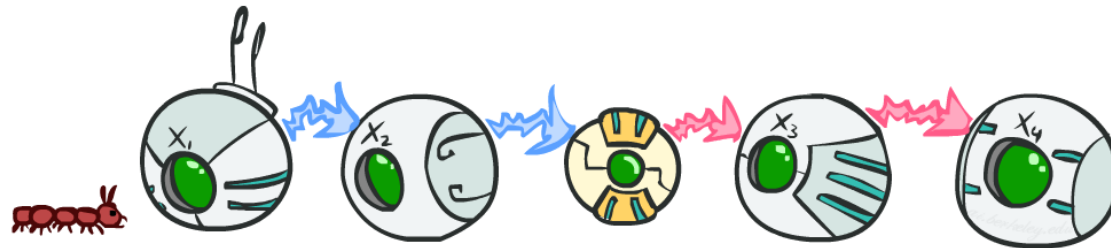# Announcements

- Midterm 2: 7/27 from 11am—2pm
  - Email the instructors *now* if you have conflicts

- Contest 2
  - Visualization bug fixed
  - Extended to Saturday night

- Homework 4
  - Due tonight!

CS 188: Artificial Intelligence

Markov Models

Instructors: Dan Klein and Pieter Abbeel --- University of California, Berkeley

Recap: Decision Networks

Umbrella = leave

$$\text{EU}(\text{leave}) = \sum_w P(w)U(\text{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

Umbrella = take

$$\text{EU}(\text{take}) = \sum_w P(w)U(\text{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\text{MEU}(\varnothing) = \max_a \text{EU}(a) = 70$$

Umbrella

U

Weather

W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70
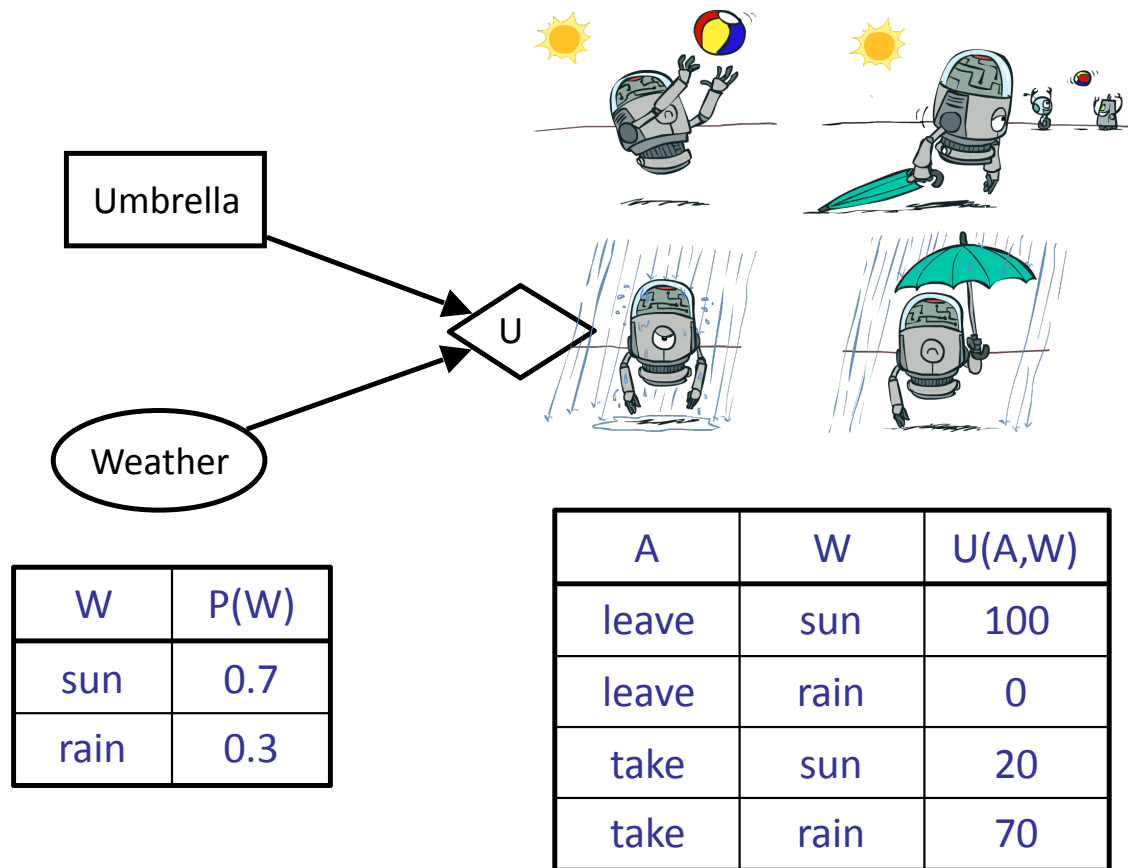
VPI Example: Weather

MEU with no evidence

$$\text{MEU}(\emptyset) = \max_a \text{EU}(a) = 70$$

MEU if forecast is bad

$$\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53$$

MEU if forecast is good

$$\text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 95$$
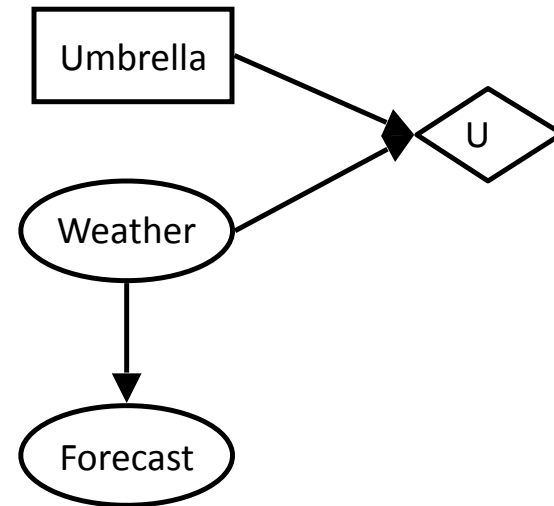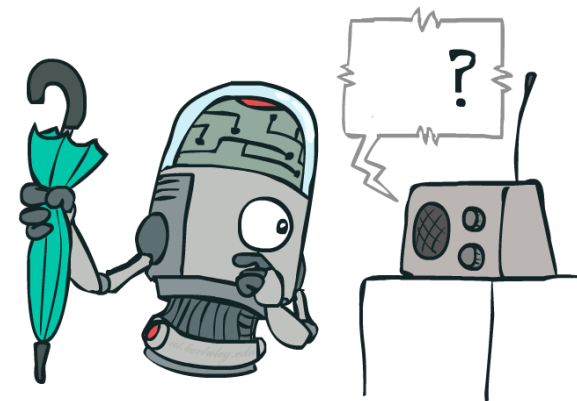
Forecast distribution

F	P(F)
good	0.59
bad	0.41

$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$
$$77.8 - 70 = 7.8$$

A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$\text{VPI}(E'|e) = \left(\sum_{e'} P(e'|e)\text{MEU}(e, e') \right) - \text{MEU}(e)$$

VPI Properties

- Nonnegative

$$\forall E', e : \mathsf{VPI}(E'|e) \geq 0$$

- Nonadditive

(think of observing E_j twice)

$$\mathsf{VPI}(E_j, E_k|e) \neq \mathsf{VPI}(E_j|e) + \mathsf{VPI}(E_k|e)$$

- Order-independent

$$\mathsf{VPI}(E_j, E_k|e) = \mathsf{VPI}(E_j|e) + \mathsf{VPI}(E_k|e, E_j)$$

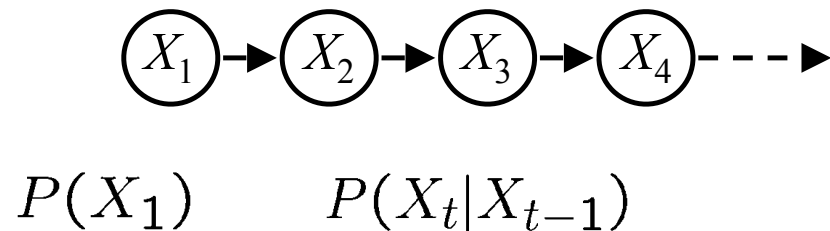$$= \mathsf{VPI}(E_k|e) + \mathsf{VPI}(E_j|e, E_k)$$
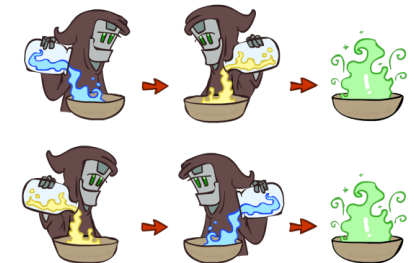
Reasoning over Time or Space

- Often, we want to reason about a sequence of observations

  - Speech recognition

  - Robot localization

  - User attention

  - Medical monitoring

- Need to introduce time (or space) into our models

Markov Models

- Value of X at a given time is called the state

$$P(X_1) \qquad P(X_t | X_{t-1})$$

- Parameters: called transition probabilities or dynamics, specify how the state evolves over time (also, initial state probabilities)
- Stationarity assumption: transition probabilities the same at all times
- Same as MDP transition model, but no choice of action
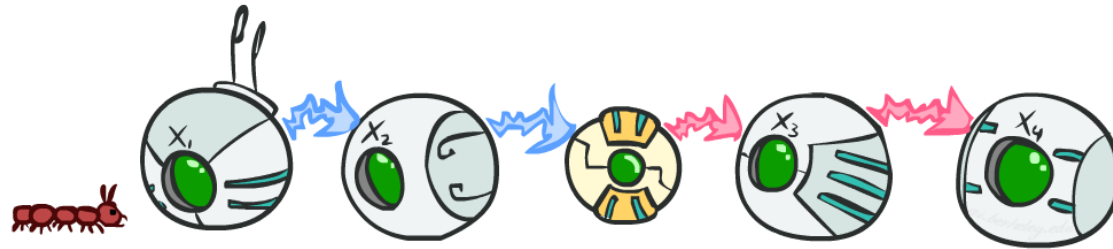
Joint Distribution of a Markov Model

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4$$
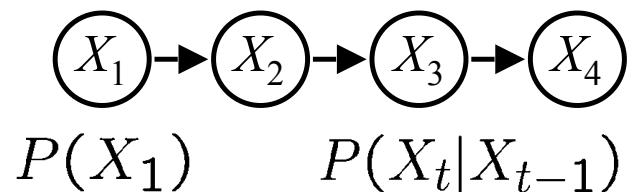
$$P(X_1) \qquad P(X_t|X_{t-1})$$

- Joint distribution:

$$P(X_1, X_2, X_3, X_4) = P(X_1)P(X_2|X_1)P(X_3|X_2)P(X_4|X_3)$$

- More generally:

$$P(X_1, X_2, \ldots, X_T) = P(X_1)P(X_2|X_1)P(X_3|X_2)\ldots P(X_T|X_{T-1})$$

$$= P(X_1)\prod_{t=2}^{T} P(X_t|X_{t-1})$$

Conditional Independence

- Basic conditional independence:
    - Past and future independent of the present
    - Each time step only depends on the previous
    - This is called the (first order) Markov property

Conditional Independence
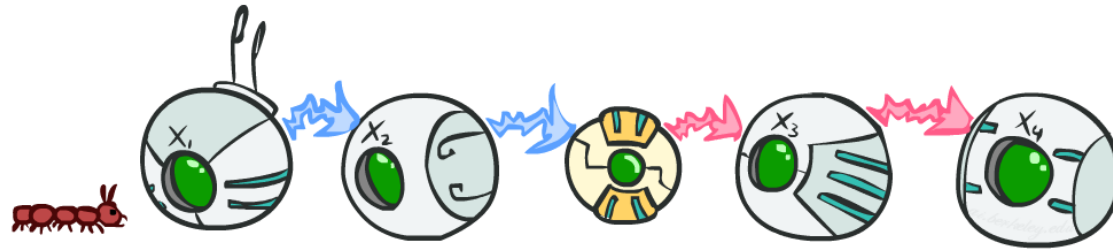
- Basic conditional independence:
  - Past and future independent of the present
  - Each time step only depends on the previous
  - This is called the (first order) Markov property

- Note that the chain is just a (growable) BN
  - We can always use generic BN reasoning on it if we truncate the chain at a fixed length
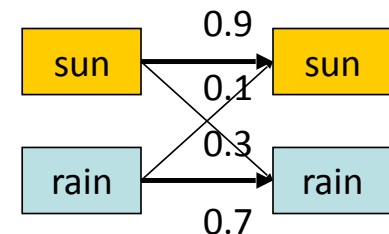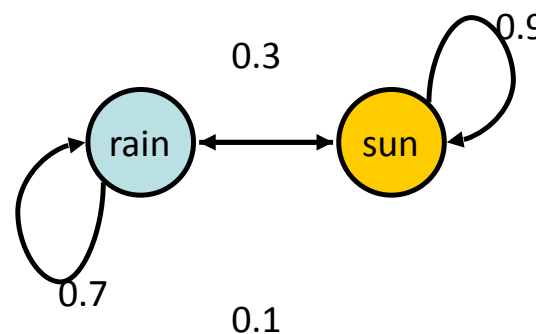
Example Markov Chain: Weather

- States: X = {rain, sun}

- Initial distribution: 1.0 sun

- CPT P(X_t | X_{t-1}):

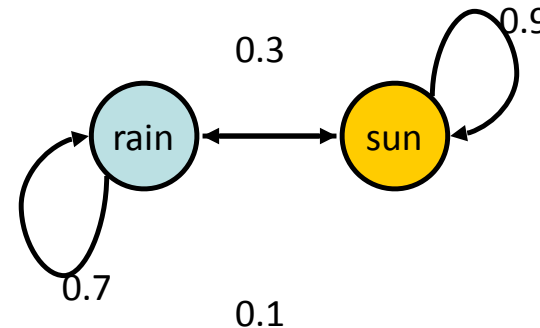| X_{t-1}	X_t	P(X_t|X_{t-1})
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7
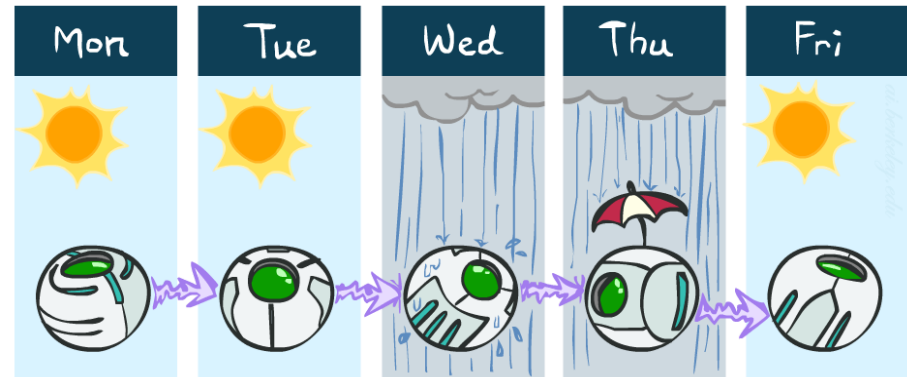
Two new ways of representing the same CPT

Example Markov Chain: Weather
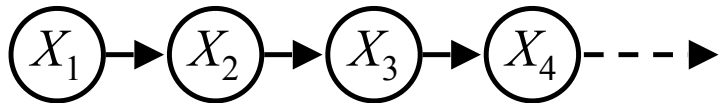
- Initial distribution: 1.0 sun

- What is the probability distribution after one step?

$$P(X_2 = \text{sun}) = \quad P(X_2 = \text{sun}|X_1 = \text{sun})P(X_1 = \text{sun}) +$$

$$P(X_2 = \text{sun}|X_1 = \text{rain})P(X_1 = \text{rain})$$

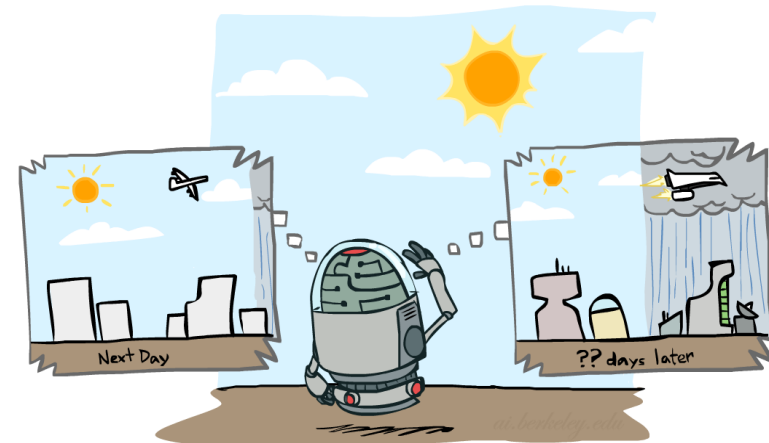$$0.9 \cdot 1.0 + 0.3 \cdot 0.0 = 0.9$$

Mini-Forward Algorithm

- Question: What's P(X) on some day t?

$$P(x_1) = \text{known}$$

$$P(x_t) = \sum_{x_{t-1}} P(x_{t-1}, x_t)$$

$$= \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$

Forward simulation

Example Run of Mini-Forward Algorithm

- From initial observation of sun

$$\left\langle \begin{array}{c} 1.0 \\ 0.0 \end{array} \right\rangle \quad \left\langle \begin{array}{c} 0.9 \\ 0.1 \end{array} \right\rangle \quad \left\langle \begin{array}{c} 0.84 \\ 0.16 \end{array} \right\rangle \quad \left\langle \begin{array}{c} 0.804 \\ 0.196 \end{array} \right\rangle \longrightarrow \left\langle \begin{array}{c} 0.75 \\ 0.25 \end{array} \right\rangle$$

$P(X_1) \qquad P(X_2) \qquad P(X_3) \qquad P(X_4) \qquad P(X_\infty)$

- From initial observation of rain

$$\left\langle \begin{array}{c} 0.0 \\ 1.0 \end{array} \right\rangle \quad \left\langle \begin{array}{c} 0.3 \\ 0.7 \end{array} \right\rangle \quad \left\langle \begin{array}{c} 0.48 \\ 0.52 \end{array} \right\rangle \quad \left\langle \begin{array}{c} 0.588 \\ 0.412 \end{array} \right\rangle \longrightarrow \left\langle \begin{array}{c} 0.75 \\ 0.25 \end{array} \right\rangle$$

$P(X_1) \qquad P(X_2) \qquad P(X_3) \qquad P(X_4) \qquad P(X_\infty)$

- From yet another initial distribution $P(X_1)$:

$$\left\langle \begin{array}{c} p \\ 1-p \end{array} \right\rangle \qquad \ldots \qquad \longrightarrow \left\langle \begin{array}{c} 0.75 \\ 0.25 \end{array} \right\rangle$$

$P(X_1) \qquad\qquad\qquad\qquad\qquad P(X_\infty)$

[Demo: L13D1,2,3]

Stationary Distributions

- For most chains:
  - Influence of the initial distribution gets less and less over time.
  - The distribution we end up in is independent of the initial distribution
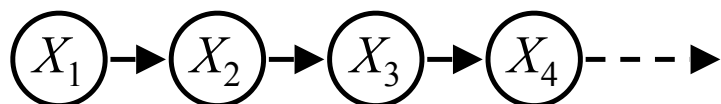
- Stationary distribution:
  - The distribution we end up with is called the stationary distribution P_∞ of the chain
  - It satisfies

$$P_\infty(X) = P_{\infty+1}(X) = \sum_x P(X|x)P_\infty(x)$$

Example: Stationary Distributions

- Question: What's P(X) at time t = infinity?

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4 \dashrightarrow$$

$$P_\infty(sun) = P(sun|sun)P_\infty(sun) + P(sun|rain)P_\infty(rain)$$
$$P_\infty(rain) = P(rain|sun)P_\infty(sun) + P(rain|rain)P_\infty(rain)$$

$$P_\infty(sun) = 0.9P_\infty(sun) + 0.3P_\infty(rain)$$
$$P_\infty(rain) = 0.1P_\infty(sun) + 0.7P_\infty(rain)$$

$$P_\infty(sun) = 3P_\infty(rain)$$
$$P_\infty(rain) = 1/3P_\infty(sun)$$

Also: $P_\infty(sun) + P_\infty(rain) = 1$
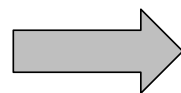
$$\Rightarrow \quad P_\infty(sun) = 3/4$$
$$P_\infty(rain) = 1/4$$
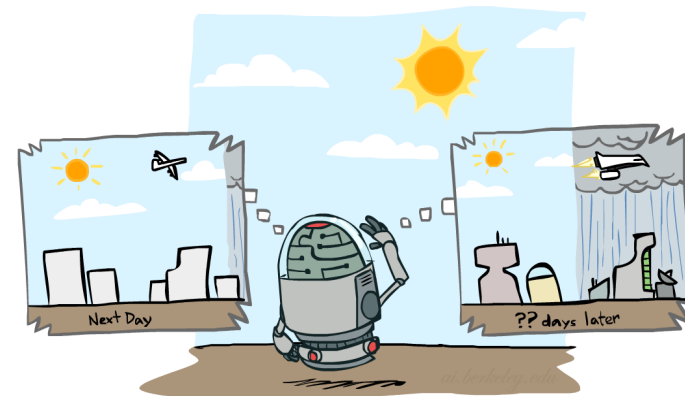
X_{t-1}	X_t	$P(X_t|X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

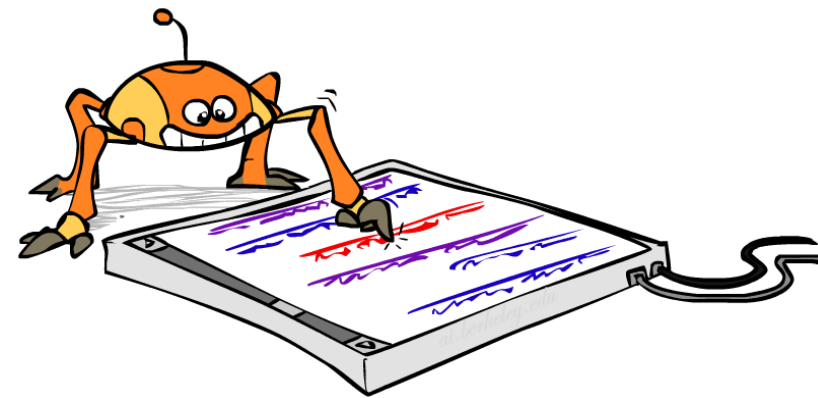# Application of Stationary Distribution: Web Link Analysis

- PageRank over a web graph
    - Each web page is a state
    - Initial distribution: uniform over pages
    - Transitions:
        - With prob. c, uniform jump to a random page (dotted lines, not all shown)
        - With prob. 1-c, follow a random outlink (solid lines)
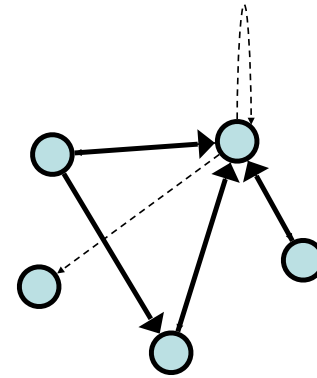
- Stationary distribution
    - Will spend more time on highly reachable pages
    - E.g. many ways to get to the Acrobat Reader download page
    - Somewhat robust to link spam
    - Google 1.0 returned the set of pages containing all your keywords in decreasing rank, now all search engines use link analysis along with many other factors (rank actually getting less important over time)

Next Time: Hidden Markov Models!