Problem Set #4

Help

The **due date** for this quiz is **Mon 4 Aug 2014 2:59 PM CST**.

- [ ] **In accordance with the Coursera Honor Code, I (朱顺) certify that the answers here are my own work.**

Question 1

Consider a directed graph with real-valued edge lengths and no negative-cost cycles. Let s be a source vertex. Assume that there is a unique shortest path from s to every other vertex. What can you say about the subgraph of G that you get by taking the union of these shortest paths? [Pick the strongest statement that is guaranteed to be true.]

- It is a directed acyclic subgraph in which s has no incoming arcs.
- It has no strongly connected component with more than one vertex.
- It is a path, directed away from s.
- It is a tree, with all edges directed away from s.

Question 2

Consider the following optimization to the Bellman-Ford algorithm. Given a graph $G = (V, E)$ with real-valued edge lengths, we label the vertices $V = \{1, 2, 3, \ldots, n\}$. The source vertex s should be labeled "1", but the rest of the labeling can be arbitrary. Call an edge $(u, v) \in E$ *forward* if $u < v$ and *backward* if $u > v$. In every odd iteration of the outer loop (i.e., when $i = 1, 3, 5, \ldots$), we visit the vertices in the order from 1 to n. In every even iteration of the outer loop (when $i = 2, 4, 6, \ldots$), we visit the vertices in the order from n to 1. In every odd iteration, we update the value of $A[i, v]$ using only the forward edges of the form (w, v), using the *most recent* subproblem value for w (that from the current iteration rather than the previous one). That is, we compute $A[i, v] = \min\{A[i-1, v], \min_{(w,v)} A[i, w] + c_{wv}\}$, where the inner minimum ranges only over forward edges sticking into v (i.e., with $w < v$). Note that all relevant subproblems from the current round ($A[i,w]$ for all $w < v$ with $(w,v) \in E$) are available for constant-time lookup. In even iterations, we compute this same recurrence using only the backward edges (again, all relevant subproblems from the current round are available for constant-time lookup). Which of the following is true about this modified Bellman-Ford algorithm?

- This algorithm has an asymptotically superior running time to the original Bellman-Ford algorithm.
- It correctly computes shortest paths if and only if the input graph is a directed acyclic graph.
- It correctly computes shortest paths if and only if the input graph has no negative edges.
- It correctly computes shortest paths if and only if the input graph has no negative-cost cycle.

Question 3

Consider a directed graph in which every edge has length 1. Suppose we run the Floyd-Warshall algorithm with the following modification: instead of using the recurrence A[i,j,k] = min{A[i,j,k-1], A[i,k,k-1] + A[k,j,k-1]}, we use the recurrence A[i,j,k] = A[i,j,k-1] + A[i,k,k-1] * A[k,j,k-1]. For the base case, set A[i,j,0] = 1 if (i,j) is an edge and 0 otherwise. What does this modified algorithm compute -- specificially, what is A[i,j,n] at the conclusion of the algorithm?

- None of the other answers are correct.
- The number of shortest paths from i to j.
- The length of a longest path from i to j.
- The number of simple (i.e., cycle-free) paths from i to j.

Question 4

Suppose we run the Floyd-Warshall algorithm on a directed graph $G = (V, E)$ in which every edge's length is either -1, 0, or 1. Suppose further that G is strongly connected, with at least one u-v path for every pair u, v of vertices. The graph G may or may not have a negative-cost cycle. How large can the final entries A[i,j,n] be, in absolute value? Choose the smallest number that is guaranteed to be a valid upper bound. (As usual, n denotes $|V|$.) [WARNING: for this question, make sure you refer to the implementation of the Floyd-Wardshall algorithm given in lecture, rather than to some alternative source.]

- $+\infty$
- n^2
- 2^n
- $n-1$

Question 5

Which of the following events cannot possibly occur during the reweighting step of Johnson's algorithm for the all-pairs shortest-paths problem? (Assume that the input graph has no negative-cost cycles.)

- In a directed acyclic graph, reweighting causes the length of every path to strictly increase.
- Reweighting strictly increases the length of some s-t path, while strictly decreasing the length of some t-s path.
- The length of some edge strictly decreases after the reweighting.
- In a directed graph with at least one cycle, reweighting causes the length of every path to strictly increase.

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