APPROXIMATION BY A CONSTANT VECTOR (30 points possible)

What is the solution of the norm approximation problem with one scalar variable $x \in \mathbf{R}$,

$$\text{minimize} \quad \|x\mathbf{1} - b\|,$$

for the following cases?

ℓ_1 norm:

- The average $(\mathbf{1}^T b)/m$ of the entries of b
- The (or a) median of the entries of b
- The largest magnitude entry of b
- The midrange point $(\max b_i + \min b_i)/2$

ℓ_2 norm:

- The average $(\mathbf{1}^T b)/m$ of the entries of b
- The (or a) median of the entries of b
- The largest magnitude entry of b
- The midrange point $(\max b_i + \min b_i)/2$

ℓ_∞ norm:

- The average $(\mathbf{1}^T b)/m$ of the entries of b
- The (or a) median of the entries of b
- The largest magnitude entry of b
- The midrange point $(\max b_i + \min b_i)/2$

Show Answer *You have used 0 of 2 submissions*

FITTING WITH CENSORED DATA (20 points possible)

In some experiments there are two kinds of measurements or data available: The usual ones, in which you get a number (say), and *censored data*, in which you don't get the specific number, but are told something about it, such as a lower bound. A classic example is a study of lifetimes of a set of subjects (say, laboratory mice). For those who have died by the end of data collection, we get the lifetime. For those who have not died by the end of data collection, we do not have the lifetime, but we do have a lower bound, i.e., the length of the study. These are the censored data values.

We wish to fit a set of data points,

$$(x^{(1)}, y^{(1)}), \ldots, (x^{(K)}, y^{(K)}),$$

with $x^{(k)} \in \mathbf{R}^n$ and $y^{(k)} \in \mathbf{R}$, with a linear model of the form $y \approx c^T x$. The vector $c \in \mathbf{R}^n$ is the model parameter, which we want to choose. We will use a least-squares criterion, i.e., choose c to minimize

$$J = \sum_{k=1}^{K} \left(y^{(k)} - c^T x^{(k)} \right)^2.$$

Here is the tricky part: some of the values of $y^{(k)}$ are censored; for these entries, we have only a (given) lower bound. We will re-order the data so that $y^{(1)}, \ldots, y^{(M)}$ are given (i.e., uncensored), while $y^{(M+1)}, \ldots, y^{(K)}$ are all censored, i.e., unknown, but larger than D, a given number. All the values of $x^{(k)}$ are known.

Develop a method to how to find c (the model parameter) and $y^{(M+1)}, \ldots, y^{(K)}$ (the censored data values) that minimize J. Carry out this method on the data values in `cens_fit_data_norng.m`.

The data file contains c_{true}, the true value of c, in the vector `c_true`.

Give the value of the residual

$$\frac{\|c_{\text{true}} - \hat{c}\|_2}{\|c_{\text{true}}\|_2}$$

where $\hat{c}$ is the value of c found using your method:

Help

Give the value of the residual

$$\frac{\|c_{\text{true}} - \hat{c}_{\text{ls}}\|_2}{\|c_{\text{true}}\|_2},$$

where $\hat{c}_{\text{ls}}$ is the least-squares estimate of c obtained by simply ignoring the censored data samples, i.e., the least-squares estimate based on the data

$$(x^{(1)}, y^{(1)}), \ldots, (x^{(M)}, y^{(M)}).$$

Show Answer

You have used 0 of 2 submissions

MINIMAX RATIONAL FIT TO THE EXPONENTIAL (60 points possible)

See exercise 6.9 of *Convex Optimization* for background. Here, we consider the specific problem instance with data

$$t_i = -3 + 6(i-1)/(k-1), \quad y_i = e^{t_i}, \quad i = 1, \ldots, k,$$

where $k = 201$. (In other words, the data are obtained by uniformly sampling the exponential function over the interval $[-3, 3]$.) Find a function of the form

$$f(t) = \frac{a_0 + a_1 t + a_2 t^2}{1 + b_1 t + b_2 t^2}$$

that minimizes $\max_{i=1,\ldots,k} |f(t_i) - y_i|$. (We require that $1 + b_1 t_i + b_2 t_i^2 > 0$ for $i = 1, \ldots, k$.)

Find optimal values of a_0, a_1, a_2, b_1, b_2, and give the optimal objective value, computed to an accuracy of 0.001.

Hint: You can use `strcmp(cvx_status,'Solved')` after cvx_end to check if a feasibility problem is feasible.

a_0 =

a_1 =

a_2 =

b_1 =

b_2 =

Optimal objective value =

Show Answer *You have used 0 of 2 submissions*