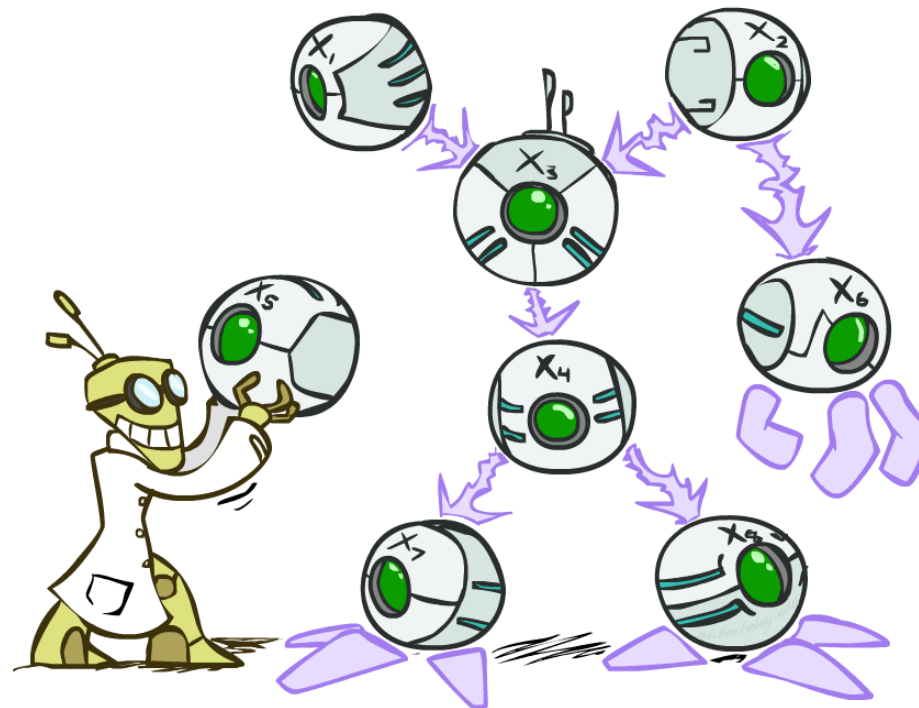


CS 188: Artificial Intelligence

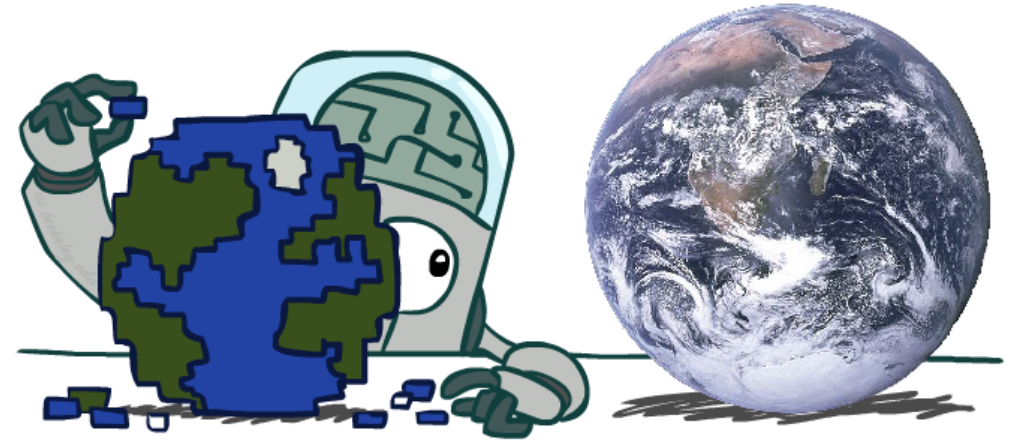
Bayes' Nets



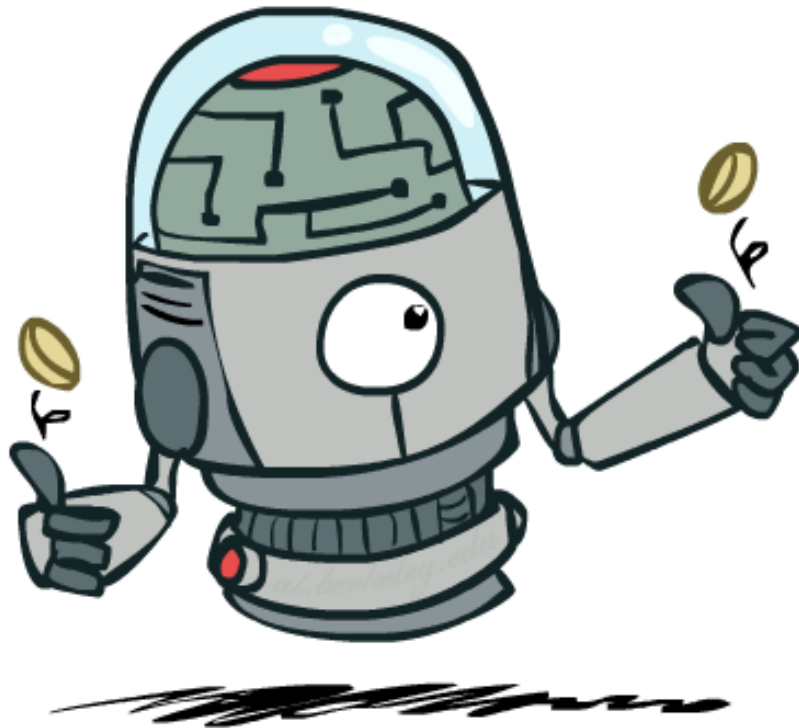
Lecturer: Davis Foote --- University of California, Berkeley

Probabilistic Models

- Models describe how (a portion of) the world works
- **Models are always simplifications**
 - May not account for every variable
 - May not account for all interactions between variables
 - “All models are wrong; but some are useful.”
 - George E. P. Box
- What do we do with probabilistic models?
 - We (or our agents) need to reason about unknown variables, given evidence
 - Example: explanation (diagnostic reasoning)
 - Example: prediction (causal reasoning)
 - Example: value of information



Independence



Independence

- Two variables are *independent* if:

$$\forall x, y : P(x, y) = P(x)P(y)$$

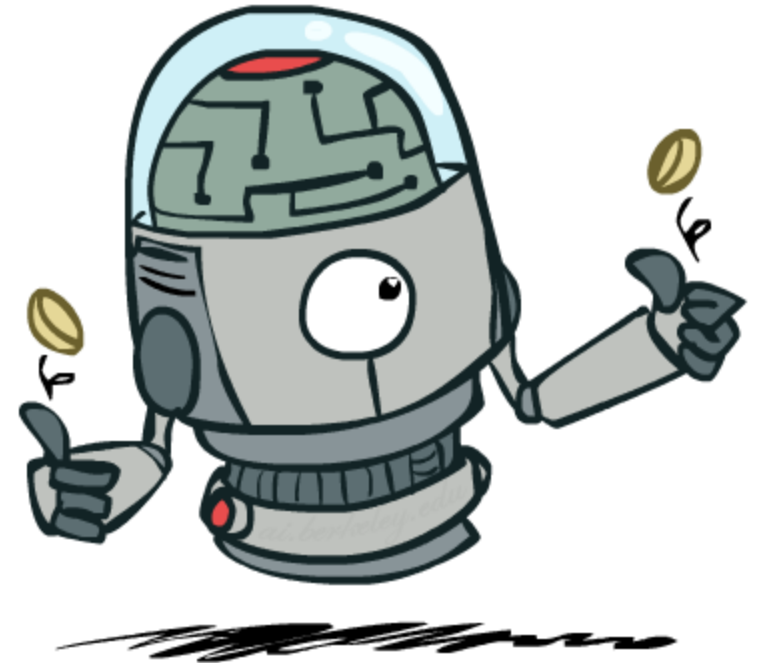
- This says that their joint distribution *factors* into a product two simpler distributions
- Another form:

$$\forall x, y : P(x|y) = P(x)$$

- We write: $X \perp\!\!\!\perp Y$

- Independence is a *simplifying modeling assumption*

- Empirical* joint distributions: at best “close” to independent
- What could we assume for {Weather, Traffic, Cavity, Toothache}?



Example: Independence?

$P_1(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$P(T)$

T	P
hot	0.5
cold	0.5

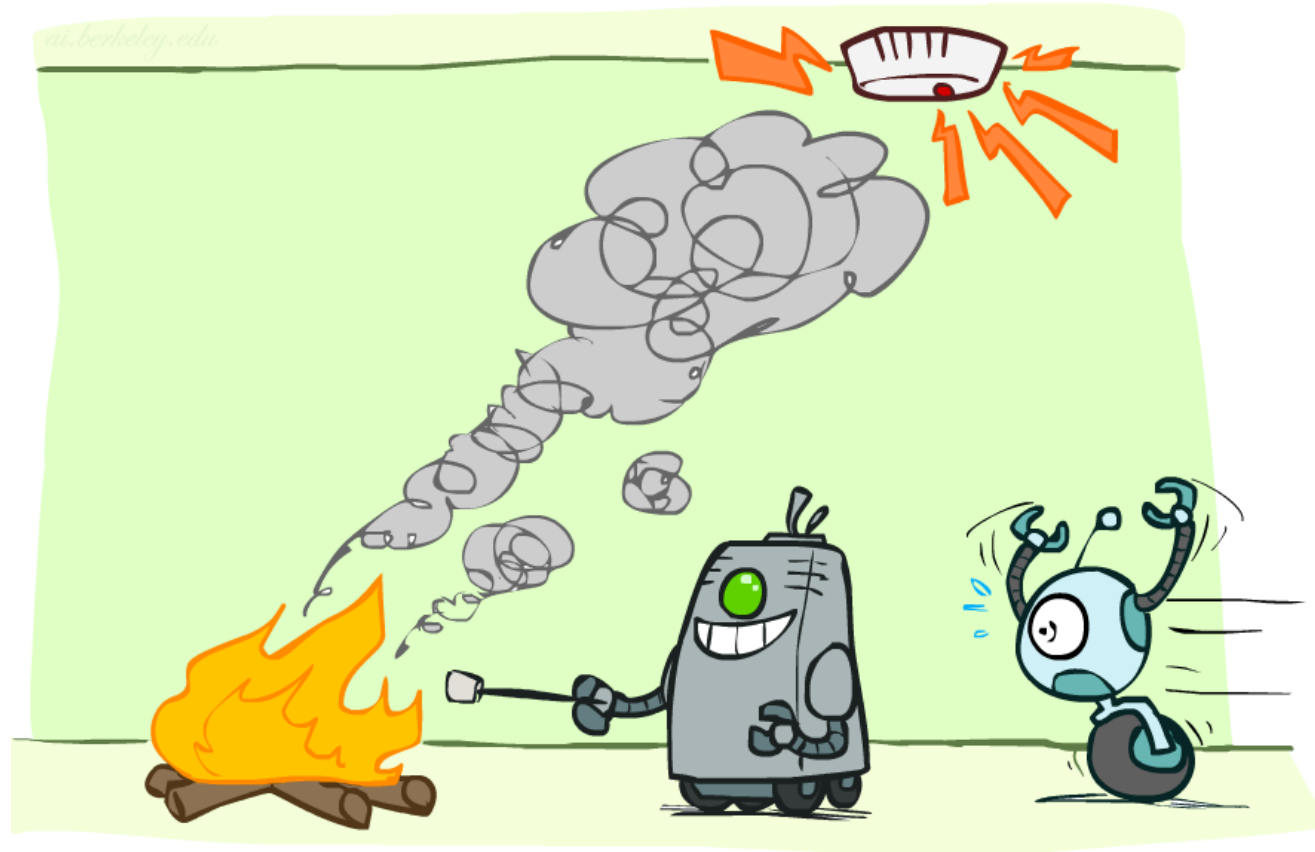
$P_2(T, W)$

T	W	P
hot	sun	0.3
hot	rain	0.2
cold	sun	0.3
cold	rain	0.2

$P(W)$

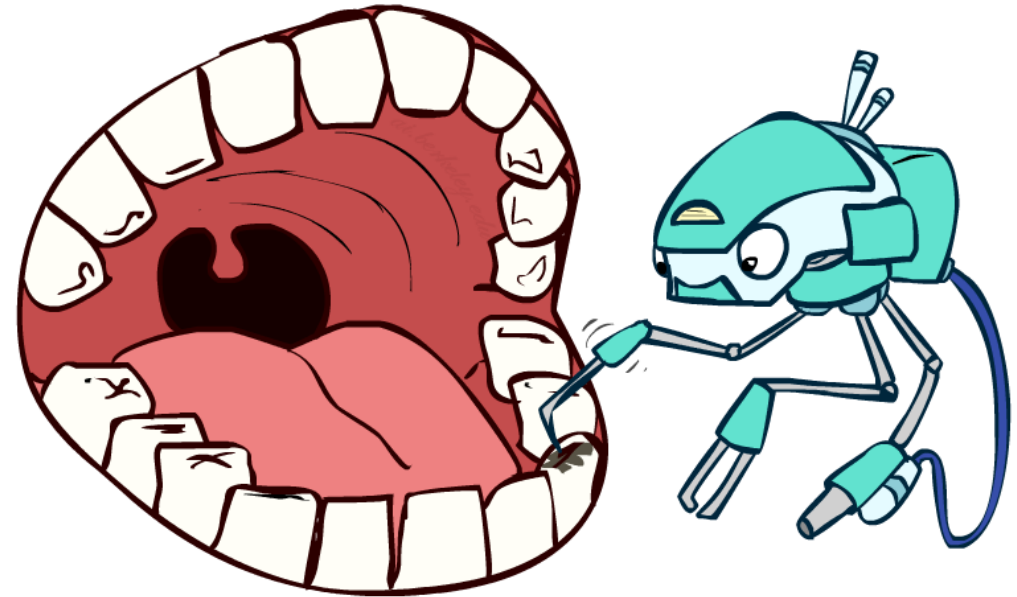
W	P
sun	0.6
rain	0.4

Conditional Independence



Conditional Independence

- $P(\text{Toothache}, \text{Cavity}, \text{Catch})$
- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
 - $P(+\text{catch} \mid +\text{toothache}, +\text{cavity}) = P(+\text{catch} \mid +\text{cavity})$
- The same independence holds if I don't have a cavity:
 - $P(+\text{catch} \mid +\text{toothache}, -\text{cavity}) = P(+\text{catch} \mid -\text{cavity})$
- Catch is *conditionally independent* of Toothache given Cavity:
 - $P(\text{Catch} \mid \text{Toothache}, \text{Cavity}) = P(\text{Catch} \mid \text{Cavity})$



Conditional Independence

- Unconditional (absolute) independence very rare (why?)
- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments.

- X is conditionally independent of Y given Z

$$X \perp\!\!\!\perp Y | Z$$

if and only if:

$$\forall x, y, z : P(x, y | z) = P(x | z)P(y | z)$$

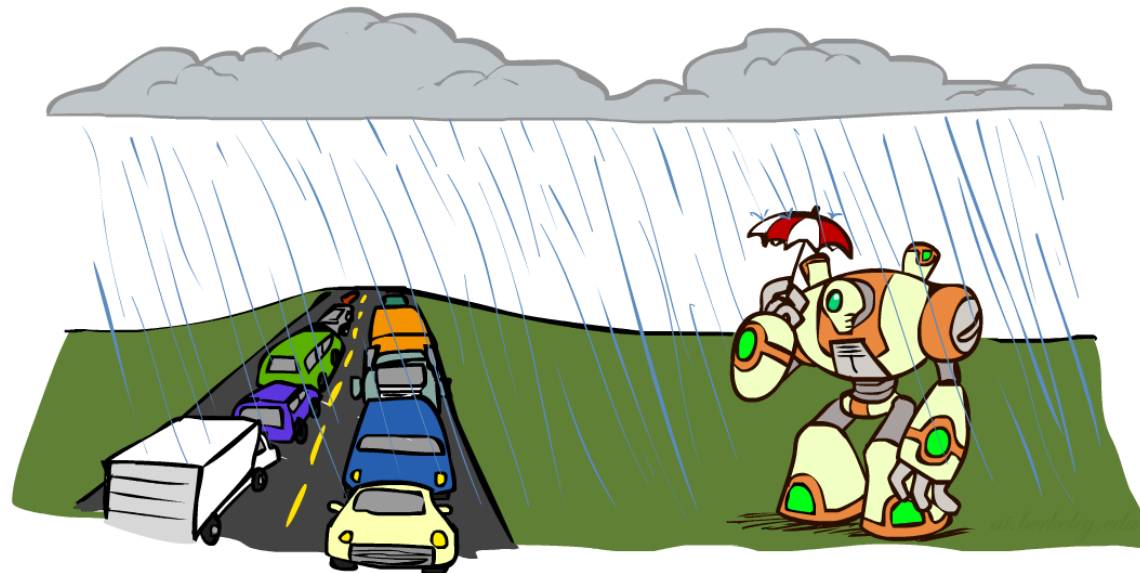
or, equivalently, if and only if

$$\forall x, y, z : P(x | z, y) = P(x | z)$$

Conditional Independence

- What about this domain:

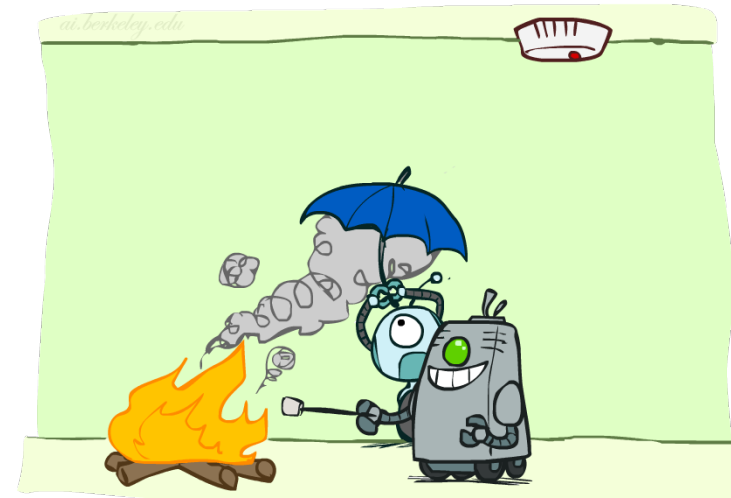
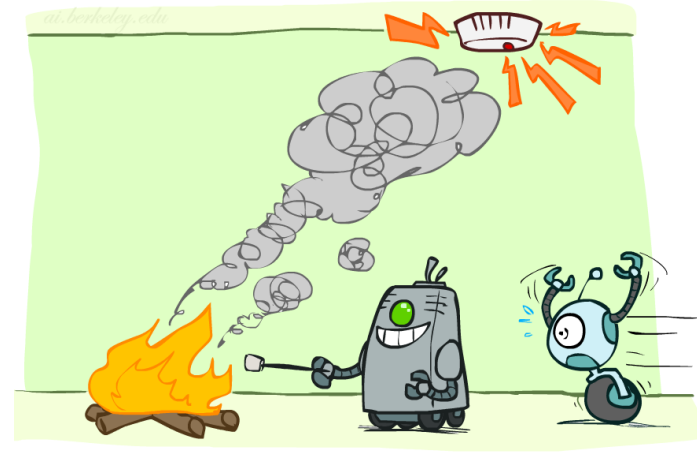
- Traffic
- Umbrella
- Raining



Conditional Independence

- What about this domain:

- Fire
- Smoke
- Alarm



Conditional Independence and the Chain Rule

- Chain rule: $P(X_1, X_2, \dots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2) \dots$

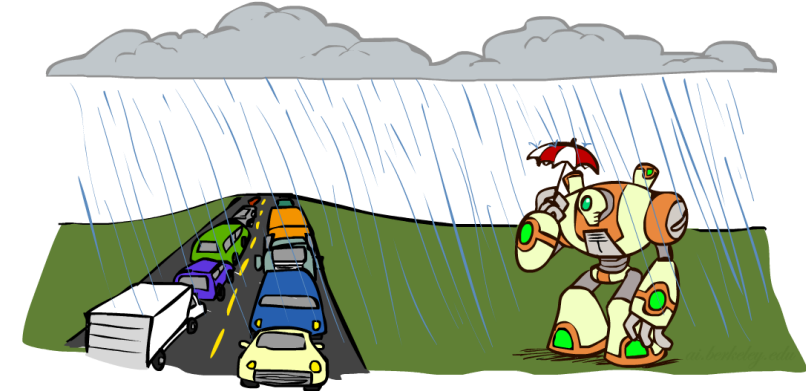
- Trivial decomposition:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain}, \text{Traffic})$$

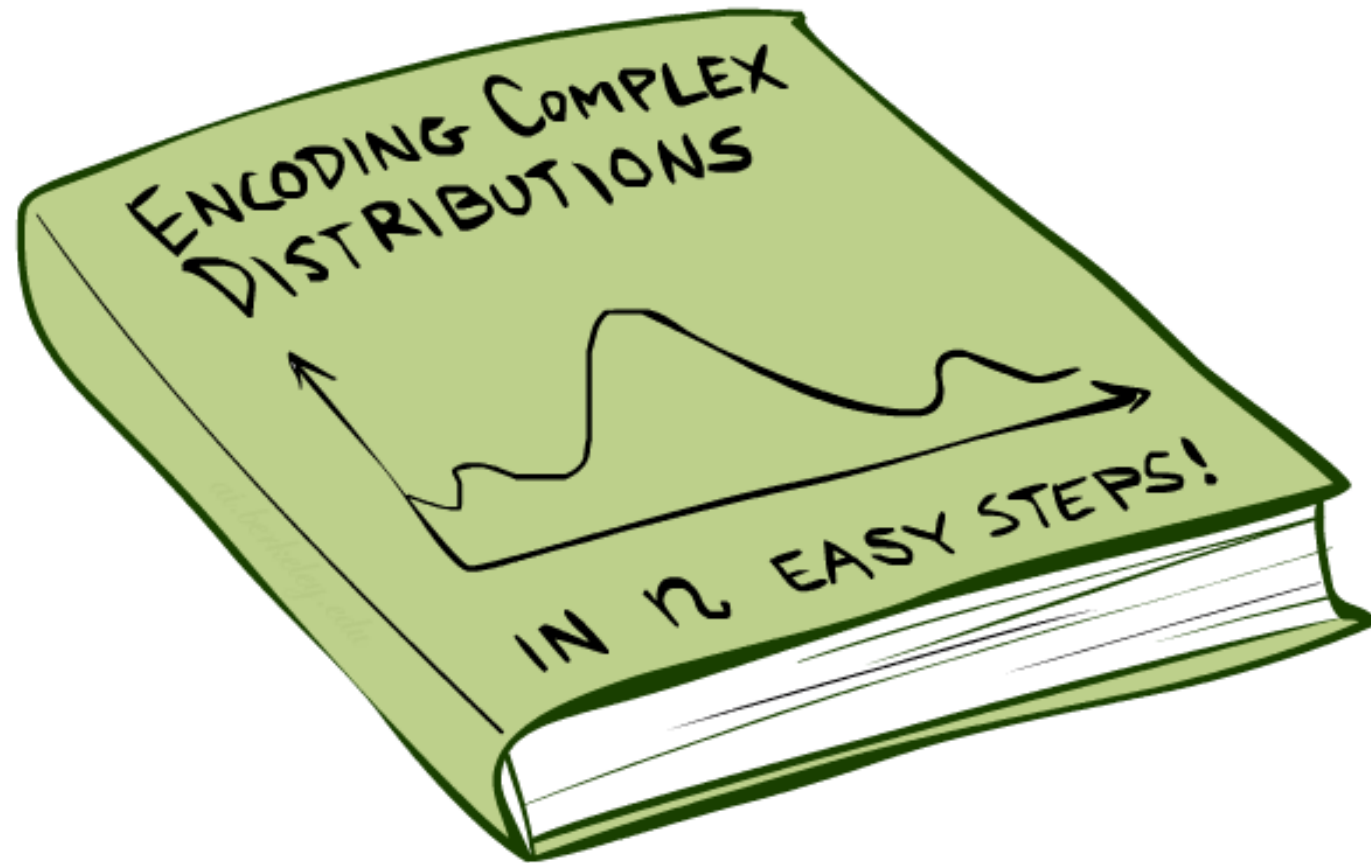
- With assumption of conditional independence:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain})$$

- Bayes' nets / graphical models help us express conditional independence assumptions

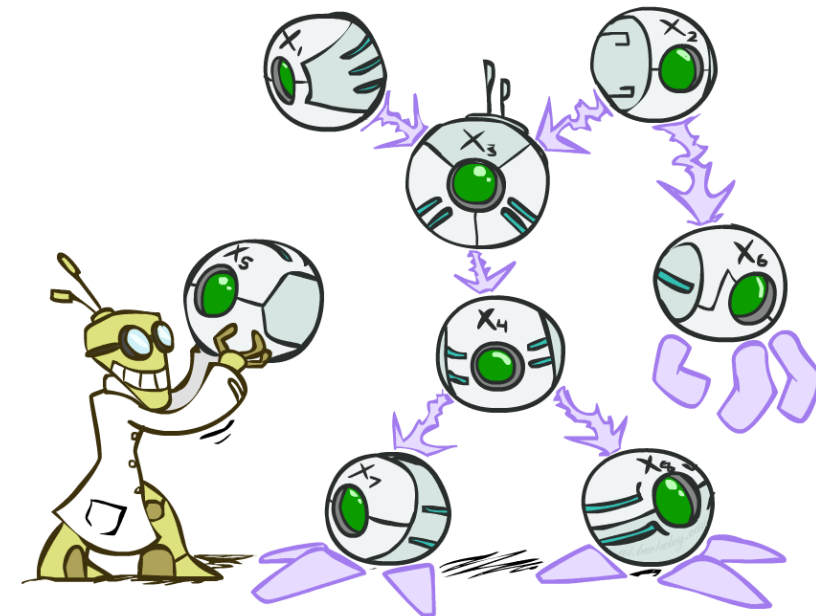


Bayes' Nets: Big Picture

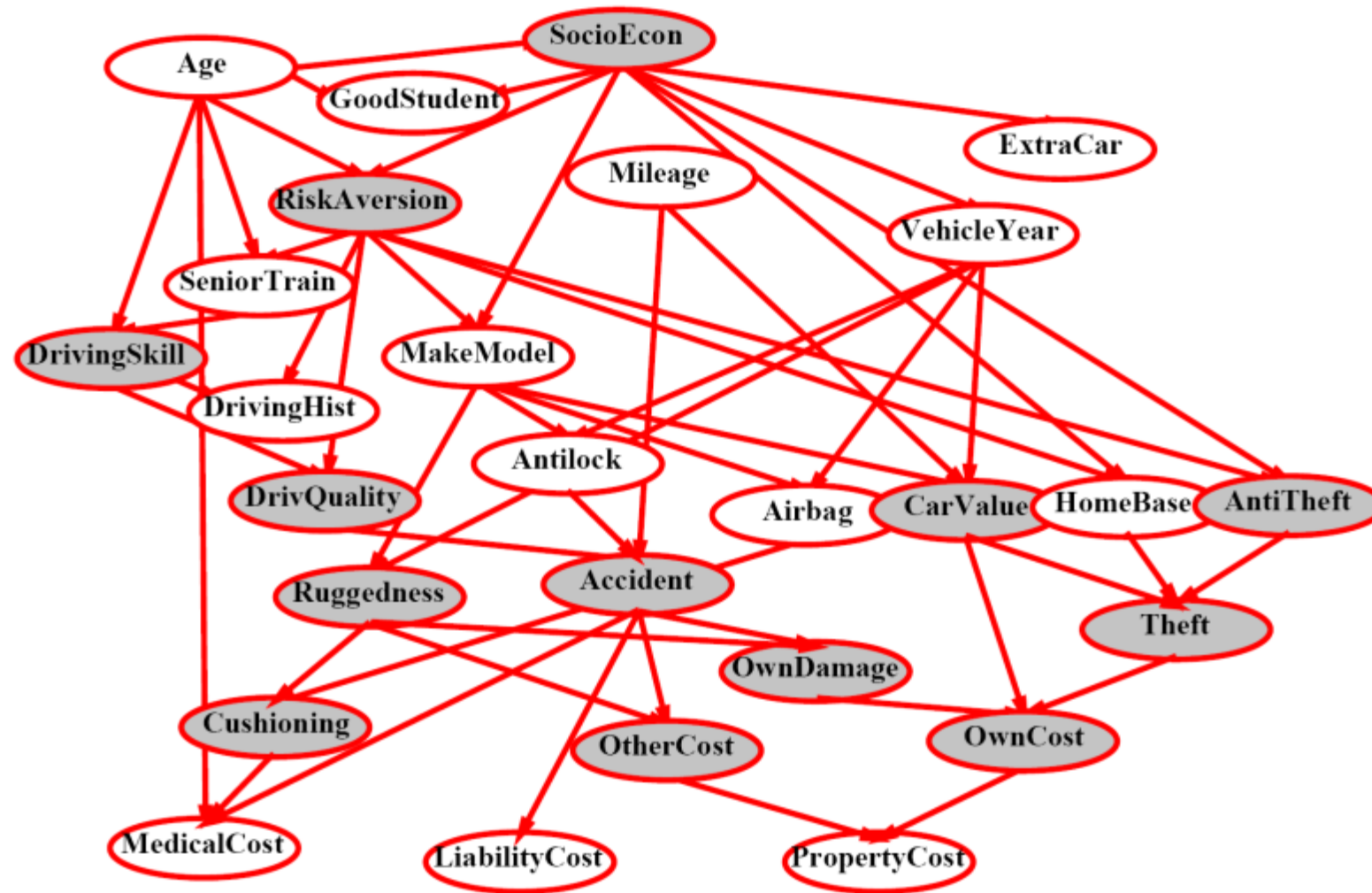


Bayes' Nets: Big Picture

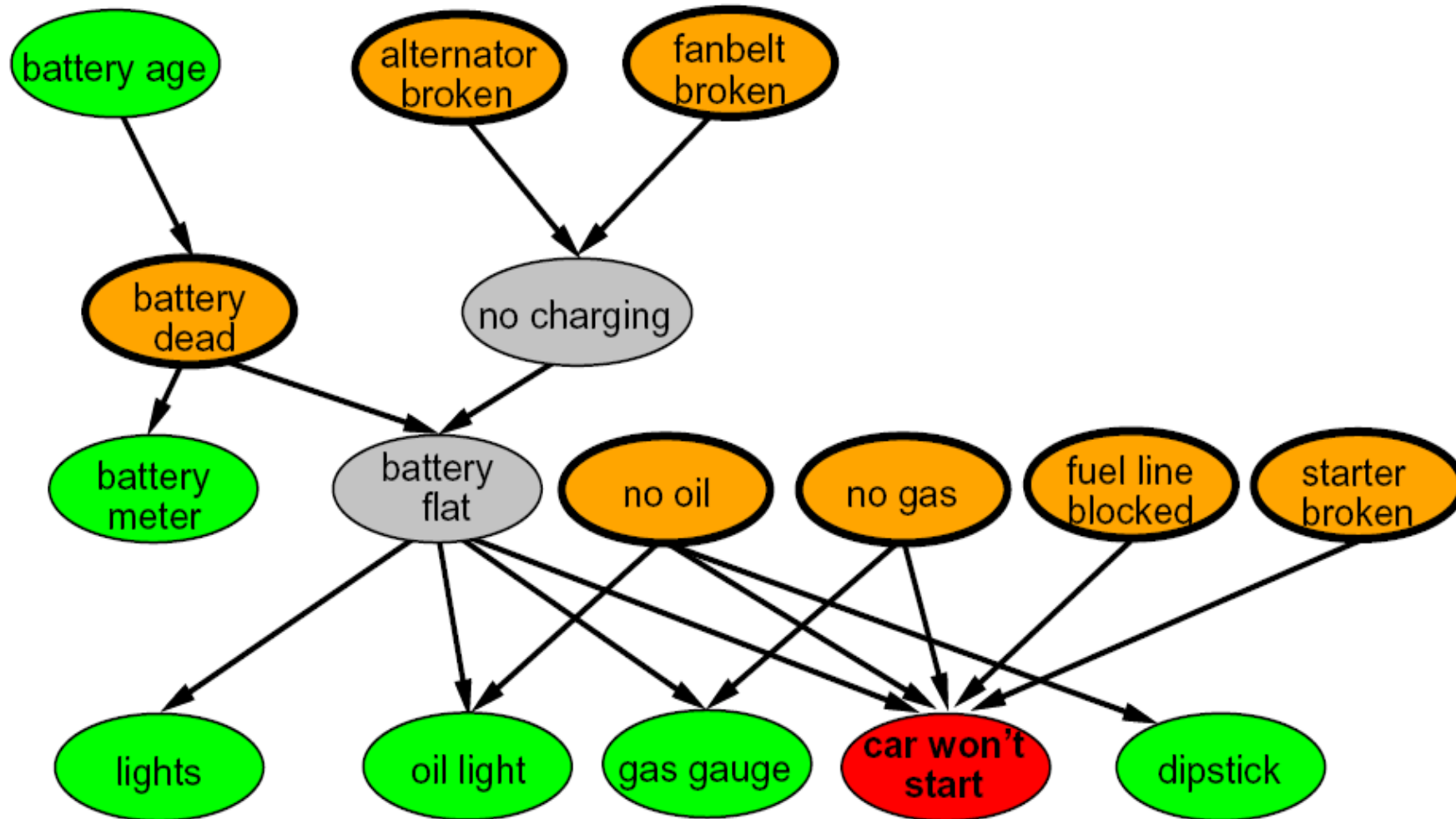
- Two problems with using full joint distribution tables as our probabilistic models:
 - Unless there are only a few variables, the joint is WAY too big to represent explicitly
 - Hard to learn (estimate) anything empirically about more than a few variables at a time
- **Bayes' nets:** a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
 - More properly called **graphical models**
 - We describe how variables locally interact
 - Local interactions chain together to give global, indirect interactions
 - For about 10 min, we'll be vague about how these interactions are specified



Example Bayes' Net: Insurance

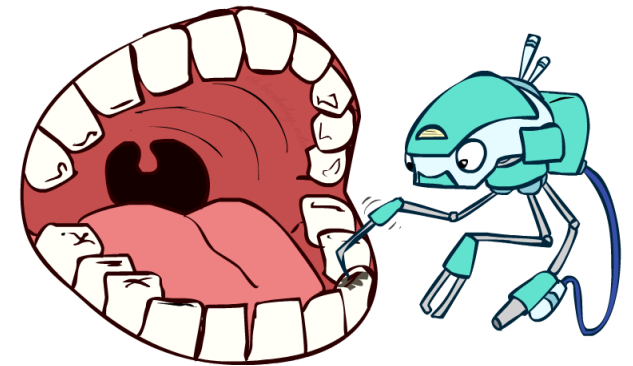
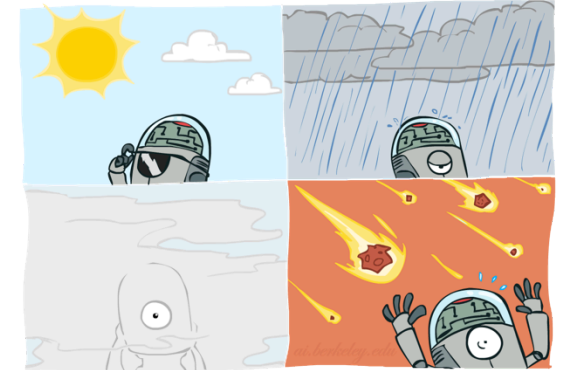
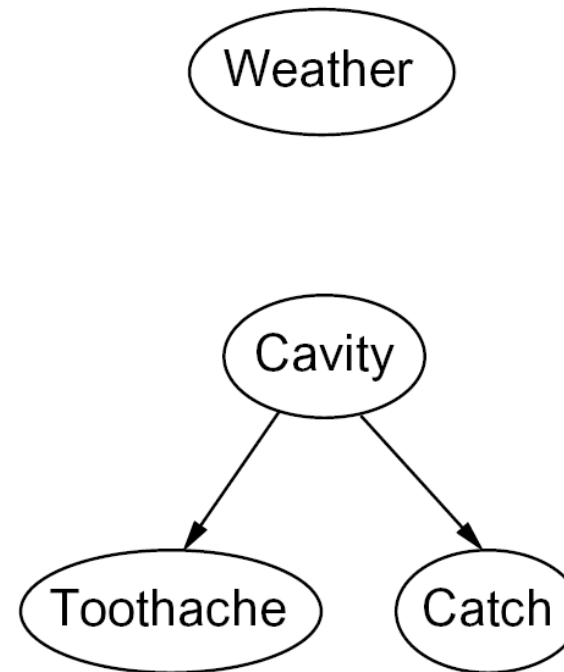


Example Bayes' Net: Car



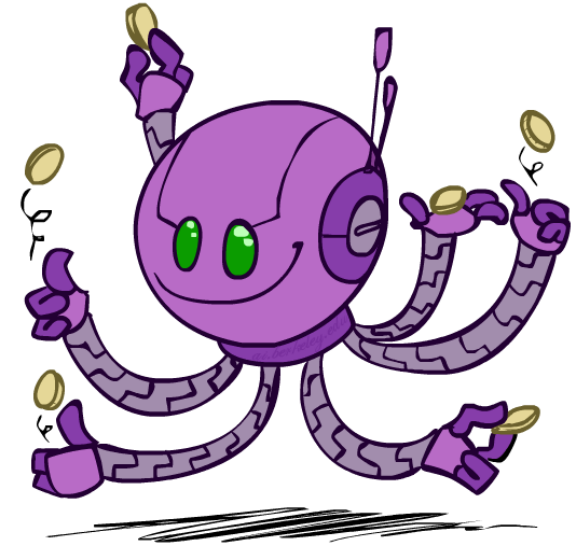
Graphical Model Notation

- **Nodes: variables (with domains)**
 - Can be assigned (observed) or unassigned (unobserved)
- **Arcs: interactions**
 - Similar to CSP constraints
 - Indicate “direct influence” between variables
 - Formally: encode conditional independence (more later)
- For now: imagine that arrows mean direct causation (in general, they don't!)



Example: Coin Flips

- N independent coin flips



- No interactions between variables: **absolute independence**

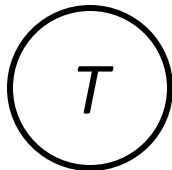
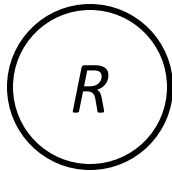
Example: Traffic

- Variables:

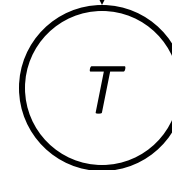
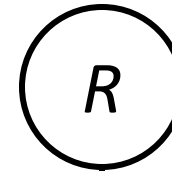
- R: It rains
- T: There is traffic



- Model 1: independence



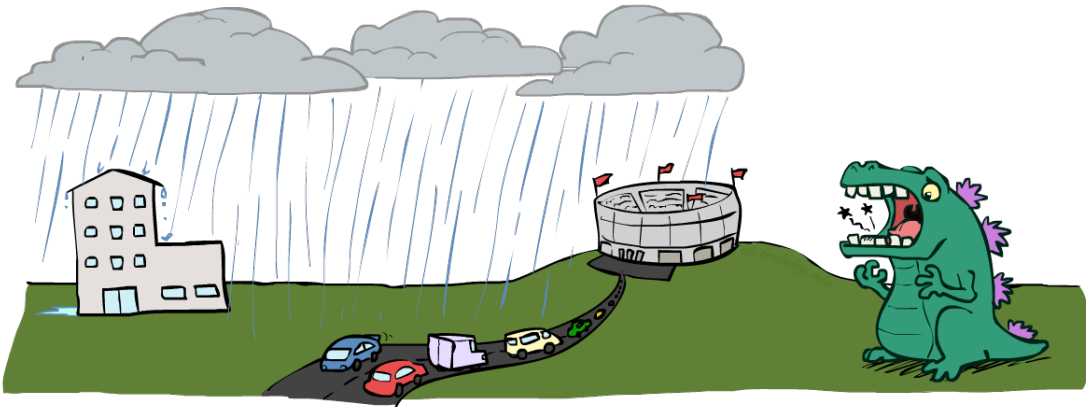
- Model 2: rain causes traffic



- Why is an agent using model 2 better?

Example: Traffic II

- Let's build a causal graphical model!
- Variables
 - T: Traffic
 - R: It rains
 - L: Low pressure
 - D: Roof drips
 - B: Ballgame
 - C: Cavity



Example: Alarm Network

- Variables

- B: Burglary
- A: Alarm goes off
- M: Mary calls
- J: John calls
- E: Earthquake!

