Announcements

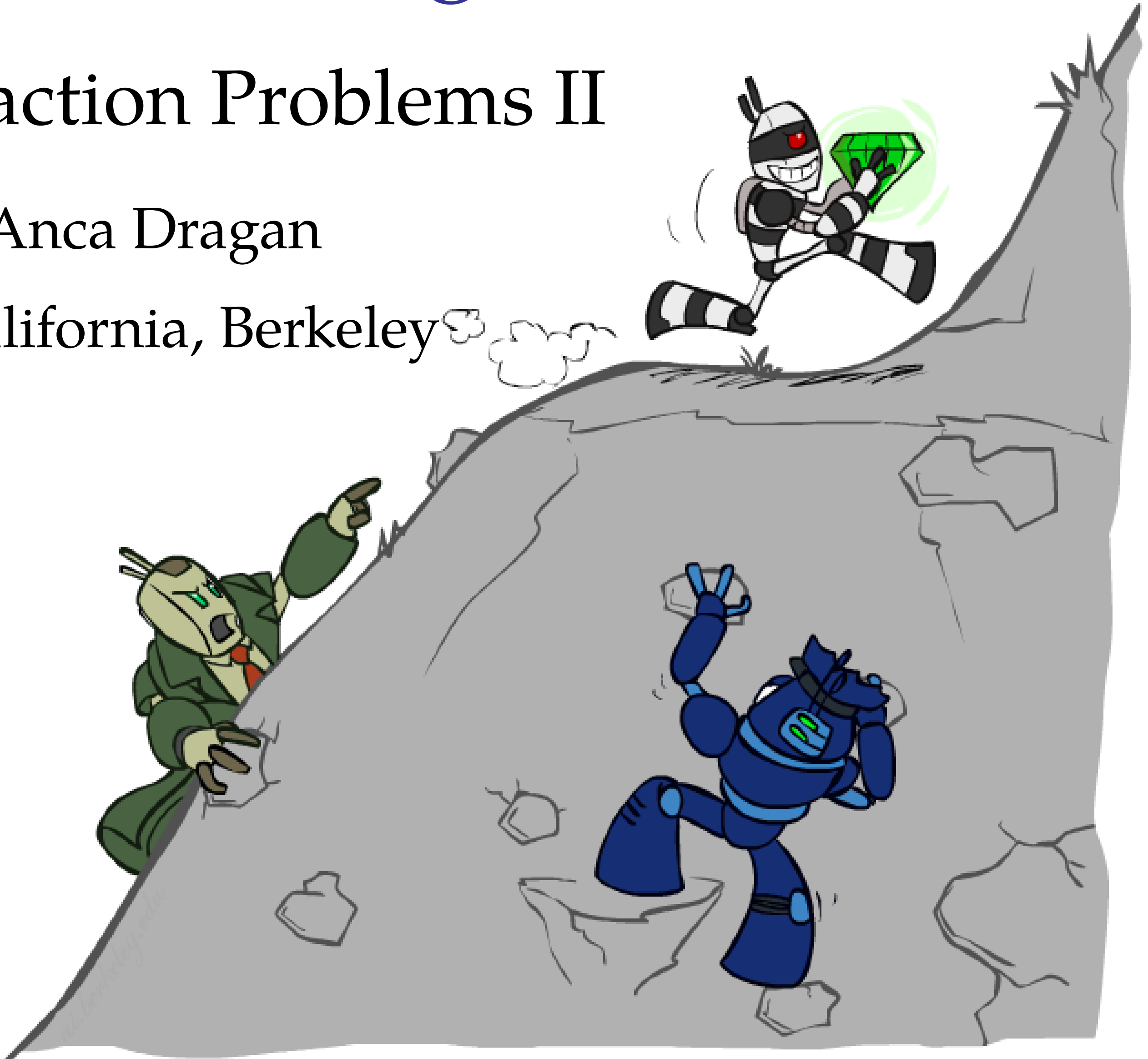
- Homework 1: Search
  - Due yesterday
  - "Show Answer"
- Project 1: Search
  - due Friday 5pm
- Contest 1: Search – optional but fun
  - due Sunday
- State space practice on piazza – coming up
- Homework 2: CSPs
  - due Monday

CS 188: Artificial Intelligence

Constraint Satisfaction Problems II

Instructor: Anca Dragan

University of California, Berkeley



Slides by Dan Klein, Pieter Abbeel, Anca Dragan (ai.berkeley.edu)

Today

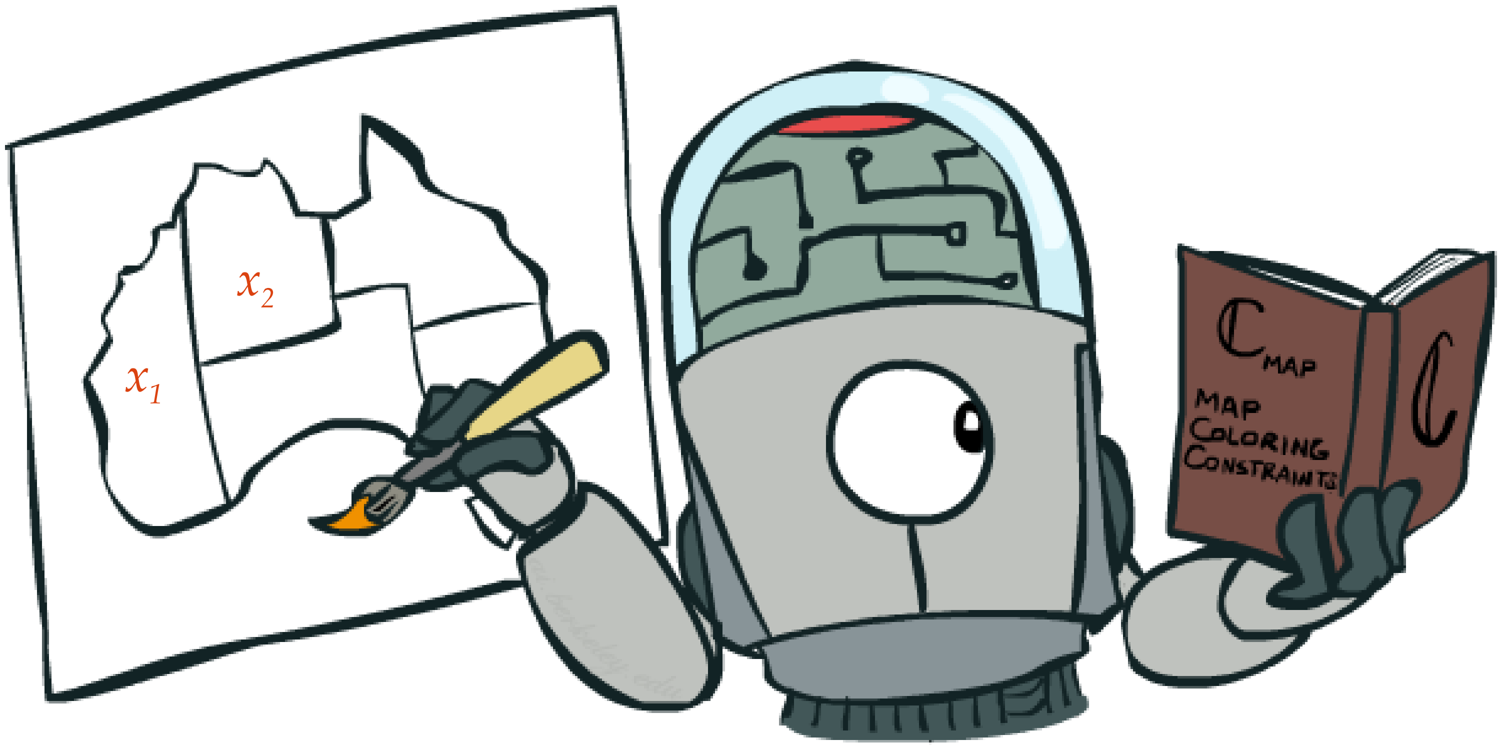
- Efficient Solution of CSPs
- Local Search

Constraint Satisfaction Problems

N variables

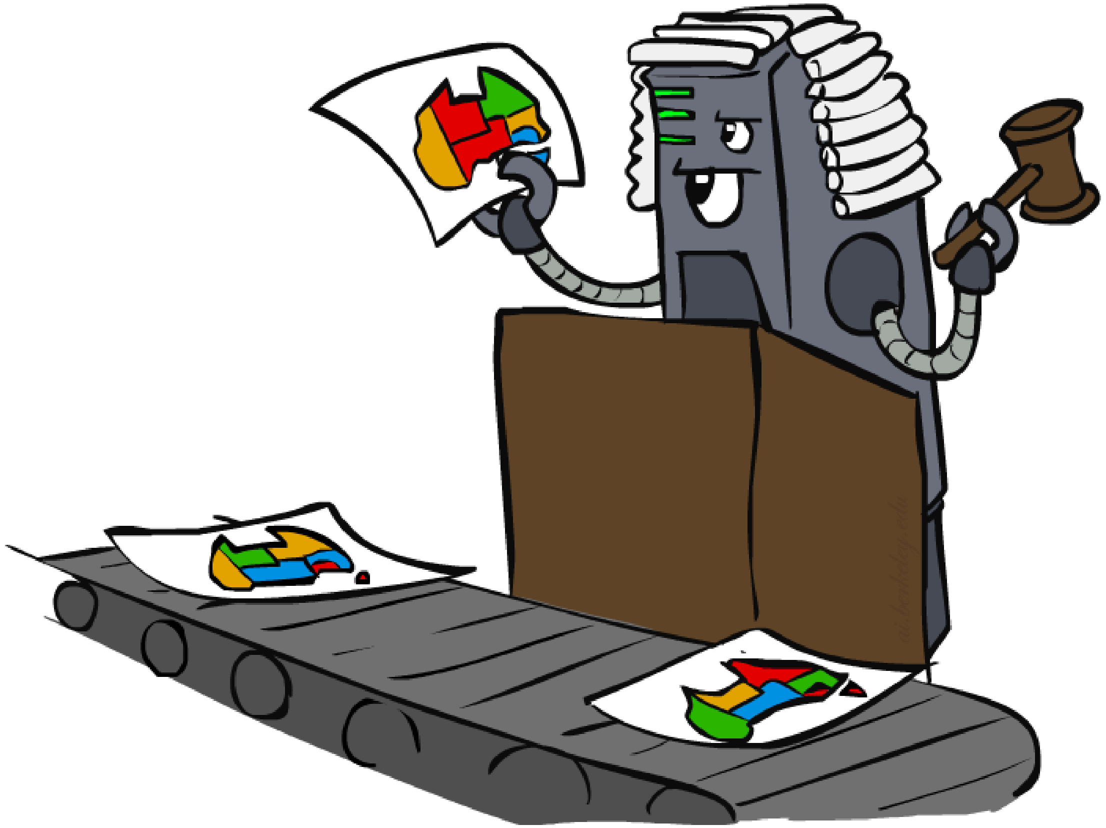
domain D

constraints

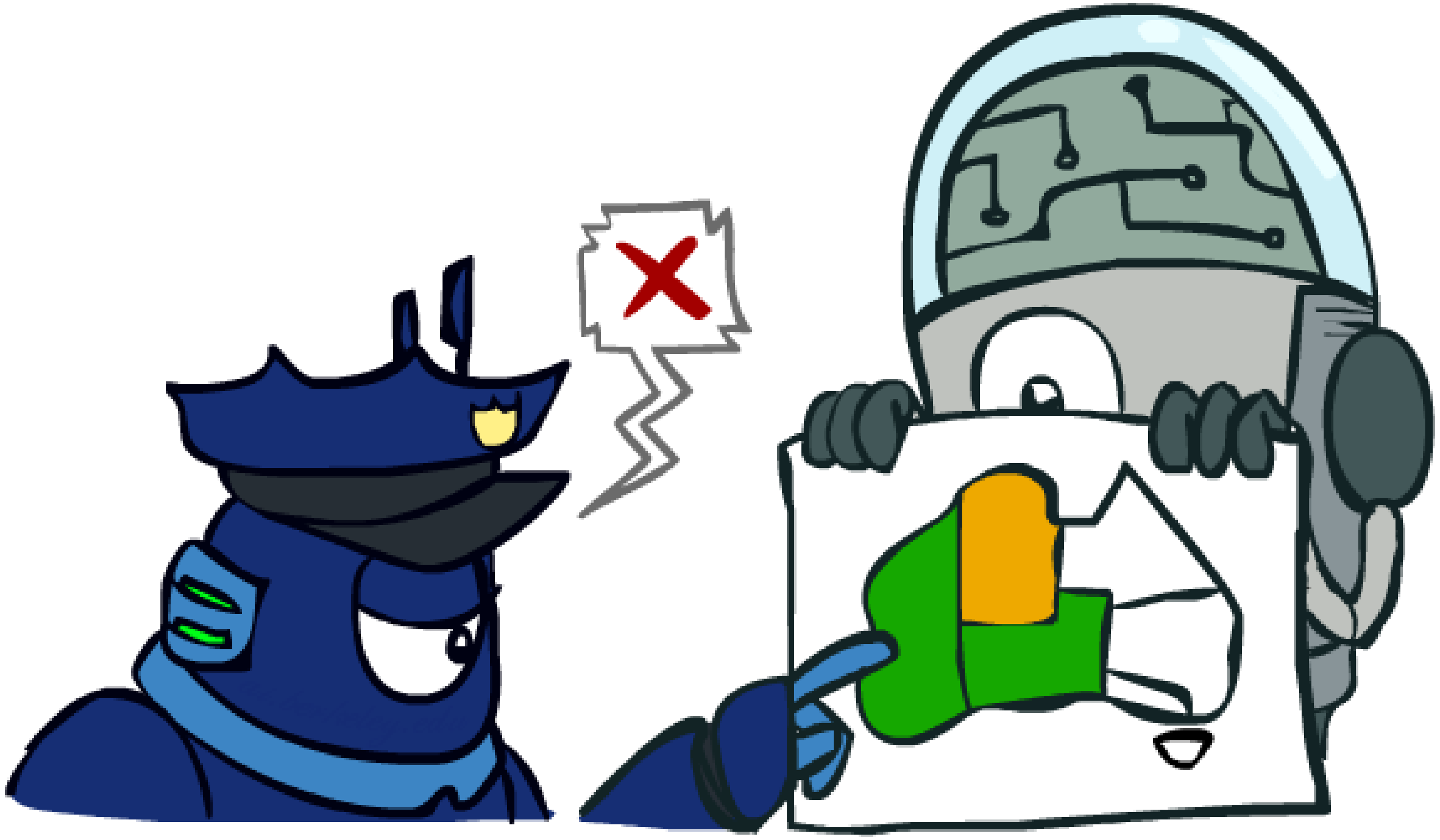


Standard Search Formulation

- Standard search formulation of CSPs
- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints
- We'll start with the straightforward, naïve approach, then improve it



Backtracking Search



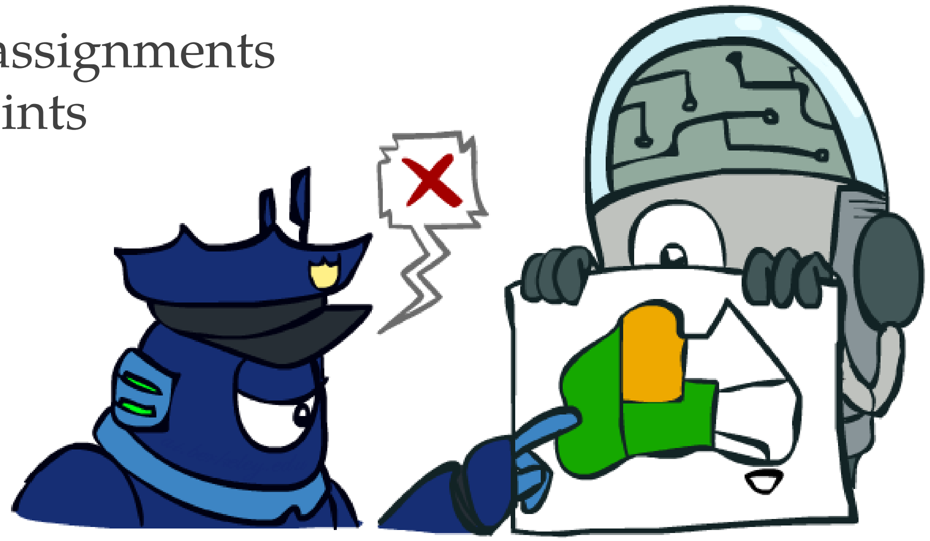
Backtracking Search

- Backtracking search is the basic uninformed algorithm for solving CSPs

- Idea 1: One variable at a time
  - Variable assignments are commutative, so fix ordering -> better branching factor!
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step

- Idea 2: Check constraints as you go
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to check the constraints
  - "Incremental goal test"

- Depth-first search with these two improvements is called *backtracking search* (not the best name)



- Can solve n-queens for $n \approx 25$

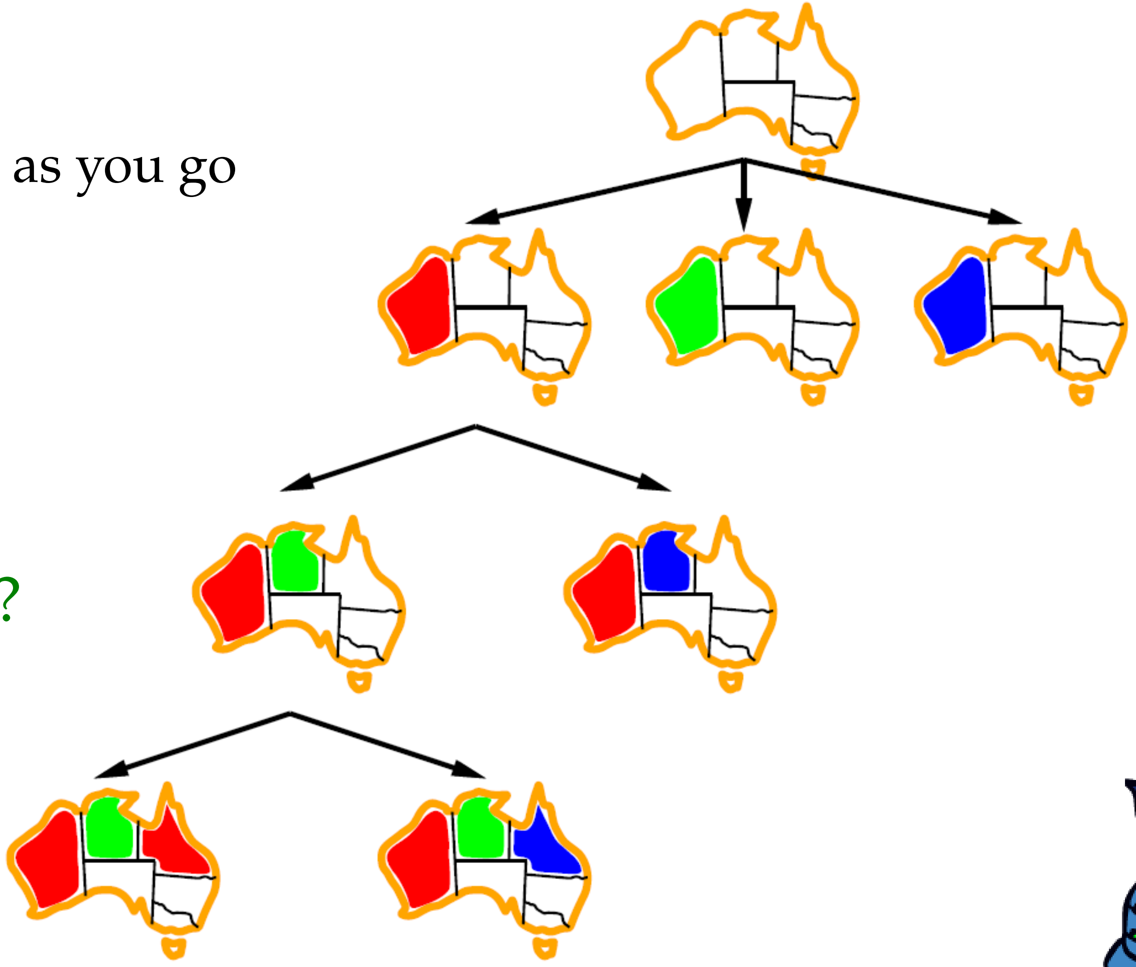
Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

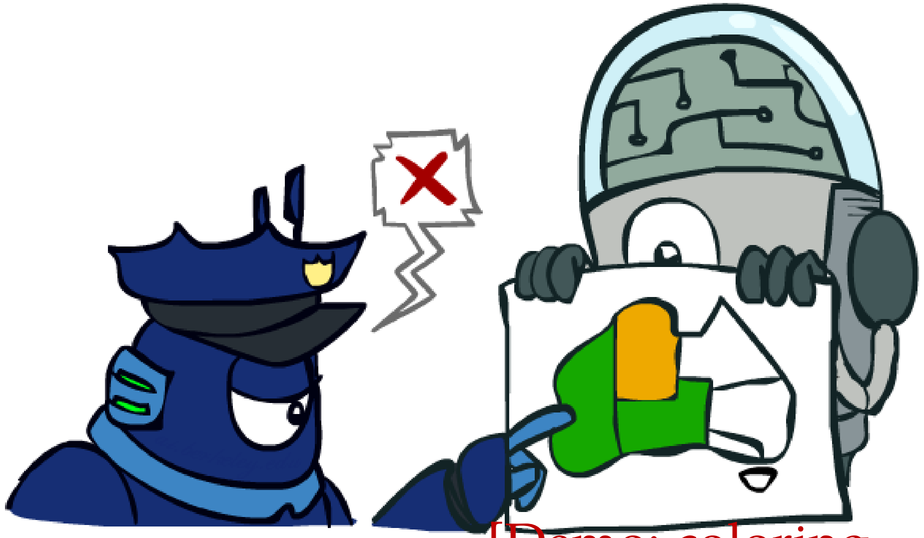
- Backtracking = DFS + variable-ordering + fail-on-violation
- What are the choice points?

Backtracking Search

1. fix ordering
2. check constraints as you go



how should
we improve it?



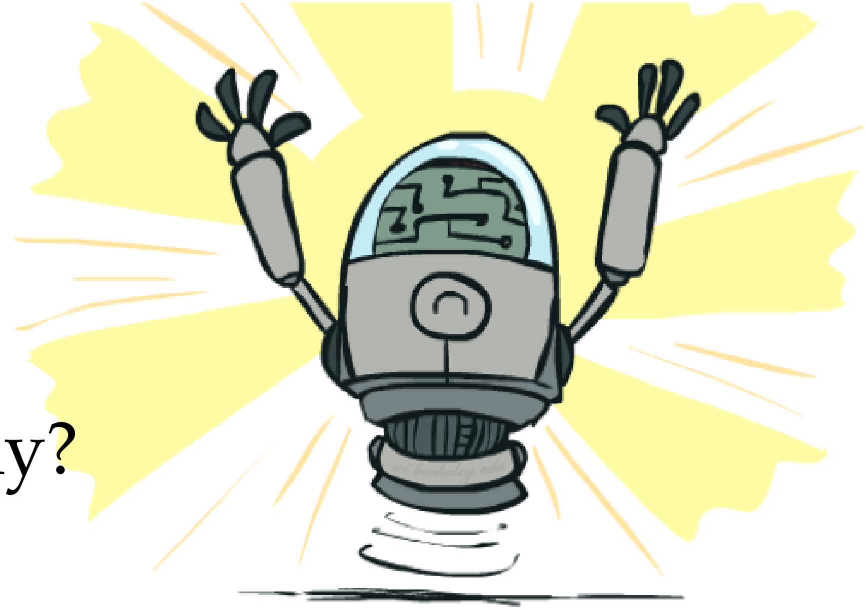
[Demo: coloring --

Improving Backtracking

- General-purpose ideas give huge gains in speed

- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?

- Filtering: Can we detect inevitable failure early?

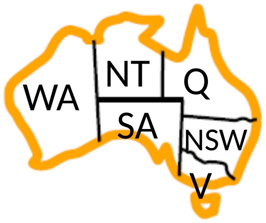


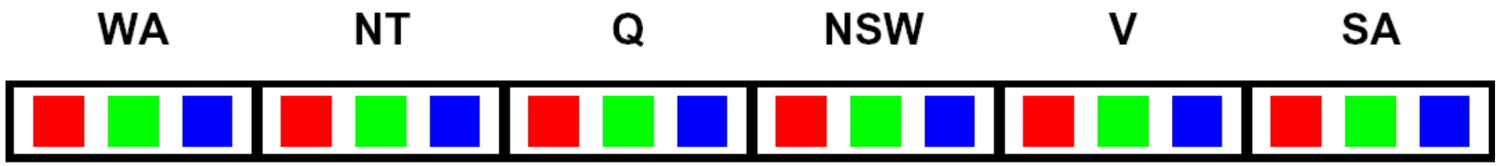
Filtering

Keep track of domains for unassigned variables and cross off bad options

Filtering: Forward Checking

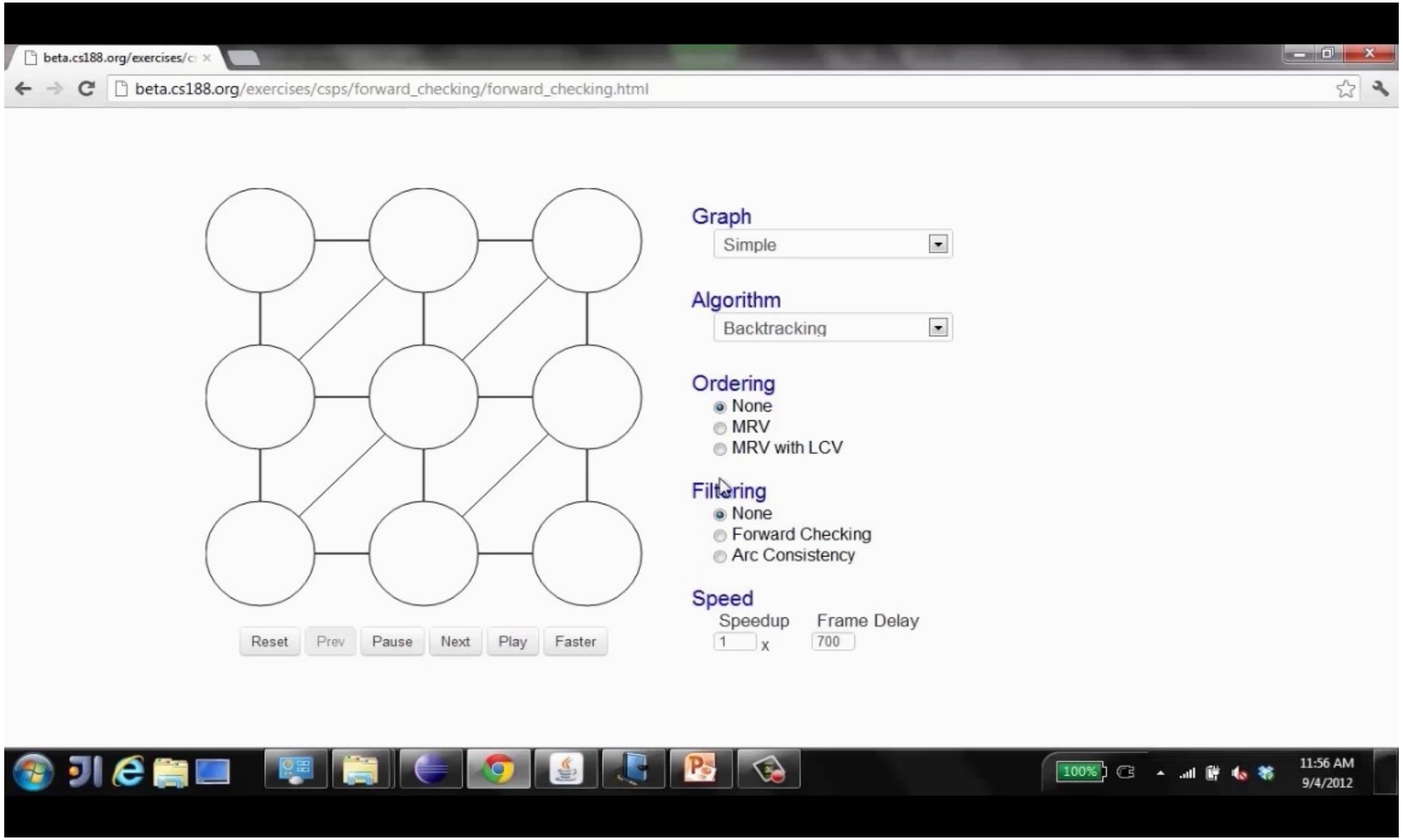
- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment





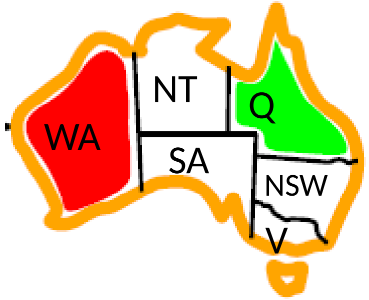
[Demo: coloring -- forward checking]

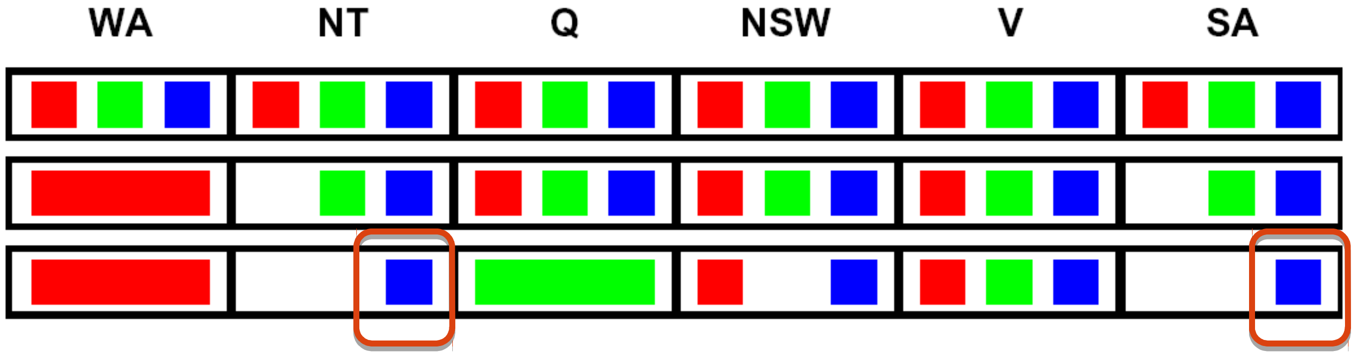
Video of Demo Coloring – Backtracking with Forward Checking



Filtering: Constraint Propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

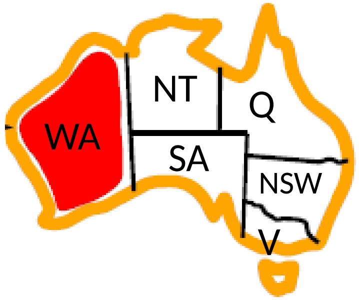


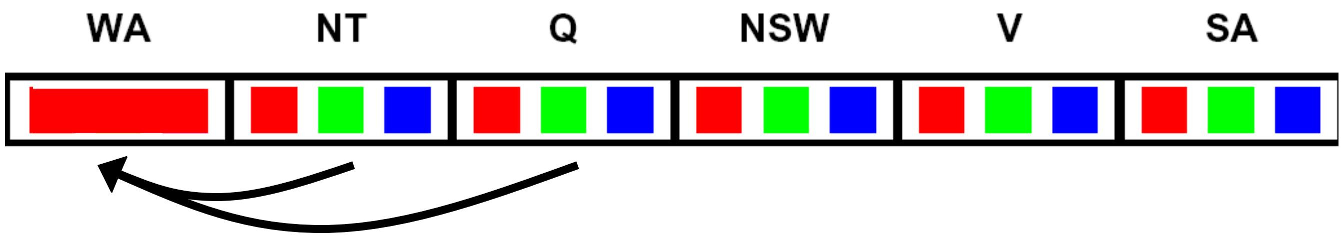


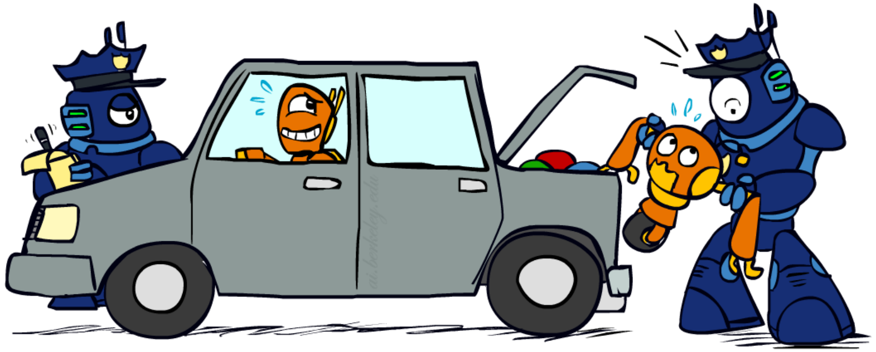
- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

Consistency of A Single Arc

- An arc X → Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint







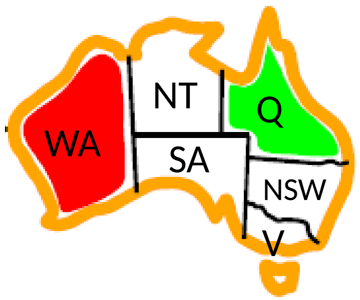
Delete from the tail!

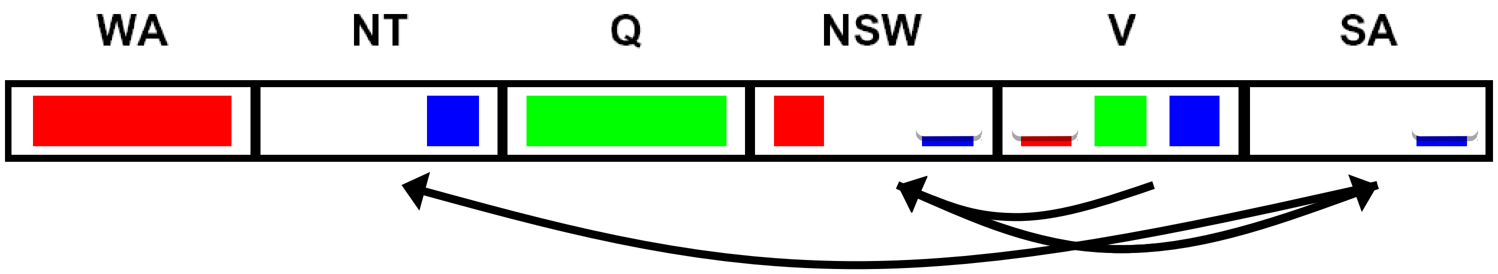
Forward checking?

Enforcing consistency of arcs pointing to each new assignment

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1,\ X_2,\ \ldots,\ X_n\}$
  local variables: $\boxed{\textit{queue},}$ a queue of arcs, initially all the arcs in *csp*

  while *queue* is not empty **do**
    $(X_i,\ X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if $\boxed{\text{REMOVE-INCONSISTENT-VALUES}(X_i,\ X_j)}$ **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add $\boxed{(X_k,\ X_i)}$ to *queue*

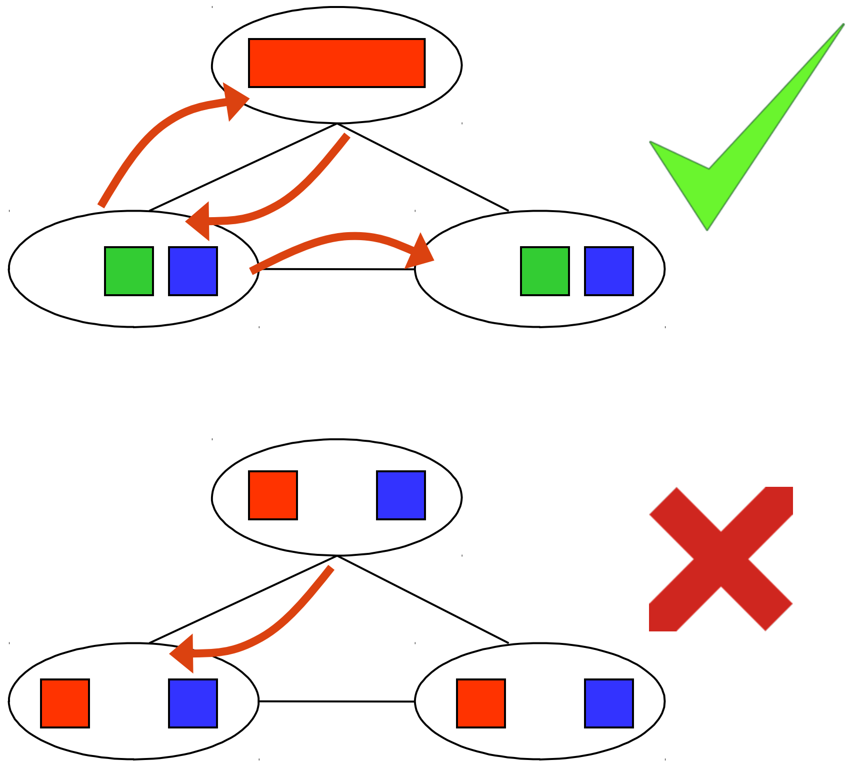
---

function REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **returns** true iff succeeds
  removed ← *false*
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN[X_i]; *removed* ← *true*
  return *removed*

- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting all possible future problems is NP-hard – why?

Limitations of Arc Consistency

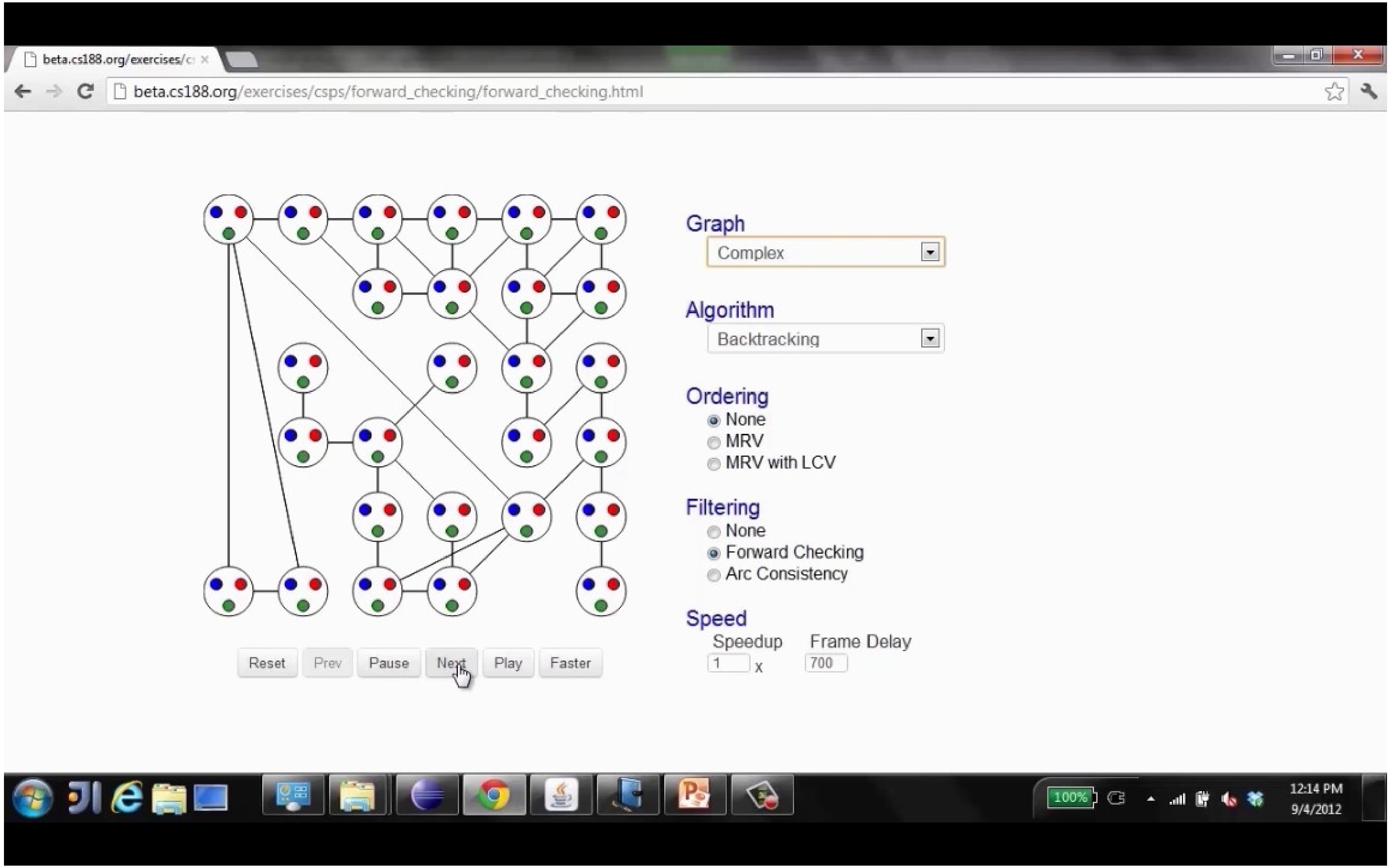
- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)



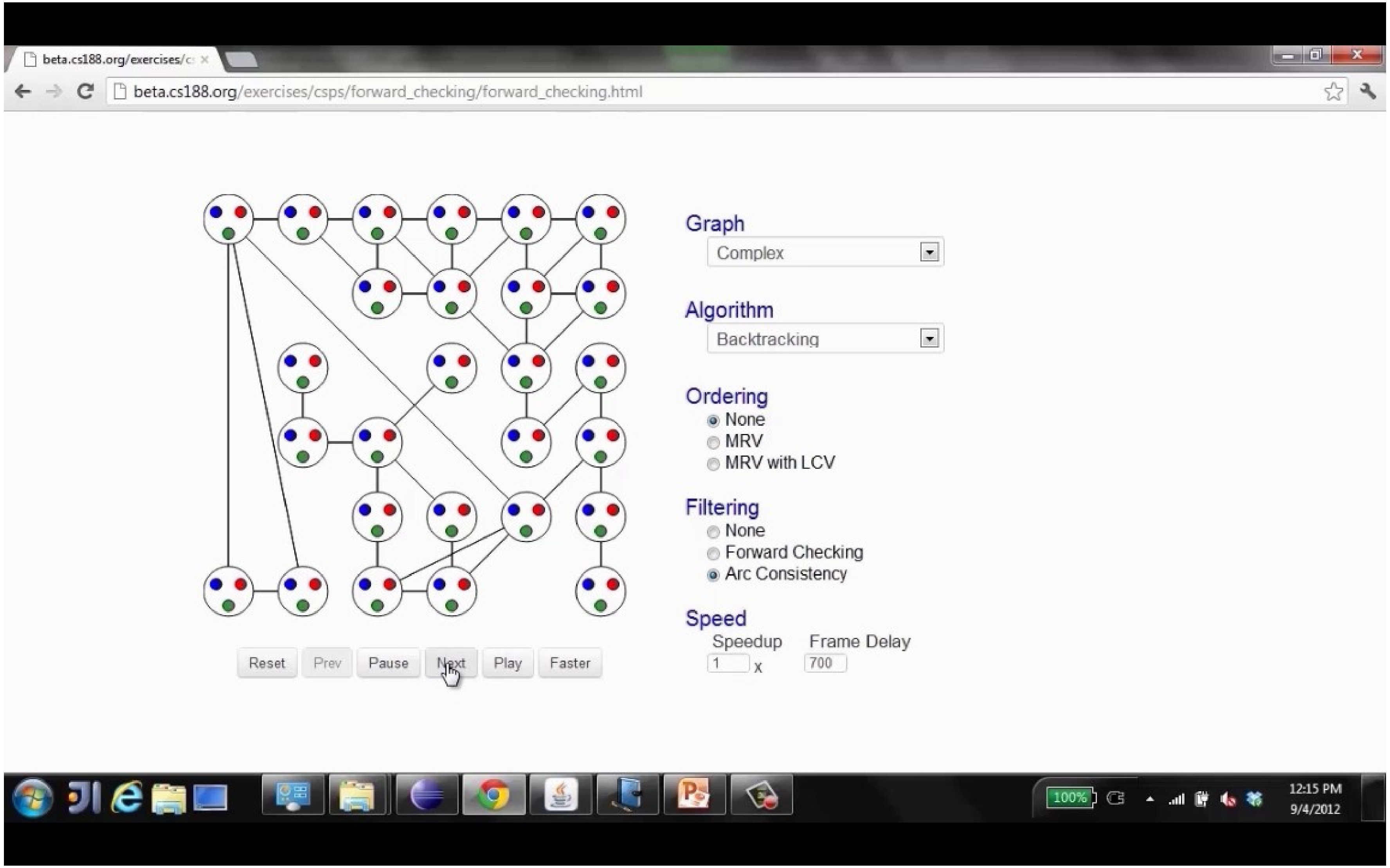
- Arc consistency still runs inside a backtracking search!

[Demo: coloring -- forward checking]
[Demo: coloring -- arc consistency]

Video of Demo Coloring – Backtracking with Forward Checking – Complex Graph



Video of Demo Coloring – Backtracking with Arc Consistency – Complex Graph

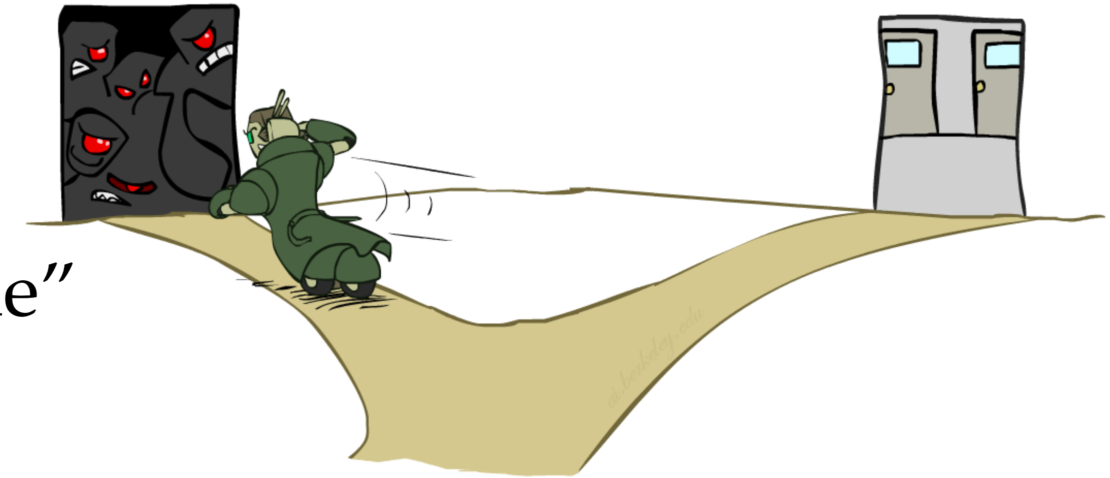


Ordering

Ordering: Minimum Remaining Values

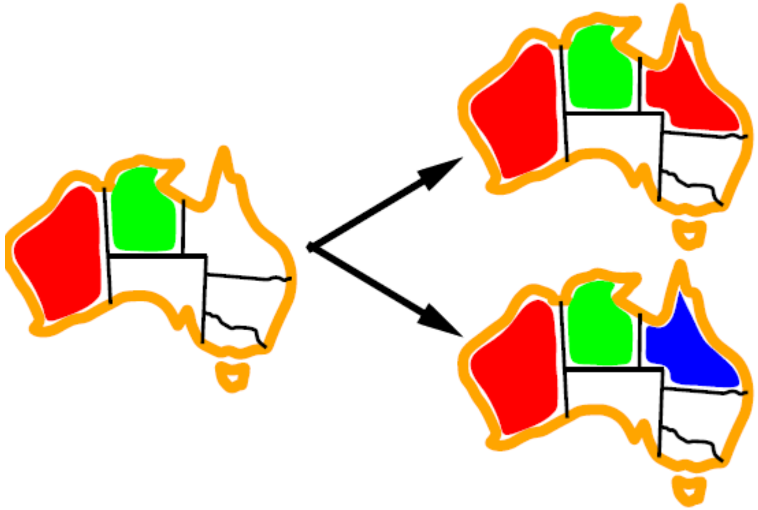
- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain

- Why min rather than max?
- Also called "most constrained variable"
- "Fail-fast" ordering



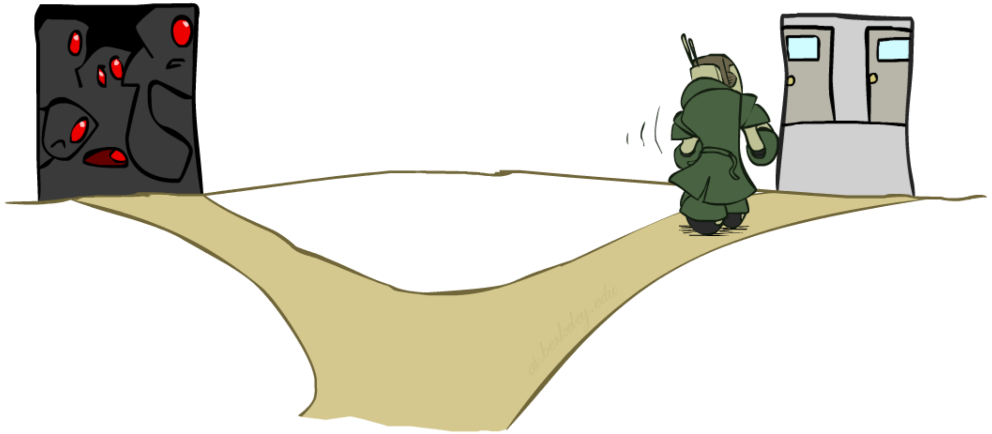
Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)



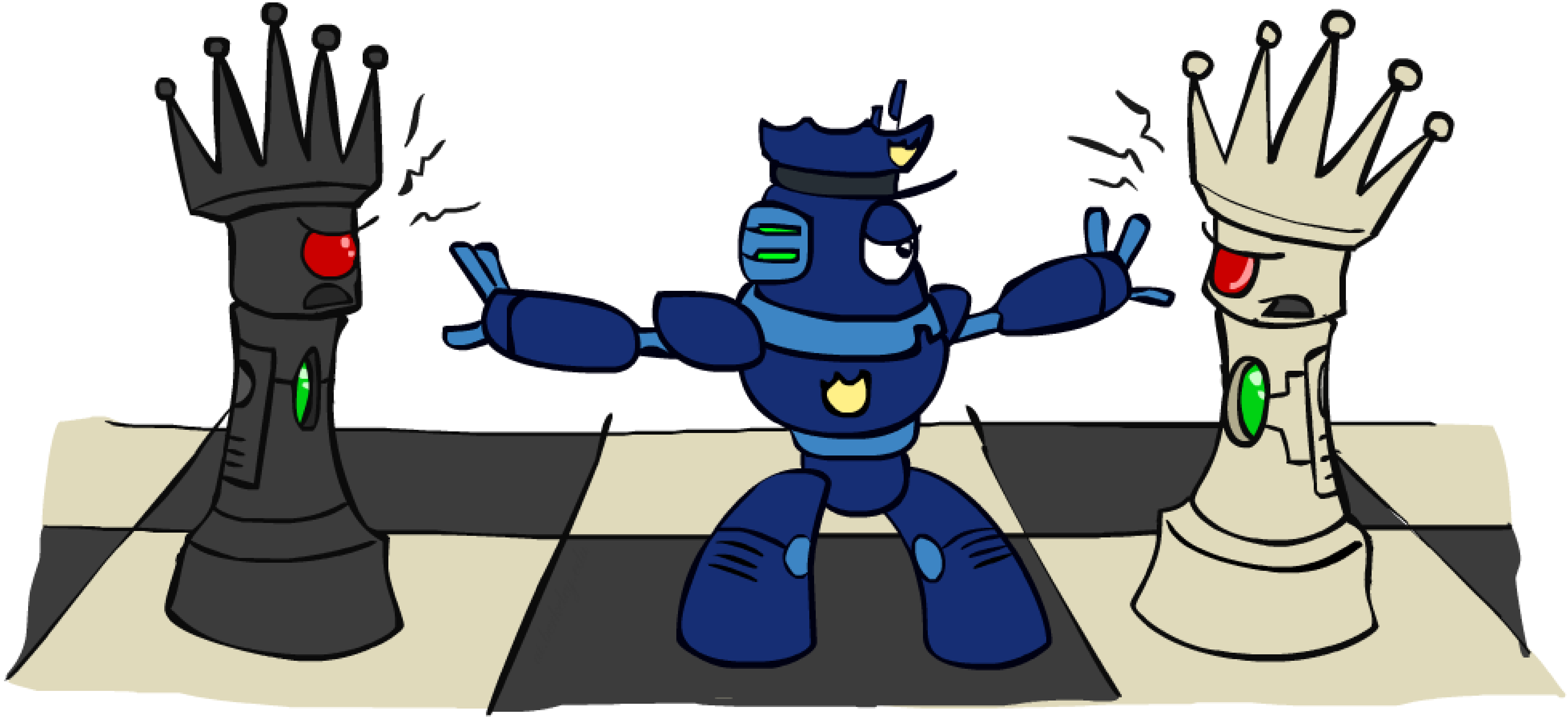
- Why least rather than most?

- Combining these ordering ideas makes 1000 queens feasible



Demo: Coloring -- Backtracking + Forward Checking + Ordering

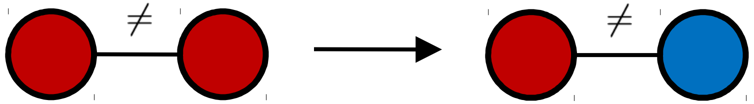
Iterative Improvement



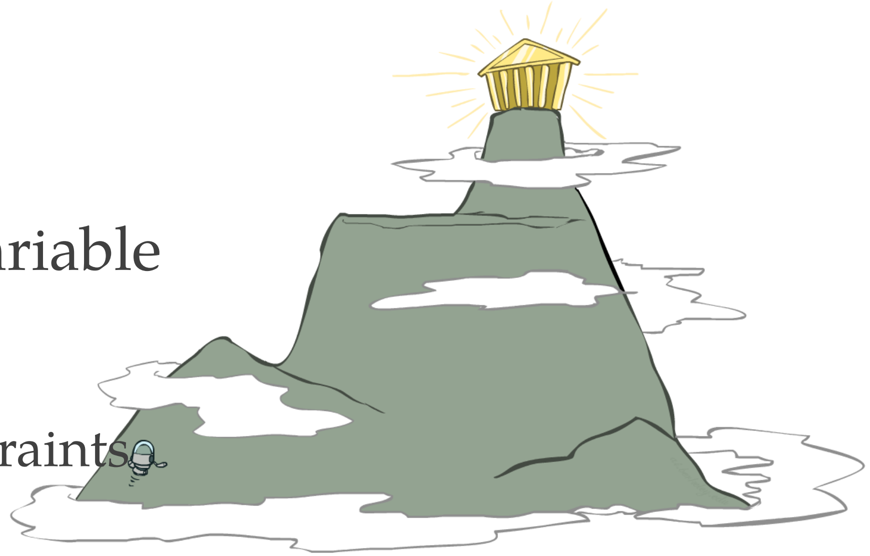
Iterative Algorithms for CSPs

- Local search methods typically work with "complete" states, i.e., all variables assigned

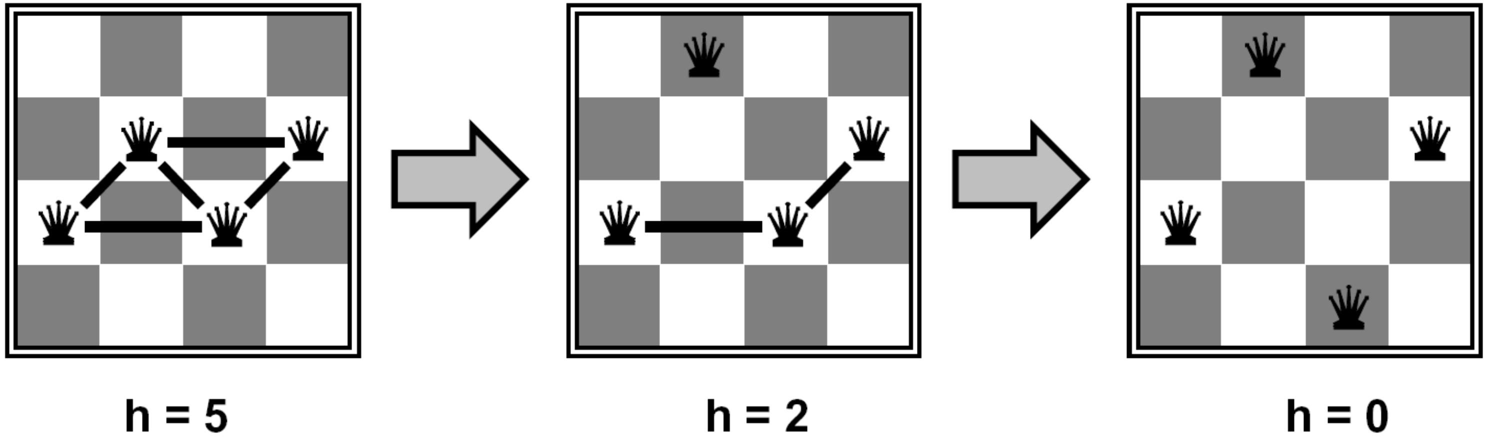
- To apply to CSPs:
  - Take an assignment with unsatisfied constraints
  - Operators *reassign* variable values
  - No fringe! Live on the edge.



- Algorithm: While not solved,
  - Variable selection: randomly select any conflicted variable
  - Value selection: min-conflicts heuristic:
    - Choose a value that violates the fewest constraints
    - I.e., hill climb with h(x) = total number of violated constraints



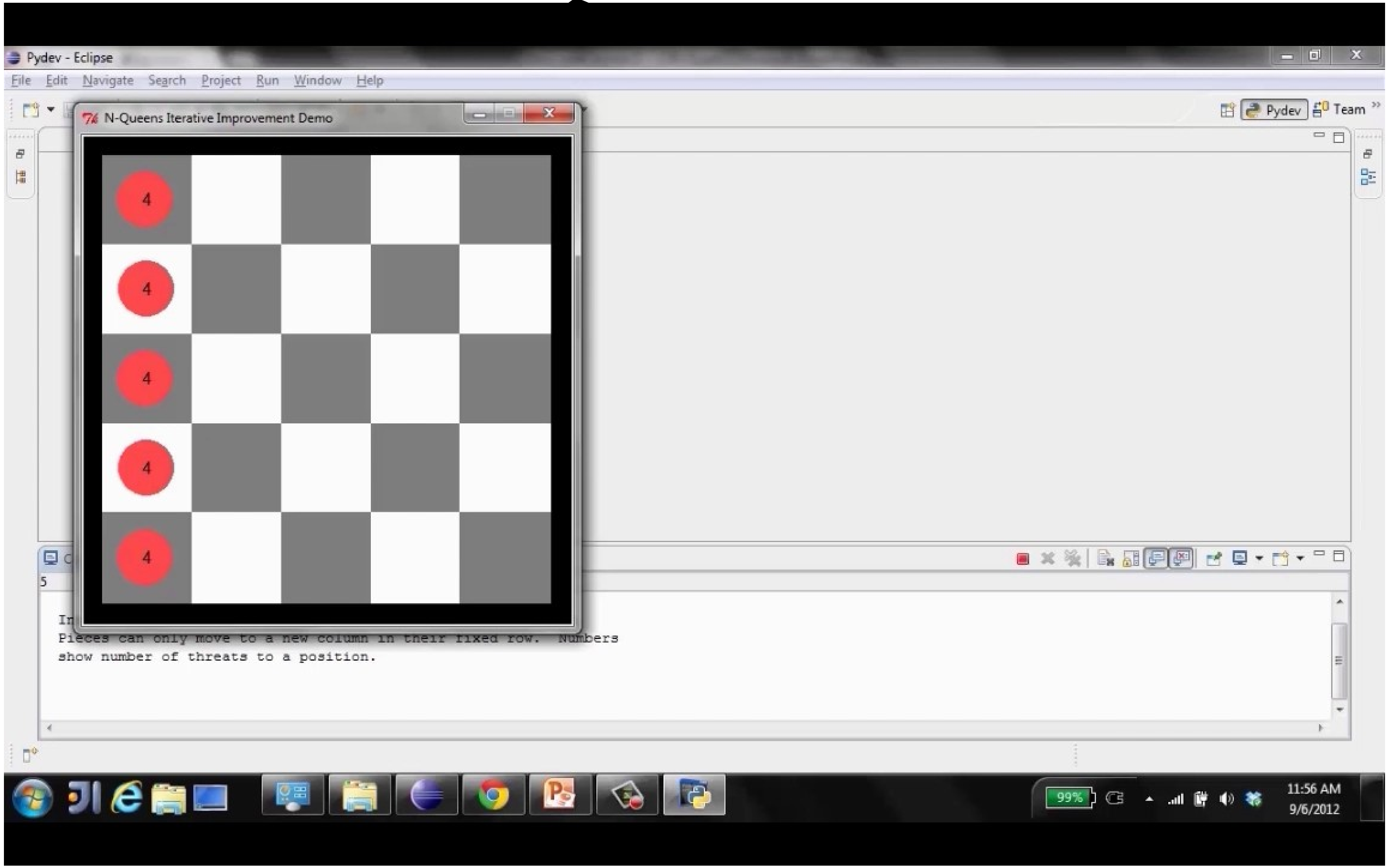
Example: 4-Queens



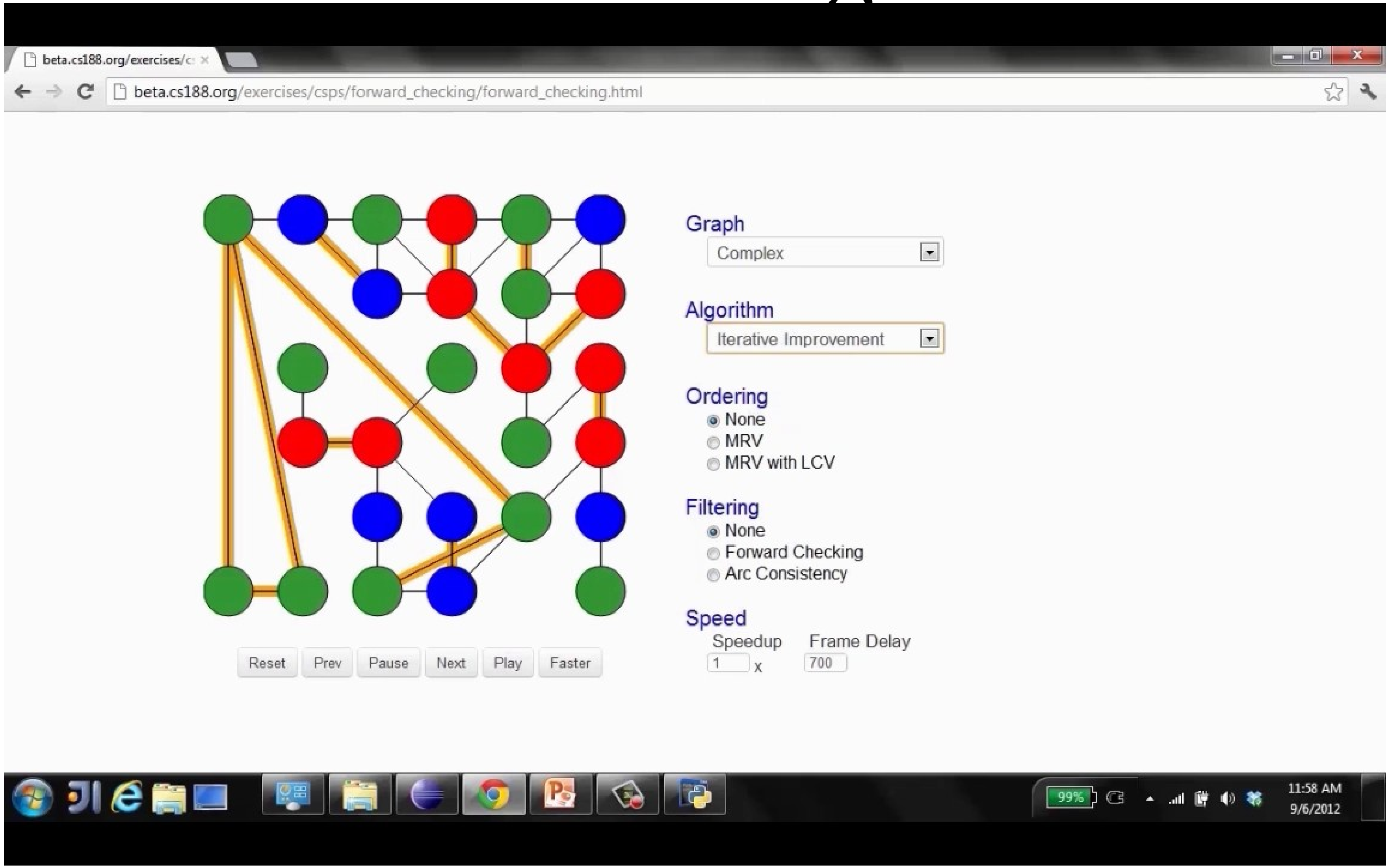
- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: c(n) = number of attacks

[Demo: n-queens – iterative improvement (L5D1)]
[Demo: coloring – iterative improvement]

Video of Demo Iterative Improvement – n Queens



Video of Demo Iterative Improvement – Coloring

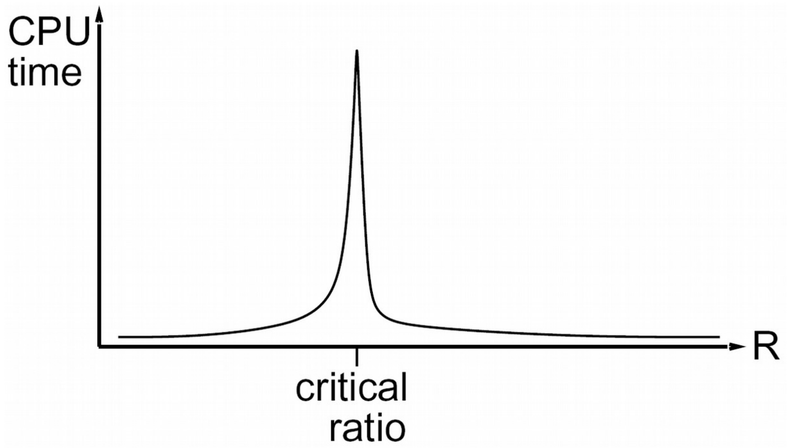


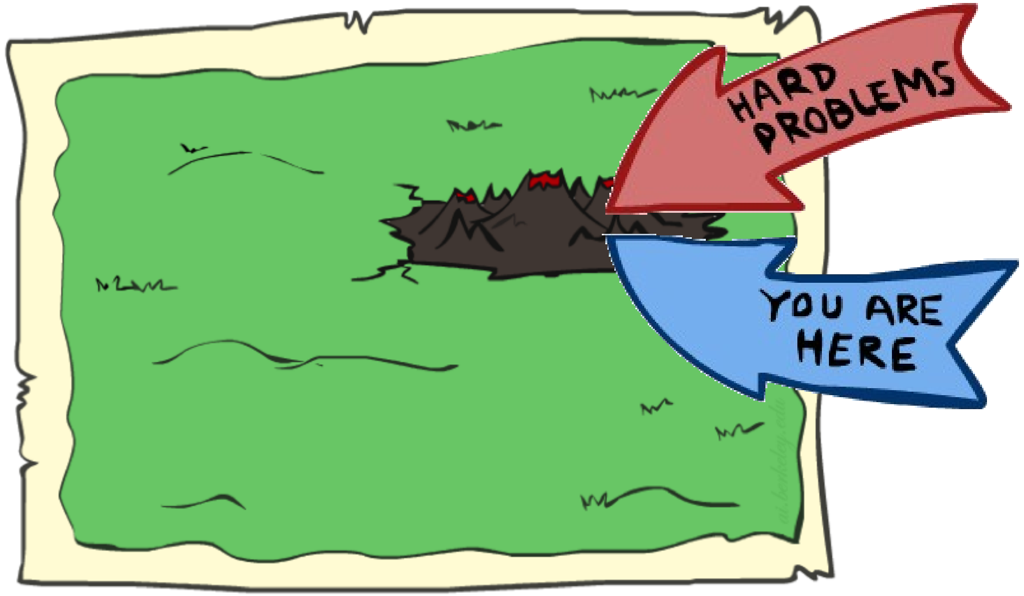
Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)!

- The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

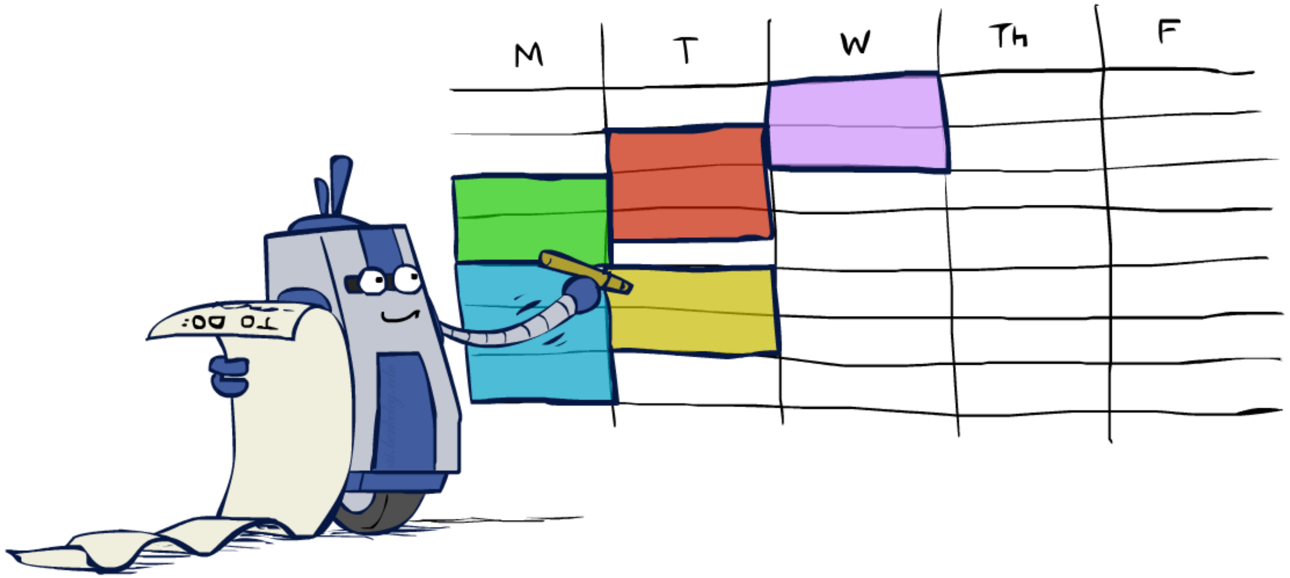
$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$





Summary: CSPs

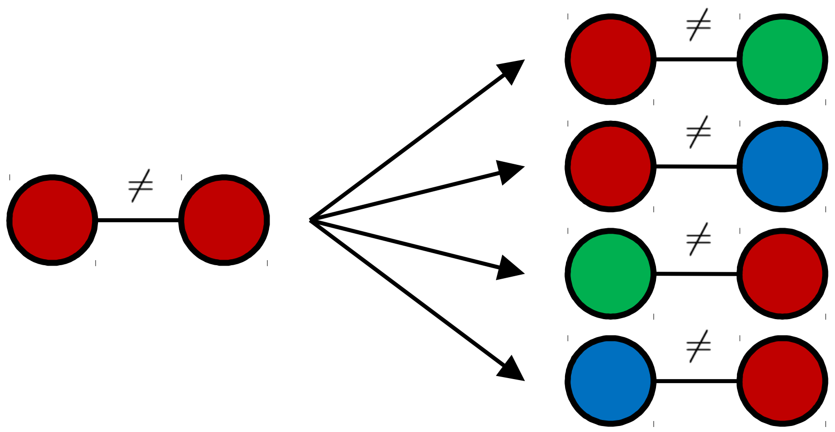
- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constrain
- Basic solution: backtracking sear
- Speed-ups:
  - Ordering
  - Filtering
  - Structure – turns out trees are easy!
- Iterative min-conflicts is often effective in practice



Local Search

Local Search

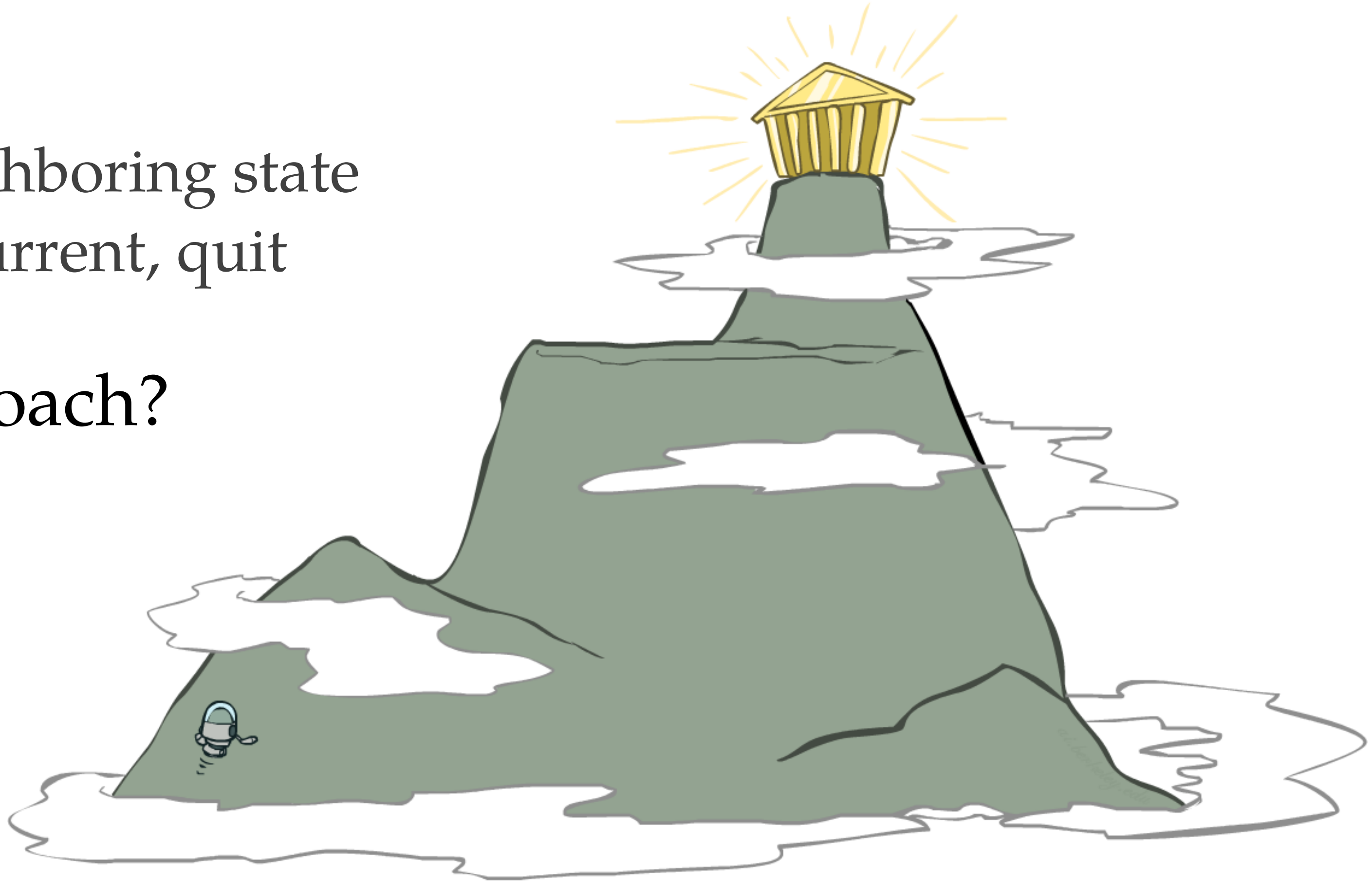
- Tree search keeps unexplored alternatives on the fringe (ensures completeness)
- Local search: improve a single option until you can't make it better (no fringe!)
- New successor function: local changes



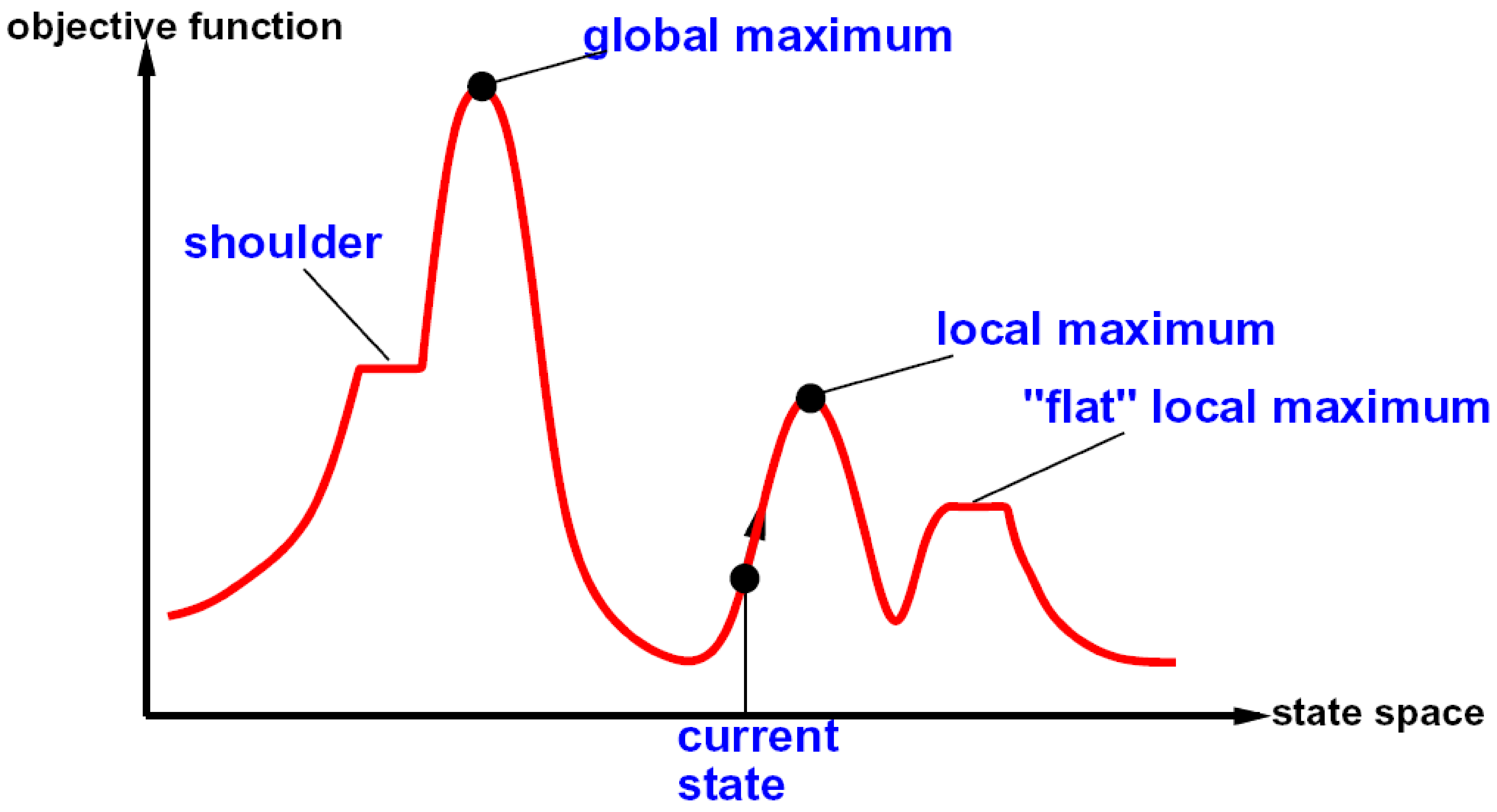
- Generally much faster and more memory efficient (but incomplete and suboptimal)

Hill Climbing

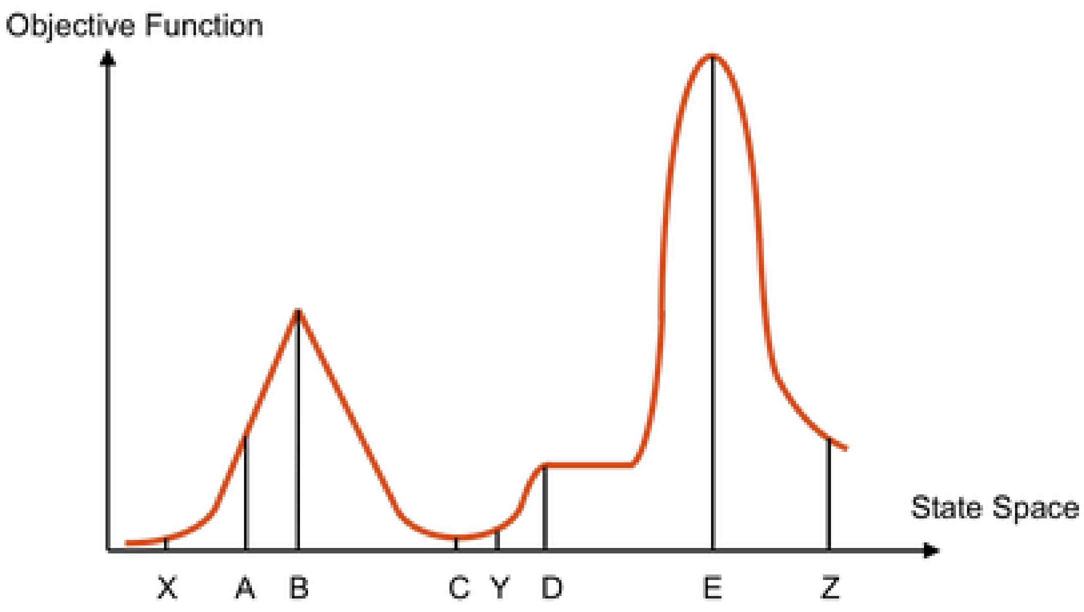
- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit
- What's bad about this approach?
- What's good about it?



Hill Climbing Diagram



Hill Climbing Quiz



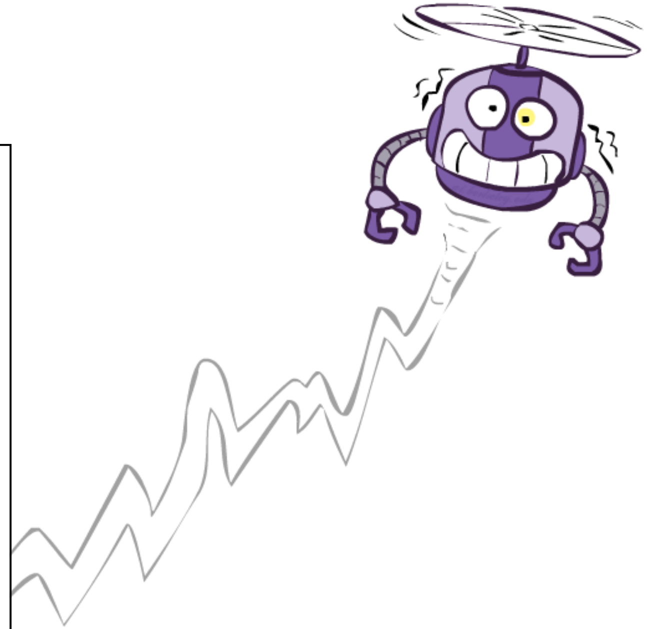
Starting from X, where do you end up ?

Starting from Y, where do you end up ?

Starting from Z, where do you end up ?

Simulated Annealing

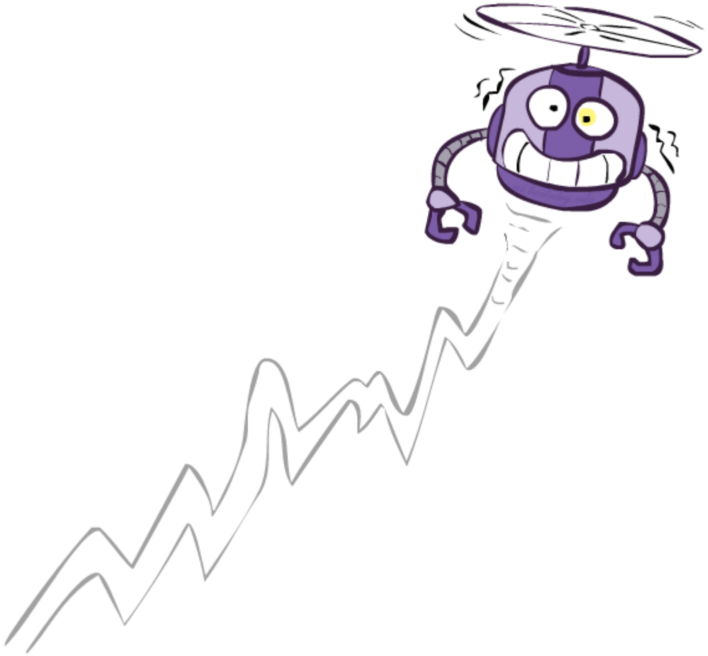
- Idea: Escape local maxima by allowing downhill move
  - But make them rarer as time goes on



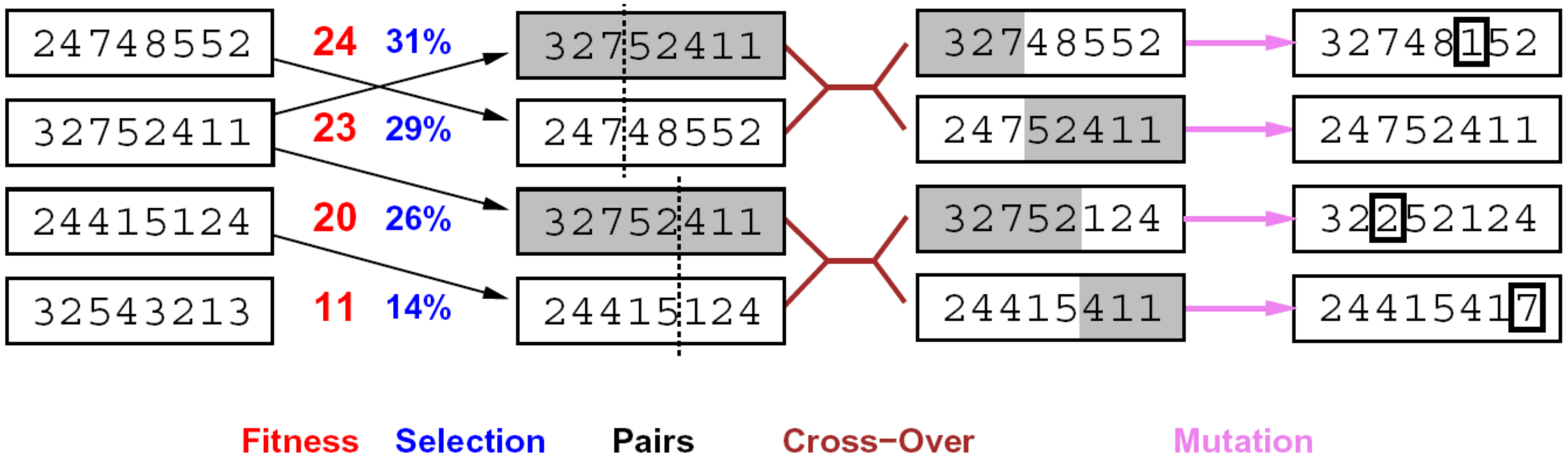
```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(Δ E/T)
```

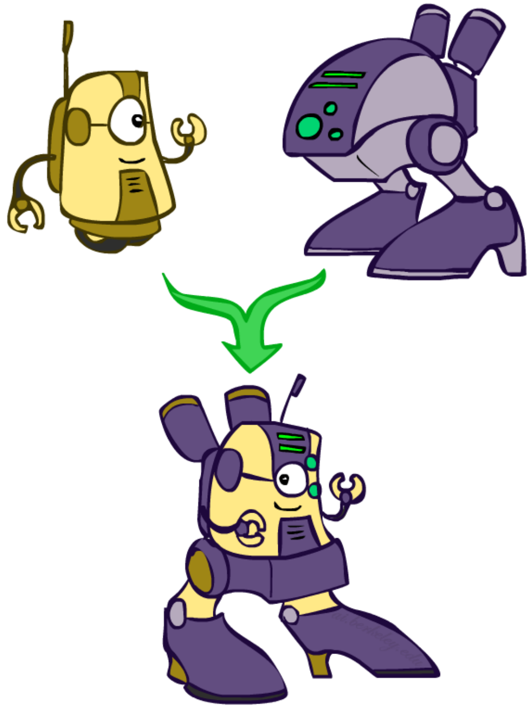
Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough, will converge to optimal state!
- Is this an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways



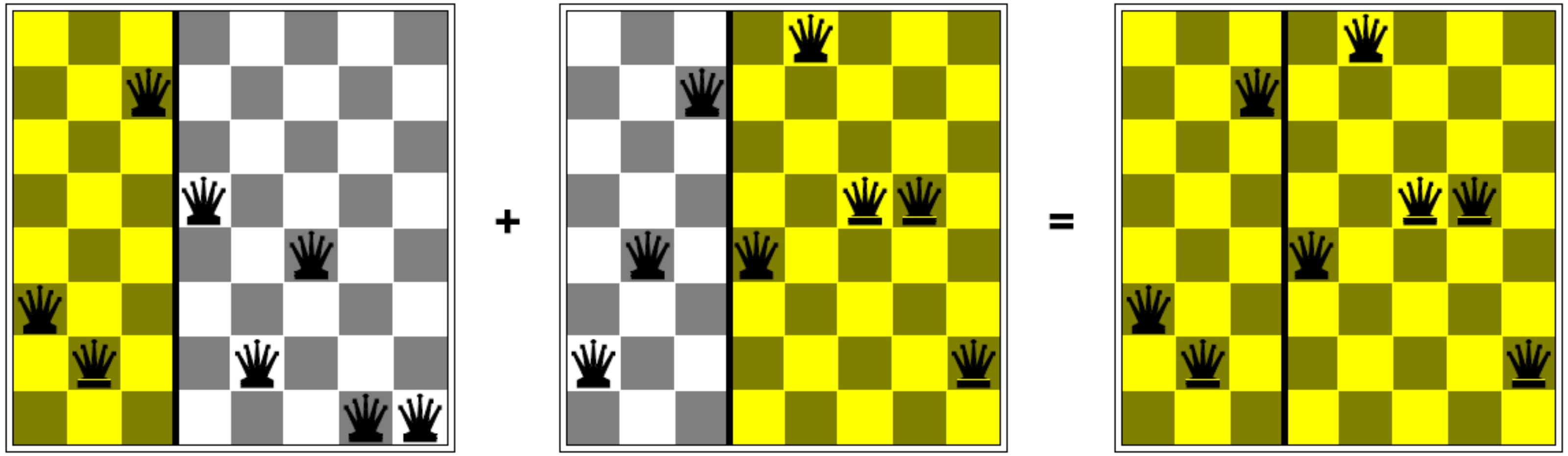
Genetic Algorithms





- Genetic algorithms use a natural selection metaphor
  - Keep best N hypotheses at each step (selection) based on a fitness function
  - Also have pairwise crossover operators, with optional mutation to give variety
- Possibly the most misunderstood, misapplied (and even maligned) technique around

Example: N-Queens



- Why does crossover make sense here?
- When wouldn’t it make sense?
- What would mutation be?
- What would a good fitness function be?

Next Time: Adversarial Search!