. Amato. 2002. On the optimal solution definition for many-criteria optimization problems. In *Proceedings of the 2002 Annual Meeting of the North American Fuzzy Information Processing Society (NAFIPS'02)*. IEEE, Los Alamitos, CA, 233–238.

J. Figueira, S. Greco, and M. Ehrgott. 2005. *Multiple Criteria Decision Analysis: State of the Art Surveys*. Vol. 78. Springer, New York, NY.

P. J. Fleming, R. C. Purshouse, and R. J. Lygoe. 2005. Many-objective optimization: An engineering design perspective. In *Evolutionary Multi-Criterion Optimization*. Springer, 14–32.

C. M. Fonseca and P. J. Fleming. 1998. Multiobjective optimization and multiple constraint handling with evolutionary algorithms. I. A unified formulation. *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans* 28, 1, 26–37.

G. Fu, Z. Kapelan, J. R. Kasprzyk, and P. M. Reed. 2012. Optimal design of water distribution systems using many-objective visual analytics. *Journal of Water Resources Planning and Management* 139, 6, 624–633.

M. Garza-Fabre, G. T. Pulido, and C. A. C. Coello. 2009. Ranking methods for many-objective optimization. In *MICAI 2009: Advances in Artificial Intelligence*. Springer, 633–645.

M. Garza-Fabre, G. T. Pulido, and C. A. C. Coello. 2010a. Alternative fitness assignment methods for many-objective optimization problems. In *Artificial Evolution*. Springer, 146–157.

M. Garza-Fabre, G. T. Pulido, and C. A. C. Coello. 2010b. Two novel approaches for many-objective optimization. In *Proceedings of the 2010 IEEE Congress on Evolutionary Computation (CEC'10)*. IEEE, Los Alamitos, CA, 1–8.

M. Garza-Fabre, G. T. Pulido, C. A. C. Coello, and E. Rodriguez-Tello. 2011. Effective ranking + speciation = many-objective optimization. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 2115–2122.

I. Giagkiozis, R. C. Purshouse, and P. J. Fleming. 2013. Generalized decomposition. In *Evolutionary Multi-Criterion Optimization*. Springer, 428–442.

R. H. Gómez and C. A. C. Coello. 2013. MOMBI: A new metaheuristic for many-objective optimization based on the R2 indicator. In *Proceedings of the 2013 IEEE Congress on Evolutionary Computation (CEC'13)*. IEEE, Los Alamitos, CA, 2488–2495.

D. Gong, J. Sun, and X. Ji. 2013a. Evolutionary algorithms with preference polyhedron for interval multi-objective optimization problems. *Information Sciences* 233, 141–161.

D. Gong, G. Wang, and X. Sun. 2013b. Set-based genetic algorithms for solving many-objective optimization problems. In *Proceedings of the 2013 13th UK Workshop on Computational Intelligence (UKCI'13)*. IEEE, Los Alamitos, CA, 96–103.

F. Güneş and F. Tokan. 2010. Pattern search optimization with applications on synthesis of linear antenna arrays. *Expert Systems with Applications* 37, 6, 4698–4705.

X. Guo, X. Wang, M. Wang, and Y. Wang. 2012. A new objective reduction algorithm for many-objective problems: Employing mutual information and clustering algorithm. In *Proceedings of the 2012 8th International Conference on Computational Intelligence and Security (CIS'12)*. IEEE, Los Alamitos, CA, 11–16.

X. Guo, Y. Wang, and X. Wang. 2013. Using objective clustering for solving many-objective optimization problems. *Mathematical Problems in Engineering* 2013, Article No. 584909.

D. Hadka and P. M. Reed. 2012. Diagnostic assessment of search controls and failure modes in many-objective evolutionary optimization. *Evolutionary Computation* 20, 3, 423–452.

D. Hadka, P. M. Reed, and T. W. Simpson. 2012. Diagnostic assessment of the Borg MOEA for many-objective product family design problems. In *Proceedings of the 2012 IEEE Congress on Evolutionary Computation (CEC'12)*. IEEE, Los Alamitos, CA, 1–10.

N. Hamada, Y. Nagata, S. Kobayashi, and I. Ono. 2011a. Adaptive weighted aggregation 2: More scalable AWA for multiobjective function optimization. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 2375–2382.

N. Hamada, Y. Nagata, S. Kobayashi, and I. Ono. 2011b. On scalability of adaptive weighted aggregation for multiobjective function optimization. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 669–678.

M. P. Hansen and A. Jaszkiewicz. 1998. *Evaluating the Quality of Approximations to the Non-Dominated Set*. IMM, Department of Mathematical Modelling, Technical University of Denmark.

J. G. Herrero, A. Berlanga, and J. M. M. López. 2009. Effective evolutionary algorithms for many-specifications attainment: Application to air traffic control tracking filters. *IEEE Transactions on Evolutionary Computation* 13, 1, 151–168.

H. Hirano and T. Yoshikawa. 2012. A study on two-step search using global-best in PSO for multi-objective optimization problems. In *Proceedings of the 2012 Joint 6th International Conference on Soft Computing* *and Intelligent Systems (SCIS) and the 13th International Symposium on Advanced Intelligent Systems (ISIS)*. IEEE, Los Alamitos, CA, 1894–1897.

S. Huband, P. Hingston, L. Barone, and L. While. 2006. A review of multiobjective test problems and a scalable test problem toolkit. *IEEE Transactions on Evolutionary Computation* 10, 5, 477–506.

E. J. Hughes. 2003. Multiple single objective Pareto sampling. In *Proceedings of the 2003 IEEE Congress on Evolutionary Computation (CEC'13)*, Vol. 4. IEEE, Los Alamitos, CA, 2678–2684.

E. J. Hughes. 2007a. MSOPS-II: A general-purpose many-objective optimiser. In *Proceedings of the 2007 IEEE Congress on Evolutionary Computation (CEC'07)*. 3944–3951.

E. J. Hughes. 2007b. Radar waveform optimisation as a many-objective application benchmark. In *Evolutionary Multi-Criterion Optimization*. Springer, 700–714.

E. J. Hughes. 2008. Fitness assignment methods for many-objective problems. In *Multiobjective Problem Solving from Nature*. Springer, 307–329.

E. J. Hughes. 2011. Many-objective directed evolutionary line search. In *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 761–768.

K. Ikeda, H. Kita, and S. Kobayashi. 2001. Failure of Pareto-based MOEAs: Does non-dominated really mean near to optimal? In *Proceedings of the 2001 Congress on Evolutionary Computation*, Vol. 2. 957–962.

H. Ishibuchi, N. Akedo, and Y. Nojima. 2011. A many-objective test problem for visually examining diversity maintenance behavior in a decision space. In *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation (GECCO'11)*. 649–656.

H. Ishibuchi, N. Akedo, and Y. Nojima. 2013. Relation between neighborhood size and MOEA/D performance on many-objective problems. In *Evolutionary Multi-Criterion Optimization*. Springer, 459–474.

H. Ishibuchi, N. Akedo, and Y. Nojima. 2015. Behavior of multiobjective evolutionary algorithms on many-objective knapsack problems. *IEEE Transactions on Evolutionary Computation* 19, 2, 264–283.

H. Ishibuchi, N. Akedo, H. Ohyanagi, and Y. Nojima. 2011. Behavior of EMO algorithms on many-objective optimization problems with correlated objectives. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 1465–1472.

H. Ishibuchi, Y. Hitotsuyanagi, N. Tsukamoto, and Y. Nojima. 2010. Many-objective test problems to visually examine the behavior of multiobjective evolution in a decision space. In *Parallel Problem Solving from Nature, PPSN XI*. Springer, 91–100.

H. Ishibuchi, N. Tsukamoto, and Y. Nojima. 2008. Evolutionary many-objective optimization: A short review. In *Proceedings of the IEEE World Congress on Evolutionary Computation (CEC'08)*. IEEE, Los Alamitos, CA, 2419–2426.

A. L. Jaimes and C. A. C. Coello. 2009. Study of preference relations in many-objective optimization. In *Proceedings of the 11th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 611–618.

A. L. Jaimes, C. A. C. Coello, H. E. Aguirre, and K. Tanaka. 2011a. Adaptive objective space partitioning using conflict information for many-objective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 151–165.

A. L. Jaimes, C. A. C. Coello, and J. E. U. Barrientos. 2009. Online objective reduction to deal with many-objective problems. In *Evolutionary Multi-Criterion Optimization*. Springer, 423–437.

A. L. Jaimes, C. A. C. Coello, and D. Chakraborty. 2008. Objective reduction using a feature selection technique. In *Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 673–680.

A. L. Jaimes, C. A. C. Coello, A. Oyama, and K. Fujii. 2013. An alternative preference relation to deal with many-objective optimization problems. In *Evolutionary Multi-Criterion Optimization*. Springer, 291–306.

A. L. Jaimes, A. A. Montano, and C. A. C. Coello. 2011b. Preference incorporation to solve many-objective airfoil design problems. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 1605–1612.

H. Jain and K. Deb. 2013. An improved adaptive approach for elitist nondominated sorting genetic algorithm for many-objective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 307–321.

H. Jain and K. Deb. 2014. An evolutionary many-objective optimization algorithm using reference-point based nondominated sorting approach, part II: Handling constraints and extending to an adaptive approach. *IEEE Transactions on Evolutionary Computation* 18, 4, 602–622.

A. Jaszkiewicz. 2002. On the performance of multiple-objective genetic local search on the 0/1 knapsack problem-A comparative experiment. *IEEE Transactions on Evolutionary Computation* 6, 4, 402–412.

Yaochu Jin and B. Sendhoff. 2003. Connectedness, regularity and the success of local search in evolutionary multi-objective optimization. In *Proceedings of the 2003 Congress on Evolutionary Computation (CEC'03)*. Vol. 3. 1910–1917. DOI:http://dx.doi.org/10.1109/CEC.2003.1299907

D. S. Johnson, L. McGeogh, F. Glover, and C. Rego. 2000. 8th DIMACS Implementation Challenge: The Traveling Salesman Problem. Retrieved July 27, 2015, from http://dimacs.rutgers.edu/Challenges/TSP/.

S. Kalboussi, S. Bechikh, M. Kessentini, and L. B. Said. 2013. Preference-based many-objective evolutionary testing generates harder test cases for autonomous Agents. In *Search Based Software Engineering*. Springer, 245–250.

J. R. Kasprzyk, P. M. Reed, G. W. Characklis, and B. R. Kirsch. 2012. Many-objective de Novo water supply portfolio planning under deep uncertainty. *Environmental Modelling and Software* 34, 87–104.

L. Kaufman and P. Rousseeuw. 2009. *Finding Groups in Data: An Introduction to Cluster Analysis*. Vol. 344. John Wiley and Sons.

J. H. Kim, J. H. Han, Y. H. Kim, S. H. Choi, and E. S. Kim. 2012. Preference-based solution selection algorithm for evolutionary multiobjective optimization. *IEEE Transactions on Evolutionary Computation* 16, 1, 20–34.

J. D. Knowles and D. W. Corne. 2003a. Instance generators and test suites for the multiobjective quadratic assignment problem. In *Evolutionary Multi-Criterion Optimization*. Springer, 295–310.

J. D. Knowles and D. W. Corne. 2003b. Properties of an adaptive archiving algorithm for storing nondominated vectors. *IEEE Transactions on Evolutionary Computation* 7, 2, 100–116.

J. D. Knowles and D. W. Corne. 2007. Quantifying the effects of objective space dimension in evolutionary multiobjective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 757–771.

J. B. Kollat and P. M. Reed. 2006. Comparing state-of-the-art evolutionary multi-objective algorithms for long-term groundwater monitoring design. *Advances in Water Resources* 29, 6, 792–807.

M. Köppen, R. Vicente-Garcia, and B. Nickolay. 2005. Fuzzy-Pareto-dominance and its application in evolutionary multi-objective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 399–412.

M. Köppen and K. Yoshida. 2007. Substitute distance assignments in NSGA-II for handling many-objective optimization problems. In *Evolutionary Multi-Criterion Optimization*. Springer, 727–741.

A. Kostin, G. Guillén-Gosálbez, F. D. Mele, and L. Jiménez. 2012. Identifying key life cycle assessment metrics in the multiobjective design of bioethanol supply chains using a rigorous mixed-integer linear programming approach. *Industrial and Engineering Chemistry Research* 51, 14, 5282–5291.

N. Kowatari, A. Oyama, H. E. Aguirre, and K. Tanaka. 2012. Analysis on population size and neighborhood recombination on many-objective optimization. In *Parallel Problem Solving from Nature, PPSN XII*. Springer, 22–31.

R. Kudikala, A. R. Mills, P. J. Fleming, G. F. Tanner, and J. E. Holt. 2013. Aero engine health management system architecture design using multi-criteria optimization. In *Proceedings of the 15th Annual Conference Companion on Genetic and Evolutionary Computation (GECCO'13 Companion)*. ACM, New York, NY, 185–186.

S. Kukkonen and J. Lampinen. 2007. Ranking-dominance and many-objective optimization. In *Proceedings of the 2007 IEEE Congress on Evolutionary Computation (CEC'07)*. IEEE, Los Alamitos, CA, 3983–3990.

N. Kusuno, H. E. Aguirre, K. Tanaka, and M. Koishi. 2013. Evolutionary multi-objective optimization to attain practically desirable solutions. In *Proceedings of the 15th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 639–646.

M. Laumanns, L. Thiele, K. Deb, and E. Zitzler. 2002. Combining convergence and diversity in evolutionary multiobjective optimization. *Evolutionary Computation* 10, 3, 263–282.

M. Laumanns and R. Zenklusen. 2011. Stochastic convergence of random search methods to fixed size Pareto front approximations. *European Journal of Operational Research* 213, 2, 414–421.

K. Le, D. L. Silva, and H. Li. 2009. An improved version of volume dominance for multi-objective optimisation. In *Evolutionary Multi-Criterion Optimization*. Springer, 231–245.

K. B. Lee and J. H. Kim. 2011. Multi-objective particle swarm optimization with preference-based sorting. In *Proceedings of the 2011 IEEE Congress on Evolutionary Computation (CEC'11)*. IEEE, Los Alamitos, CA, 2506–2513.

M. Li, S. Yang, and X. Liu. 2013. Shift-based density estimation for Pareto-based algorithms in many-objective optimization. *IEEE Transactions on Evolutionary Computation* PP, 99, 1.

M. Li, S. Yang, and X. Liu. 2014. Diversity comparison of Pareto front approximations in many-objective optimization. *IEEE Transactions on Cybernetics* 44, 12, 2568–2584. DOI:http://dx.doi.org/10.1109/TCYB.2014.2310651

M. Li, J. Zheng, K. Li, Q. Yuan, and R. Shen. 2010. Enhancing diversity for average ranking method in evolutionary many-objective optimization. In *Parallel Problem Solving from Nature, PPSN XI*. Springer, 647–656.

Z. Li and H. Liu. 2012. Preference-based evolutionary multi-objective optimization. In *Proceedings of the 2012 8th International Conference on Computational Intelligence and Security (CIS'12)*. IEEE, Los Alamitos, CA, 71–76.

P. Lindroth, M. Patriksson, and A. B. Strömberg. 2010. Approximating the Pareto optimal set using a reduced set of objective functions. *European Journal of Operational Research* 207, 3, 1519–1534.

E. Løken. 2007. Use of multicriteria decision analysis methods for energy planning problems. *Renewable and Sustainable Energy Reviews* 11, 7, 1584–1595.

R. J. Lygoe, M. Cary, and P. J. Fleming. 2013. A real-world application of a many-objective optimisation complexity reduction process. In *Evolutionary Multi-Criterion Optimization*. Springer, 641–655.

A. D. Manriquez, G. T. Pulido, C. A. C. Coello, and R. L. Becerra. 2013. A ranking method based on the R2 indicator for many-objective optimization. In *Proceedings of the 2013 IEEE Congress on Evolutionary Computation (CEC'13)*. IEEE, Los Alamitos, CA, 1523–1530.

A. M. Mendez and C. A. C. Coello. 2012. Solving multi-objective optimization problems using differential evolution and a maximin selection criterion. In *Proceedings of the 2012 IEEE Congress on Evolutionary Computation (CEC'12)*. IEEE, Los Alamitos, CA, 1–8.

A. M. Mendez and C. A. C. Coello. 2013. Selection operators based on maximin fitness function for multi-objective evolutionary algorithms. In *Evolutionary Multi-Criterion Optimization*. Springer, 215–229.

K. Miettinen. 1999. *Nonlinear Multiobjective Optimization*. Vol. 12. Springer.

K. Miettinen. 2003. Graphical illustration of Pareto optimal solutions. In *Multi-Objective Programming and Goal Programming.* Advances in Soft Computing, Vol. 21. Springer, 197–202.

L. L. Minku and X. Yao. 2013. Software effort estimation as a multiobjective learning problem. *ACM Transactions on Software Engineering and Methodology* 22, 4, 35:1–35:32.

P. Mitra, C. A. Murthy, and S. K. Pal. 2002. Unsupervised feature selection using feature similarity. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 24, 3, 301–312.

H. J. F. Moen, N. B. Hansen, H. Hovland, and J. Tørresen. 2013. Many-objective optimization using taxi-cab surface evolutionary algorithm. In *Evolutionary Multi-Criterion Optimization*. Springer, 128–142.

A. Mukhopadhyay, U. Maulik, S. Bandyopadhyay, and C. A. C. Coello. 2014a. A survey of multiobjective evolutionary algorithms for data mining: Part I. *IEEE Transactions on Evolutionary Computation* 18, 1, 4–19.

A. Mukhopadhyay, U. Maulik, S. Bandyopadhyay, and C. A. C. Coello. 2014b. Survey of multiobjective evolutionary algorithms for data mining: Part II. *IEEE Transactions on Evolutionary Computation* 18, 1, 20–35.

K. Musselman and J. Talavage. 1980. A tradeoff cut approach to multiple objective optimization. *Operations Research* 28, 6, 1424–1435.

K. Narukawa. 2013. Effect of dominance balance in many-objective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 276–290.

K. Narukawa and T. Rodemann. 2012. Examining the performance of evolutionary many-objective optimization algorithms on a real-world application. In *Proceedings of the 2012 6th International Conference on Genetic and Evolutionary Computing (ICGEC'12)*. IEEE, Los Alamitos, CA, 316–319.

P. R. Palmer, A. M. Cristóbal, and G. T. Parks. 2011. Coherent design methodology using modelling, simulation and optimisation. In *Proceedings of the 2011 Grand Challenges on Modeling and Simulation Conference*. 65–75.

J. Pasia, H. E. Aguirre, and K. Tanaka. 2011. Improved random one-bit climbers with adaptive ε-ranking and tabu moves for many-objective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 182–196.

K. Praditwong, M. Harman, and X. Yao. 2011. Software module clustering as a multi-objective search problem. *IEEE Transactions on Software Engineering* 37, 2, 264–282.

K. Praditwong and X. Yao. 2006. A new multi-objective evolutionary optimisation algorithm: The two-archive algorithm. In *Proceedings of the 2006 International Conference on Computational Intelligence and Security*. Vol. 1. IEEE, Los Alamitos, CA, 286–291.

R. C. Purshouse, K. Deb, M. M. Mansor, S. Mostaghim, and R. Wang. 2014. A review of hybrid evolutionary multiple criteria decision making methods. In *Proceedings of the 2014 IEEE Congress on Evolutionary Computation (CEC'14)*. 1147–1154.

R. C. Purshouse and P. J. Fleming. 2007. On the evolutionary optimization of many conflicting objectives. *IEEE Transactions on Evolutionary Computation* 11, 6, 770–784.

R. C. Purshouse, C. Jalbă, and P. J. Fleming. 2011. Preference-driven co-evolutionary algorithms show promise for many-objective optimisation. In *Evolutionary Multi-Criterion Optimization*. Springer, 136–150.

F. Qiu, Y. Wu, L. Wang, and B. Jiang. 2012. Bipolar preferences dominance based evolutionary algorithm for many-objective optimization. In *Proceedings of the 2012 IEEE Congress on Evolutionary Computation (CEC'12)*. IEEE, Los Alamitos, CA, 1–8.

L. Rachmawati and D. Srinivasan. 2006. Preference incorporation in multi-objective evolutionary algorithms: A survey. In *Proceedings of the 2006 IEEE Congress on Evolutionary Computation (CEC'06)*. IEEE, Los Alamitos, CA, 962–968.

T. Ray, M. Asafuddoula, and A. Isaacs. 2013. A steady state decomposition based quantum genetic algorithm for many objective optimization. In *Proceedings of the 2013 IEEE Congress on Evolutionary Computation (CEC'13)*. IEEE, Los Alamitos, CA, 2817–2824.

T. Ray, K. Tai, and K. C. Seow. 2001. Multiobjective design optimization by an evolutionary algorithm. *Engineering Optimization* 33, 3, 399–424.

P. M. Reed and J. B. Kollat. 2012. Save now, pay later? Multi-period many-objective groundwater monitoring design given systematic model errors and uncertainty. *Advances in Water Resources* 35, 55–68.

P. M. Reed and J. B. Kollat. 2013. Visual analytics clarify the scalability and effectiveness of massively parallel many-objective optimization: A groundwater monitoring design example. *Advances in Water Resources* 56, 1–13.

C. Reeves. 1995. A genetic algorithm for flowshop sequencing. *Computers and Operations Research* 22, 1, 5–13.

S. J. Ryu, K. B. Lee, and J. H. Kim. 2012. Improved version of a multiobjective quantum-inspired evolutionary algorithm with preference-based selection. In *Proceedings of the 2012 IEEE Congress on Evolutionary Computation (CEC'12)*. IEEE, Los Alamitos, CA, 1–7.

A. Santiago, H. J. F. Huacuja, B. Dorronsoro, J. E. Pecero, C. G. Santillan, J. J. G. Barbosa, and J. C. S. Monterrubio. 2014. A survey of decomposition methods for multi-objective optimization. In *Recent Advances on Hybrid Approaches for Designing Intelligent Systems*. Springer, 453–465.

H. Sato, H. E. Aguirre, and K. Tanaka. 2007. Controlling dominance area of solutions and its impact on the performance of MOEAs. In *Evolutionary Multi-Criterion Optimization*. Springer, 5–20.

H. Sato, H. E. Aguirre, and K. Tanaka. 2010a. Pareto partial dominance MOEA and hybrid archiving strategy included CDAS in many-objective optimization. In *Proceedings of the 2010 IEEE Congress on Evolutionary Computation (CEC'10)*. IEEE, Los Alamitos, CA, 1–8.

H. Sato, H. E. Aguirre, and K. Tanaka. 2010b. Self-controlling dominance area of solutions in evolutionary many-objective optimization. In *Simulated Evolution and Learning*. Springer, 455–465.

H. Sato, H. E. Aguirre, and K. Tanaka. 2011a. Genetic diversity and effective crossover in evolutionary many-objective optimization. In *Learning and Intelligent Optimization*. Springer, 91–105.

H. Sato, H. E. Aguirre, and K. Tanaka. 2011b. Improved S-CDAs using crossover controlling the number of crossed genes for many-objective optimization. In *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 753–760.

H. Sato, C. A. C. Coello, H. E. Aguirre, and K. Tanaka. 2012. Adaptive control of the number of crossed genes in many-objective evolutionary optimization. In *Learning and Intelligent Optimization*. Springer, 478–484.

D. K. Saxena and K. Deb. 2007. Non-linear dimensionality reduction procedures for certain large-dimensional multi-objective optimization problems: Employing correntropy and a novel maximum variance unfolding. In *Evolutionary Multi-Criterion Optimization*. Springer, 772–787.

D. K. Saxena, J. A. Duro, A. Tiwari, K. Deb, and Q. Zhang. 2013. Objective reduction in many-objective optimization: Linear and nonlinear algorithms. *IEEE Transactions on Evolutionary Computation* 17, 1, 77–99.

D. K. Saxena, Q. Zhang, J. A. Duro, and A. Tiwari. 2011. Framework for many-objective test problems with both simple and complicated Pareto-set shapes. In *Evolutionary Multi-Criterion Optimization*. Springer, 197–211.

A. S. Sayyad, T. Menzies, and H. Ammar. 2013. On the value of user preferences in search-based software engineering: A case study in software product lines. In *Proceedings of the 2013 International Conference on Software Engineering*. IEEE, Los Alamitos, CA, 492–501.

O. Schütze, X. Esquivel, A. Lara, and C. A. C. Coello. 2010. *Measuring the Averaged Hausdorff Distance to the Pareto Front of a Multi-Objective Optimization Problem*. Technical Report TR-OS-2010-02. CINVESTAV.

T. W. Simpson, W. Chen, J. K. Allen, and F. Mistree. 1996. Conceptual design of a family of products through the use of the robust concept exploration method. In *Proceedings of the 6th AIAA/NASA/USAF MultiDisciplinary Analysis and Optimization Symposium*. 1535–1545.

H. K. Singh, A. Isaacs, and T. Ray. 2011. A Pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems. *IEEE Transactions on Evolutionary Computation* 15, 4, 539–556.

A. Sinha, D. K. Saxena, K. Deb, and A. Tiwari. 2013. Using objective reduction and interactive procedure to handle many-objective optimization problems. *Applied Soft Computing* 13, 1, 415–427.

D. W. Stouch, E. Zeidman, M. Richards, K. D. McGraw, and W. Callahan. 2011. Coevolving collection plans for UAS constellations. In *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 1691–1698.

A. Sülflow, N. Drechsler, and R. Drechsler. 2007. Robust multi-objective optimization in high dimensional spaces. In *Evolutionary Multi-Criterion Optimization*. Springer, 715–726.

K. Tagawa and A. Imamura. 2013. Many-hard-objective optimization using differential evolution based on two-stage constraint-handling. In *Proceedings of the 15th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 671–678.

R. Takahashi, E. Carrano, and E. Wanner. 2011. On a stochastic differential equation approach for multi-objective optimization up to Pareto-criticality. In *Evolutionary Multi-Criterion Optimization*. Springer, 61–75.

Y. Tan, Y. Jiao, H. Li, and X. Wang. 2013. MOEA/D + uniform design: A new version of MOEA/D for optimization problems with many objectives. *Computers and Operations Research* 40, 6, 1648–1660.

J. Tang, S. Alam, C. Lokan, and H. Abbass. 2012. A multi-objective evolutionary method for dynamic airspace re-sectorization using sectors clipping and similarities. In *Proceedings of the 2012 IEEE Congress on Evolutionary Computation (CEC'12)*. IEEE, Los Alamitos, CA, 1–8.

J. Tate, B. Woolford-Lim, I. Bate, and X. Yao. 2012. Evolutionary and principled search strategies for sensornet protocol optimization. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 42, 1, 163–180.

L. Thiele, K. Miettinen, P. Korhonen, and J. Molina. 2009. A preference-based evolutionary algorithm for multi-objective optimization. *Evolutionary Computation* 17, 3, 411–436.

R. Ursem. 2010. *Centrifugal Pump Design: Three Benchmark Problems for Many-Objective Optimization*. Technical Report 2010-01. Grundfos A/S.

L. J. P. Van der Maaten, E. O. Postma, and H. J. Van Den Herik. 2009. Dimensionality reduction: A comparative review. *Journal of Machine Learning Research* 10, 1–41.

D. A. Van Veldhuizen and G. B. Lamont. 1998. Evolutionary computation and convergence to a Pareto front. In *Proceedings of the Genetic Programming Conference*. 221–228.

M. Velasquez and P. T. Hester. 2013. An analysis of multi-criteria decision making methods. *International Journal of Operations Research* 10, 56–66.

N. Venkatarayalu and T. Ray. 2004. Optimum design of Yagi-Uda antennas using computational intelligence. *IEEE Transactions on Antennas and Propagation* 52, 7, 1811–1818.

S. Verel, A. Liefooghe, and C. Dhaenens. 2011. Set-based multiobjective fitness landscapes: A preliminary study. In *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation*. 769–776.

C. A. R. Villalobos and C. A. C. Coello. 2012. A new multi-objective evolutionary algorithm based on a performance assessment indicator. In *Proceedings of the 14th International Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 505–512.

C. Von Lücken, B. Barán, and C. Brizuela. 2014. A survey on multi-objective evolutionary algorithms for many-objective problems. *Computational Optimization and Applications* 1, 1–50.

M. Wagner and F. Neumann. 2013. A fast approximation guided evolutionary multi-objective algorithm. In *Proceedings of the 15th Annual Conference on Genetic and Evolutionary Computation (GECCO'13)*. ACM, New York, NY, 687–694.

T. Wagner, N. Beume, and B. Naujoks. 2007. Pareto-, aggregation-, and indicator-based methods in many-objective optimization. In *Evolutionary Multi-Criterion Optimization*. Springer, 742–756.

D. J. Walker, R. M. Everson, and J. E. Fieldsend. 2013. Visualizing mutually nondominating solution sets in many-objective optimization. *IEEE Transactions on Evolutionary Computation* 17, 2, 165–184.

H. Wang, L. Jiao, and X. Yao. 2014. An improved two-archive algorithm for many-objective optimization. *IEEE Transactions on Evolutionary Computation* PP, 99, 1. DOI: http://dx.doi.org/10.1109/TEVC.2014.2350987

H. Wang and X. Yao. 2014. Corner sort for Pareto-based many-objective optimization. *IEEE Transactions on Cybernetics* 44, 1, 92–102.

R. Wang, R. C. Purshouse, and P. J. Fleming. 2012. Local preference-inspired co-evolutionary algorithms. In *Proceedings of the 14th International Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 513–520.

R. Wang, R. C. Purshouse, and P. J. Fleming. 2013a. On finding well-spread Pareto optimal solutions by preference-inspired co-evolutionary algorithm. In *Proceedings of the 15th Annual Conference on Genetic and Evolutionary Computation*. ACM, New York, NY, 695–702.

R. Wang, R. C. Purshouse, and P. J. Fleming. 2013b. Preference-inspired co-evolutionary algorithm using weights for many-objective optimization. In *Proceedings of the 15th Annual Conference Companion on Genetic and Evolutionary Computation*. ACM, New York, NY, 101–102.

R. Wang, R. C. Purshouse, and P. J. Fleming. 2013c. Preference-inspired coevolutionary algorithms for many-objective optimization. *IEEE Transactions on Evolutionary Computation* 17, 4, 474–494.

R. Wang, T. Zhang, and B. Guo. 2013. An enhanced MOEA/D using uniform directions and a pre-organization procedure. In *Proceedings of the 2013 IEEE Congress on Evolutionary Computation (CEC'13)*. IEEE, Los Alamitos, CA, 2390–2397.

Z. Wang, K. Tang, and X. Yao. 2010. Multi-objective approaches to optimal testing resource allocation in modular software systems. *IEEE Transactiona on Reliability* 59, 3, 563–575.

L. While, L. Bradstreet, and L. Barone. 2012. A fast way of calculating exact hypervolumes. *IEEE Transactions on Evolutionary Computation* 16, 1, 86–95.

L. While, P. Hingston, L. Barone, and S. Huband. 2006. A faster algorithm for calculating hypervolume. *IEEE Transactions on Evolutionary Computation* 10, 1, 29–38.

U. Wickramasinghe, R. Carrese, and X. Li. 2010. Designing airfoils using a reference point based evolutionary many-objective particle swarm optimization algorithm. In *Proceedings of the 2010 IEEE Congress on Evolutionary Computation (CEC'10)*. IEEE, Los Alamitos, CA, 1–8.

S. Yang, M. Li, X. Liu, and J. Zheng. 2013. A grid-based evolutionary algorithm for many-objective optimization. *IEEE Transactions on Evolutionary Computation* 17, 5, 721–736.

P. L. Yu. 1974. Cone convexity, cone extreme points, and nondominated solutions in decision problems with multiobjectives. *Journal of Optimization Theory and Applications* 14, 3, 319–377.

Q. Zhang and H. Li. 2007. MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation* 11, 6, 712–731.

Q. Zhang, W. Liu, and H. Li. 2009. The performance of a new version of MOEA/D on CEC09 unconstrained MOP test instances. In *Proceedings of the 2009 IEEE Congress on Evolutionary Computation (CEC'09)*. IEEE, Los Alamitos, CA, 203–208.

Q. Zhang, A. Zhou, S. Zhao, P. N. Suganthan, W. Liu, and S. Tiwari. 2008. *Multiobjective Optimization Test Instances for the CEC 2009 Special Session and Competition*. Technical Report. University of Essex, Colchester, UK.

A. Zhou, Y. Jin, Q. Zhang, B. Sendhoff, and E. Tsang. 2006. Combining model-based and genetics-based offspring generation for multi-objective optimization using a convergence criterion. In *Proceedings of the 2006 IEEE Congress on Evolutionary Computation (CEC'06)*. IEEE, Los Alamitos, CA, 892–899.

A. Zhou, B. Qu, H. Li, S. Zhao, P. N. Suganthan, and Q. Zhang. 2011. Multiobjective evolutionary algorithms: A survey of the state of the art. *Swarm and Evolutionary Computation* 1, 1, 32–49.

E. Zitzler and S. Künzli. 2004. Indicator-based selection in multiobjective search. In *Parallel Problem Solving from Nature, PPSN VIII*. Springer, 832–842.

E. Zitzler, M. Laumanns, and L. Thiele. 2001. *SPEA2: Improving the Strength Pareto Evolutionary Algorithm*. Technical Report 103. Eidgenössische Technische Hochschule Zürich (ETH).

E. Zitzler and L. Thiele. 1998. Multiobjective optimization using evolutionary algorithms: A comparative case study. In *Parallel Problem Solving from Nature, PPSN V*. Springer, 292–301.

E. Zitzler and L. Thiele. 1999. Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach. *IEEE Transactions on Evolutionary Computation* 3, 4, 257–271.

E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G Da Fonseca. 2003. Performance assessment of multiobjective optimizers: An analysis and review. *IEEE Transactions on Evolutionary Computation* 7, 2, 117–132.

X. Zou, Y. Chen, M. Liu, and L. Kang. 2008. A new evolutionary algorithm for solving many-objective optimization problems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 38, 5, 1402–1412.

Received April 2014; revised April 2015; accepted June 2015