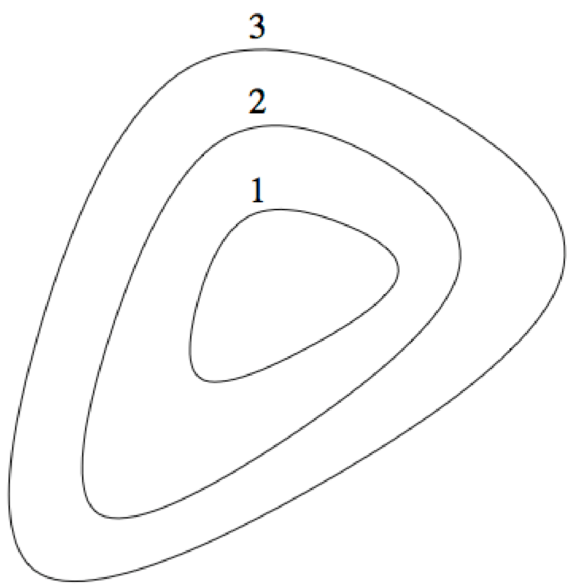
LEVEL SETS (20 points possible)

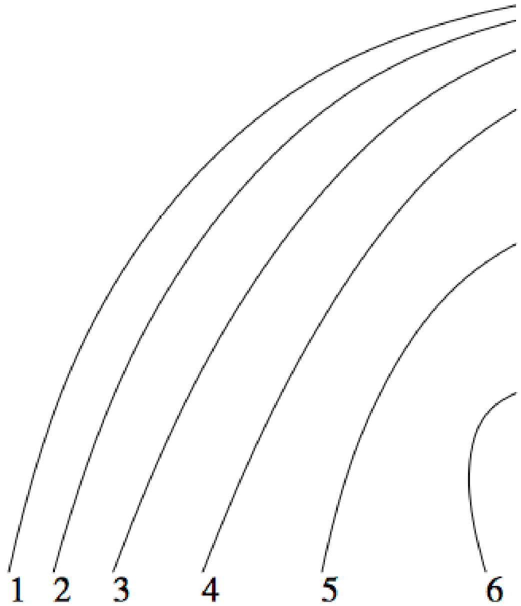
Some level sets of a function f are shown below. The curve labeled 1 shows $\{x \mid f(x) = 1\}$, etc.



Which of the following properties could f have?

- [ ] Convex
- [ ] Concave
- [ ] Quasiconvex
- [ ] Quasiconcave

Now consider the following function:



Which of the following properties could f have?

- [ ] Convex
- [ ] Concave
- [ ] Quasiconvex
- [ ] Quasiconcave

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FUNCTIONS AND EPIGRAPHS (30 points possible)

The epigraph of a function f is a halfspace if and only if

- f is convex
- f is affine
- f is quasiconvex
- f is concave.

The epigraph of a function f is a convex cone if and only if

- f is affine
- f is linear
- f is convex and positively homogeneous, i.e., $f(\alpha x) = \alpha f(x)$ for any x and any $\alpha \geq 0$.

The epigraph of a function f is a polyhedron if and only if

- f is piecewise affine
- f is convex and piecewise affine
- f is quasiconvex and piecewise affine.

(Piecewise affine is also called piecewise linear.)

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CONVEXITY AND QUASICONVEXITY (40 points possible)

For each of the following functions, determine whether it is convex, concave, quasiconvex, or quasiconcave. (Check all that apply.)

$f(x) = e^x - 1$ on $\mathbf{R}$ is

- [ ] Convex
- [ ] Concave
- [ ] Quasiconvex
- [ ] Quasiconcave.

$f(x_1, x_2) = x_1 x_2$ on $\mathbf{R}^2_{++}$ is

- [ ] Convex
- [ ] Concave
- [ ] Quasiconvex
- [ ] Quasiconcave.

$f(x_1, x_2) = 1/(x_1 x_2)$ on $\mathbf{R}^2_{++}$ is

- [ ] Convex
- [ ] Concave
- [ ] Quasiconvex
- [ ] Quasiconcave.

$f(x_1, x_2) = x_1/x_2$ on $\mathbf{R}^2_{++}$ is

- [ ] Convex
- [ ] Concave
- [ ] Quasiconvex
- [ ] Quasiconcave.

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A GENERAL VECTOR COMPOSITION RULE (20 points possible)

Suppose

$$f(x) = h(g_1(x), g_2(x), \ldots, g_k(x))$$

where $h : \mathbf{R}^k \to \mathbf{R}$ is convex, and $g_i : \mathbf{R}^n \to \mathbf{R}$. Suppose that for each i, one of the following holds:

h is nondecreasing in the ith argument, and g_i is convex

h is nonincreasing in the ith argument, and g_i is concave

g_i is affine.

Fill in the blanks in the proof below to show that f is convex. (This composition rule subsumes all the ones given in the book and is the one used in software systems such as CVX.) Assume that $\mathbf{dom}\ h = \mathbf{R}^k$; the result also holds in the general case when the monotonicity conditions above are imposed on $\tilde{h}$, the extended-valued extension of h.

Proof.

Help

Fix x, y, and $\theta \in [0,1]$, and let $z = \theta x + (1-\theta)y$. Let's re-arrange the indexes so that g_i is affine for $i = 1, \ldots, p$, g_i is convex for $i = p+1, \ldots, q$, and g_i is concave for $i = q+1, \ldots, k$. Therefore we have

$$
\begin{aligned}
g_i(z) &\_\_\_\_ \theta g_i(x) + (1-\theta) g_i(y), \;\; i = 1, \ldots, p, \\
g_i(z) &\_\_\_\_ \theta g_i(x) + (1-\theta) g_i(y), \;\; i = p+1, \ldots, q, \\
g_i(z) &\_\_\_\_ \theta g_i(x) + (1-\theta) g_i(y), \;\; i = q+1, \ldots, k.
\end{aligned}
$$

The correct inequalities in the above equations are

- $=, \geq, \leq$
- $\geq, \leq, \leq$
- $=, \leq, \geq$
- $=, \geq, \geq$
- $\geq, =, =.$

We then have

$$
\begin{aligned}
f(z) &= h(g_1(z), g_2(z), \ldots, g_k(z)) \\
&\leq h(\theta g_1(x) + (1-\theta) g_1(y), \ldots, \theta g_k(x) + (1-\theta) g_k(y)) \\
&\leq \theta h(g_1(x), \ldots, g_k(x)) + (1-\theta) h(g_1(y), \ldots, g_k(y)) \\
&= \theta f(x) + (1-\theta) f(y).
\end{aligned}
$$

The second line holds since, for $i = p+1, \ldots, q$, we have _______ the ith argument of h, which is (by assumption) nondecreasing in the ith argument, and for $i = q+1, \ldots, k$, we have _______ the ith argument, and h is nonincreasing in these arguments. The third line follows from _______. □

The correct phrases that fill the blank spots are

- increased, decreased, convexity of h
- increased, increased, convexity of f
- decreased, increased, convexity of h.

Show Answer

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CONJUGATE FUNCTIONS (10 points possible)

What is the conjugate function of $f(x) = \max_{i=1,\ldots,n} x_i$ on $\mathbf{R}^n$?

$$f^*(y) = \begin{cases} 0 & \text{if } y \succeq 0,\ \mathbf{1}^T y = 1 \\ \infty & \text{otherwise} \end{cases}$$

$$f^*(y) = \begin{cases} 0 & \text{if } y \succeq 0,\ \mathbf{1}^T y \leq 1 \\ \infty & \text{otherwise} \end{cases}$$

$$f^*(y) = \begin{cases} 0 & \text{if } 0 \preceq y \preceq 1 \\ \infty & \text{otherwise} \end{cases}$$

$$f^*(y) = \begin{cases} 0 & \text{if } y \succeq 0,\ \mathbf{1}^T y = n \\ \infty & \text{otherwise} \end{cases}$$

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