

Problem Set #5

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The **due date** for this quiz is **Mon 11 Aug 2014 2:59 PM CST**.

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Question 1

Which of the following statements *cannot* be true, given the current state of knowledge?

- ☐ There is an NP-complete problem that is polynomial-time solvable.
- ☐ There is an NP-complete problem that can be solved in $O(n^{\log n})$ time, where n is the size of the input.
- ☐ There is no NP-complete problem that can be solved in $O(n^{\log n})$ time, where n is the size of the input.
- ☐ Some NP-complete problems are polynomial-time solvable, and some NP-complete problems are not polynomial-time solvable.

Question 2

Choose the strongest true statement.

- ☐ All three of the other assertions are true.
- ☐ If the maximum-size independent set problem can be solved in time $O(T(n))$ in general graphs, then the minimum-size vertex cover problem can be solved in time $O(T(n))$ in general graphs.
- ☐

If the minimum-size vertex cover problem can be solved in time $O(T(n))$ in bipartite graphs, then the maximum-size independent set problem can be solved in time $O(T(n))$ in bipartite graphs.



If the minimum-size vertex cover problem can be solved in time $O(T(n))$ in general graphs, then the maximum-size independent set problem can be solved in time $O(T(n))$ in general graphs.

Question 3

Which of the following statements is true?



Consider a TSP instance in which every edge cost is negative. The dynamic programming algorithm covered in the video lectures might not correctly compute the optimal (i.e., minimum sum of edge lengths) tour of this instance.



Consider a TSP instance in which every edge cost is either 1 or 2. Then an optimal tour can be computed in polynomial time.



Consider a TSP instance in which every edge cost is negative. Deleting a vertex and all of its incident edges cannot increase the cost of the optimal (i.e., minimum sum of edge lengths) tour.



Consider a TSP instance in which every edge cost is the Euclidean distance between two points in the plane (just like in Programming Assignment #5). Deleting a vertex and all of its incident edges cannot increase the cost of the optimal (i.e., minimum sum of edge lengths) tour.

Question 4

Let TSP1 denote the following problem: given a TSP instance in which all edge costs are positive integers, compute the value of an optimal TSP tour. Let TSP2 denote: given a TSP instance in which all edge costs are positive integers, and a positive integer T , decide whether or not there is a TSP tour with total length at most T . Let HAM1 denote: given an undirected graph, either return the edges of a Hamiltonian cycle (a cycle that visits every vertex exactly once), or correctly decide

that the graph has no such cycle. Let HAM2 denote: given an undirected graph, decide whether or not the graph contains at least one Hamiltonian cycle.



Polynomial-time solvability of TSP2 does not necessarily imply polynomial-time solvability of TSP1. Polynomial-time solvability of HAM2 does not necessarily imply polynomial-time solvability of HAM1.



If TSP2 is polynomial-time solvable, then so is TSP1. If HAM2 is polynomial-time solvable, then so is HAM1.



Polynomial-time solvability of TSP2 does not necessarily imply polynomial-time solvability of TSP1. But, if HAM2 is polynomial-time solvable, then so is HAM1.



If TSP2 is polynomial-time solvable, then so is TSP1. But, polynomial-time solvability of HAM2 does not necessarily imply polynomial-time solvability of HAM1.

Question 5

Assume that $P \neq NP$. Consider undirected graphs with nonnegative edge lengths. Which of the following problems can be solved in polynomial time?

Hint: The Hamiltonian path problem is: given an undirected graph with n vertices, decide whether or not there is a (cycle-free) path with $n - 1$ edges that visits every vertex exactly once. You can use the fact that the Hamiltonian path problem is NP-complete. There are relatively simple reductions from the Hamiltonian path problem to 3 of the 4 problems below.



For a given source s and destination t , compute the length of a shortest s - t path that has exactly $n - 1$ edges (or $+\infty$, if no such path exists). The path is not allowed to contain cycles.



For a given source s and destination t , compute the length of a shortest s - t path that has exactly $n - 1$ edges (or $+\infty$, if no such path exists). The path is allowed to contain cycles.



Amongst all spanning trees of the graph, compute one with the smallest-possible number of leaves.



Amongst all spanning trees of the graph, compute one with the minimum-possible maximum degree. (Recall the degree of a vertex is the number of incident edges.)

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