DISTANCE BETWEEN HYPERPLANES (10/10 points)

What is the distance between the two parallel hyperplanes $\{x \in \mathbf{R}^n \mid a^T x = b_1\}$ and $\{x \in \mathbf{R}^n \mid a^T x = b_2\}$?

- ○ $|b_1 - b_2|/\|a\|_2^2$
- ◉ $|b_1 - b_2|/\|a\|_2$ ✔
- ○ $|b_1 + b_2|/(b_1 b_2 \|a\|_2)$
- ○ $|b_1 - b_2|$

Show Answer *You have used 1 of 2 submissions*

VORONOI DESCRIPTION OF A HALFSPACE (10/10 points)

Let a and b be distinct points in $\mathbf{R}^n$ and consider the set of points that are closer (in Euclidean norm) to a than b, i.e., $\mathcal{C} = \{x \mid \|x - a\|_2 \leq \|x - b\|_2\}$. The set $\mathcal{C}$ is a halfspace; which of the following inequalities characterize it?

- ○ $2(b-a)^T x \leq \|b\|_2 - \|a\|_2$
- ○ $(b-a)^T x \leq \|b\|_2^2 - \|a\|_2^2$
- ◉ $2(b-a)^T x \leq \|b\|_2^2 - \|a\|_2^2$ ✔
- ○ $(1/2)(b-a)^T x \leq \|b\|_2 - \|a\|_2$

Show Answer *You have used 2 of 2 submissions*

COMMON CONVEX SETS (10 points possible)

Which of the following sets is convex?

- ☑ A slab, i.e., a set of the form $\{x \in \mathbf{R}^n \mid \alpha \leq a^T x \leq \beta\}$
- ☑ A rectangle, i.e., a set of the form $\{x \in \mathbf{R}^n \mid \alpha_i \leq x_i \leq \beta_i,\ i = 1, \ldots, n\}$. A rectangle is sometimes called a hyperrectangle when $n > 2$
- ☑ A wedge, i.e., $\{x \in \mathbf{R}^n \mid a_1^T x \leq b_1,\ a_2^T x \leq b_2\}$
- ☐ The set of points closer to a given point than a given set, i.e.,$\{x \mid \|x - x_0\|_2 \leq \|x - y\|_2 \text{ for all } y \in S\}$ where $S \subseteq \mathbf{R}^n$
- ☐ The set of points closer to one set than another, i.e, $\{x \mid \mathbf{dist}(x, S) \leq \mathbf{dist}(x, T)\}$, where $S, T \subseteq \mathbf{R}^n$, and $\mathbf{dist}(x, S) = \inf\{\|x - z\|_2 \mid z \in S\}$

Check all that apply.

Show Answer You have used 0 of 2 submissions

SOME SETS OF PROBABILITY DISTRIBUTIONS (10 points possible)

Let x be a real-valued random variable with $\mathbf{prob}(x = a_i) = p_i$, $i = 1, \ldots, n$, where $a_1 < a_2 < \cdots < a_n$. Of course $p \in \mathbf{R}^n$ lies in the standard probability simplex $P = \{p \mid \mathbf{1}^T p = 1, p \succeq 0\}$. Which of the following conditions are convex in p? (That is, for which of the following conditions is the set of $p \in P$ that satisfy the condition convex?)

- [ ] $\alpha \leq \mathbf{E} f(x) \leq \beta$
- [ ] $\mathbf{prob}(x > \alpha) \leq \beta$
- [ ] $\mathbf{E}|x^3| \leq \alpha \mathbf{E}|x|$
- [ ] $\mathbf{E} x^2 \leq \alpha$

Check all that apply.

Here, $\mathbf{E} f(x)$ is the expected value of $f(x)$, i.e., $\mathbf{E} f(x) = \sum_{i=1}^n p_i f(a_i)$. The function $f : \mathbf{R} \to \mathbf{R}$ is given.

Show Answer You have used 0 of 2 submissions

POSITIVE SEMIDEFINITE CONES (30 points possible)

Which conditions, in terms of the ordinary inequalities on matrix coefficents, must hold true for the elements of the positive semidefinite cone $\mathbf{S}_+^n$ for $n = 1, 2, 3$?

$$n = 1, \qquad [x_1]$$

- [ ] $x_1 > 0$
- [ ] $x_1 \geq 0$
- [ ] $x_1 \leq 0$

$$n = 2, \qquad \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \end{bmatrix}$$

- [ ] $x_1 \geq 0$
- [ ] $x_1 x_3 \leq 0$
- [ ] $x_1 x_2 \geq 0$
- [ ] $x_3 \geq 0$
- [ ] $x_2 \geq 0$
- [ ] $x_2 \leq 0$
- [ ] $x_1 x_3 - x_2^2 \geq 0$

$$n = 3, \qquad \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & x_4 & x_5 \\ x_3 & x_5 & x_6 \end{bmatrix}$$

- [ ] $x_1 \geq 0$
- [ ] $x_4 \geq 0$

Help

- [ ] $x_1 x_3 \leq 0$
- [ ] $x_1 x_2 - x_3^2 \geq 0$
- [ ] $x_6 \geq 0$
- [ ] $x_2 \geq 0$
- [ ] $x_1 x_3 - x_2^2 \geq 0$
- [ ] $x_1 x_4 - x_2^2 \geq 0$
- [ ] $x_4 x_6 - x_5^2 \geq 0$
- [ ] $x_4 x_6 - x_2^2 \geq 0$
- [ ] $x_1 x_6 - x_3^2 \geq 0$
- [ ] $x_1^2 - x_4^2 + x_2^2 \leq 0$
- [ ] $x_1 x_4 x_6 + 2 x_2 x_3 x_5 - x_1 x_5^2 - x_6 x_2^2 - x_4 x_3^2 \geq 0$

Check all that apply.

Show Answer

You have used 0 of 2 submissions

DUAL CONES (40 points possible)

The dual cone of $K = \{0\} \subseteq \mathbf{R}^2$ is

- ○ $K^* = \mathbf{R}^2$
- ○ $K^* = \{0\}$
- ○ $K^* = \{(x, y) \mid x + y = 0\}$
- ○ $K^* = \{(x, y) \mid xy = 0\}$.

The dual cone of $K = \mathbf{R}^2$ is:

- ○ $K^* = \mathbf{R}^2$
- ○ $K^* = \{0\}$
- ○ $K^* = \mathbf{R}^2_+$
- ○ $K^* = \{(x, y) \mid x = 0 \text{ or } y = 0\}$.

The dual cone of $K = \{(x, y) \mid |x| \leq y\}$ is

- ○ $K^* = \{(x, y) \mid x \leq |y|\}$
- ○ $K^* = \{(x, y) \mid |x| \leq -y\}$
- ○ $K^* = \{(x, y) \mid |x| \leq y\}$
- ○ $K^* = \{(x, y) \mid |x| \geq y\}$.

The dual cone of $K = \{(x, y) \mid x + y = 0\}$ is

- ○ $K^* = \{(x, y) \mid x + y = 0\}$
- ○ $K^* = \{(x, y) \mid x - y = 0\}$
- ○ $K^* = \{(x, y) \mid x = 0 \text{ or } y = 0\}$
- ○ $K^* = \{(x, y) \mid |x| \leq y\}$.

Show Answer *You have used 0 of 2 submissions*