# CS 188: Artificial Intelligence

## Particle Filters and Applications of HMMs

Instructors: Jacob Andreas and Davis Foote

University of California, Berkeley

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.  All CS188 materials are available at http://ai.berkeley.edu.]

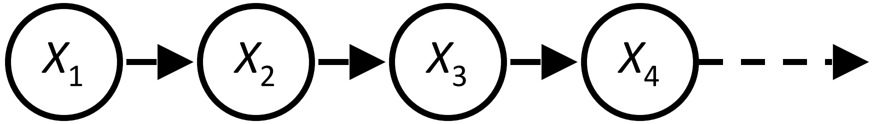
# Today

- HMMs
  - Particle filters
  - Demo bonanza!
  - Most-likely-explanation queries

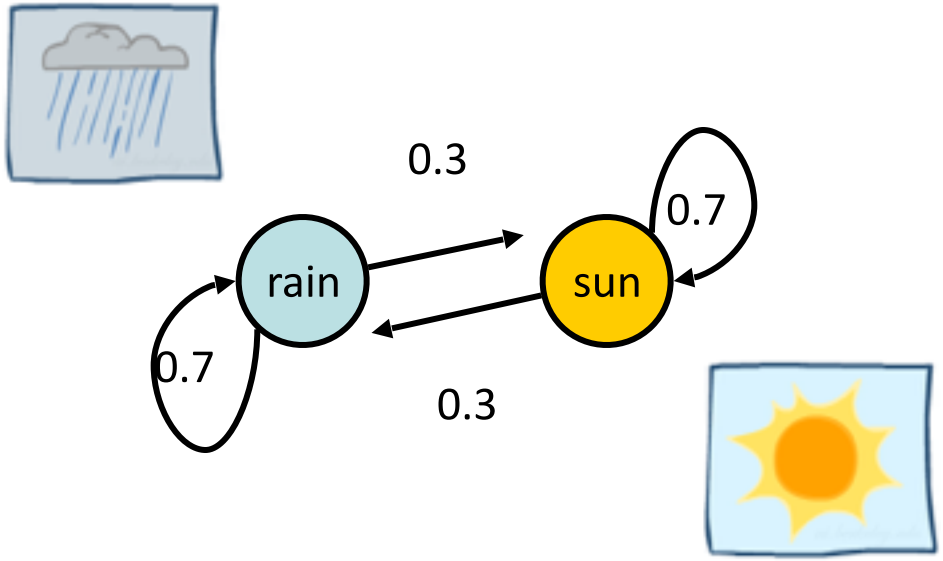
[Demo: Ghostbusters Markov Model (L15D1)]

# Recap: Reasoning Over Time

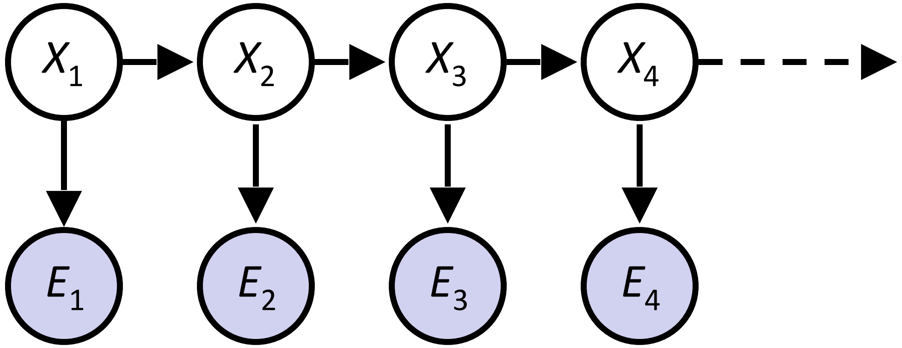
- Markov models




$P(X_1)$ $\quad$ $P(X|X_{-1})$



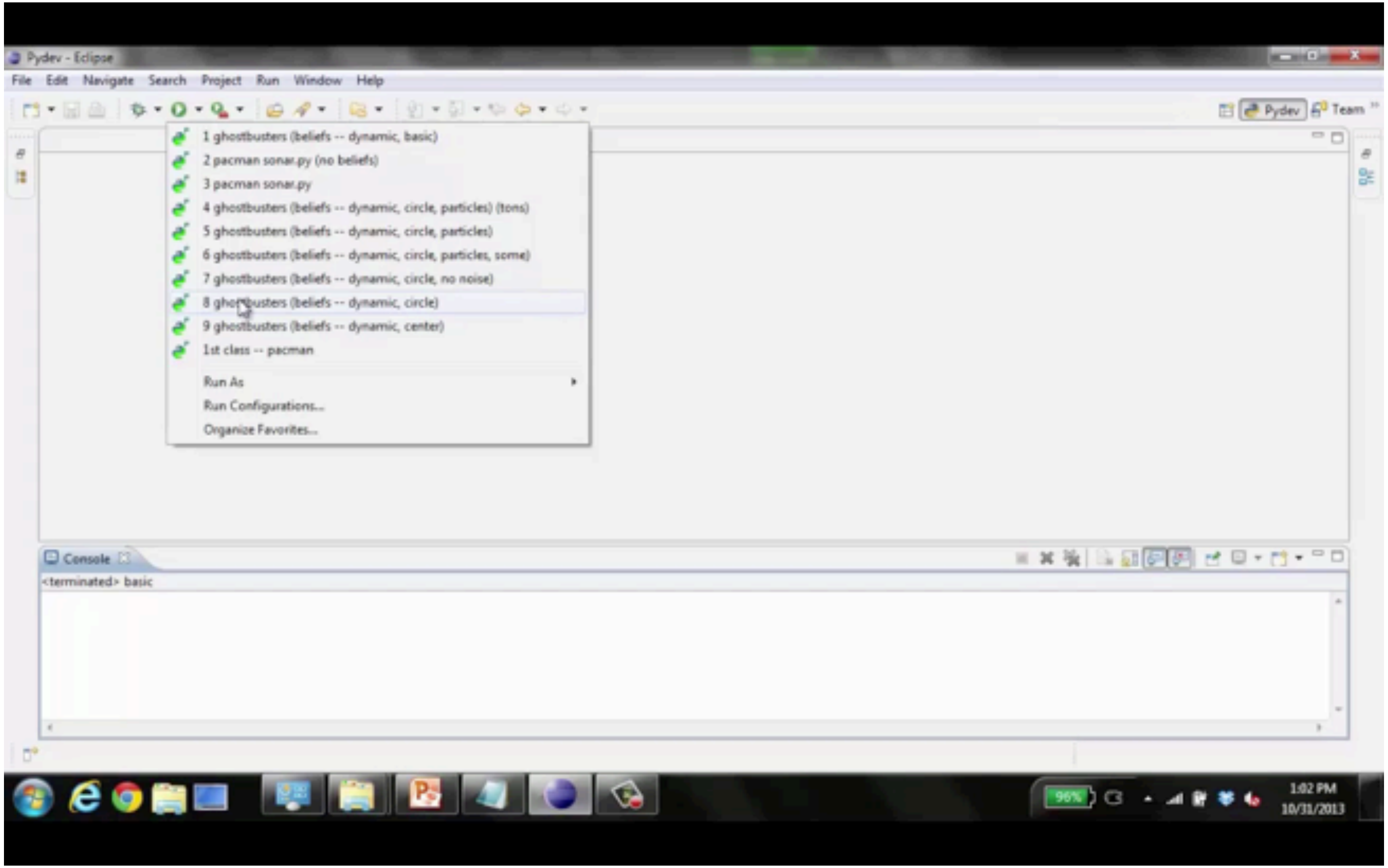

- Hidden Markov models




$P(E|X)$

| X | E | P |
| --- | --- | --- |
| rain | umbrella | 0.9 |
| rain | no umbrella | 0.1 |
| sun | umbrella | 0.2 |
| sun | no umbrella | 0.8 |

# Video of Demo Ghostbusters Markov Model (Reminder)



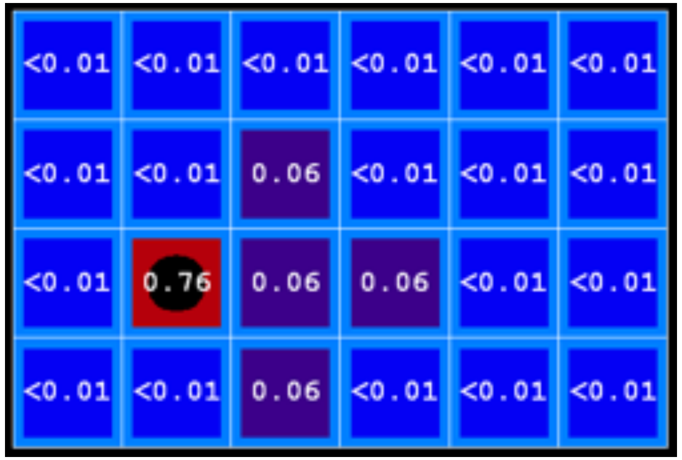
# Recap: Filtering

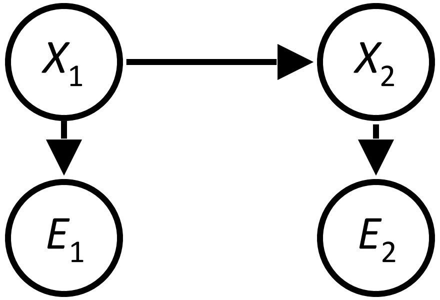
**Elapse time:** compute P( $X_t$ | $e_{1:t-1}$ )

$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

**Observe:** compute P( $X_t$ | $e_{1:t}$ )

$$P(x_t|e_{1:t}) \propto P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$



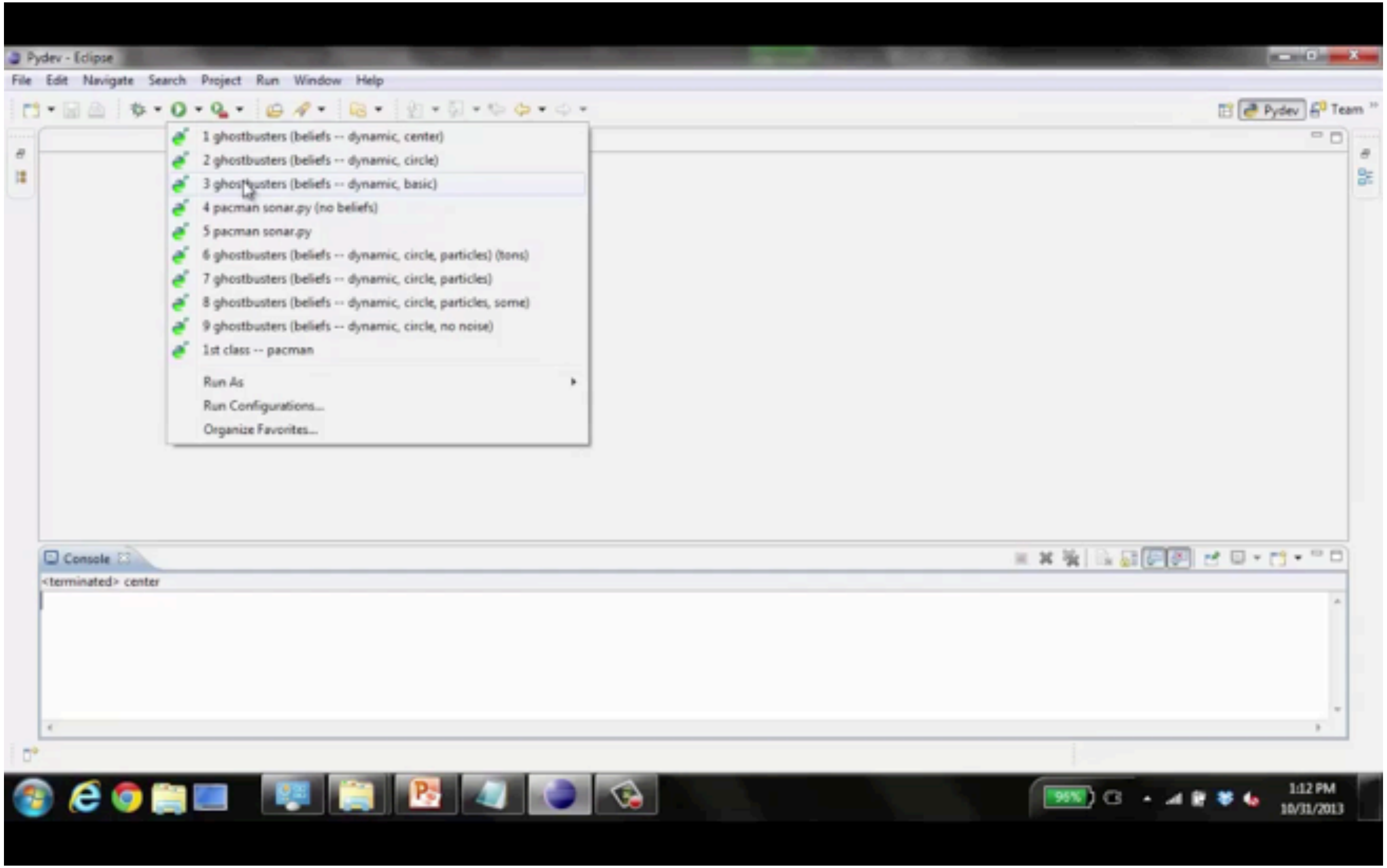




**Belief: <P(rain), P(sun)>**

| | | |
|---|---|---|
| $P(X_1)$ | <0.5, 0.5> | *Prior on $X_1$* |
| $P(X_1 \mid E_1 = umbrella)$ | <0.82, 0.18> | *Observe* |
| $P(X_2 \mid E_1 = umbrella)$ | <0.63, 0.37> | *Elapse time* |
| $P(X_2 \mid E_1 = umb, E_2 = umb)$ | <0.88, 0.12> | *Observe* |

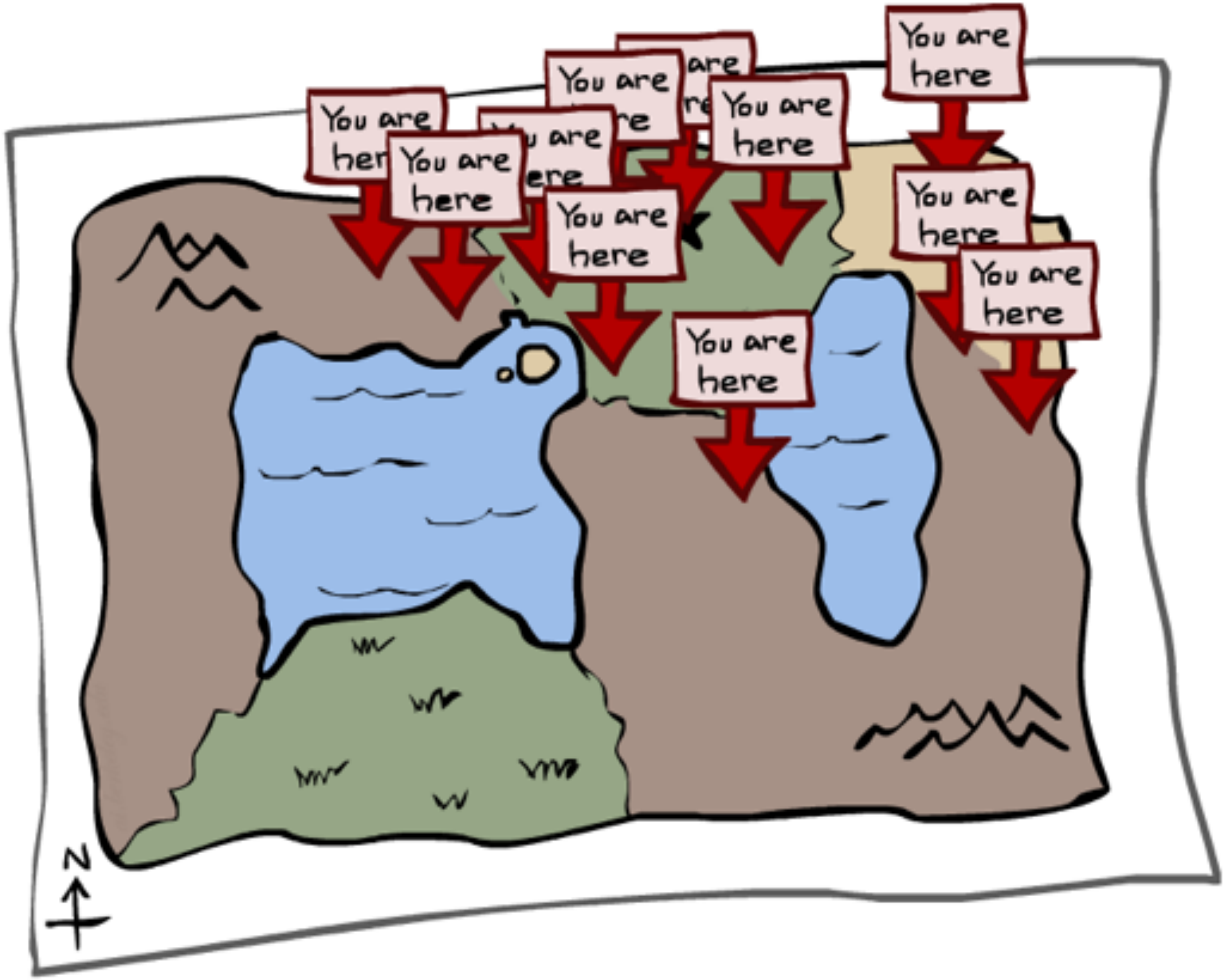
[Demo: Ghostbusters Exact Filtering (L15D2)]

# Video of Ghostbusters Exact Filtering (Reminder)



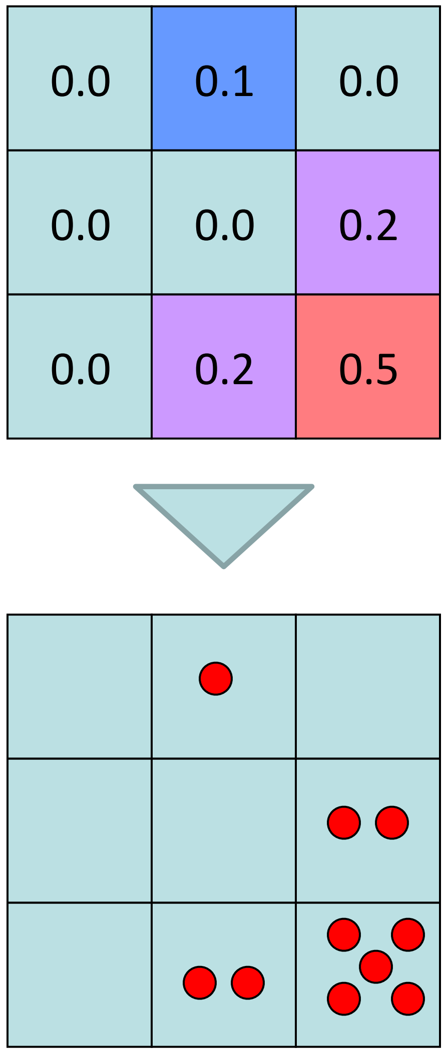
# Likelihood weighting in HMMs

# Particle Filtering



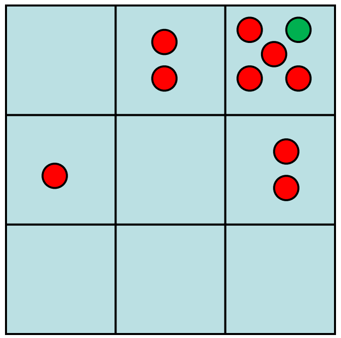
# Particle Filtering

- Filtering: approximate solution
- Sometimes |X| is too big to use exact inference
  - |X| may be too big to even store B(X)
  - E.g. X is continuous
- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called particles
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not states
- This is how robot localization works in practice
- Particle is just new name for sample



# Representation: Particles

- Our representation of P(X) is now a list of N particles (samples)
  - Generally, N << |X|
  - Storing map from X to counts would defeat the point
- P(x) approximated by number of particles with value x
  - So, many x may have P(x) = 0!
  - More particles, more accuracy
- For now, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

# Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

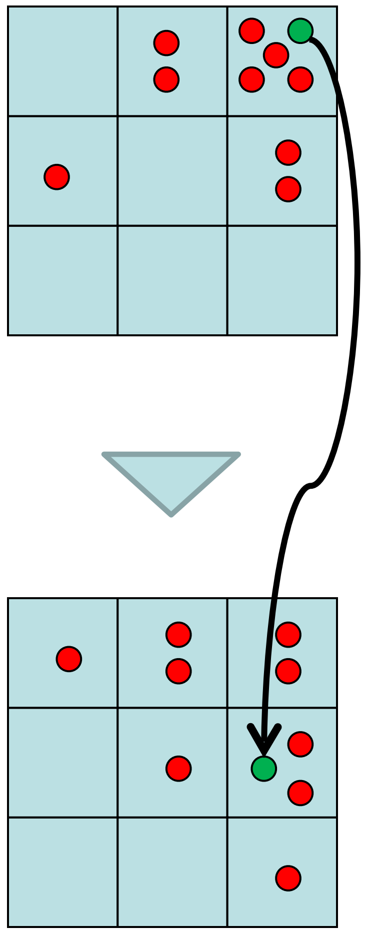
$$x' = \text{sample}(P(X'|x))$$

  - This is like prior sampling – samples' frequencies reflect the transition probabilities
  - Here, most samples move clockwise, but some move in another direction or stay in place

- This captures the passage of time
  - If enough samples, close to exact values before and after (consistent)

Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)



# Particle Filtering: Observe

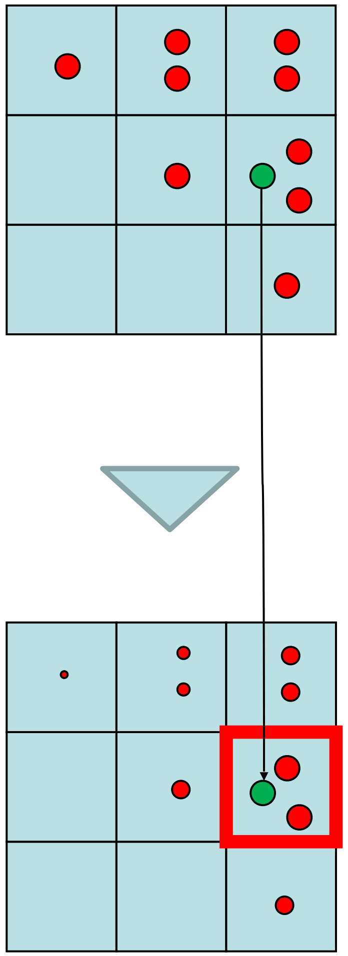
- Slightly trickier:
  - Don't sample observation, fix it
  - Similar to likelihood weighting, downweight samples based on the evidence

$$w(x) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

  - As before, the probabilities don't sum to one, since all have been downweighted (in fact they now sum to (N times) an approximation of P(e))

Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)

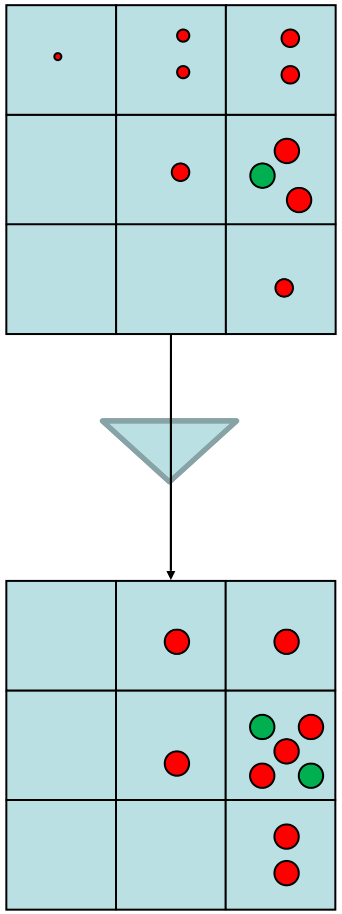


Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

# Particle Filtering: Resample

- Rather than tracking weighted samples, we resample
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is equivalent to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one

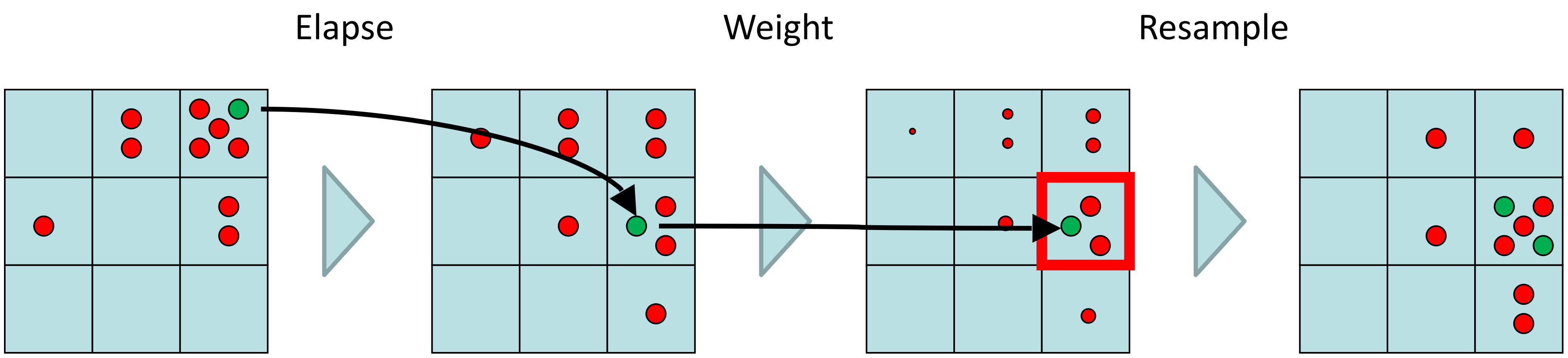
Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4



(New) Particles:
(3,2)
(2,2)
(3,2)
(2,3)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(3,2)

# Recap: Particle Filtering

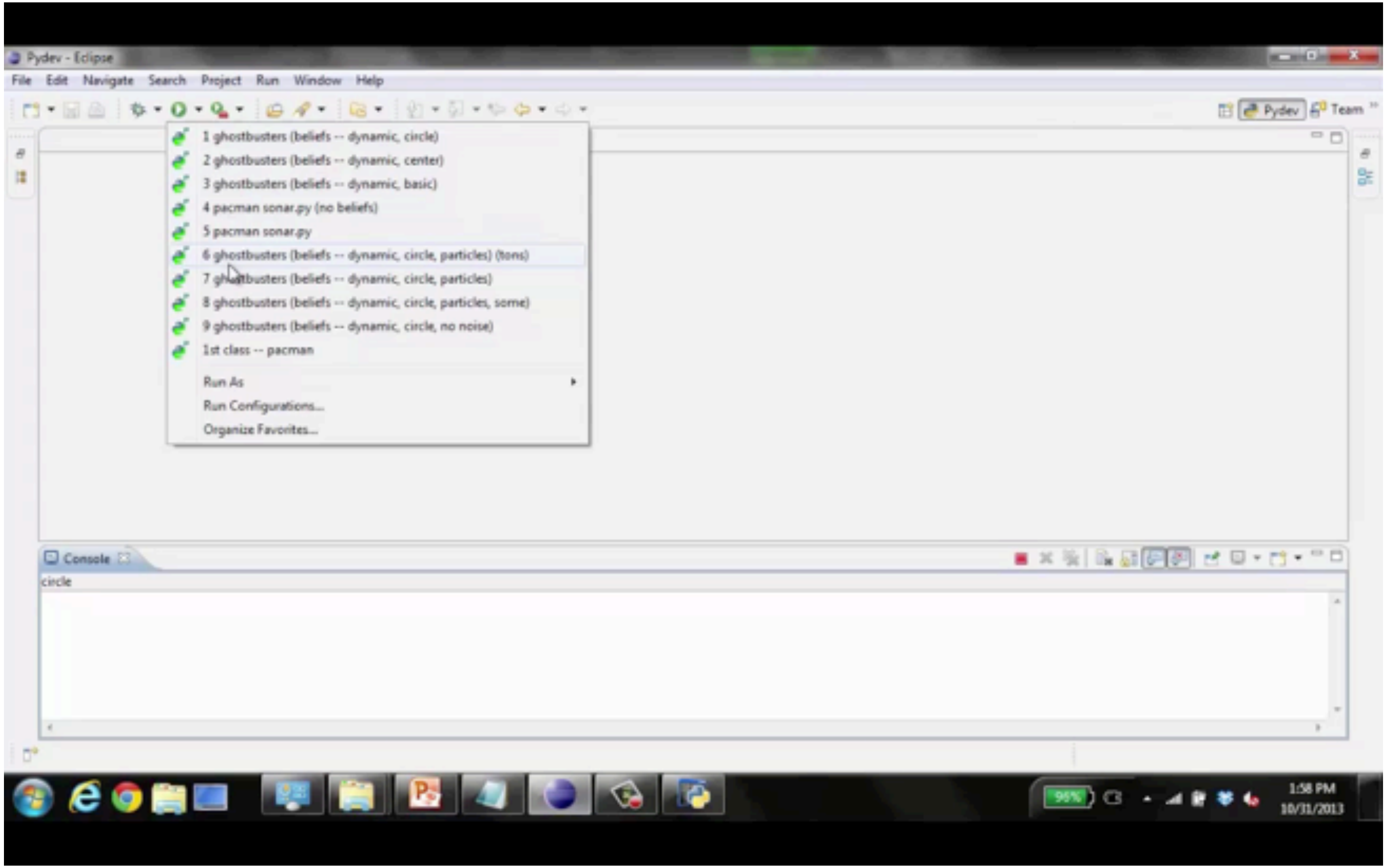
- Particles: track samples of states rather than an explicit distribution



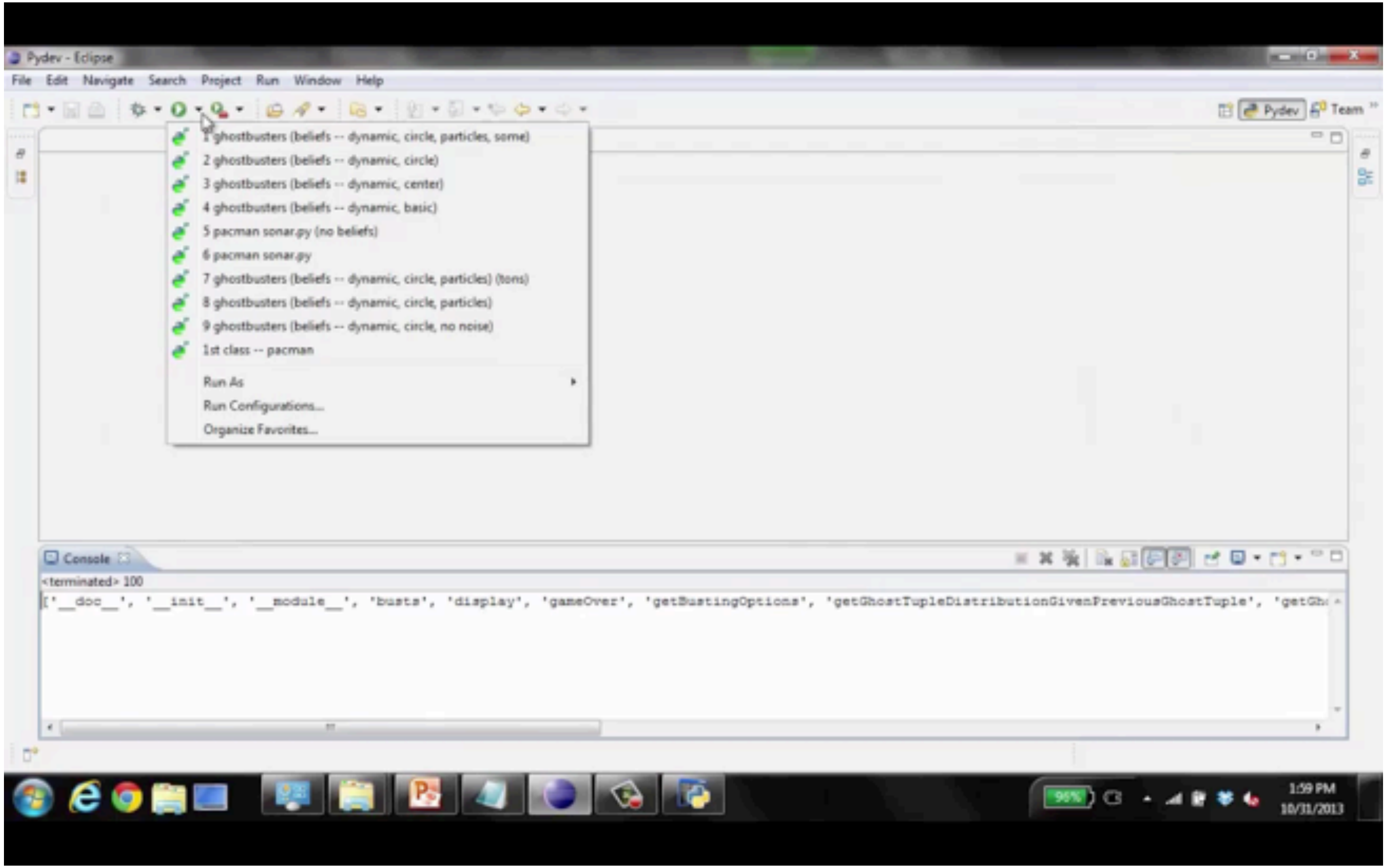

| Particles: | Particles: | Particles: | (New) Particles: |
|---|---|---|---|
| (3,3) | (3,2) | (3,2) w=.9 | (3,2) |
| (2,3) | (2,3) | (2,3) w=.2 | (2,2) |
| (3,3) | (3,2) | (3,2) w=.9 | (3,2) |
| (3,2) | (3,1) | (3,1) w=.4 | (2,3) |
| (3,3) | (3,3) | (3,3) w=.4 | (3,3) |
| (3,2) | (3,2) | (3,2) w=.9 | (3,2) |
| (1,2) | (1,3) | (1,3) w=.1 | (1,3) |
| (3,3) | (2,3) | (2,3) w=.2 | (2,3) |
| (3,3) | (3,2) | (3,2) w=.9 | (3,2) |
| (2,3) | (2,2) | (2,2) w=.4 | (3,2) |

[Demos: ghostbusters particle filtering (L15D3,4,5)]

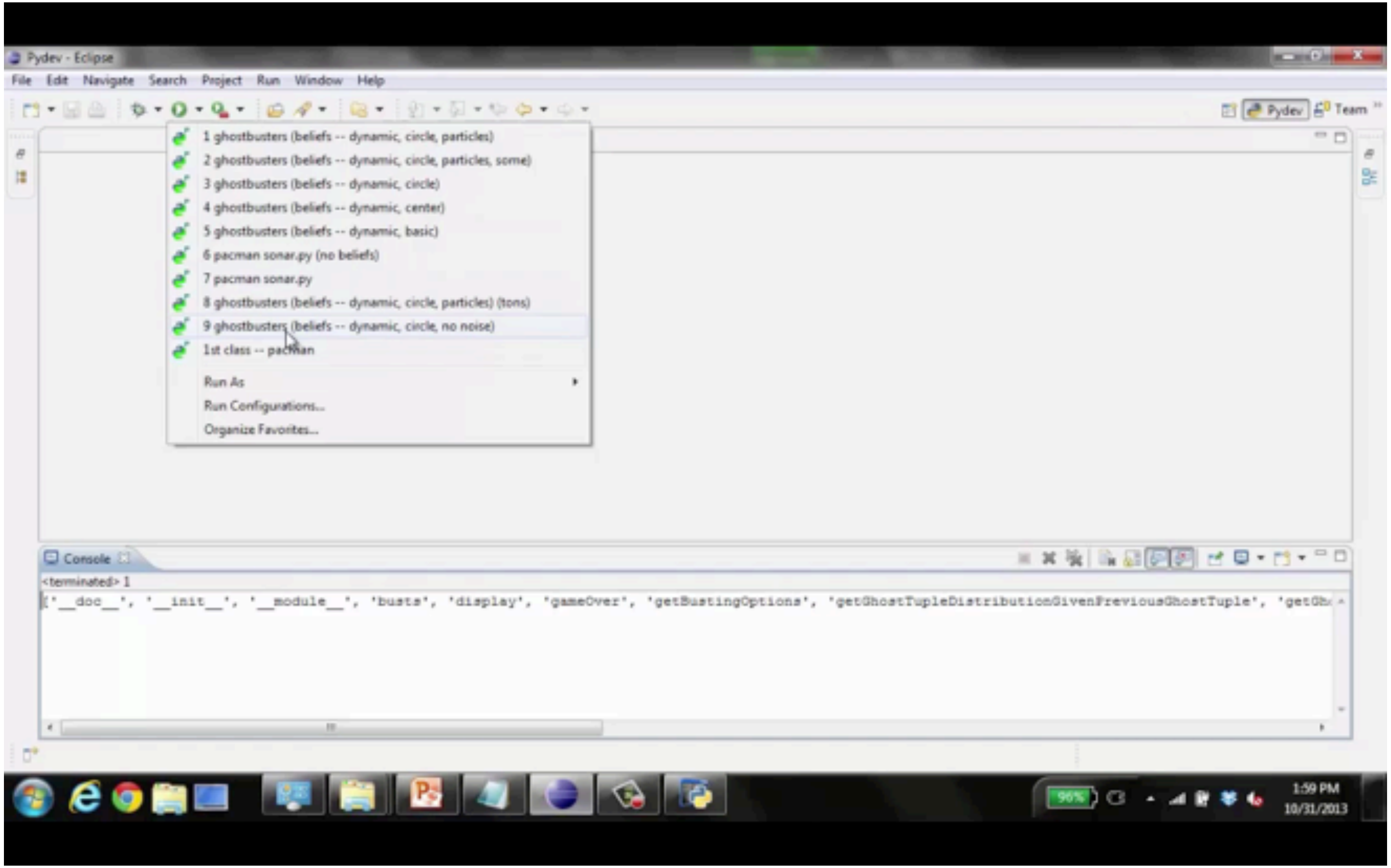
# Video of Demo – Moderate Number of Particles



# Video of Demo – One Particle

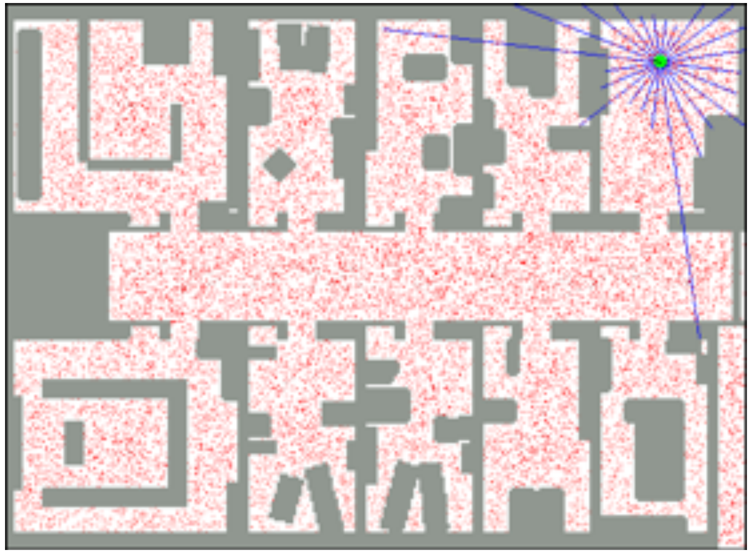


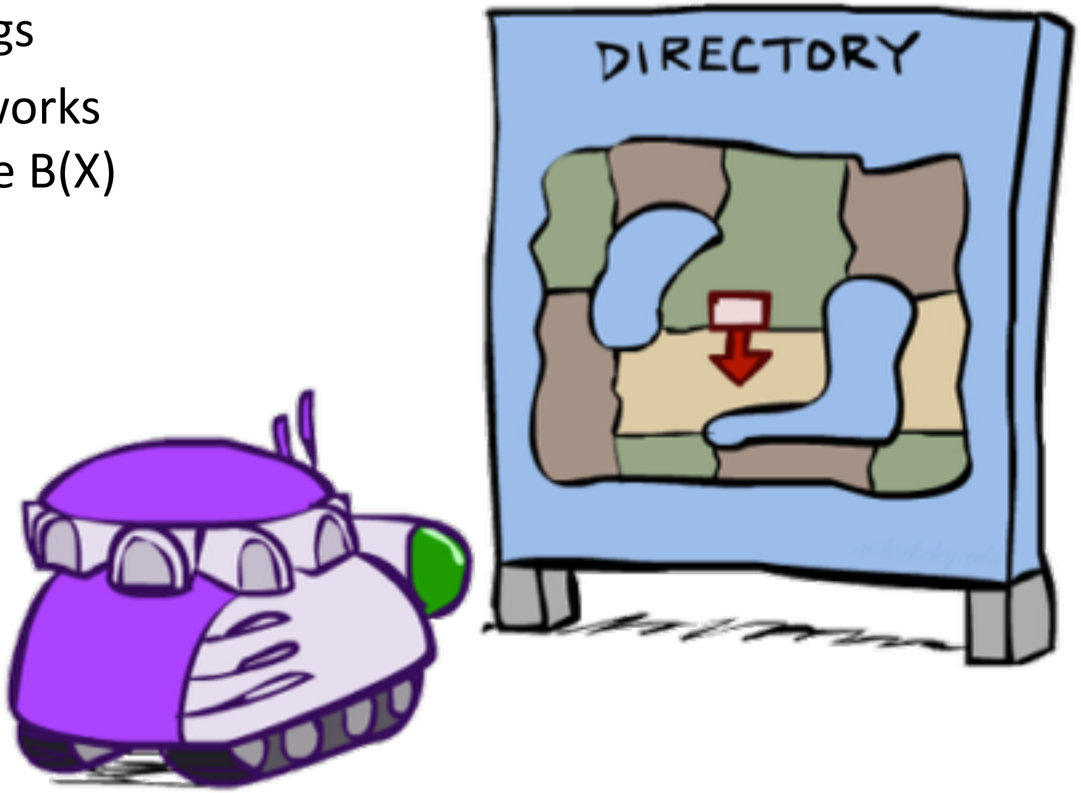
# Video of Demo – Huge Number of Particles



# Robot Localization

- In robot localization:
  - We know the map, but not the robot's position
  - Observations may be vectors of range finder readings
  - State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store B(X)
  - Particle filtering is a main technique

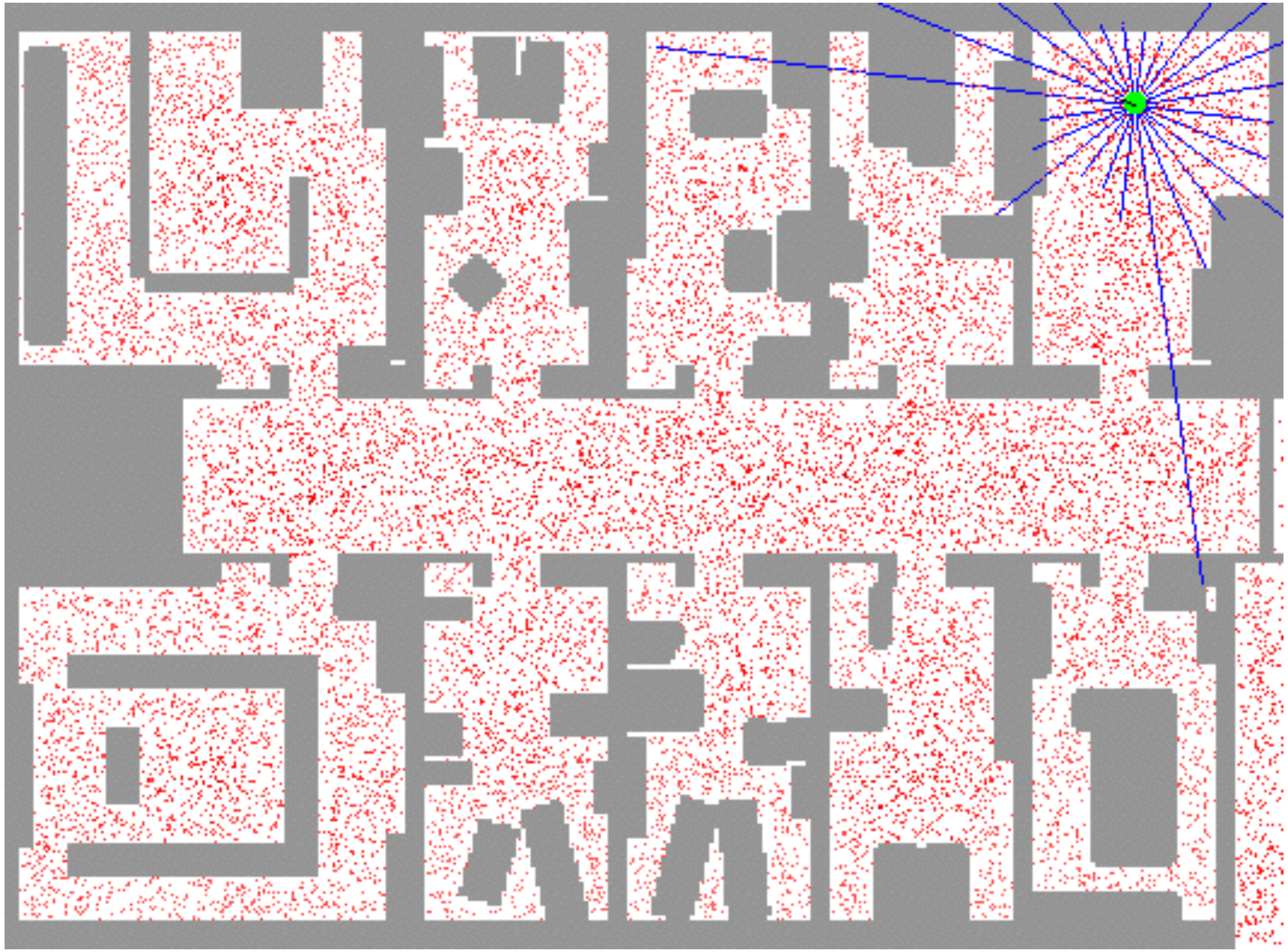




# Particle Filter Localization (Sonar)


[Video: global-sonar-uw-annotated.avi]

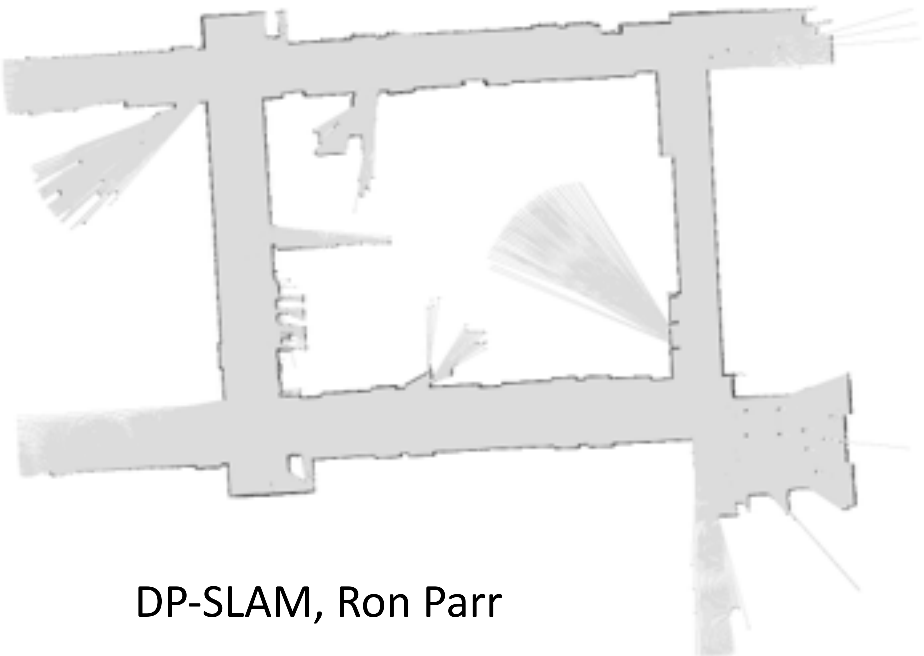
# Particle Filter Localization (Laser)



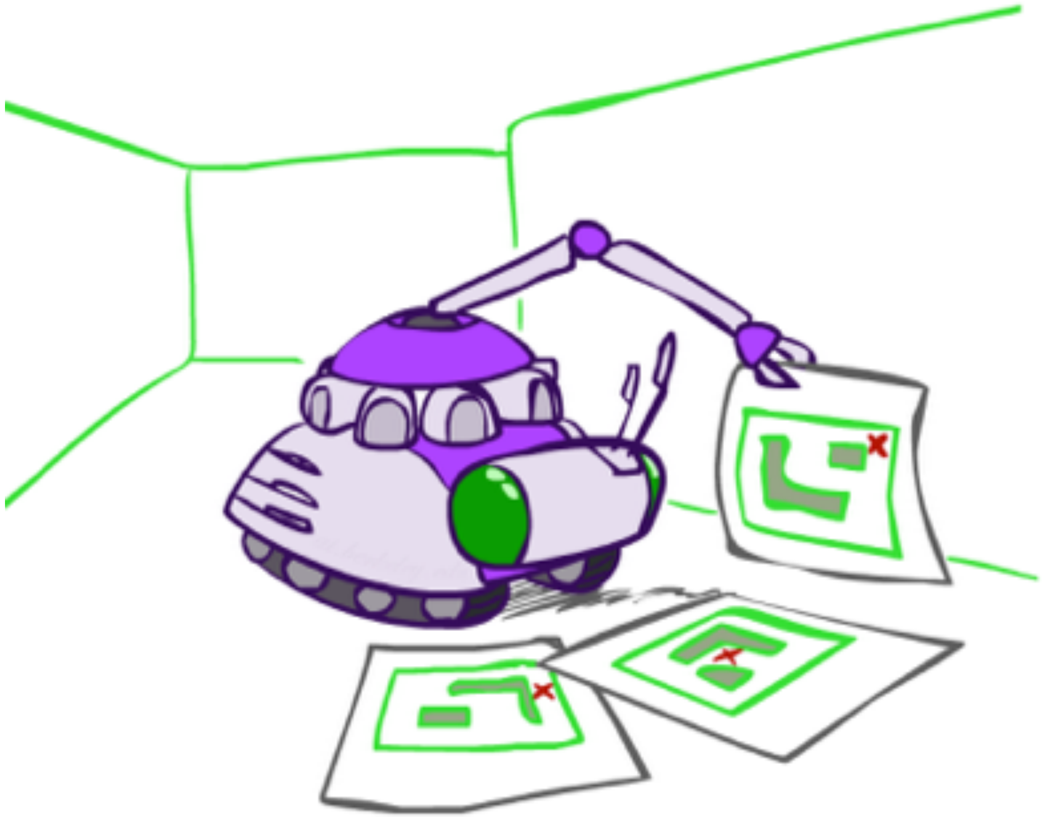
[Video: global-floor.gif]

# Robot Mapping

- SLAM: Simultaneous Localization And Mapping
  - We do not know the map or our location
  - State consists of position AND map!
  - Main techniques: Kalman filtering (Gaussian HMMs) and particle methods

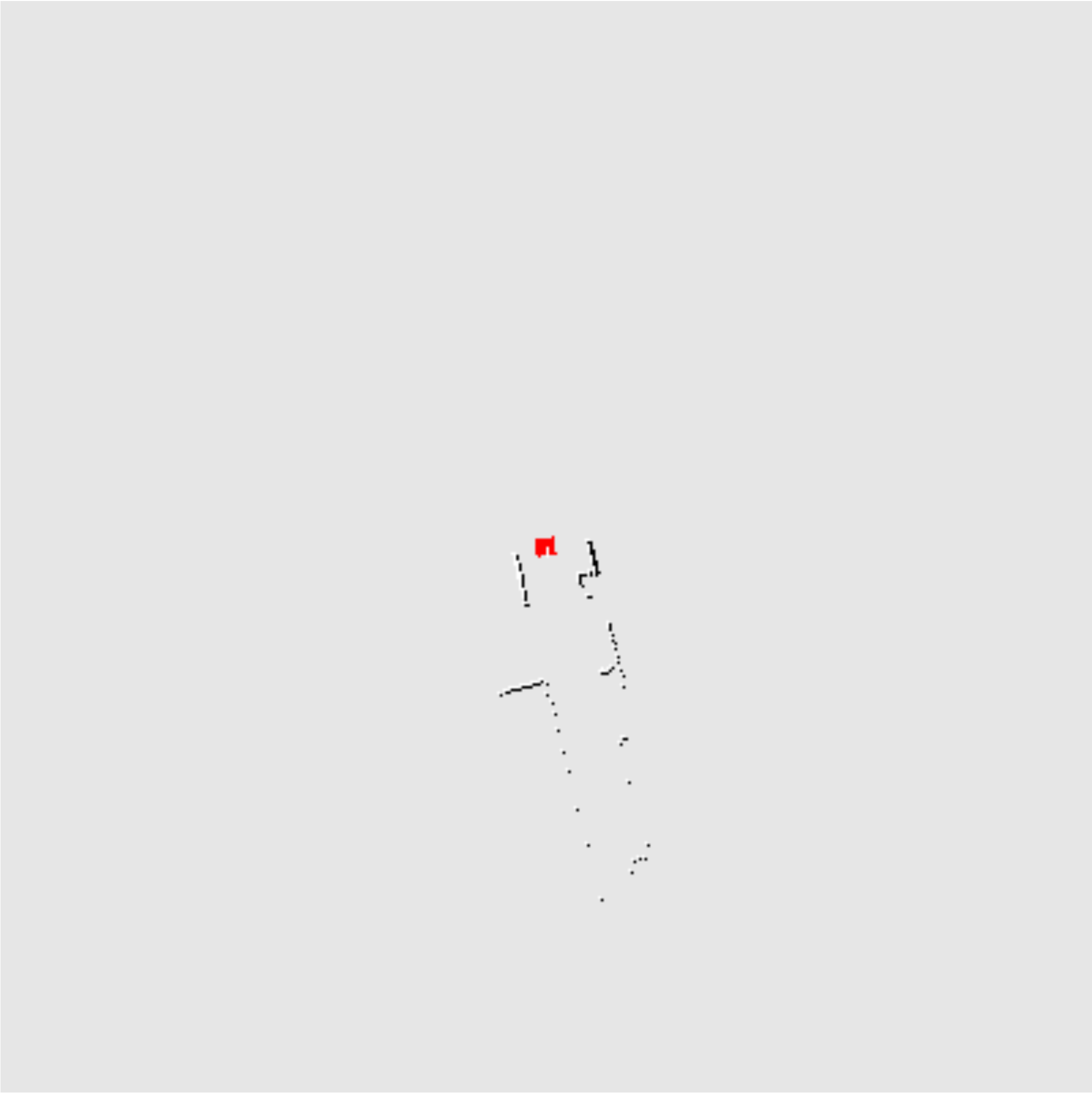


DP-SLAM, Ron Parr



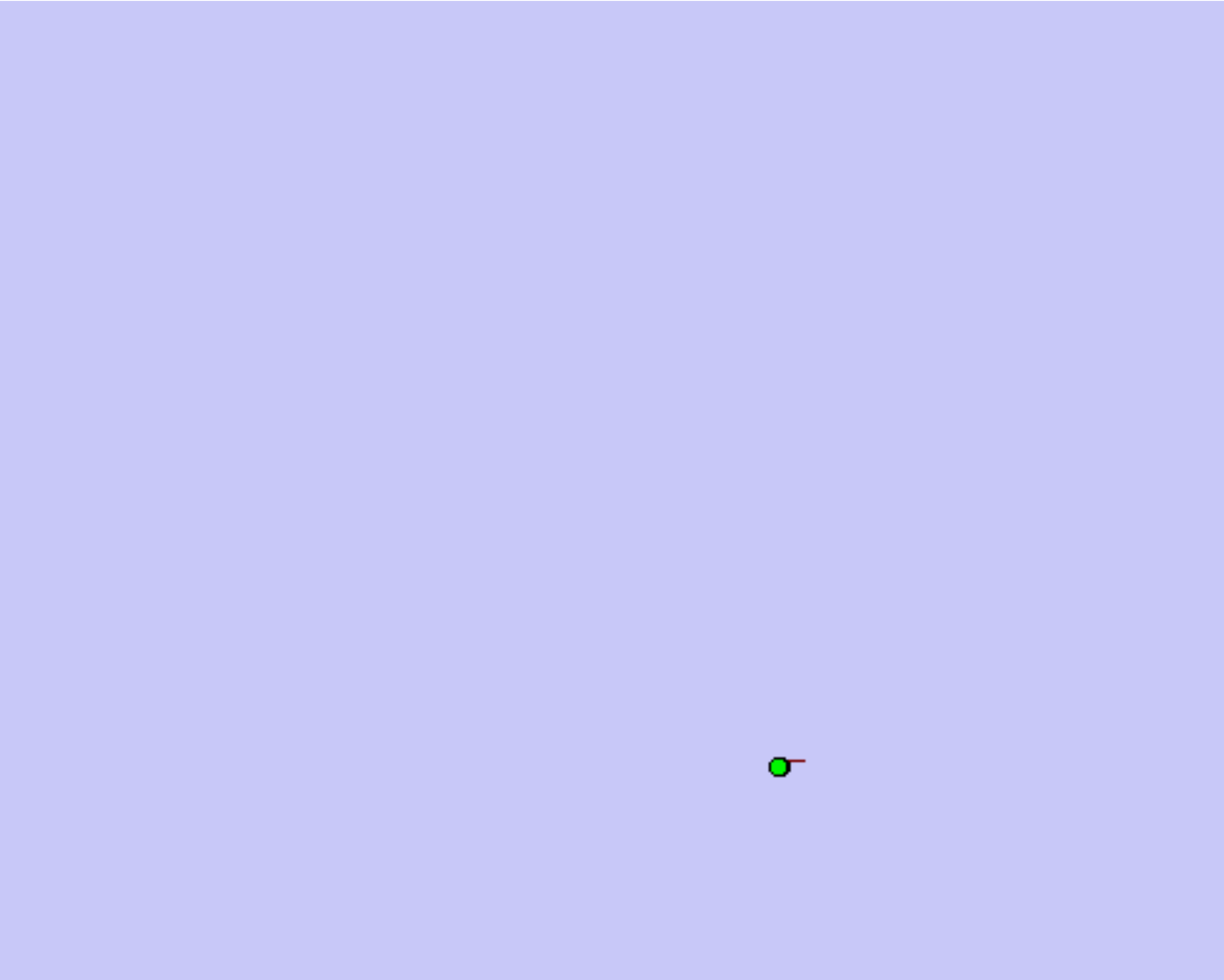
[Demo: PARTICLES-SLAM-mapping1-new.avi]

# Particle Filter SLAM – Video 1



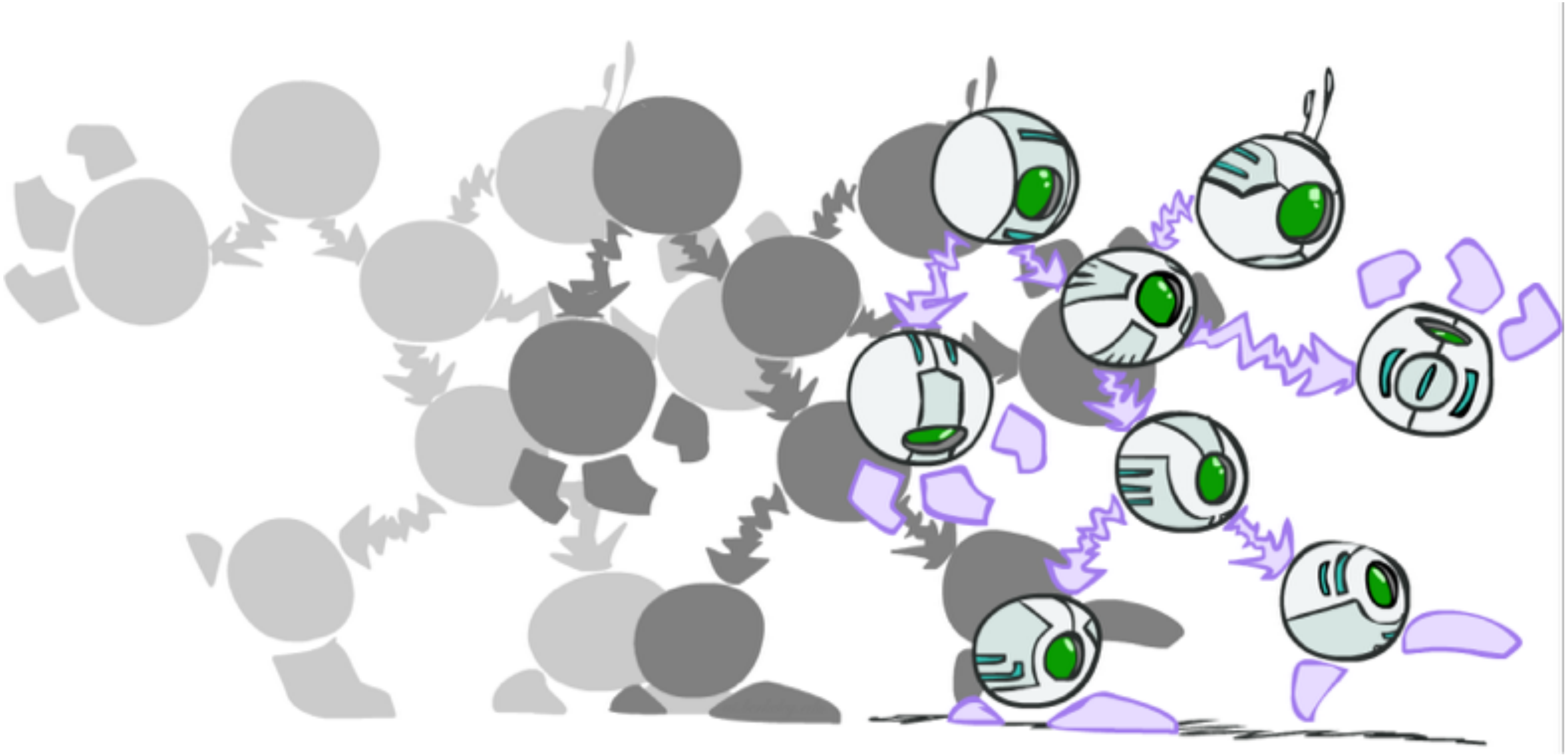
[Demo: PARTICLES-SLAM-mapping1-new.avi]

# Particle Filter SLAM – Video 2



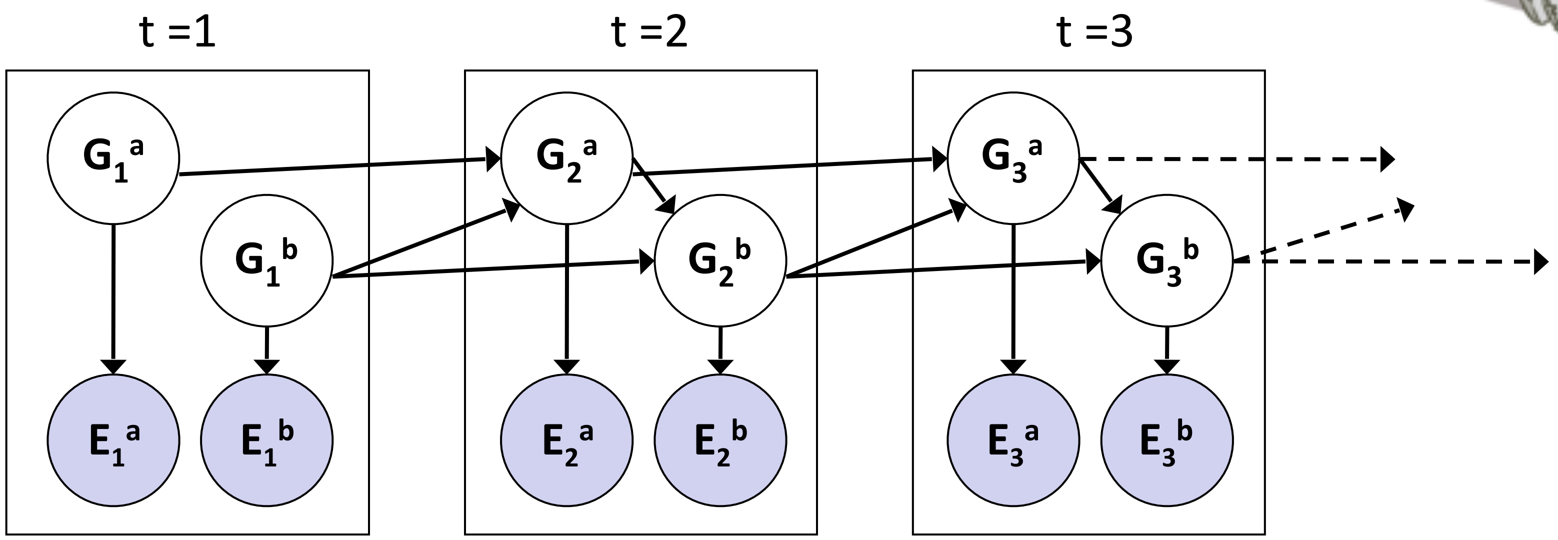
[Demo: PARTICLES-SLAM-fastslam.avi]

# Dynamic Bayes Nets



# Dynamic Bayes Nets (DBNs)

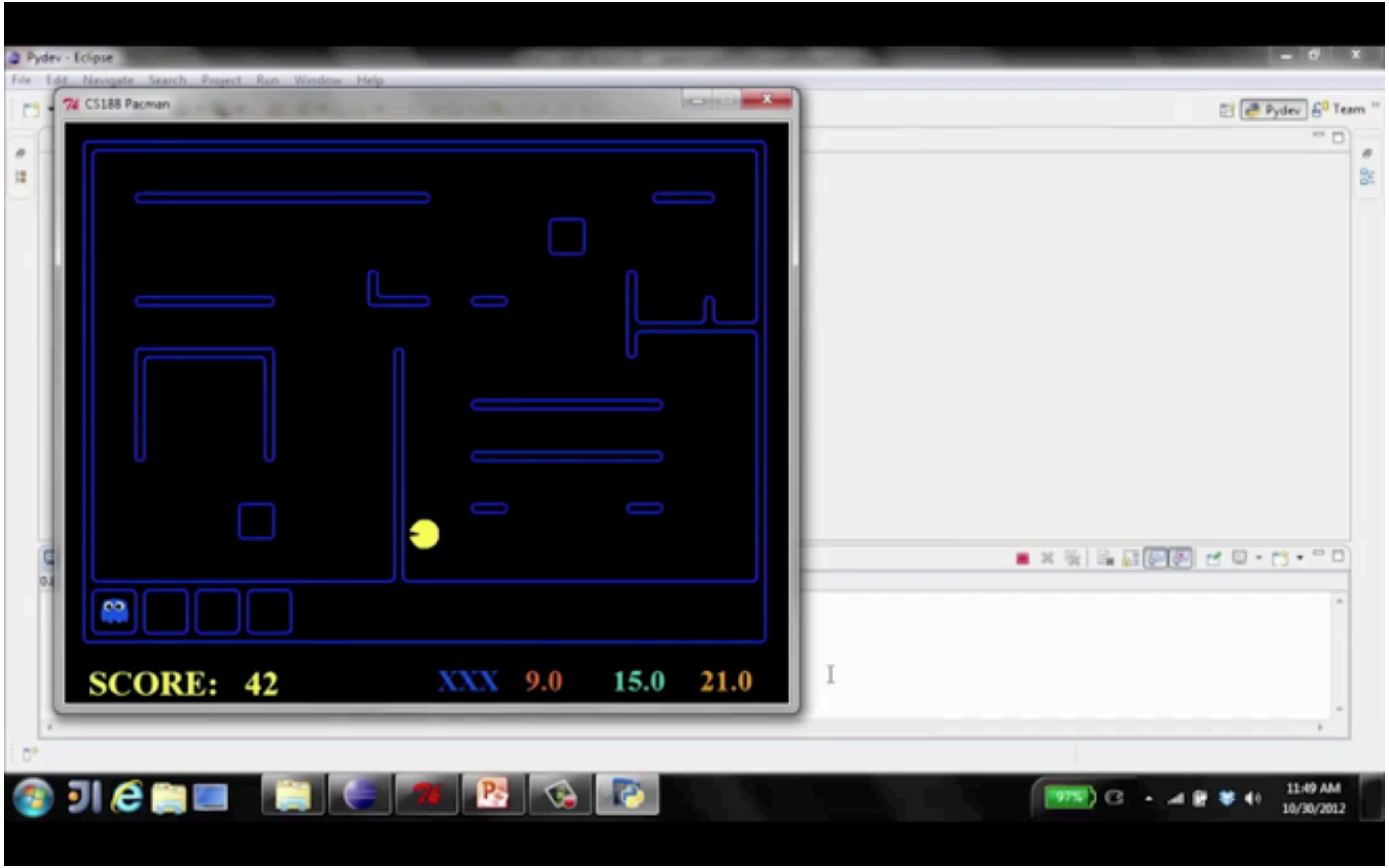
- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables from time *t* can condition on those from *t-1*




- Dynamic Bayes nets are a generalization of HMMs

[Demo: pacman sonar ghost DBN model (L15D6)]

# Video of Demo Pacman Sonar Ghost DBN Model



# DBN Particle Filters

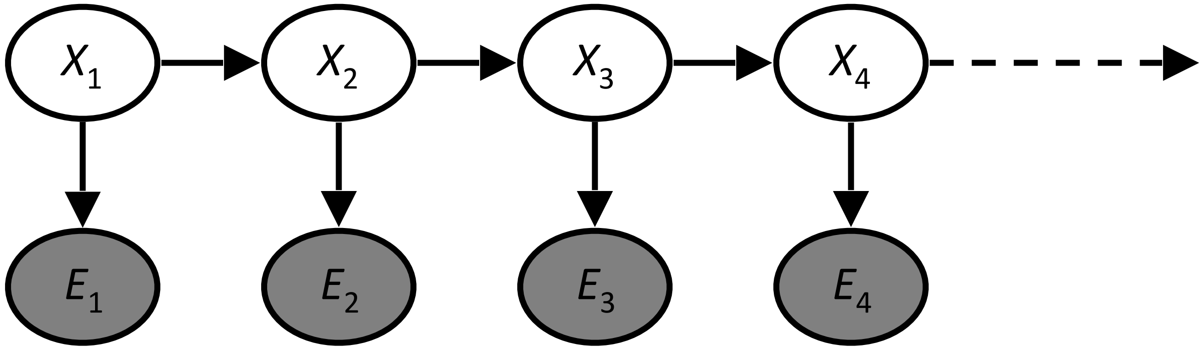
- A particle is a complete sample for a time step
- **Initialize**: Generate prior samples for the t=1 Bayes net
  - Example particle: $\mathbf{G_1^a} = (3,3)$ $\mathbf{G_1^b} = (5,3)$
- **Elapse time**: Sample a successor for each particle
  - Example successor: $\mathbf{G_2^a} = (2,3)$ $\mathbf{G_2^b} = (6,3)$
- **Observe**: Weight each *entire* sample by the likelihood of the evidence conditioned on the sample
  - Likelihood: $P(\mathbf{E_1^a} \mid \mathbf{G_1^a}) * P(\mathbf{E_1^b} \mid \mathbf{G_1^b})$
- **Resample:** Select prior samples (tuples of values) in proportion to their likelihood

# Most Likely Explanation

# HMMs: MLE Queries

- HMMs defined by
  - States X
  - Observations E
  - Initial distribution:
  - Transitions:
  - Emissions:

$$P(X_1)$$
$$P(X|X_{-1})$$
$$P(E|X)$$

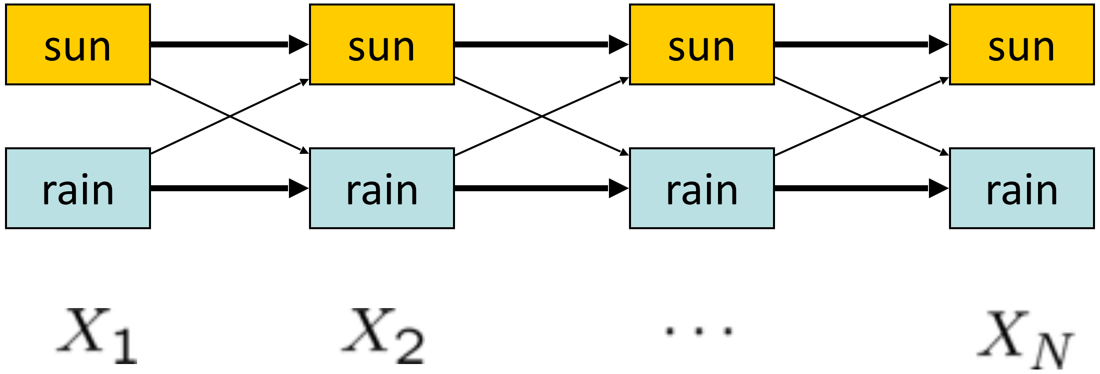



- New query: most likely explanation:

$$\arg\max_{x_{1:t}} P(x_{1:t}|e_{1:t})$$

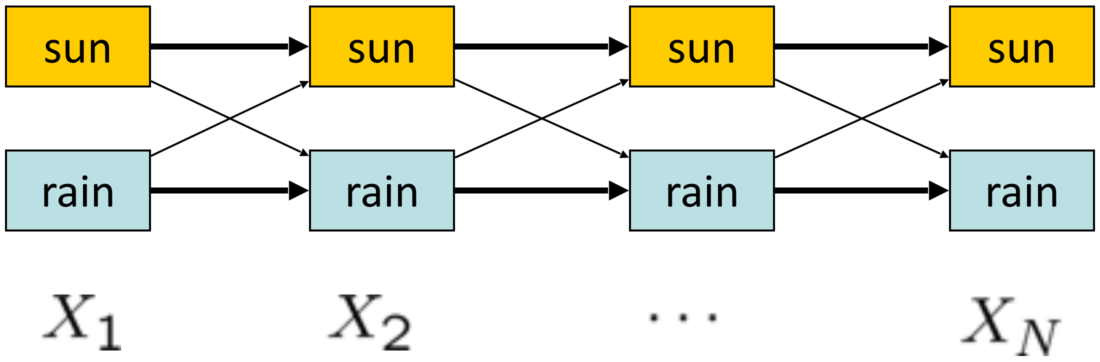
# State Trellis

- State trellis: graph of states and transitions over time




- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t|x_{t-1})P(e_t|x_t)$
- Each path is a sequence of states
- The product of weights on a path is that sequence's probability along with the evidence
- Forward algorithm computes sums of paths, Viterbi computes best paths

# Forward / Viterbi Algorithms




## Forward Algorithm (Sum)

$$f_t[x_t] = P(x_t, e_{1:t})$$

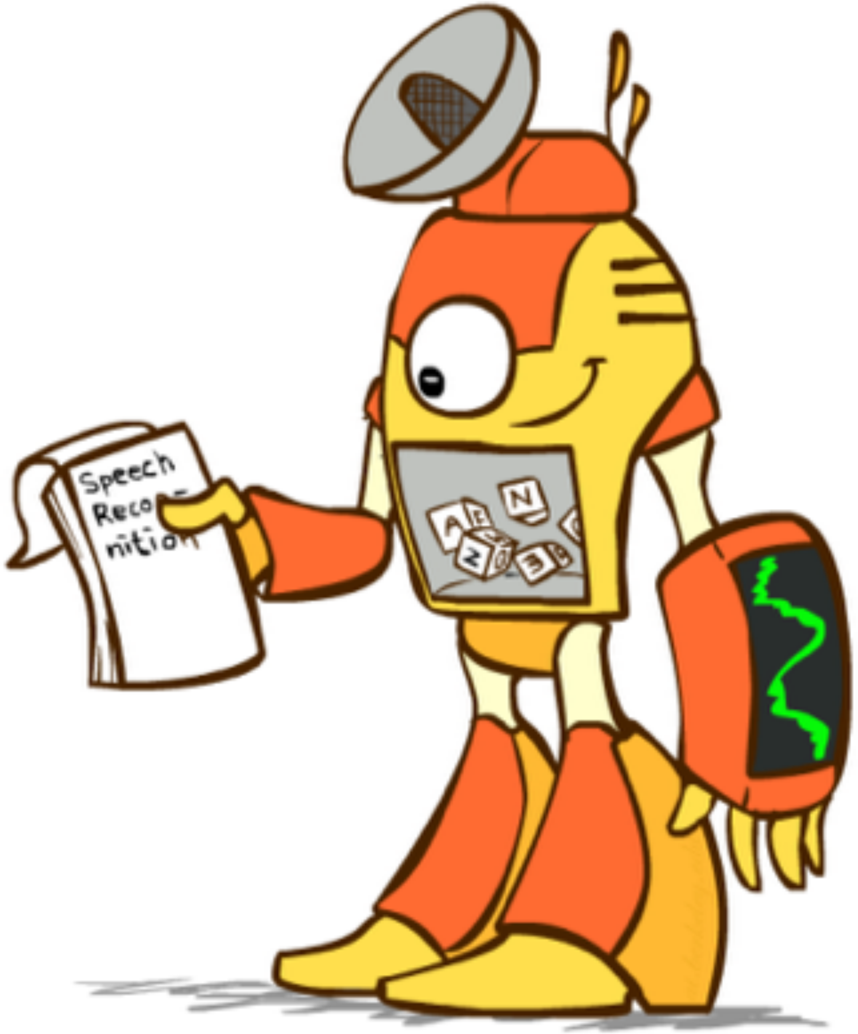
$$= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}) f_{t-1}[x_{t-1}]$$

## Viterbi Algorithm (Max)

$$m_t[x_t] = \max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t})$$

$$= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]$$

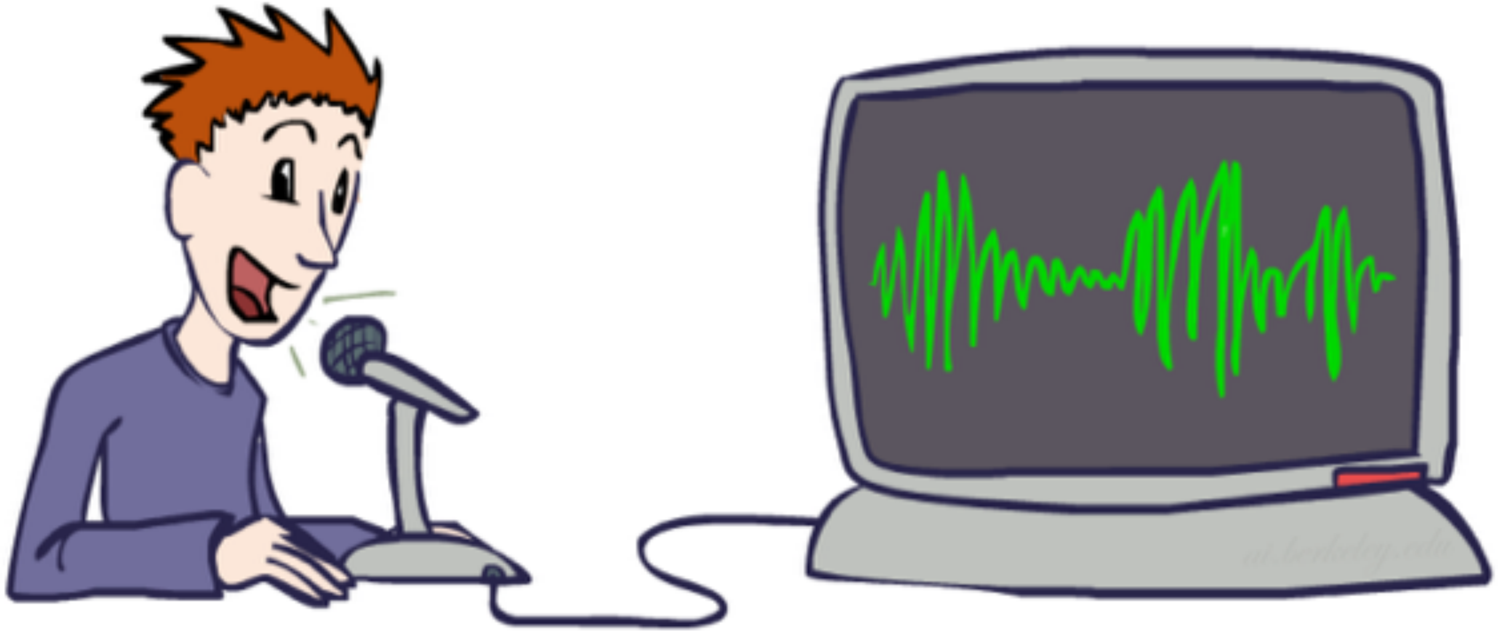
# Speech Recognition



# Speech Recognition in Action

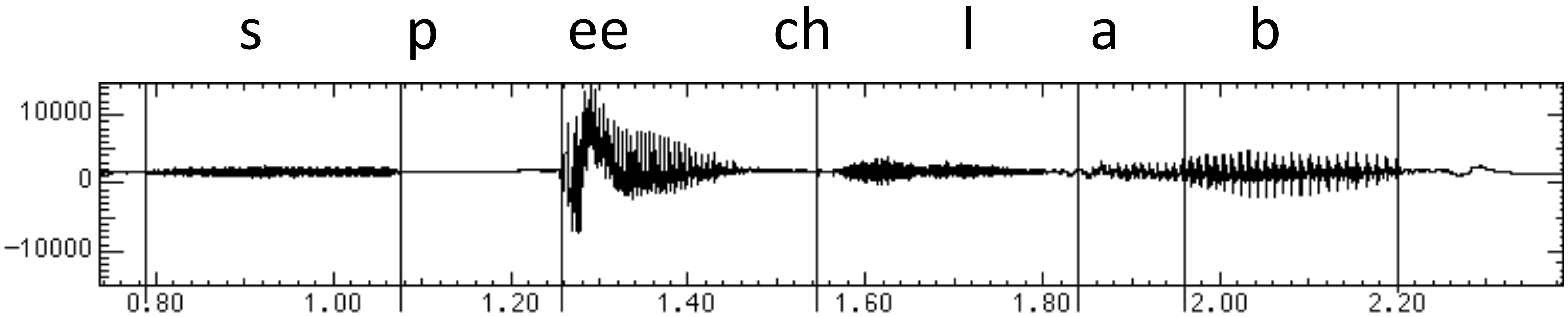
[Video: NLP – ASR tvsample.avi (from Lecture 1)]

# Digitizing Speech



# Speech in an Hour

- Speech input is an acoustic waveform




"l" to "a" transition:

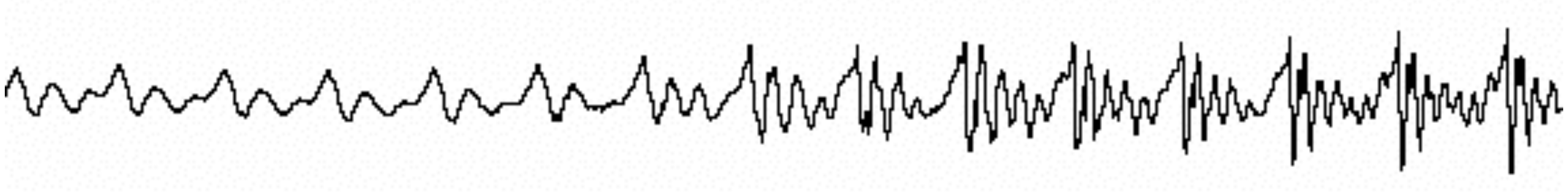
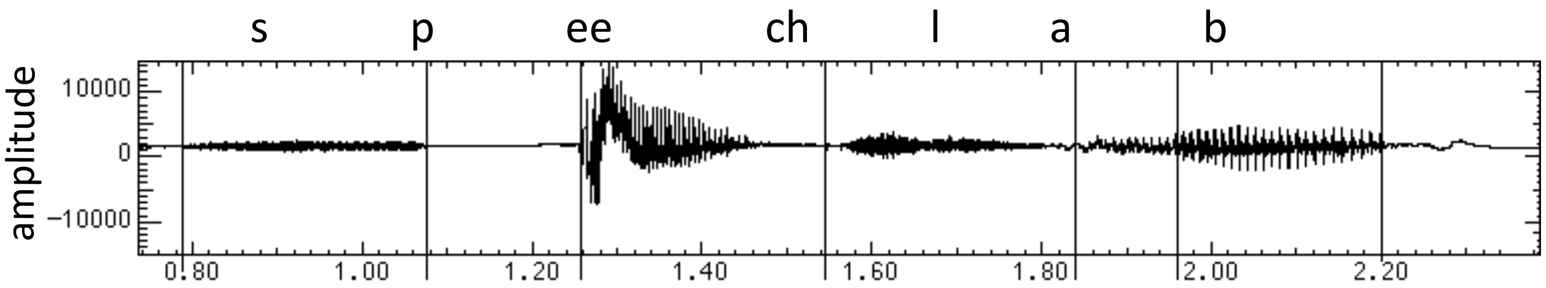
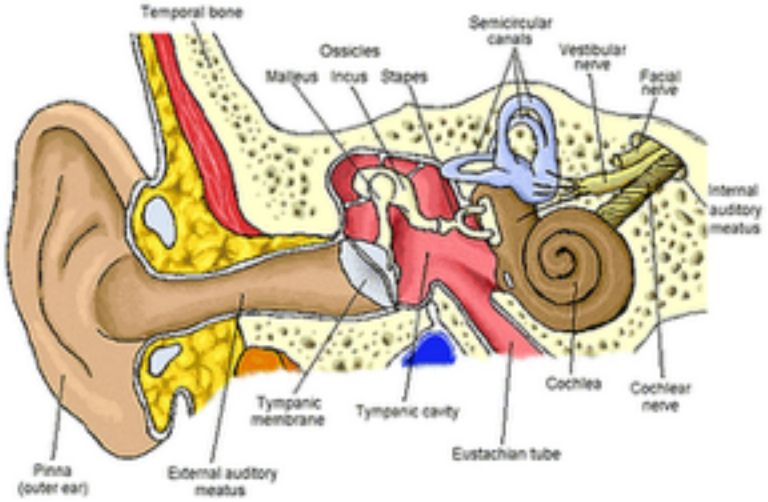


Figure: Simon Arnfield, http://www.psyc.leeds.ac.uk/research/cogn/speech/tutorial/

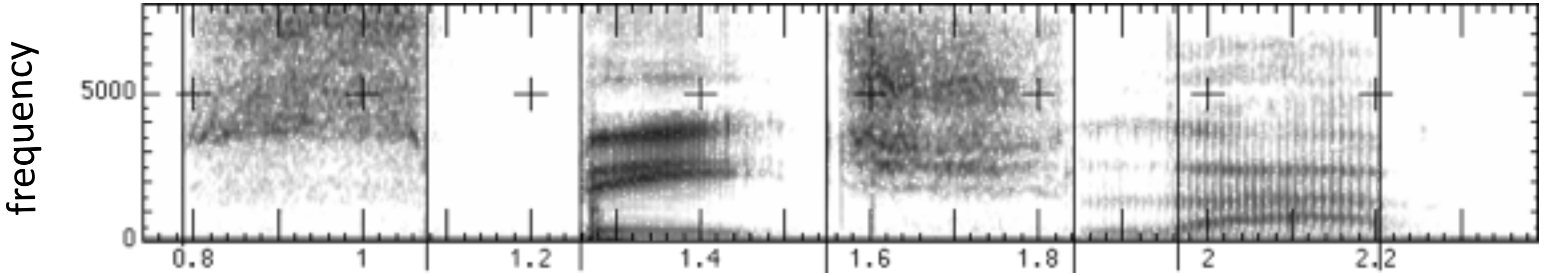
# Spectral Analysis

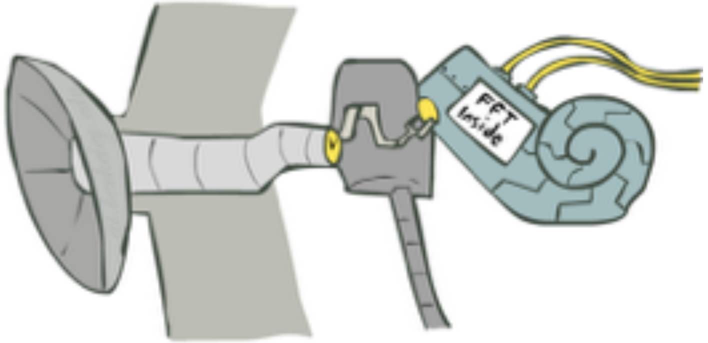
- Frequency gives pitch; amplitude gives volume
  - Sampling at ~8 kHz (phone), ~16 kHz (mic) (kHz=1000 cycles/sec)



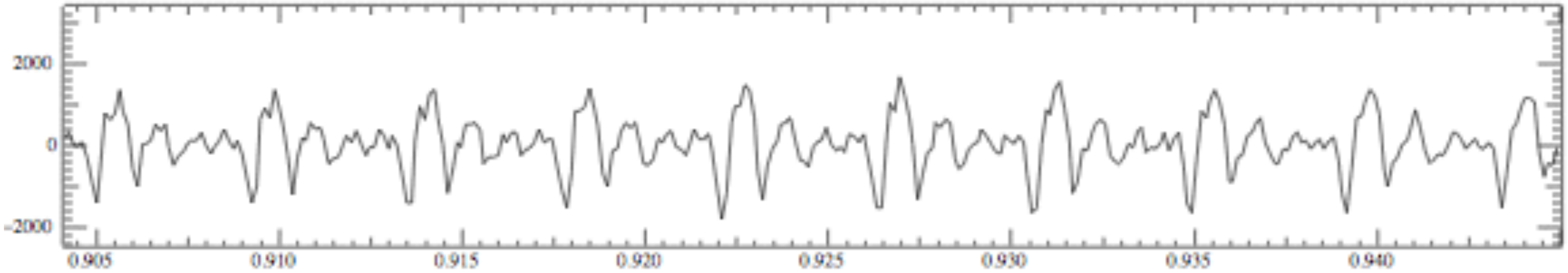

- Fourier transform of wave displayed as a spectrogram
  - Darkness indicates energy at each frequency



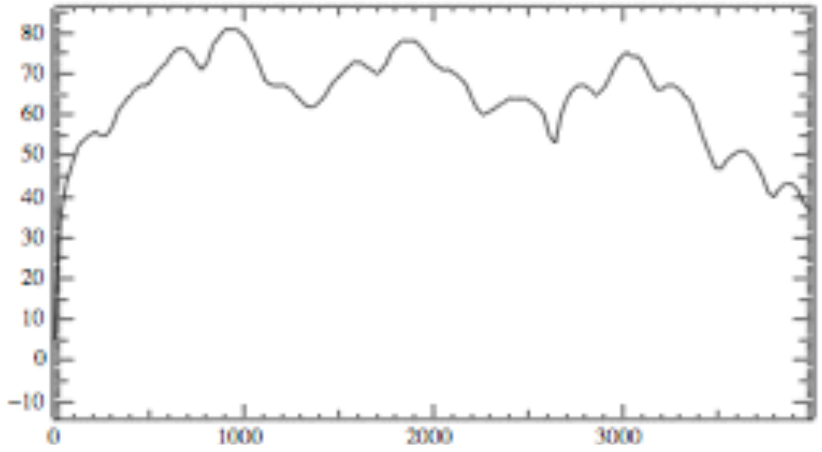




Human ear figure: depion.blogspot.com

# Part of [ae] from "lab"

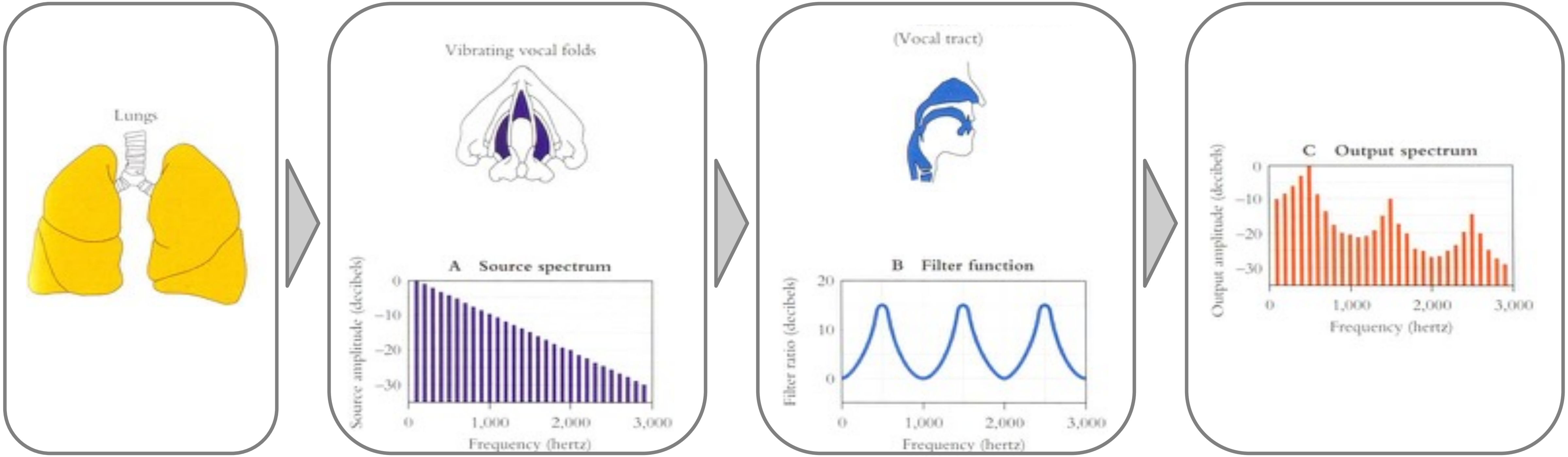



- Complex wave repeating nine times
  - Plus smaller wave that repeats 4x for every large cycle
  - Large wave: freq of 250 Hz (9 times in . 036 seconds)
  - Small wave roughly 4 times this, or roughly 1000 Hz



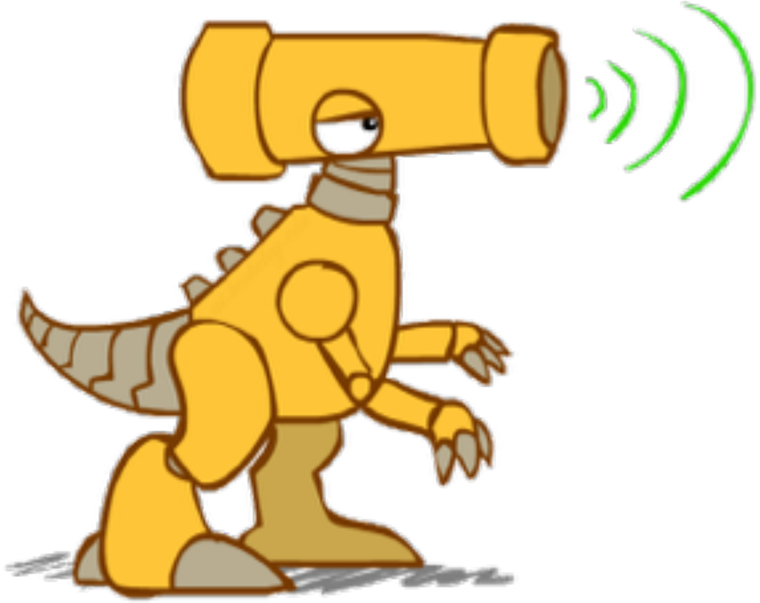
# Why These Peaks?

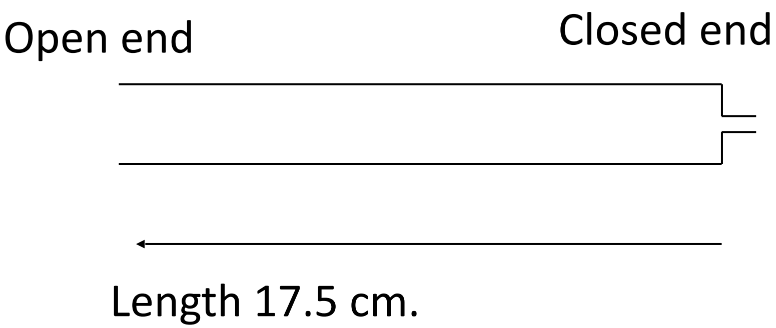
- Articulator process:
  - Vocal cord vibrations create harmonics
  - The mouth is an amplifier
  - Depending on shape of mouth, some harmonics are amplified more than others

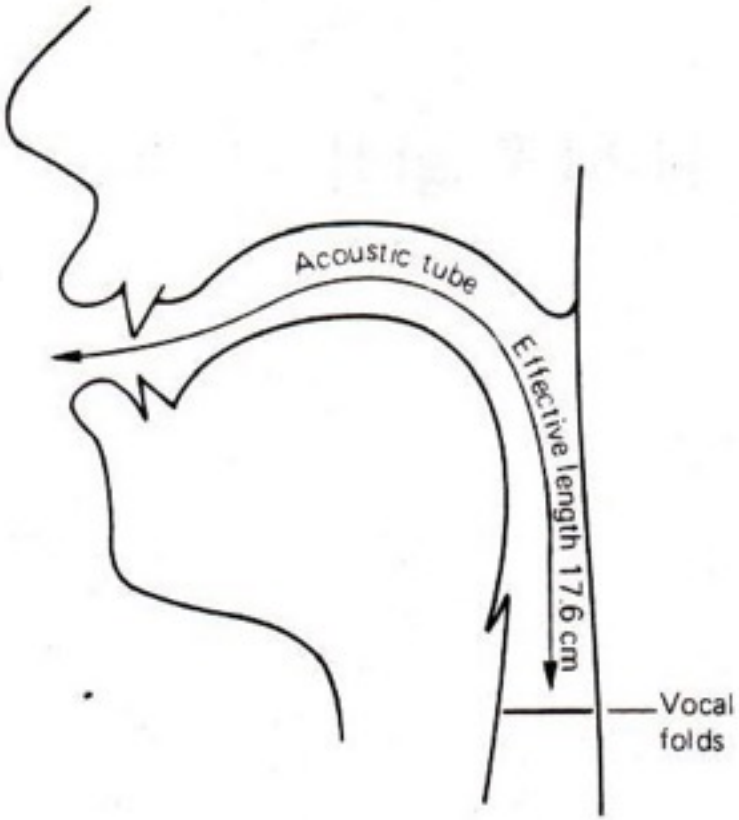


# Resonances of the Vocal Tract

- The human vocal tract as an open tube






- Air in a tube of a given length will tend to vibrate at resonance frequency of tube
- Constraint: Pressure differential should be maximal at (closed) glottal end and minimal at (open) lip end

Figure: W. Barry Speech Science slides

# Spectrum Shapes

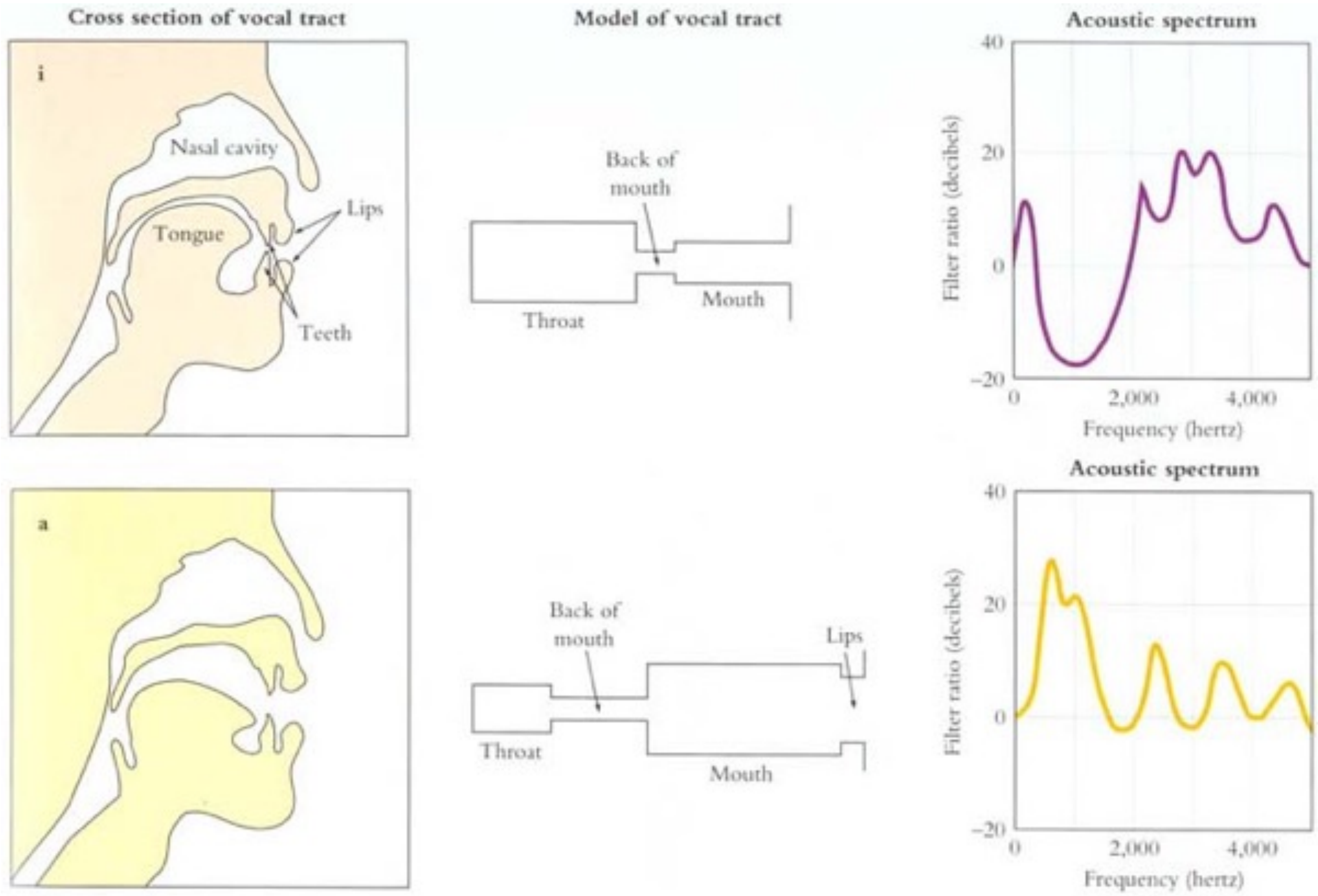
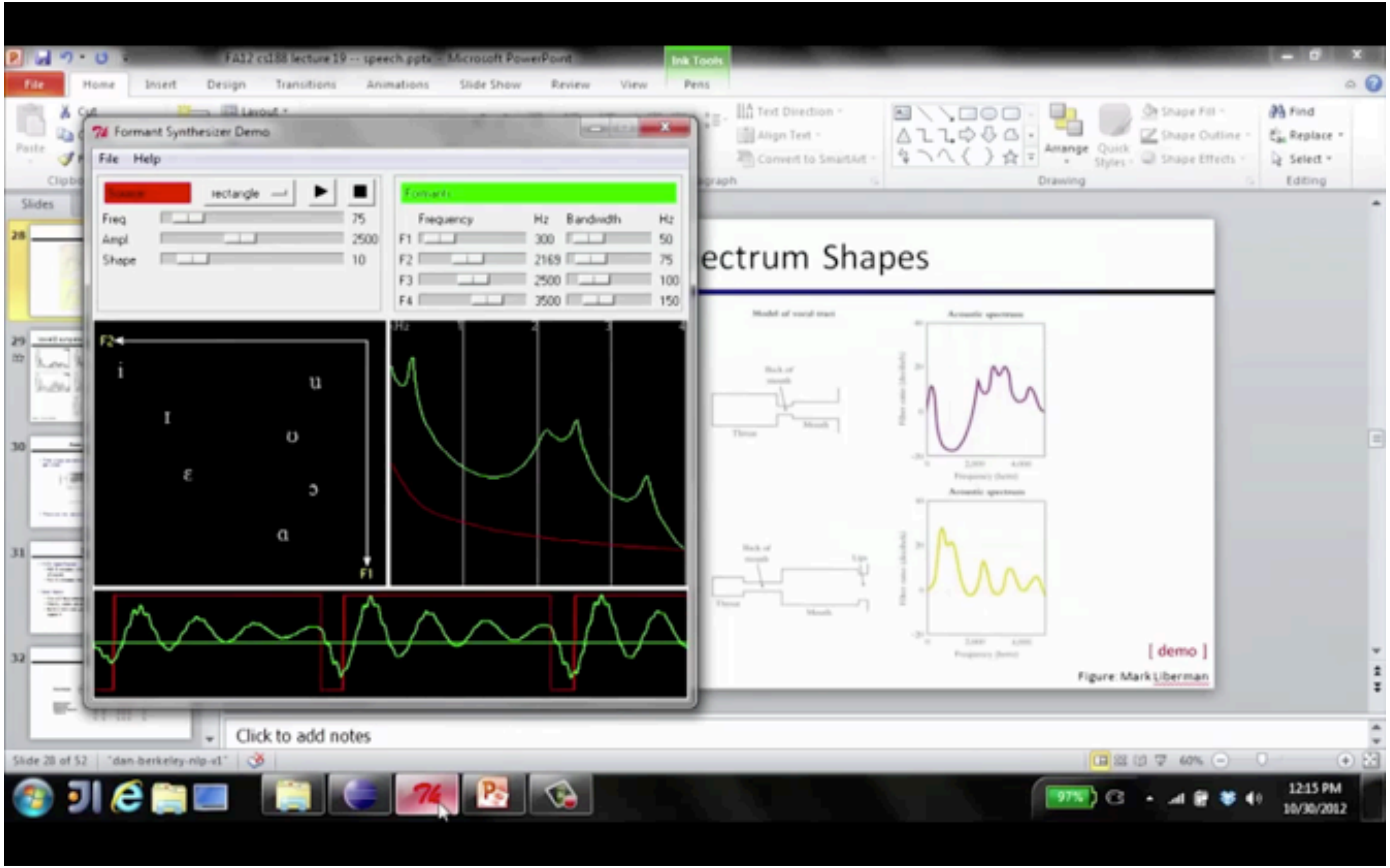


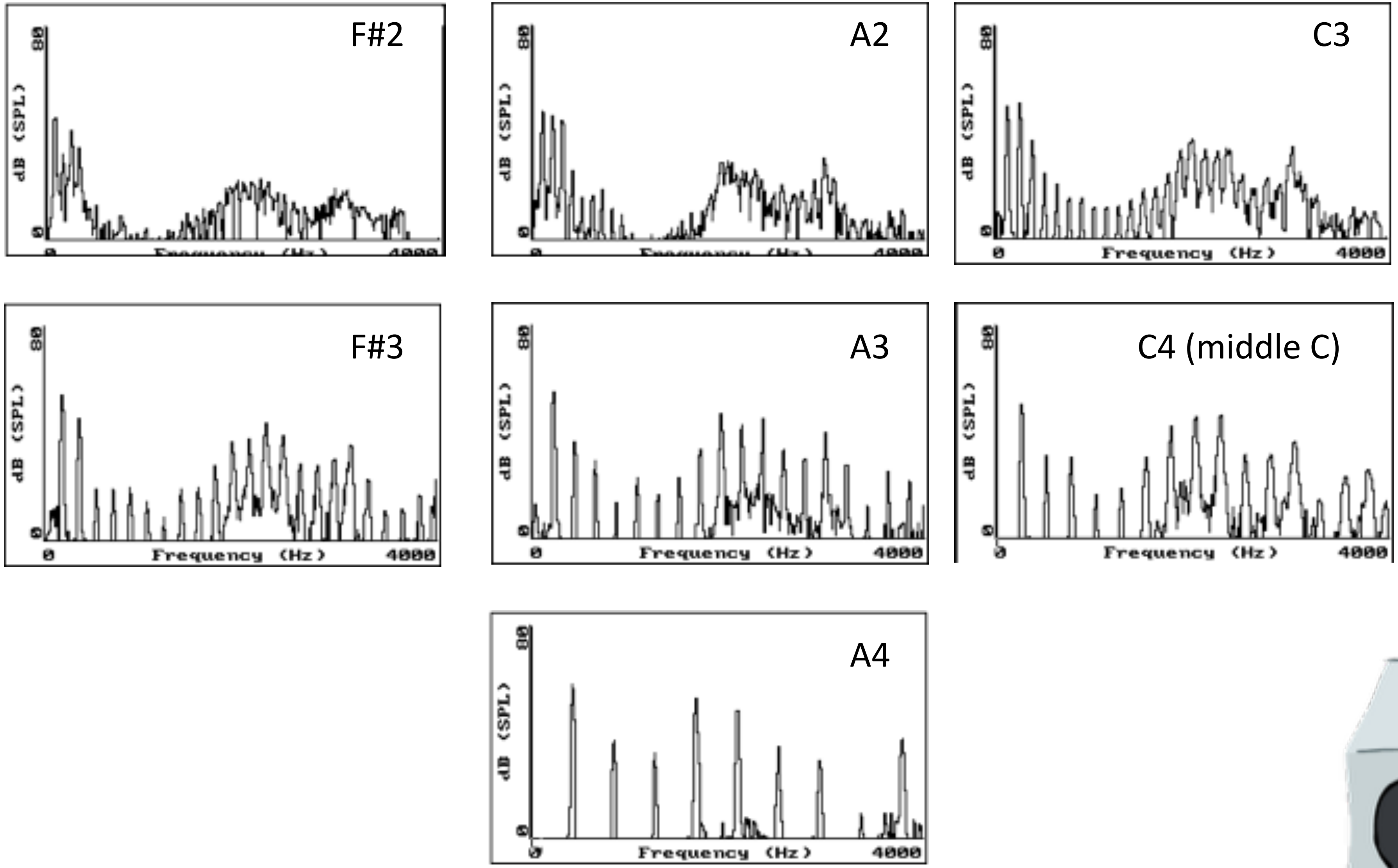

Figure: Mark Liberman

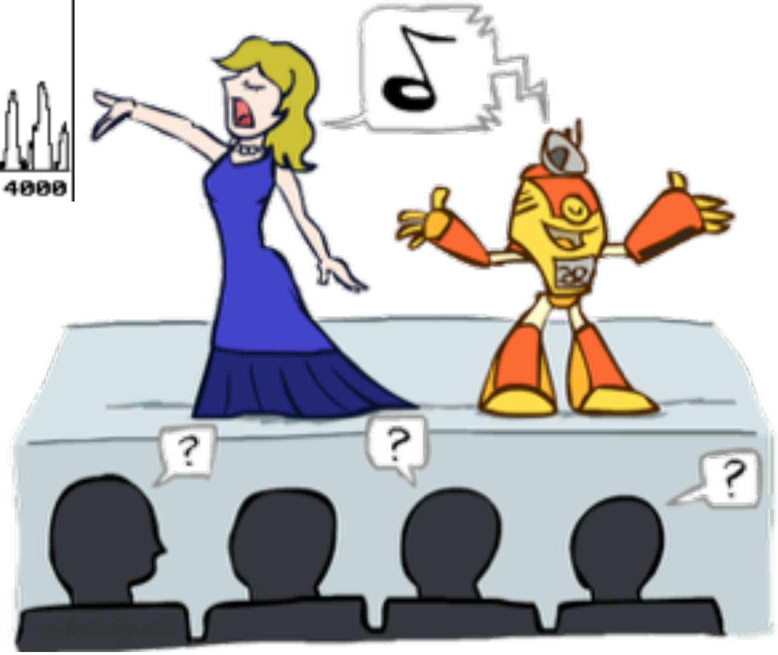
[Demo: speech synthesis ]

# Video of Demo Speech Synthesis



# Vowel [i] sung at successively higher pitches

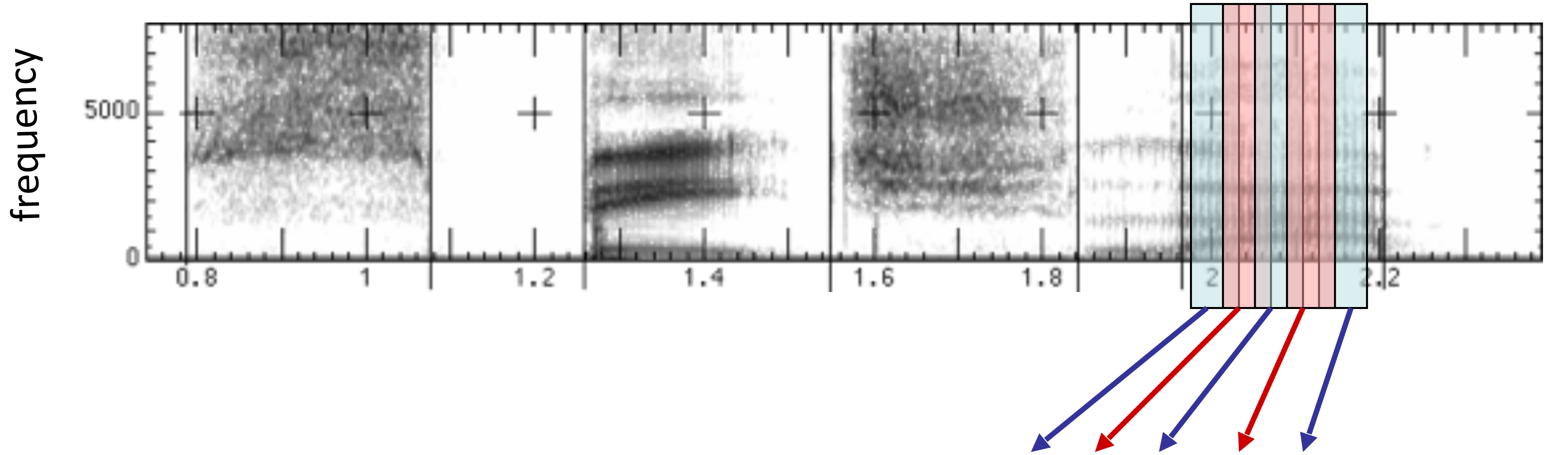


Graphs: Ratree Wayland

# Acoustic Feature Sequence

- Time slices are translated into acoustic feature vectors (~39 real numbers per slice)



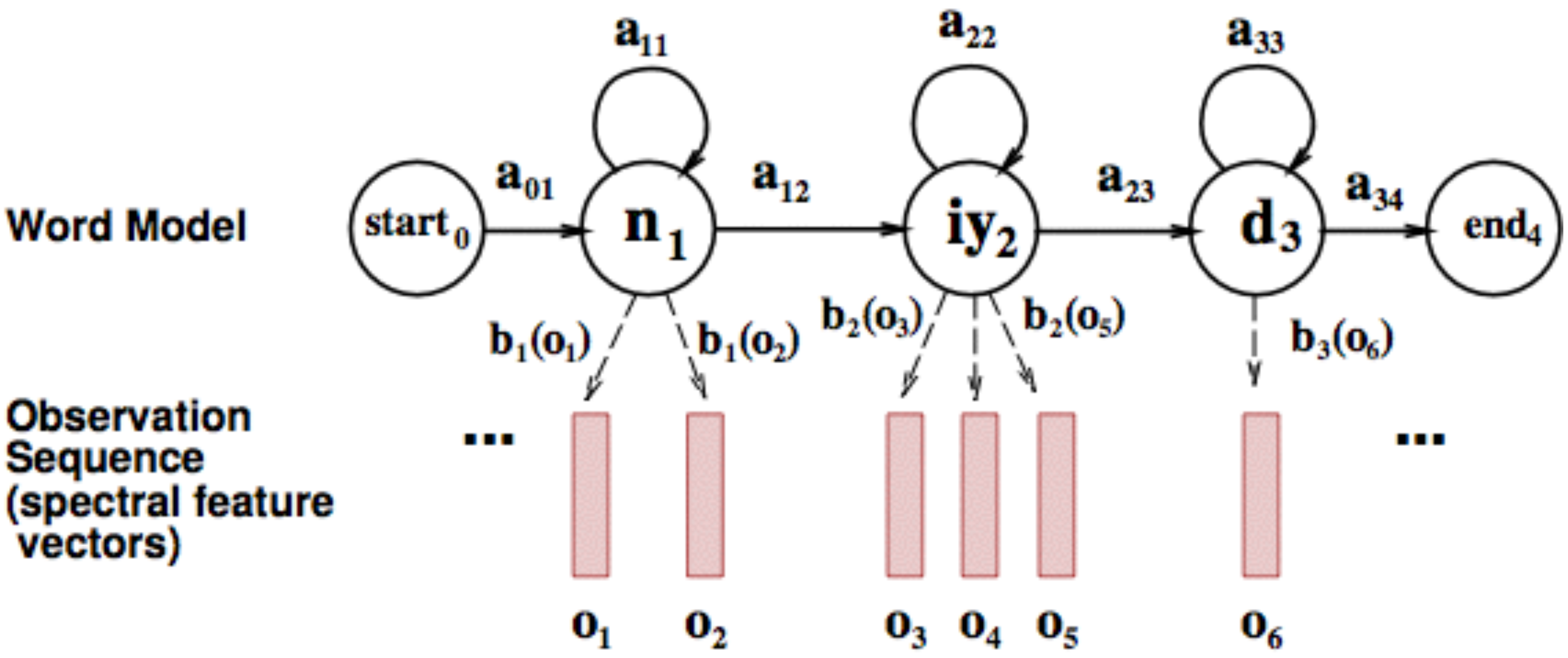

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots e_{12}e_{13}e_{14}e_{15}e_{16}\ldots\ldots$

- These are the observations E, now we need the hidden states X

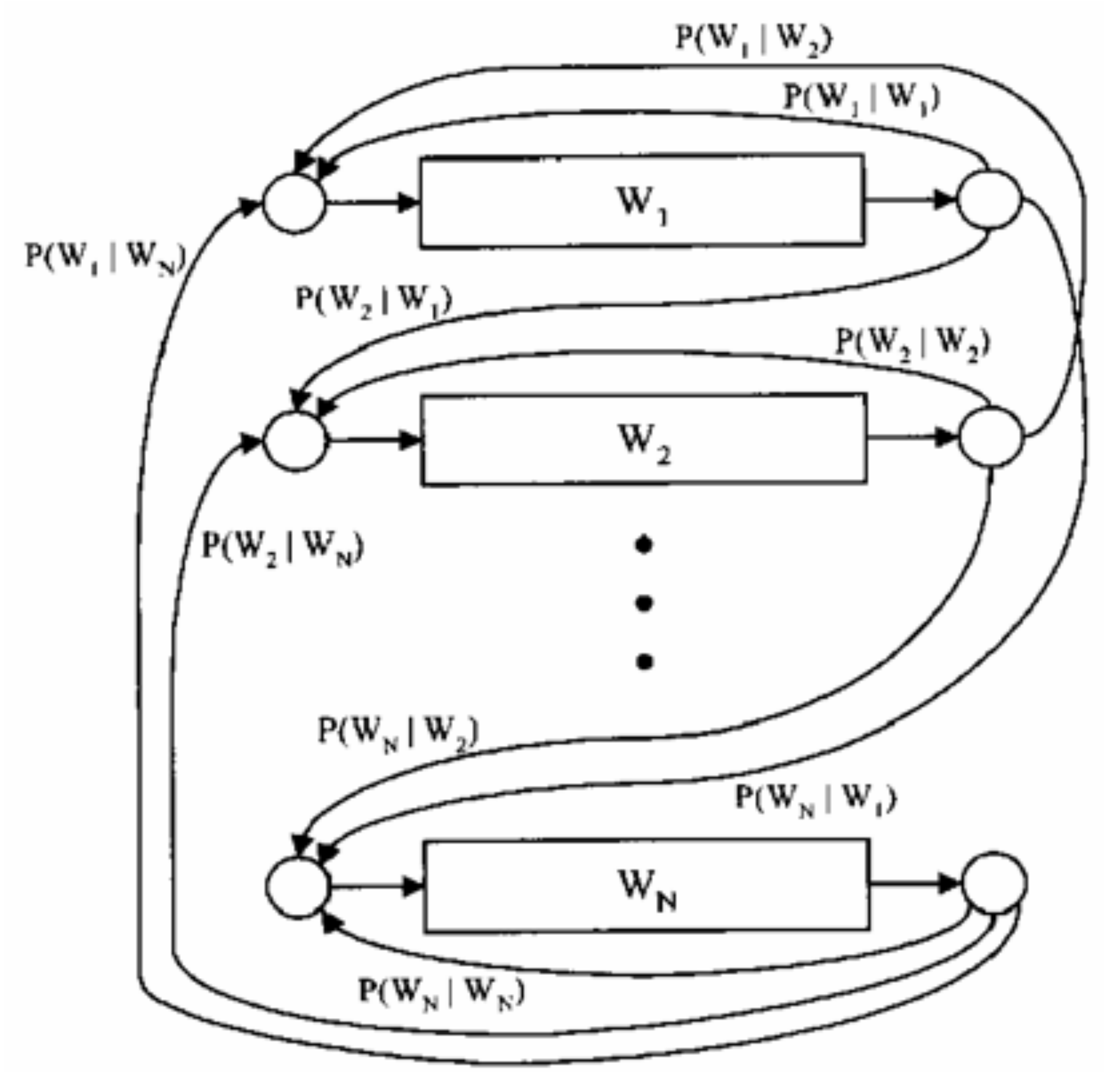
# Speech State Space

- HMM Specification
  - P(E|X) encodes which acoustic vectors are appropriate for each phoneme (each kind of sound)
  - P(X|X’) encodes how sounds can be strung together
- State Space
  - We will have one state for each sound in each word
  - Mostly, states advance sound by sound
  - Build a little state graph for each word and chain them together to form the state space X

# States in a Word



# Transitions with a Bigram Model




Training Counts

| |
|---|
| 198015222 the first |
| 194623024 the same |
| 168504105 the following |
| 158562063 the world |
| … |
| 14112454 the door |
| ----------------------------------- |
| 23135851162 the * |

$$\hat{P}(\text{door}|\text{the}) = \frac{14112454}{23135851162}$$

$$= 0.0006$$

Figure: Huang et al, p. 618

# Decoding

- Finding the words given the acoustics is an HMM inference problem
- Which state sequence $x_{1:T}$ is most likely given the evidence $e_{1:T}$?

$$x^*_{1:T} = \arg\max_{x_{1:T}} P(x_{1:T}|e_{1:T}) = \arg\max_{x_{1:T}} P(x_{1:T}, e_{1:T})$$

- From the sequence x, we can simply read off the words