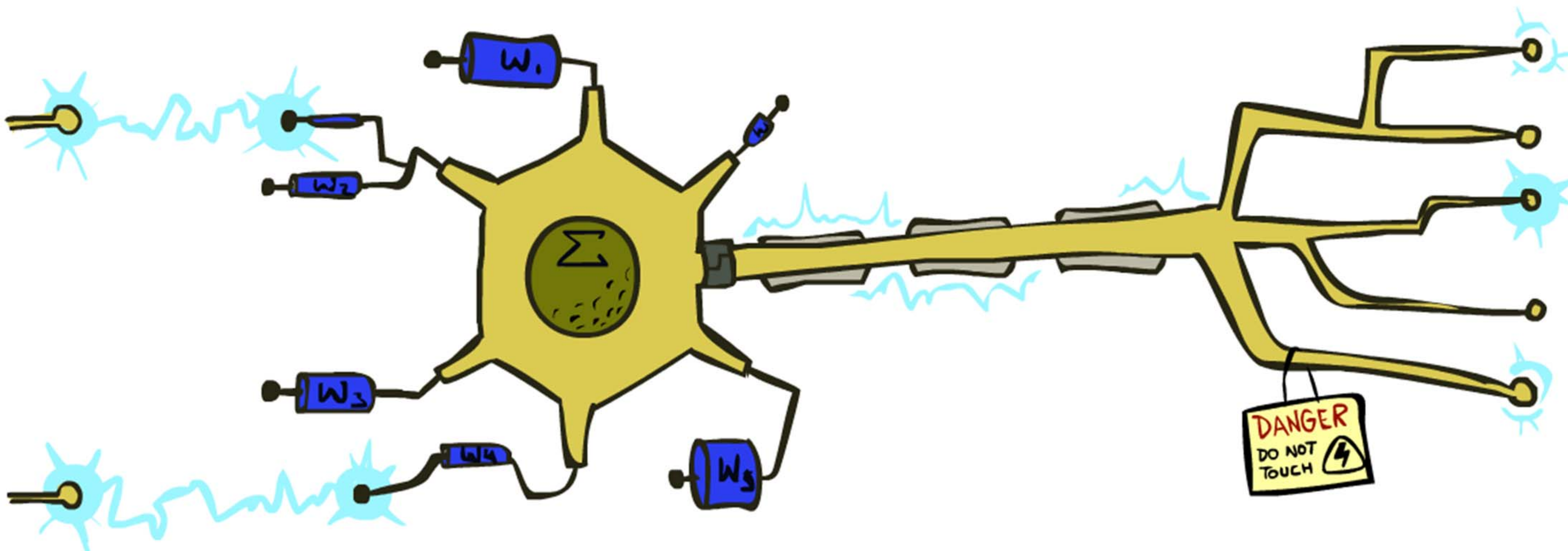
CS 188: Artificial Intelligence

Perceptrons



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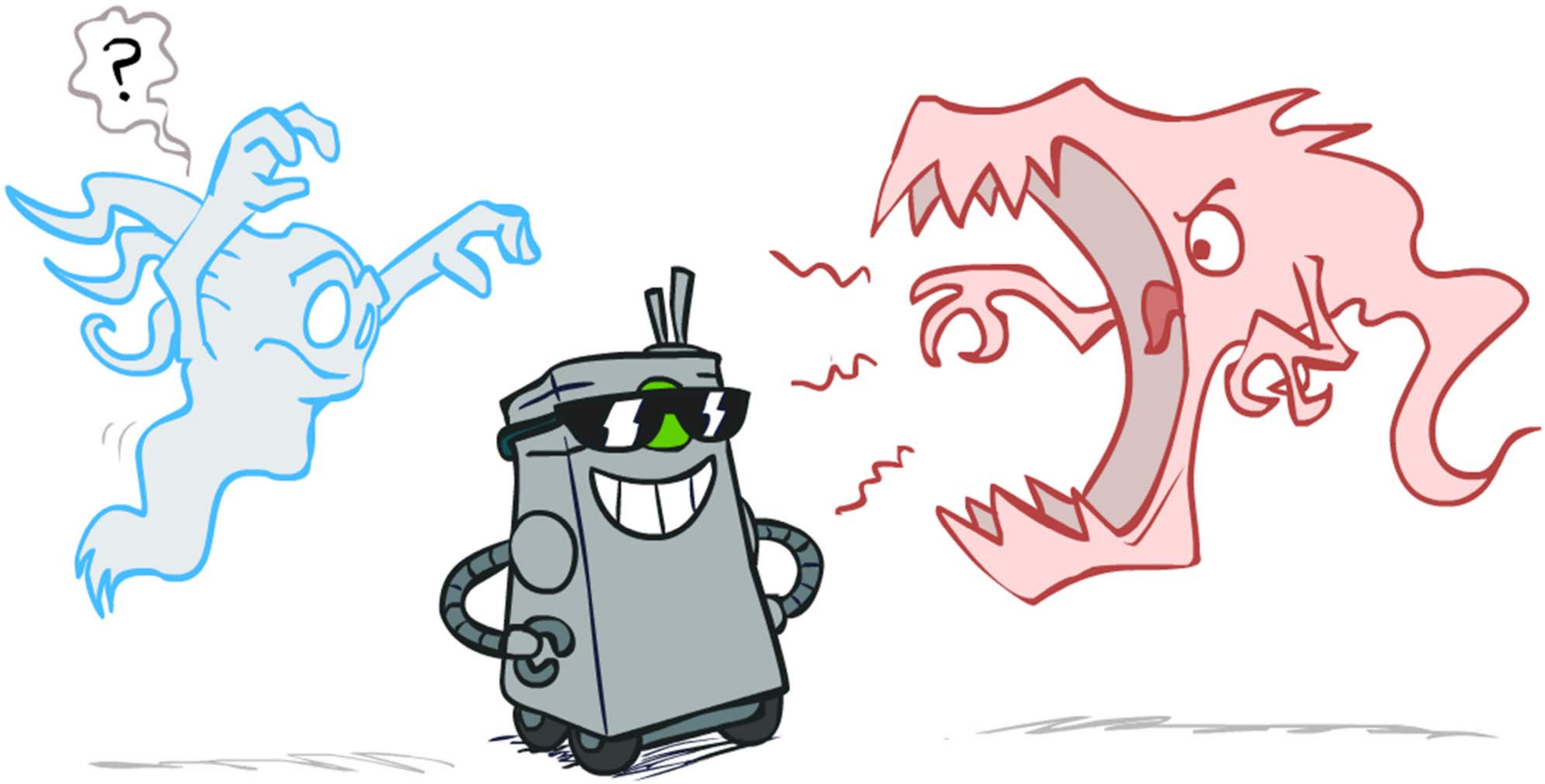
Smoothing



Laplace Smoothing

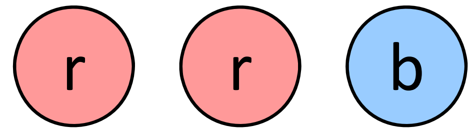
- Laplace's estimate (extended):
  - Pretend you saw every outcome k extra times

$$P_{LAP,k}(x) = \frac{c(x) + k}{N + k|X|}$$

  - What's Laplace with k = 0?
  - k is the strength of the prior

- Laplace for conditionals:
  - Smooth each condition independently:

$$P_{LAP,k}(x|y) = \frac{c(x,y) + k}{c(y) + k|X|}$$



$$P_{LAP,0}(X) =$$

$$P_{LAP,1}(X) =$$

$$P_{LAP,100}(X) =$$

Real NB: Smoothing

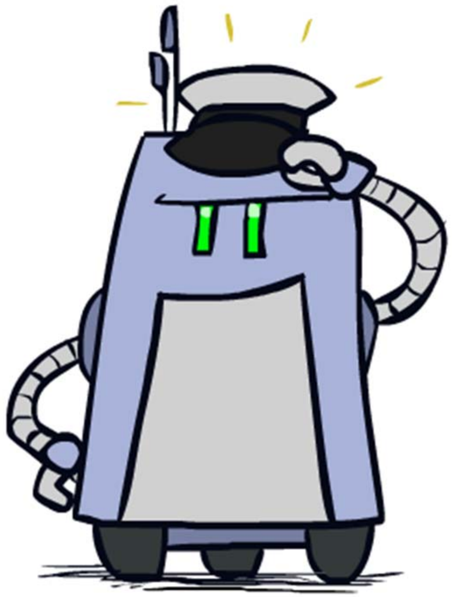
- For real classification problems, smoothing is critical
- New odds ratios:

$$\frac{P(W|\text{ham})}{P(W|\text{spam})}$$

```
helvetica : 11.4
seems     : 10.8
group     : 10.2
ago       :  8.4
areas     :  8.3
...
```

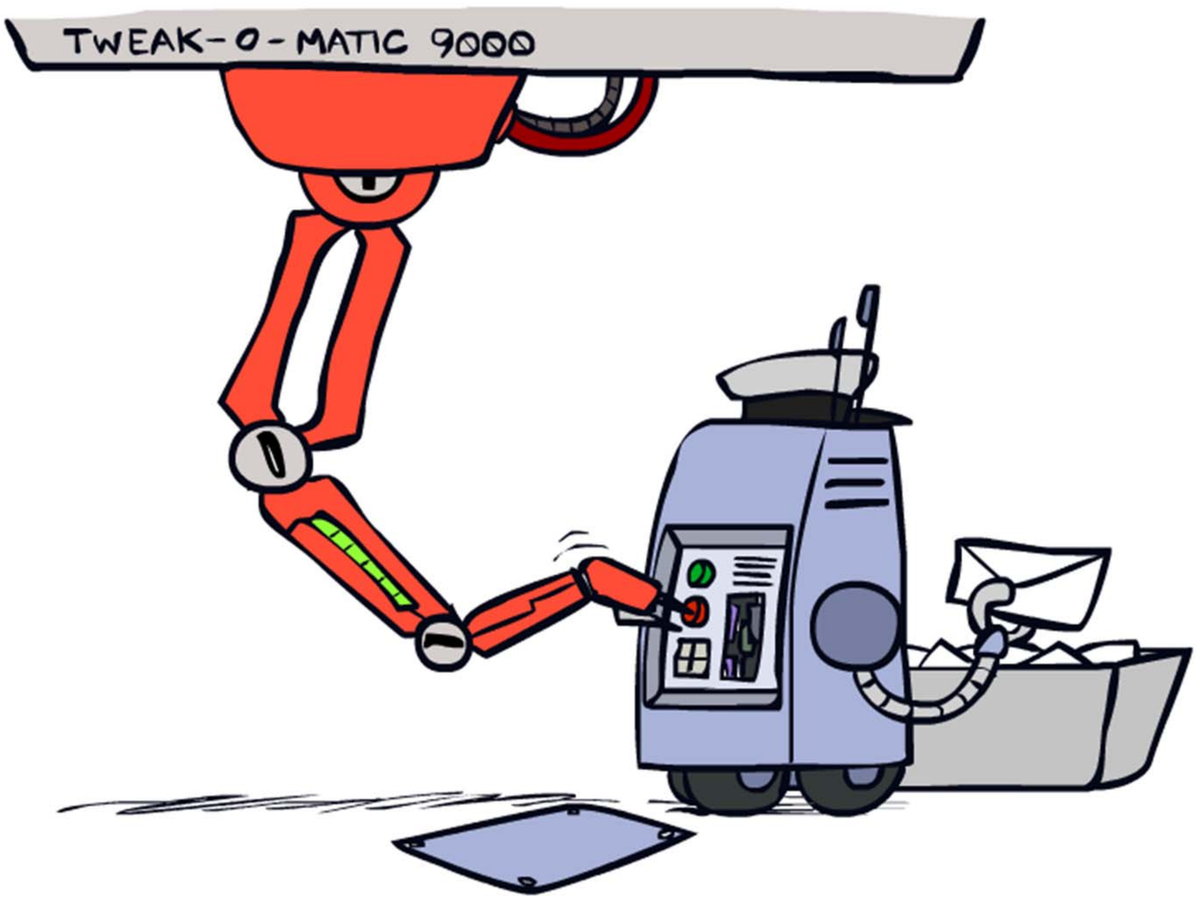
$$\frac{P(W|\text{spam})}{P(W|\text{ham})}$$

```
verdana : 28.8
Credit  : 28.4
ORDER   : 27.2
<FONT>  : 26.9
money   : 26.5
...
```



Do these make more sense?

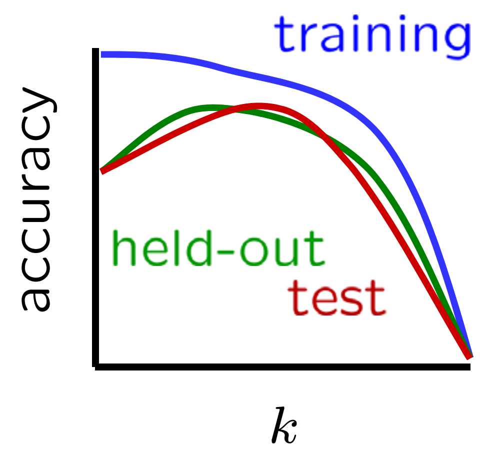
Tuning

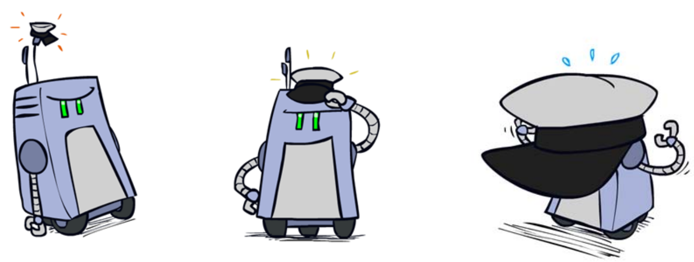


Tuning on Held-Out Data

- Now we've got two kinds of unknowns
  - Parameters: the probabilities P(X|Y), P(Y)
  - Hyperparameters: e.g. the amount / type of smoothing to do, k, α

- What should we learn where?
  - Learn parameters from training data
  - Tune hyperparameters on different data
    - Why?
  - For each value of the hyperparameters, train and test on the held-out data
  - Choose the best value and do a final test on the test data





Error-Driven Classification

Errors, and What to Do

- Examples of errors

```
Dear GlobalSCAPE Customer,

GlobalSCAPE has partnered with ScanSoft to offer you the
latest version of OmniPage Pro, for just $99.99* - the regular
list price is $499! The most common question we've received
about this offer is - Is this genuine? We would like to assure
you that this offer is authorized by ScanSoft, is genuine and
valid. You can get the . . .
```

```
. . . To receive your $30 Amazon.com promotional certificate,
click through to

  http://www.amazon.com/apparel

and see the prominent link for the $30 offer. All details are
there. We hope you enjoyed receiving this message. However, if
you'd rather not receive future e-mails announcing new store
launches, please click . . .
```

What to Do About Errors

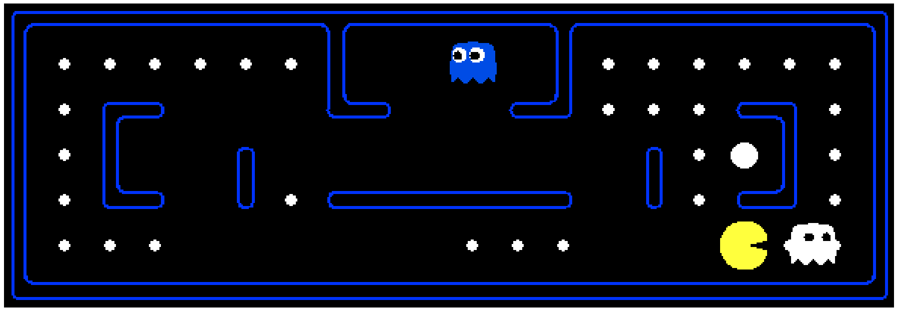
- Problem: there's still spam in your inbox

- Need more features – words aren't enough!
  - Have you emailed the sender before?
  - Have 1M other people just gotten the same email?
  - Is the sending information consistent?
  - Is the email in ALL CAPS?
  - Do inline URLs point where they say they point?
  - Does the email address you by (your) name?

- Naïve Bayes models can incorporate a variety of features, but tend to do best in homogeneous cases (e.g. all features are word occurrences)

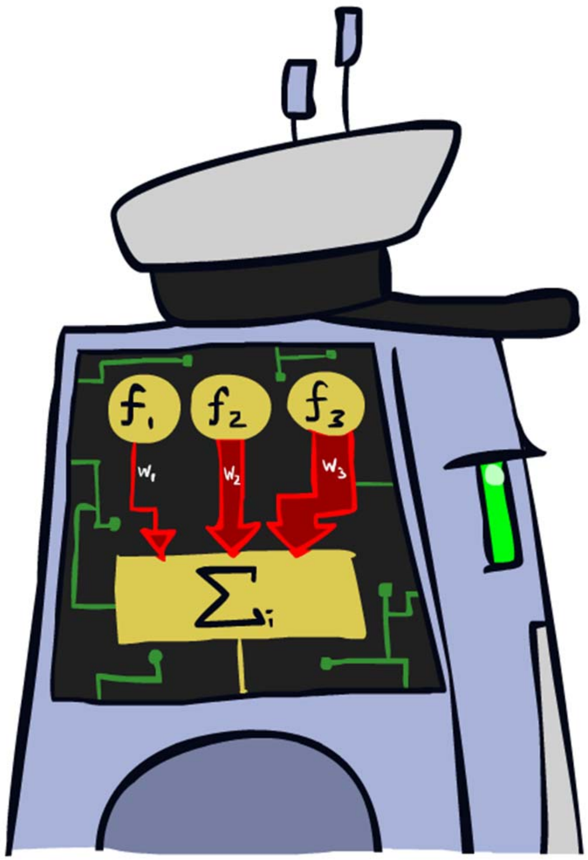
Later On...

Web Search



Decision
Problems

Linear Classifiers



Feature Vectors

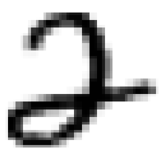
x $\quad$ $f(x)$ $\quad$ y

```
Hello,

Do you want free printr
cartriges?  Why pay more
when you can get them
ABSOLUTELY FREE!  Just
```

```
# free      : 2
YOUR_NAME   : 0
MISSPELLED  : 2
FROM_FRIEND : 0
...
```

SPAM
or
+

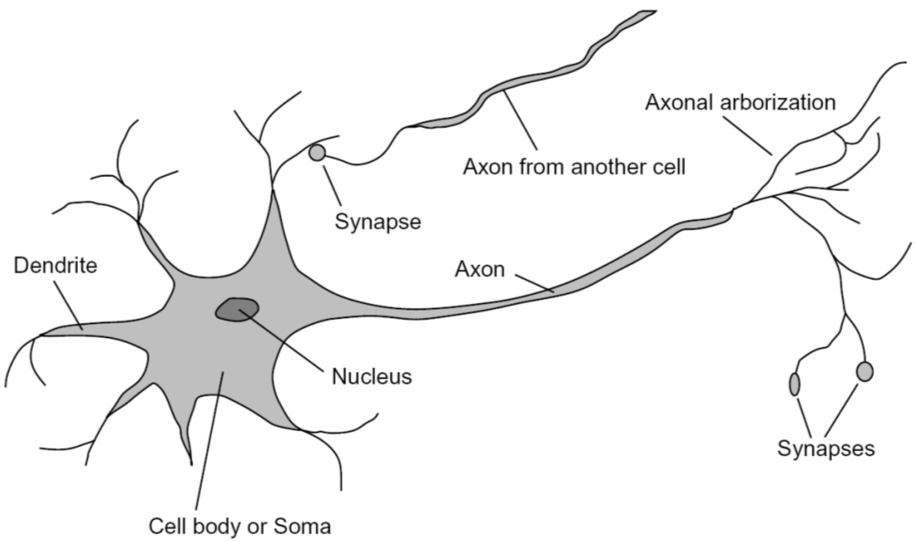


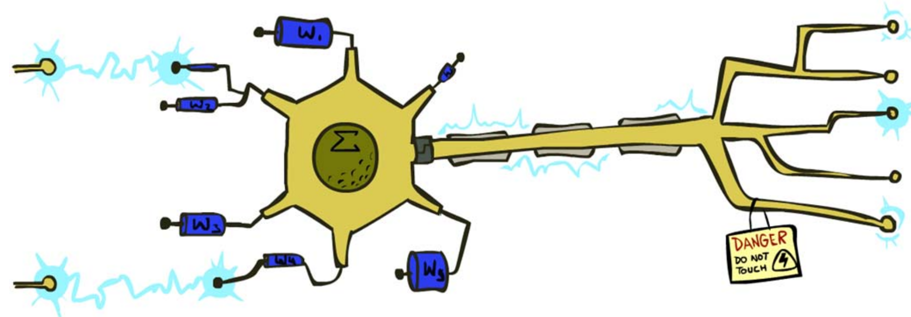
```
PIXEL-7,12  : 1
PIXEL-7,13  : 0
...
NUM_LOOPS   : 1
...
```

"2"

Some (Simplified) Biology

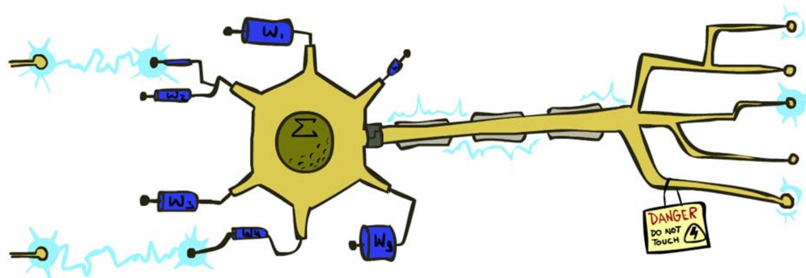
- Very loose inspiration: human neurons





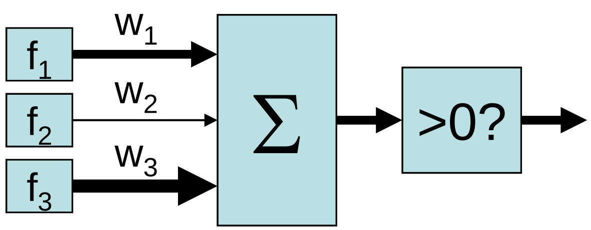
Linear Classifiers

- Inputs are feature values
- Each feature has a weight
- Sum is the activation



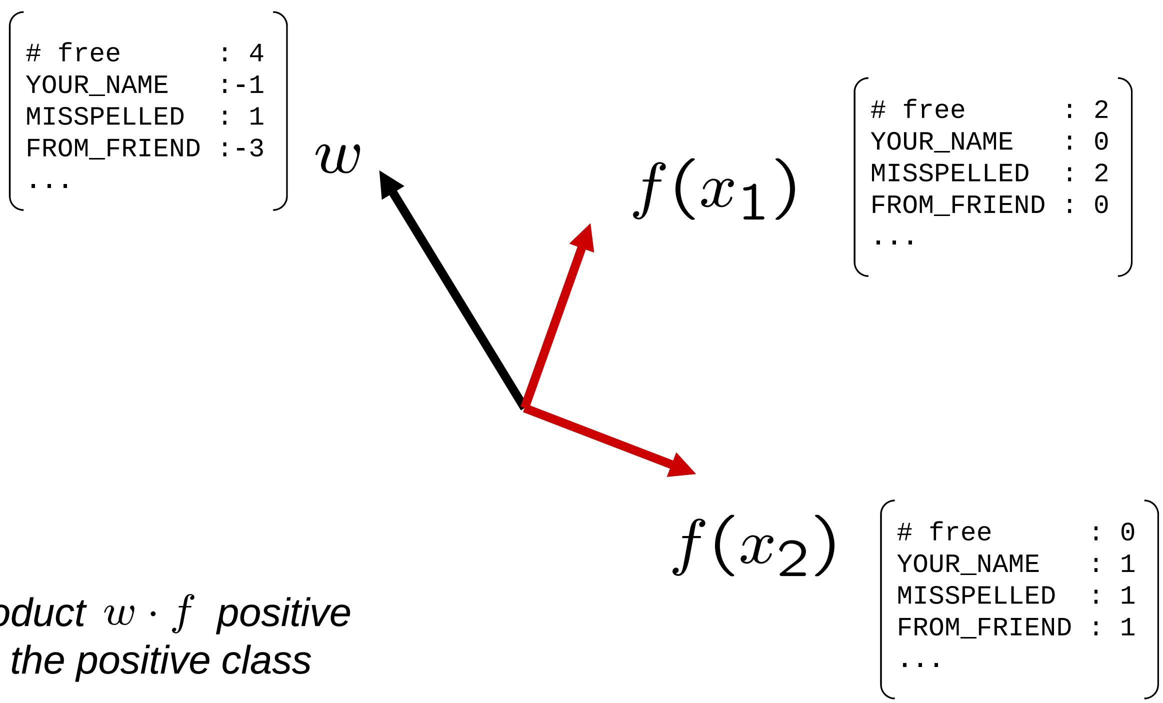
$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x) = w \cdot f(x)$$

- If the activation is:
  - Positive, output +1
  - Negative, output -1



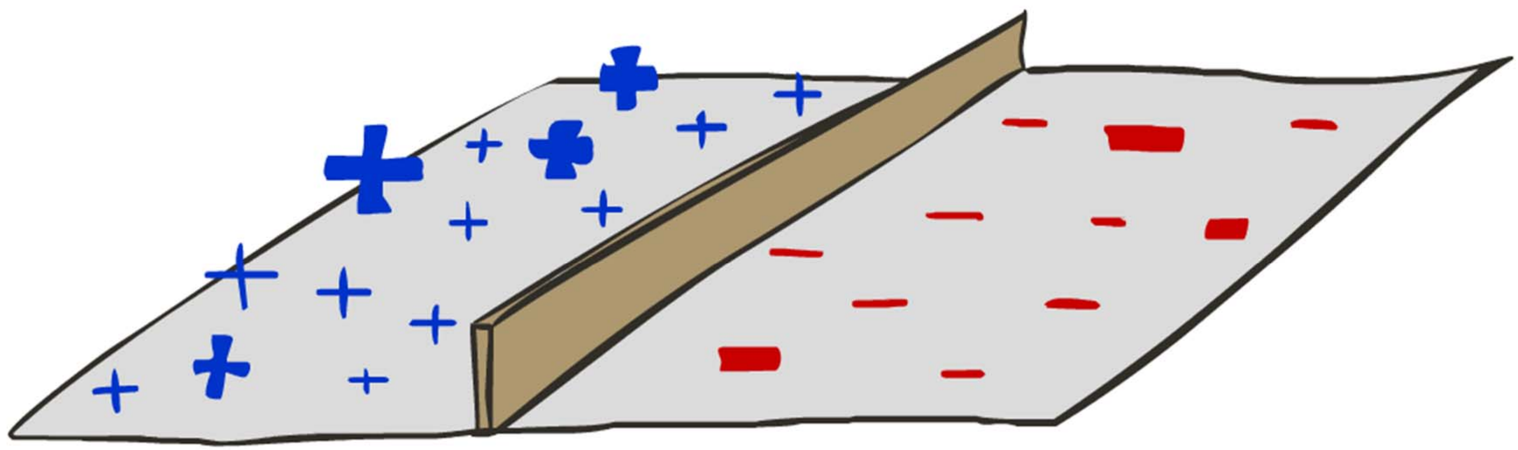
Weights

- Binary case: compare features to a weight vector
- Learning: figure out the weight vector from examples



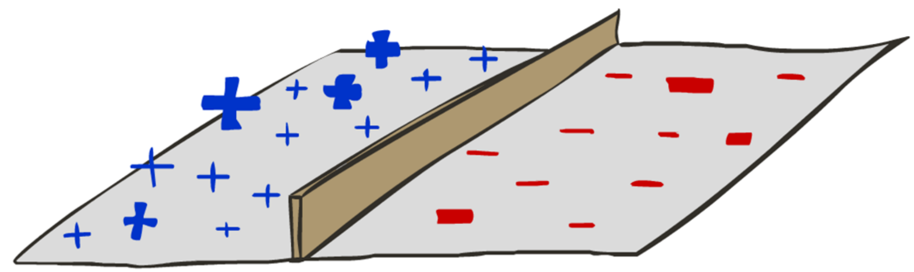
Dot product $w \cdot f$ positive means the positive class

Decision Rules



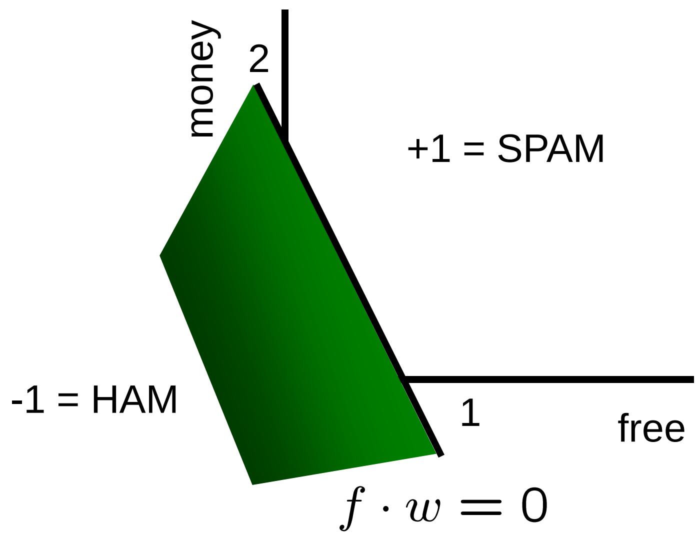
Binary Decision Rule

- In the space of feature vectors
  - Examples are points
  - Any weight vector is a hyperplane
  - One side corresponds to Y=+1
  - Other corresponds to Y=-1

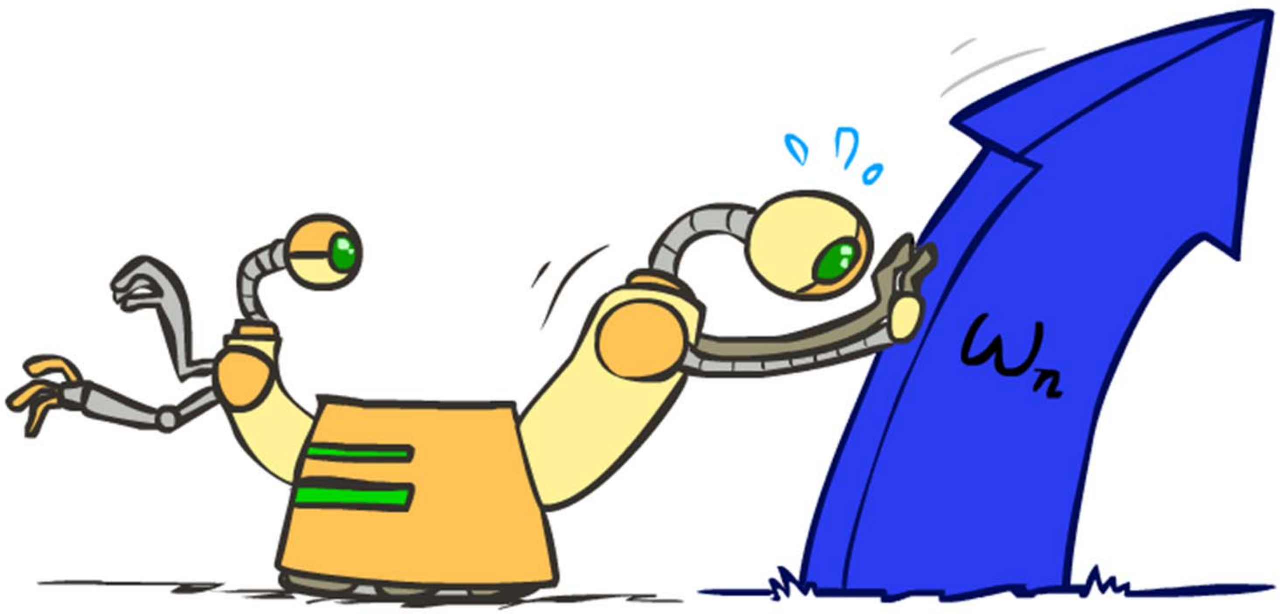


$$w$$

```
BIAS  : -3
free  :  4
money :  2
...
```

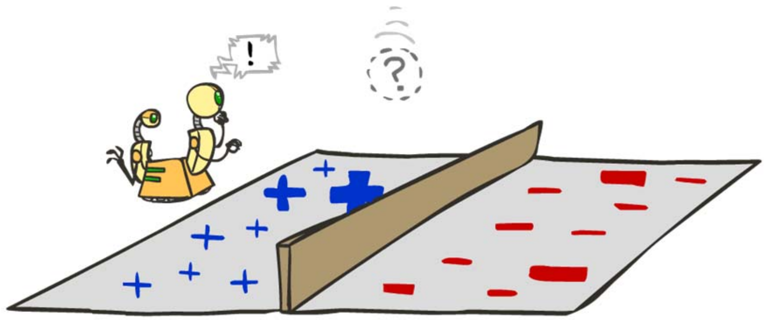


Weight Updates

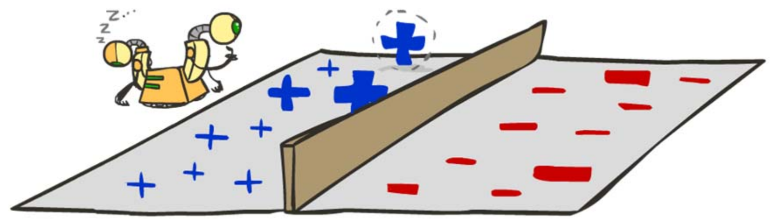


Learning: Binary Perceptron

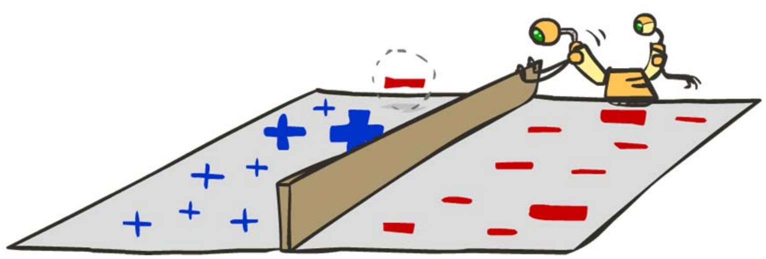
- Start with weights = 0
- For each training instance:
  - Classify with current weights



  - If correct (i.e., y=y*), no change!



  - If wrong: adjust the weight vector



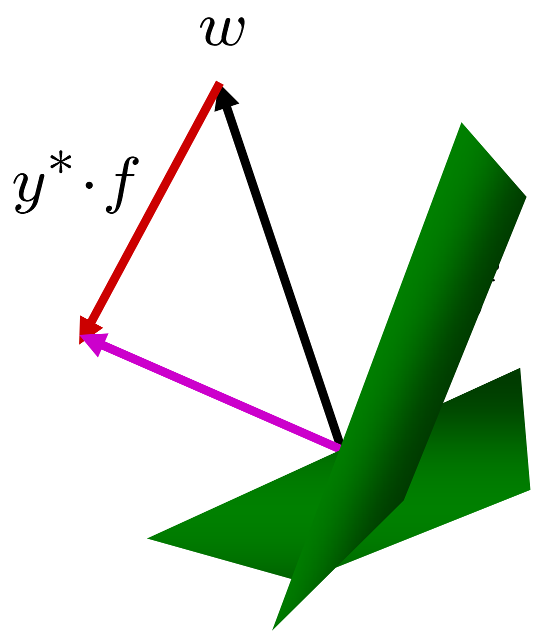
Learning: Binary Perceptron

- Start with weights = 0
- For each training instance:
  - Classify with current weights

$$y = \begin{cases} +1 & \text{if } \; w \cdot f(x) \geq 0 \\ -1 & \text{if } \; w \cdot f(x) < 0 \end{cases}$$

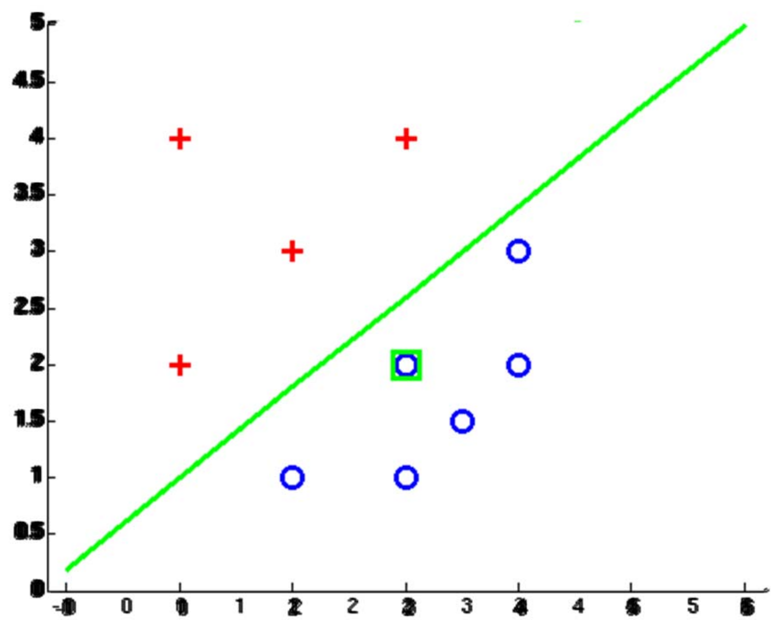
  - If correct (i.e., y=y*), no change!
  - If wrong: adjust the weight vector by adding or subtracting the feature vector. Subtract if y* is -1.

$$w = w + y^* \cdot f$$



Examples: Perceptron

- Separable Case



Multiclass Decision Rule

- If we have multiple classes:
  - A weight vector for each class:

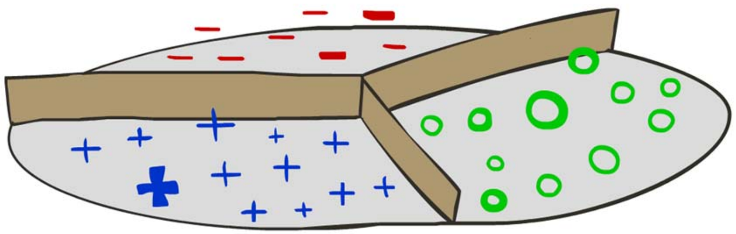
$$w_y$$

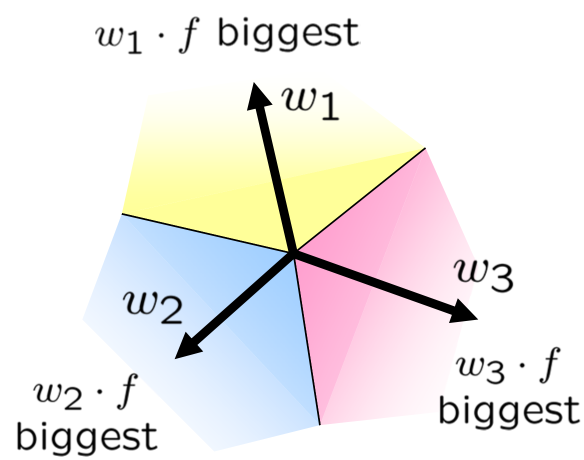
  - Score (activation) of a class y:

$$w_y \cdot f(x)$$

  - Prediction highest score wins

$$y = \arg\max_y \; w_y \cdot f(x)$$





Binary = multiclass where the negative class has weight zero

Learning: Multiclass Perceptron

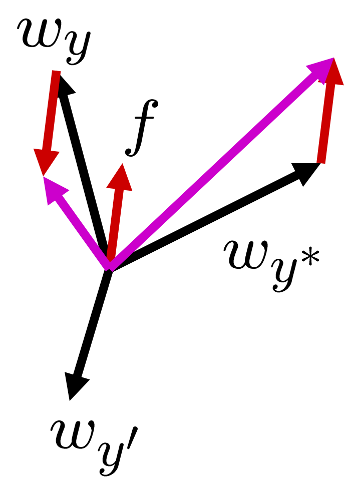
- Start with all weights = 0
- Pick up training examples one by one
- Predict with current weights

$$y = \arg\max_y \; w_y \cdot f(x)$$

- If correct, no change!
- If wrong: lower score of wrong answer, raise score of right answer

$$w_y = w_y - f(x)$$

$$w_{y^*} = w_{y^*} + f(x)$$



Example: Multiclass Perceptron

"win the vote"

"win the election"

"win the game"

w_{SPORTS}

```
BIAS  : 1
win   : 0
game  : 0
vote  : 0
the   : 0
...
```

$w_{POLITICS}$

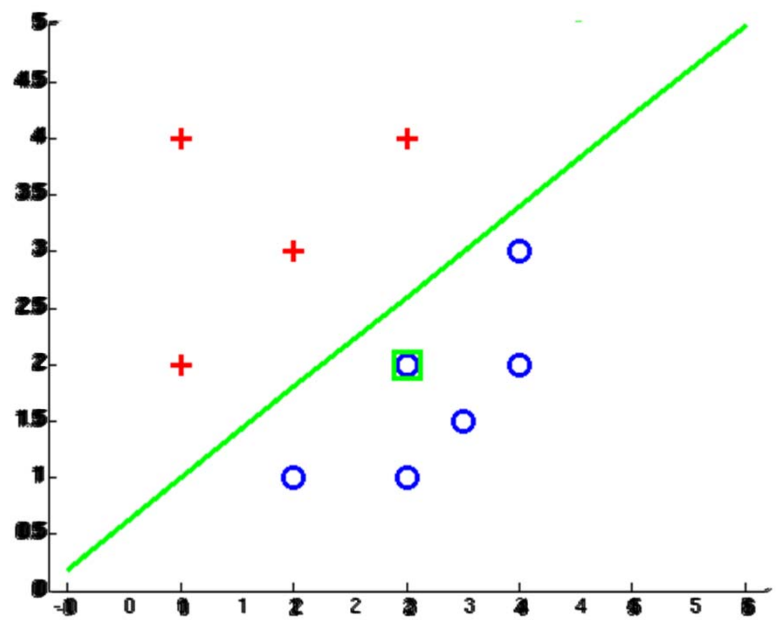
```
BIAS  : 0
win   : 0
game  : 0
vote  : 0
the   : 0
...
```

w_{TECH}

```
BIAS  : 0
win   : 0
game  : 0
vote  : 0
the   : 0
...
```

Examples: Perceptron

- Separable Case

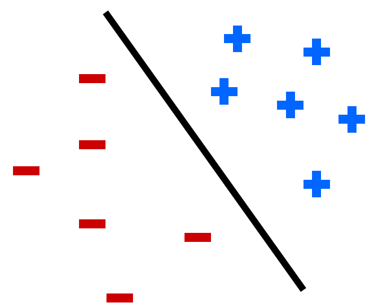


Properties of Perceptrons

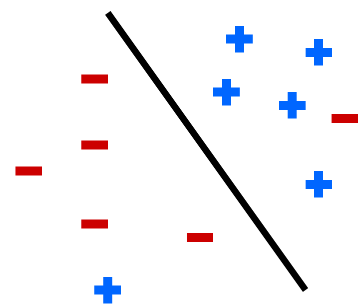
- Separability: true if some parameters get the training set perfectly correct
- Convergence: if the training is separable, perceptron will eventually converge (binary case)
- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

$$\text{mistakes} < \frac{k}{\delta^2}$$

Separable

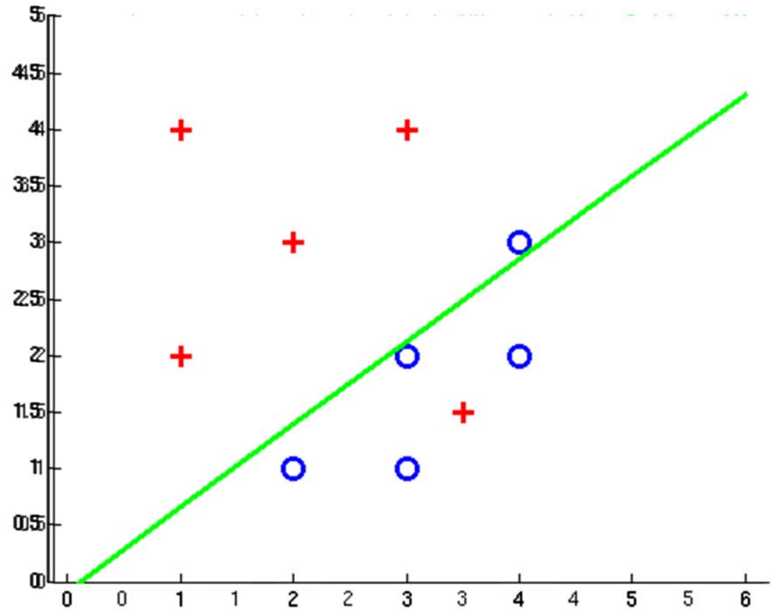


Non-Separable

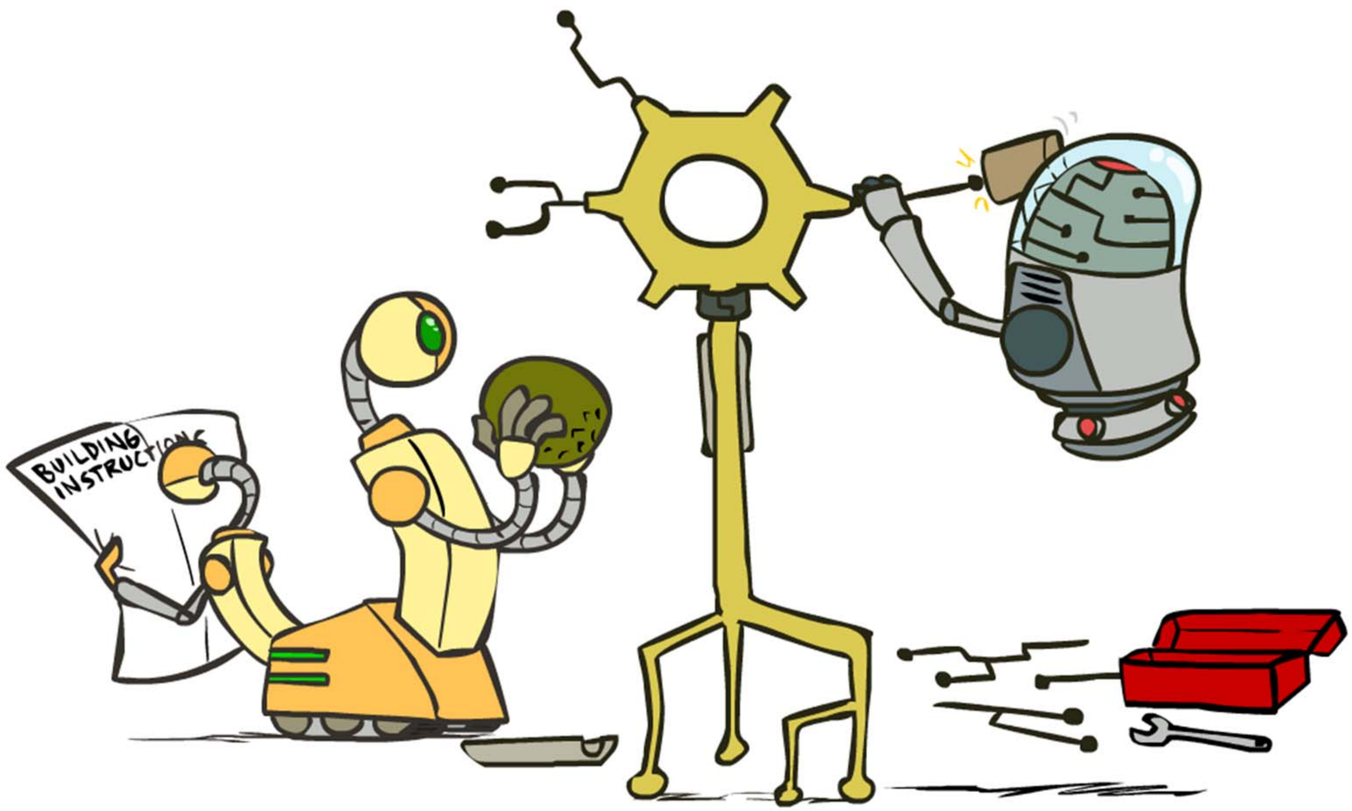


Examples: Perceptron

- Non-Separable Case

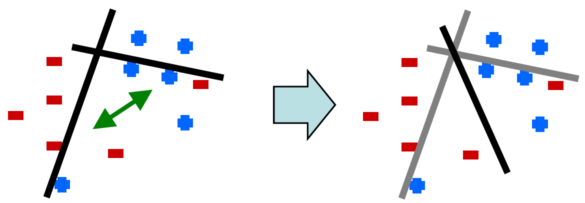


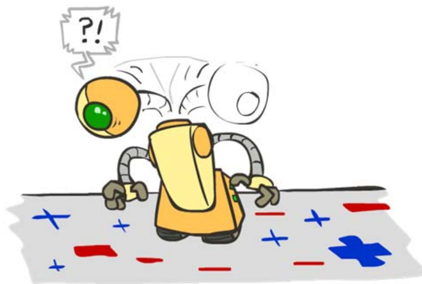
Improving the Perceptron



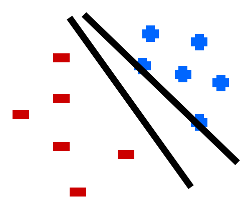
Problems with the Perceptron

- Noise: if the data isn't separable, weights might thrash
  - Averaging weight vectors over time can help (averaged perceptron)

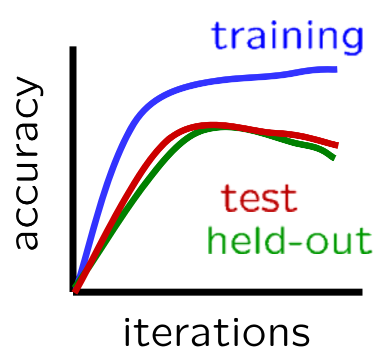


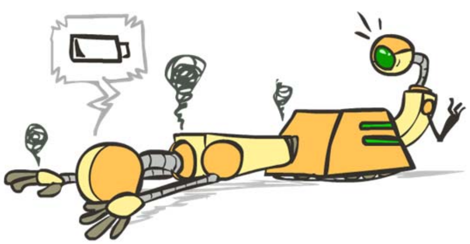


- Mediocre generalization: finds a "barely" separating solution



- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining is a kind of overfitting





Fixing the Perceptron

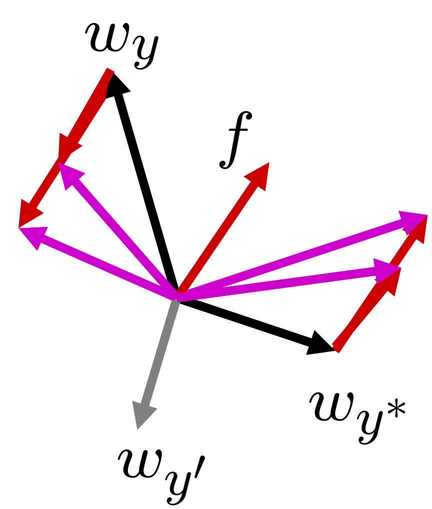
- Idea: adjust the weight update to mitigate these effects
- MIRA*: choose an update size that fixes the current mistake…
- … but, minimizes the change to w

$$\min_w \frac{1}{2} \sum_y ||w_y - w'_y||^2$$

$$w_{y^*} \cdot f(x) \geq w_y \cdot f(x) + 1$$

- The +1 helps to generalize

* Margin Infused Relaxed Algorithm



Guessed y instead of y^* on example x with features $f(x)$

$$w_y = w'_y - \tau f(x)$$
$$w_{y^*} = w'_{y^*} + \tau f(x)$$

Minimum Correcting Update

$$\min_{w} \frac{1}{2} \sum_{y} ||w_y - w'_y||^2$$

$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$

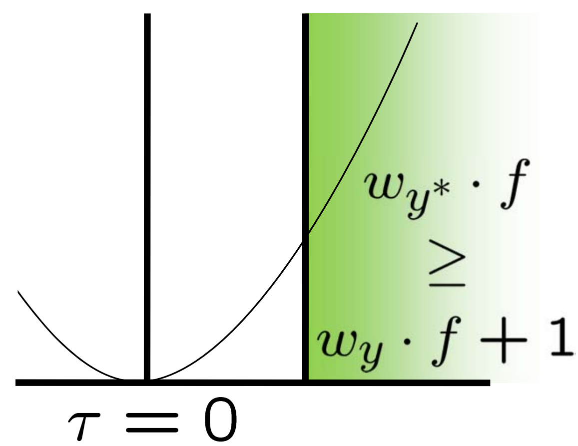
$$\min_{\tau} ||\tau f||^2$$

$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$

$$(w'_{y^*} + \tau f) \cdot f = (w'_y - \tau f) \cdot f + 1$$

$$\tau = \frac{(w'_y - w'_{y^*}) \cdot f + 1}{2 f \cdot f}$$

$$\boxed{\begin{aligned} w_y &= w'_y - \tau f(x) \\ w_{y^*} &= w'_{y^*} + \tau f(x) \end{aligned}}$$

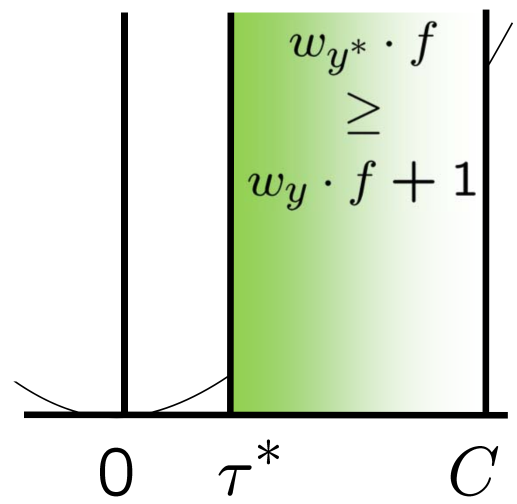


min not τ=0, or would not have made an error, so min will be where equality holds

Maximum Step Size

- In practice, it's also bad to make updates that are too large
  - Example may be labeled incorrectly
  - You may not have enough features
  - Solution: cap the maximum possible value of τ with some constant C

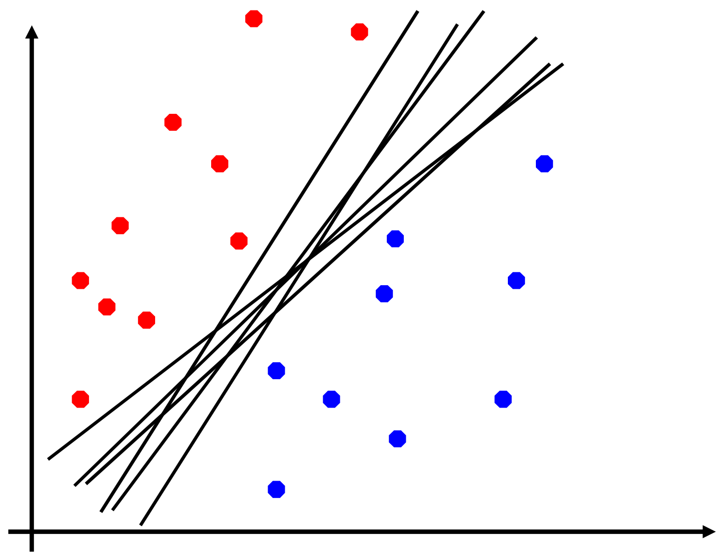
$$\tau^* = \min\left(\frac{(w'_y - w'_{y^*}) \cdot f + 1}{2f \cdot f}, C\right)$$



  - Corresponds to an optimization that assumes non-separable data
  - Usually converges faster than perceptron
  - Usually better, especially on noisy data

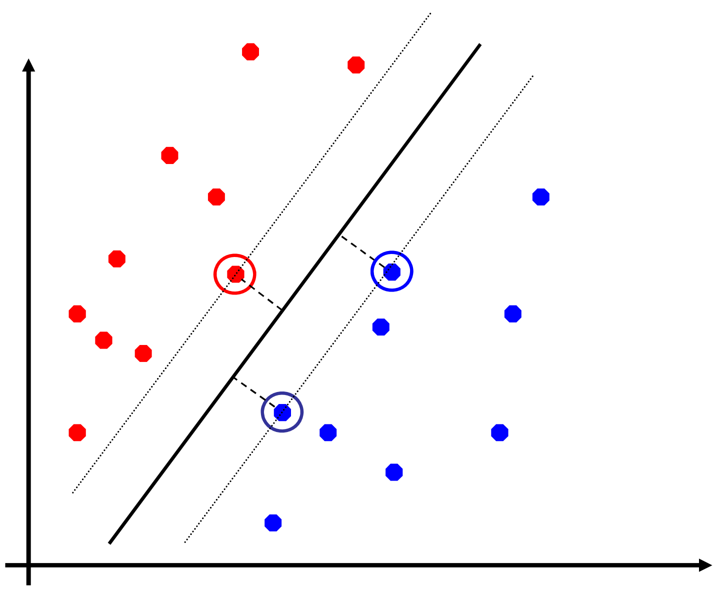
Linear Separators

- Which of these linear separators is optimal?



Support Vector Machines

- Maximizing the margin: good according to intuition, theory, practice
- Only support vectors matter; other training examples are ignorable
- Support vector machines (SVMs) find the separator with max margin
- Basically, SVMs are MIRA where you optimize over all examples at once



MIRA

$$\min_{w} \frac{1}{2}||w - w'||^2$$

$$w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

SVM

$$\min_{w} \frac{1}{2}||w||^2$$

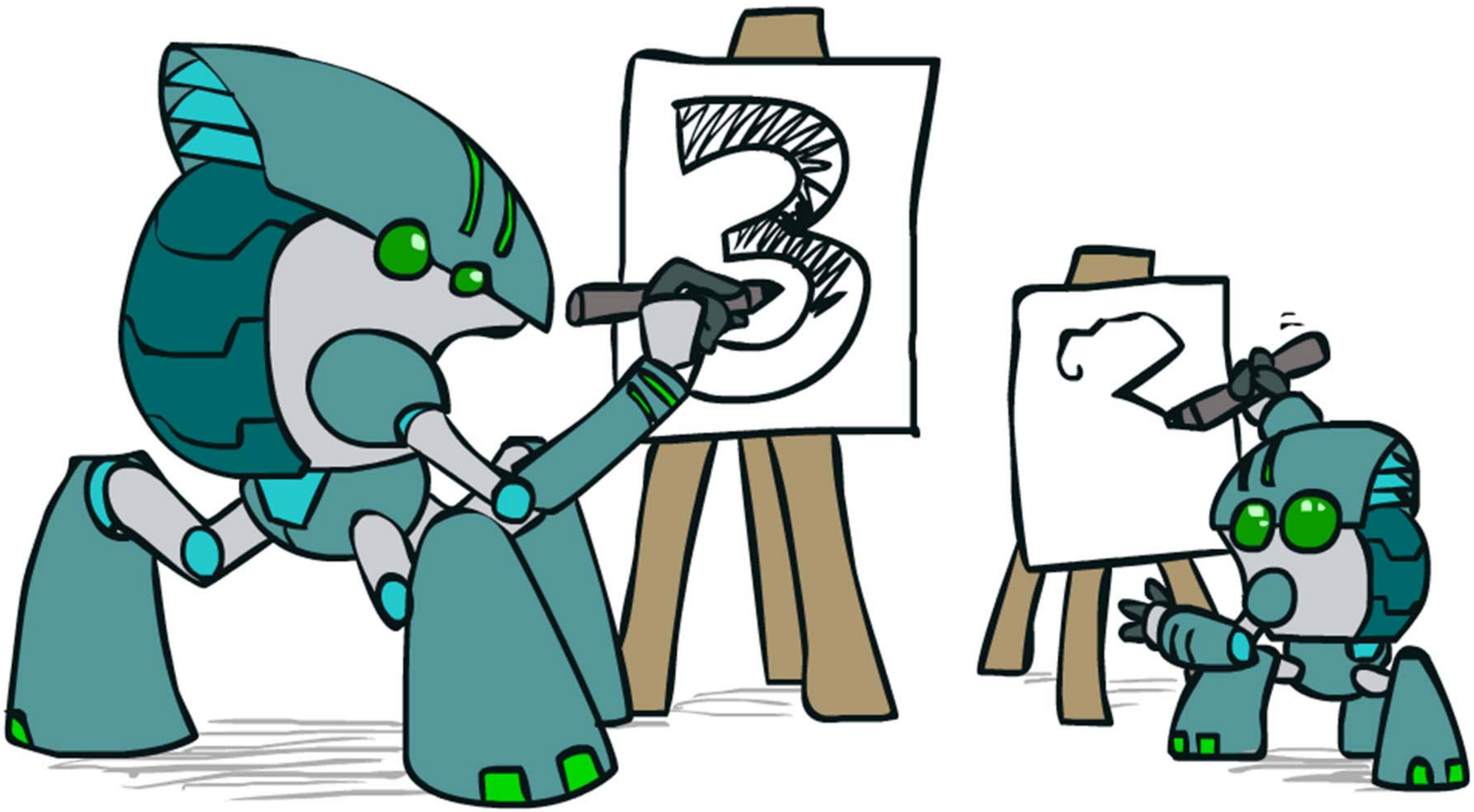
$$\forall i, y \; w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

Classification: Comparison

- Naïve Bayes
  - Builds a model training data
  - Gives prediction probabilities
  - Strong assumptions about feature independence
  - One pass through data (counting)

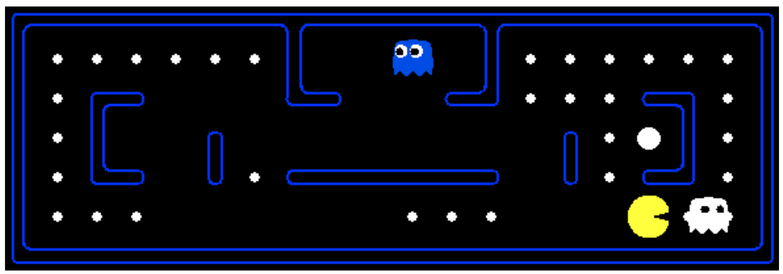
- Perceptrons / MIRA:
  - Makes less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data (prediction)
  - Often more accurate

Apprenticeship



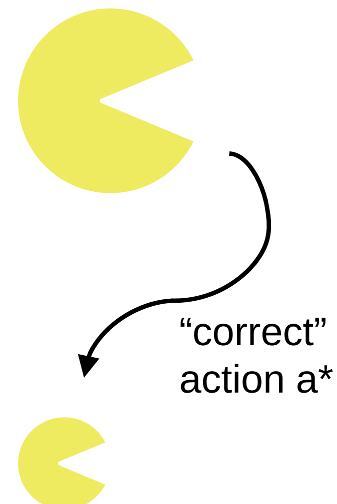
Pacman Apprenticeship!

- Examples are states s



- Candidates are pairs (s,a)
- "Correct" actions: those taken by expert
- Features defined over (s,a) pairs: f(s,a)
- Score of a q-state (s,a) given by:

$$w \cdot f(s,a)$$



$$\forall a \neq a^*,$$
$$w \cdot f(a^*) > w \cdot f(a)$$

- How is this VERY different from reinforcement learning?