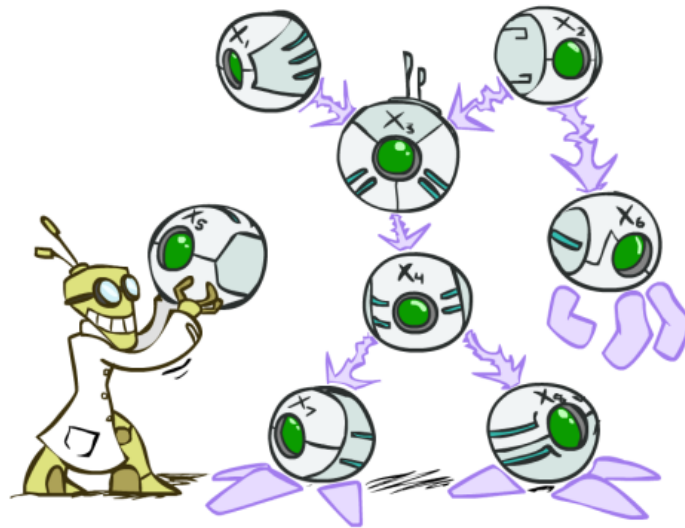


CS 188: Artificial Intelligence

Bayes' Nets



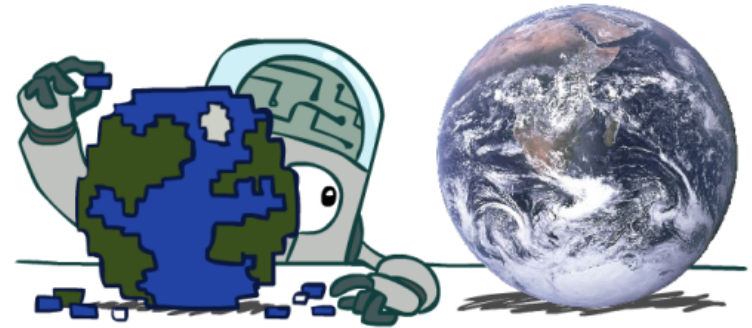
Pieter Abbeel, Dan Klein
University of California, Berkeley

Probabilistic Models

- Models describe how (a portion of) the world works

- **Models are always simplifications**

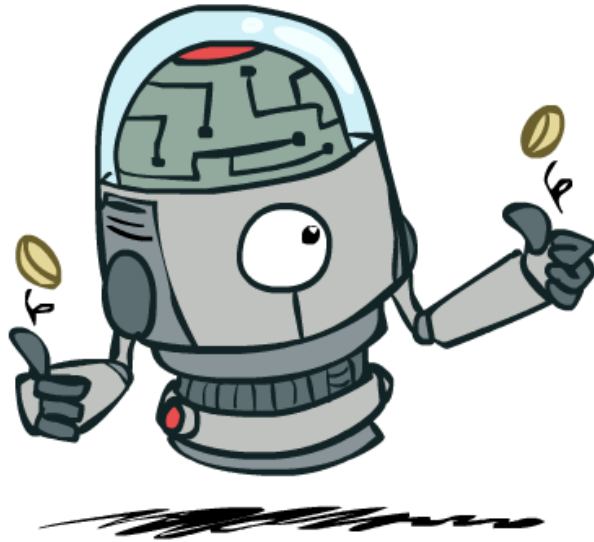
- May not account for every variable
 - May not account for all interactions between variables
 - “All models are wrong; but some are useful.”
– George E. P. Box



- **What do we do with probabilistic models?**

- We (or our agents) need to reason about unknown variables, given evidence
 - Example: explanation (diagnostic reasoning)
 - Example: prediction (causal reasoning)
 - Example: value of information

Independence



Independence

- Two variables are *independent* if:

$$\forall x, y : P(x, y) = P(x)P(y)$$

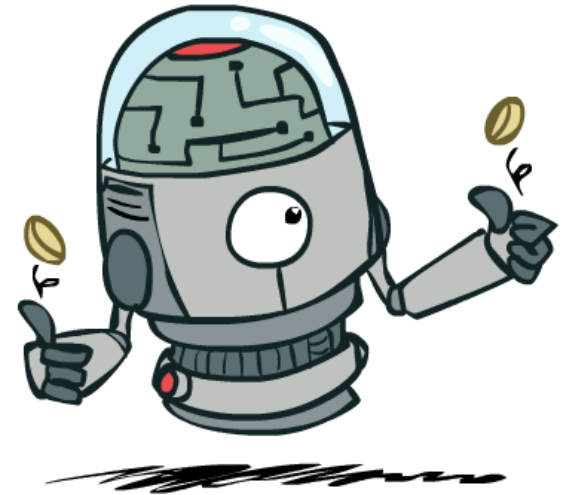
- This says that their joint distribution *factors* into a product two simpler distributions
- Another form:

$$\forall x, y : P(x|y) = P(x)$$

- We write: $X \perp\!\!\!\perp Y$

- Independence is a simplifying *modeling assumption*

- Empirical* joint distributions: at best “close” to independent
- What could we assume for {Weather, Traffic, Cavity, Toothache}?



Example: Independence?

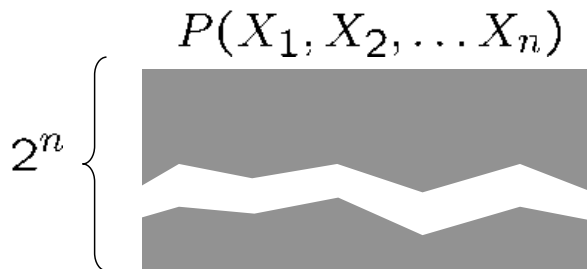
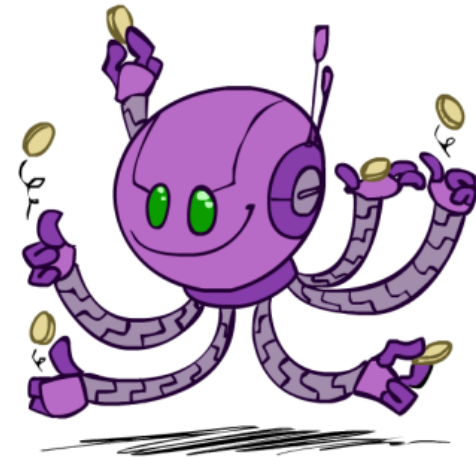
$P_1(T, W)$			$P(T)$		$P_2(T, W)$		
T	W	P	T	P	T	W	P
hot	sun	0.4	hot	0.5	hot	sun	0.3
hot	rain	0.1	cold	0.5	hot	rain	0.2
cold	sun	0.2			cold	sun	0.3
cold	rain	0.3			cold	rain	0.2

$P(W)$	
W	P
sun	0.6
rain	0.4

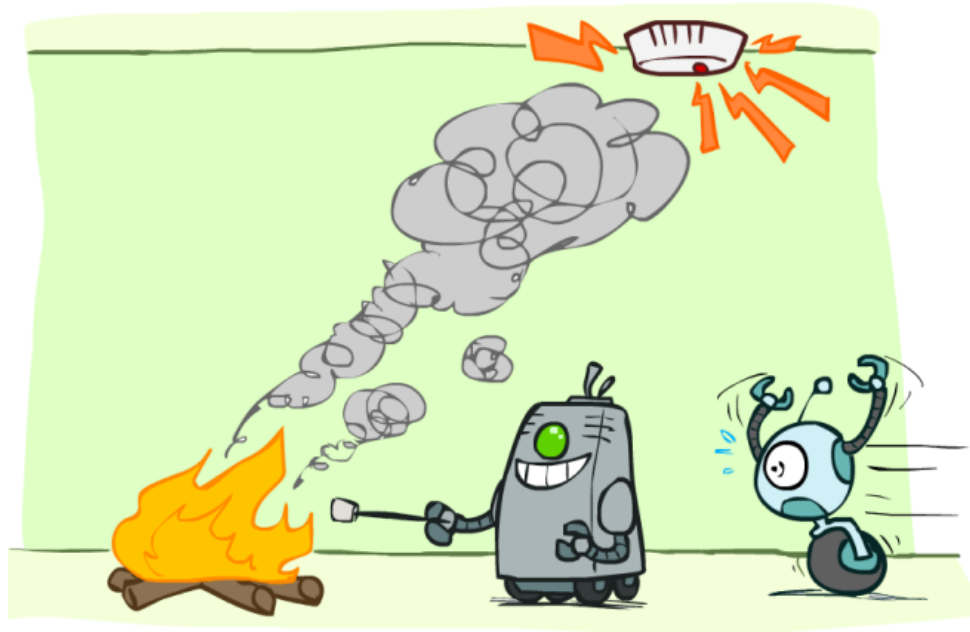
Example: Independence

- N fair, independent coin flips:

$P(X_1)$		$P(X_2)$		\dots		$P(X_n)$	
H	0.5	H	0.5			H	0.5
T	0.5	T	0.5			T	0.5

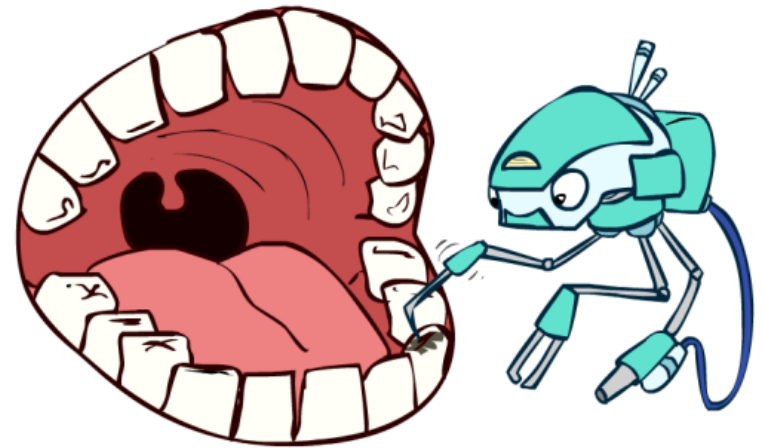


Conditional Independence



Conditional Independence

- $P(\text{Toothache}, \text{Cavity}, \text{Catch})$
- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
 - $P(+\text{catch} \mid +\text{toothache}, +\text{cavity}) = P(+\text{catch} \mid +\text{cavity})$
- The same independence holds if I don't have a cavity:
 - $P(+\text{catch} \mid +\text{toothache}, -\text{cavity}) = P(+\text{catch} \mid -\text{cavity})$
- Catch is *conditionally independent* of Toothache given Cavity:
 - $P(\text{Catch} \mid \text{Toothache}, \text{Cavity}) = P(\text{Catch} \mid \text{Cavity})$
- Equivalent statements:
 - $P(\text{Toothache} \mid \text{Catch}, \text{Cavity}) = P(\text{Toothache} \mid \text{Cavity})$
 - $P(\text{Toothache}, \text{Catch} \mid \text{Cavity}) = P(\text{Toothache} \mid \text{Cavity}) P(\text{Catch} \mid \text{Cavity})$
 - One can be derived from the other easily



Conditional Independence

- Unconditional (absolute) independence very rare (why?)
- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments.

- X is conditionally independent of Y given Z

$$X \perp\!\!\!\perp Y | Z$$

if and only if:

$$\forall x, y, z : P(x, y | z) = P(x | z)P(y | z)$$

or, equivalently, if and only if

$$\forall x, y, z : P(x | z, y) = P(x | z)$$

Conditional Independence

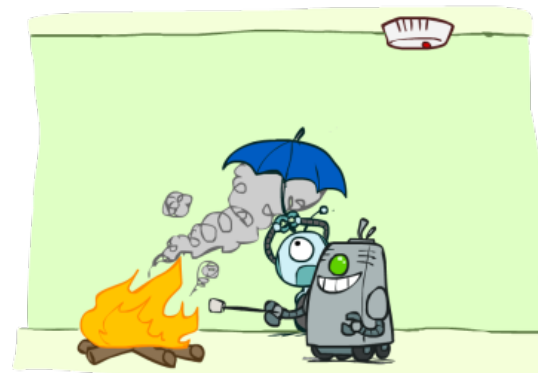
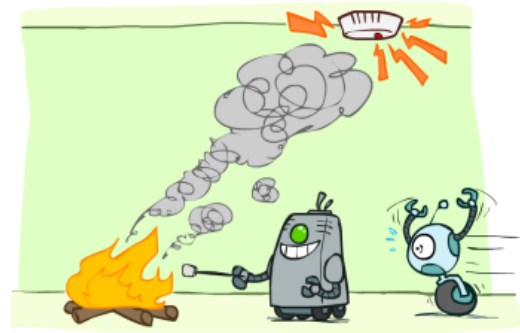
- What about this domain:
 - Traffic
 - Umbrella
 - Raining



Conditional Independence

- What about this domain:

- Fire
- Smoke
- Alarm



Conditional Independence and the Chain Rule

- Chain rule: $P(X_1, X_2, \dots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2) \dots$

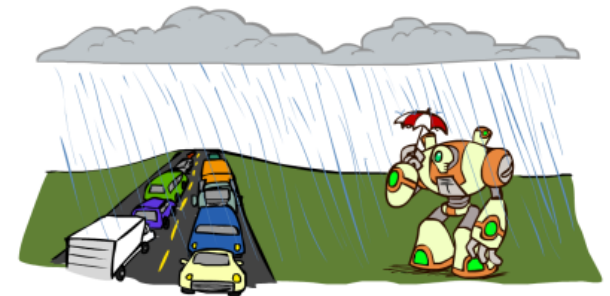
- Trivial decomposition:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) = \\ P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain}, \text{Traffic})$$

- With assumption of conditional independence:

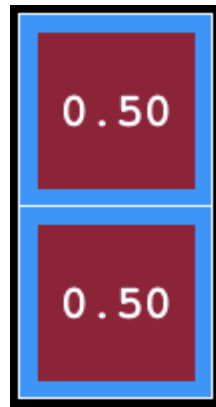
$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) = \\ P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain})$$

- Bayes' nets / graphical models help us express conditional independence assumptions



Ghostbusters Chain Rule

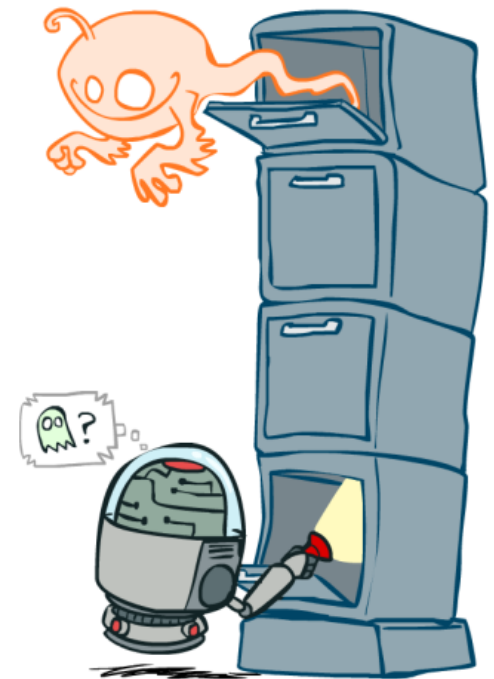
- Each sensor depends only on where the ghost is
- That means, the two sensors are conditionally independent, given the ghost position
- T: Top square is red
B: Bottom square is red
G: Ghost is in the top



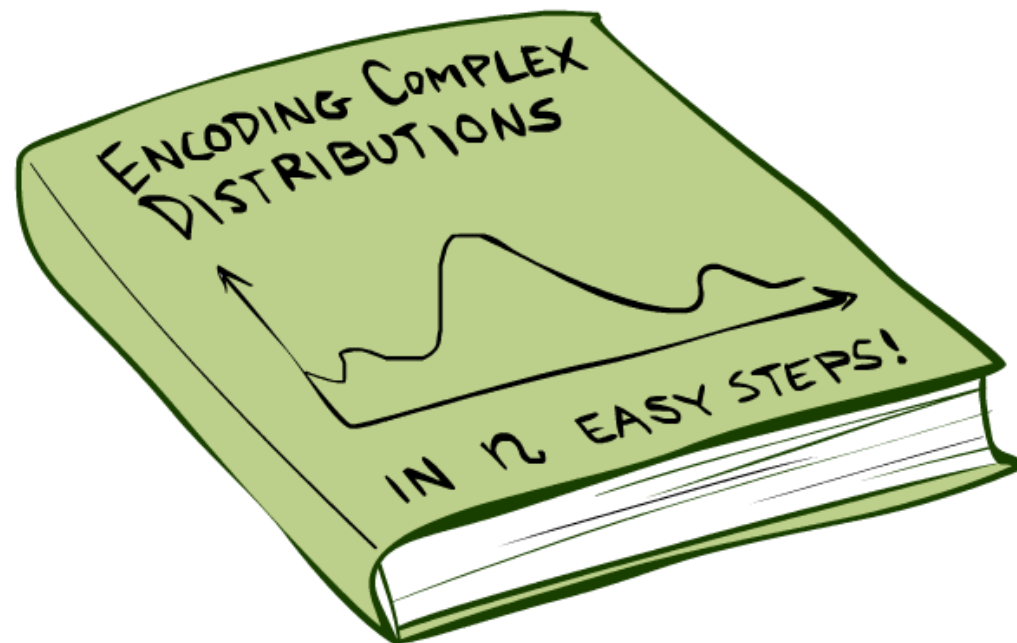
- Givens:
 $P(+g) = 0.5$
 $P(-g) = 0.5$
 $P(+t \mid +g) = 0.8$
 $P(+t \mid -g) = 0.4$
 $P(+b \mid +g) = 0.4$
 $P(+b \mid -g) = 0.8$

$$P(T,B,G) = P(G) P(T \mid G) P(B \mid G)$$

T	B	G	P(T,B,G)
+t	+b	+g	0.16
+t	+b	-g	0.16
+t	-b	+g	0.24
+t	-b	-g	0.04
-t	+b	+g	0.04
-t	+b	-g	0.24
-t	-b	+g	0.06
-t	-b	-g	0.06

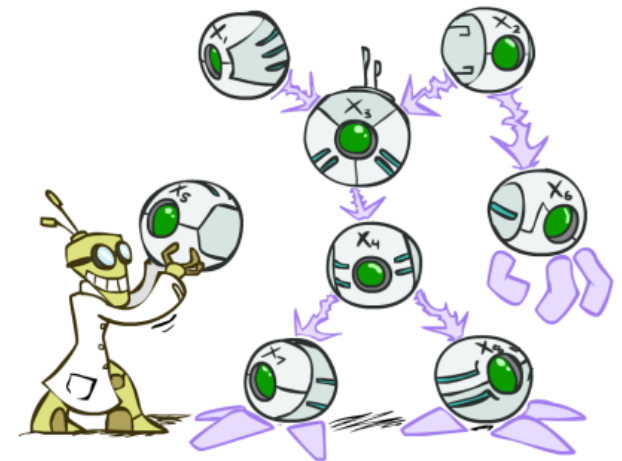


Bayes' Nets: Big Picture

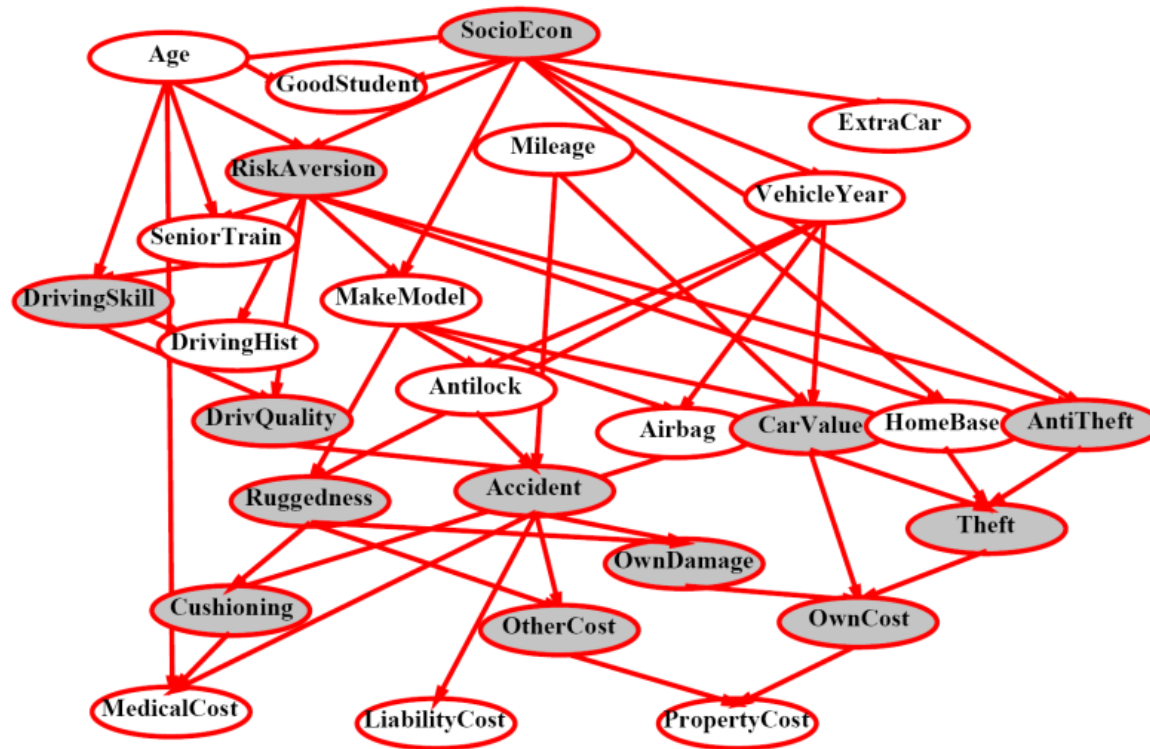


Bayes' Nets: Big Picture

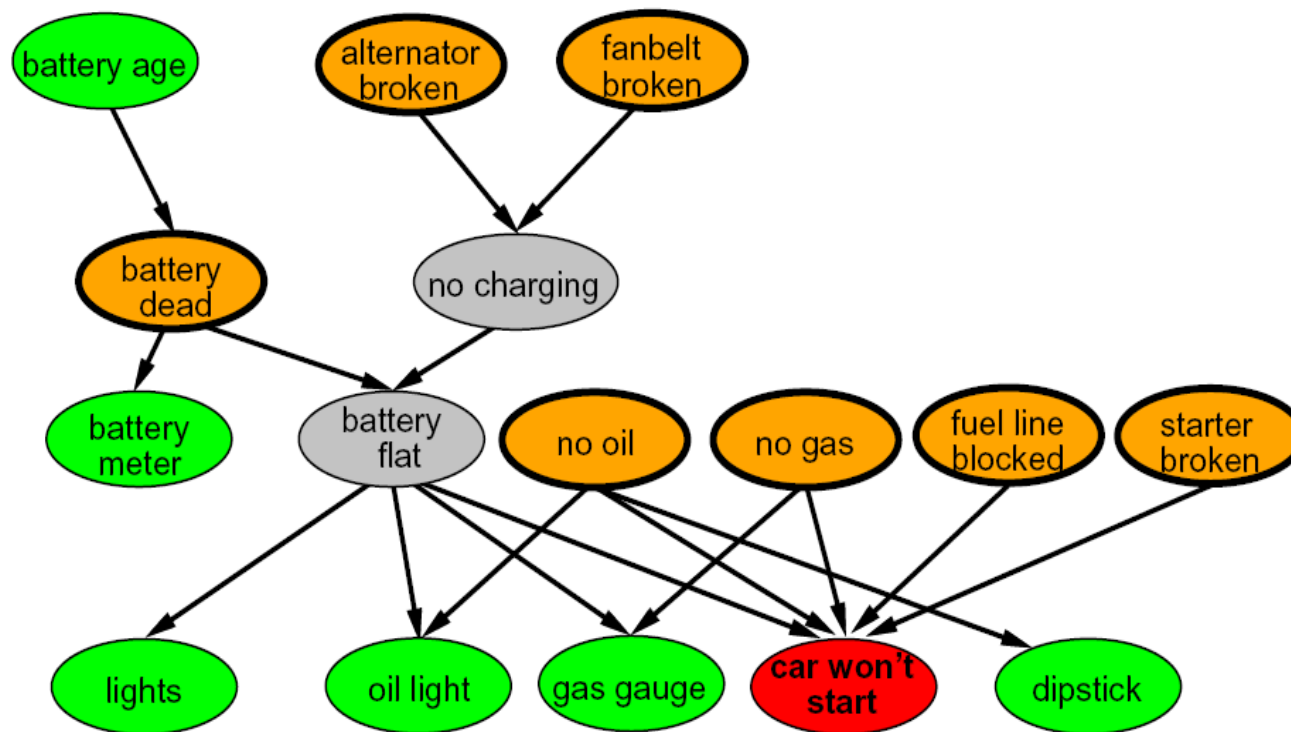
- Two problems with using full joint distribution tables as our probabilistic models:
 - Unless there are only a few variables, the joint is WAY too big to represent explicitly
 - Hard to learn (estimate) anything empirically about more than a few variables at a time
- Bayes' nets:** a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
 - More properly called **graphical models**
 - We describe how variables locally interact
 - Local interactions chain together to give global, indirect interactions
 - For about 10 min, we'll be vague about how these interactions are specified



Example Bayes' Net: Insurance

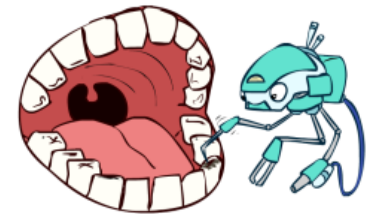
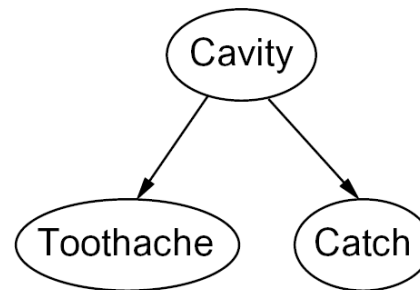


Example Bayes' Net: Car



Graphical Model Notation

- **Nodes: variables (with domains)**
 - Can be assigned (observed) or unassigned (unobserved)
- **Arcs: interactions**
 - Similar to CSP constraints
 - Indicate “direct influence” between variables
 - Formally: encode conditional independence (more later)
- For now: imagine that arrows mean direct causation (in general, they don't!)



Example: Coin Flips

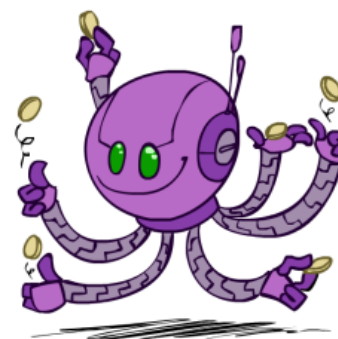
- N independent coin flips

X_1

X_2

...

X_n

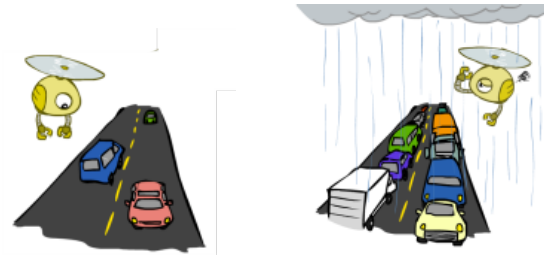


- No interactions between variables: absolute independence

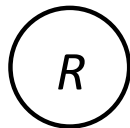
Example: Traffic

- Variables:

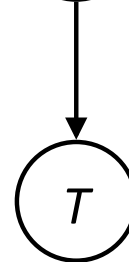
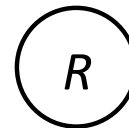
- R: It rains
- T: There is traffic



- Model 1: independence



- Model 2: rain causes traffic



- Why is an agent using model 2 better?

Example: Traffic II

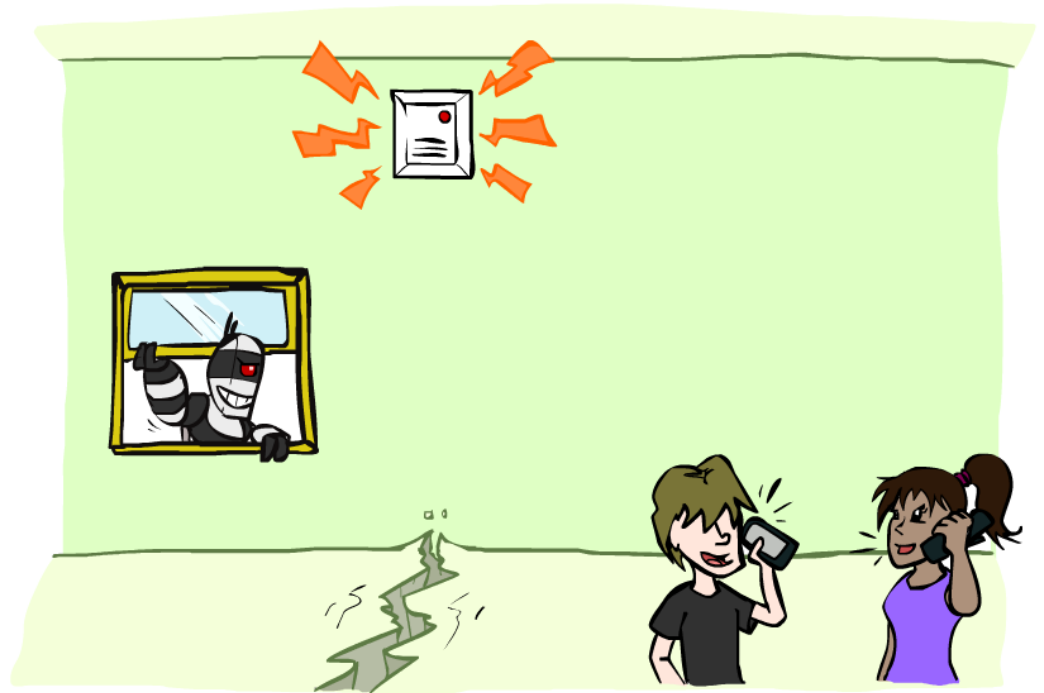
- Let's build a causal graphical model!
- Variables
 - T: Traffic
 - R: It rains
 - L: Low pressure
 - D: Roof drips
 - B: Ballgame
 - C: Cavity



Example: Alarm Network

- Variables

- B: Burglary
- A: Alarm goes off
- M: Mary calls
- J: John calls
- E: Earthquake!



Bayes' Net Semantics



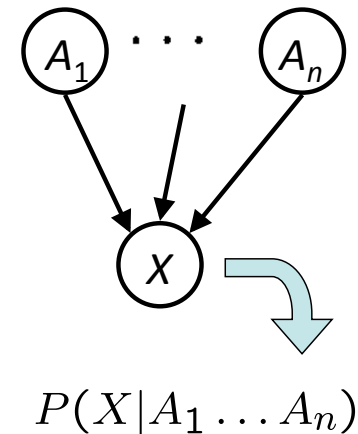
Bayes' Net Semantics



- A set of nodes, one per variable X
- A directed, acyclic graph
- A conditional distribution for each node
 - A collection of distributions over X , one for each combination of parents' values

$$P(X|a_1 \dots a_n)$$

- CPT: conditional probability table
- Description of a noisy “causal” process



A Bayes net = Topology (graph) + Local Conditional Probabilities

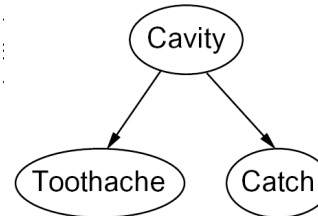
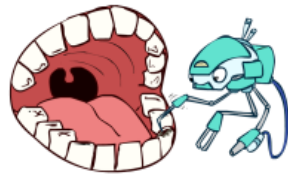
Probabilities in BNs



- Bayes' nets **implicitly** encode joint distributions
 - As a product of local conditional distributions
 - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

- Example:



$$P(+cavity, +catch, -toothache)$$

Probabilities in BNs



- Why are we guaranteed that setting

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

results in a proper joint distribution?

- Chain rule (valid for all distributions): $P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | x_1 \dots x_{i-1})$

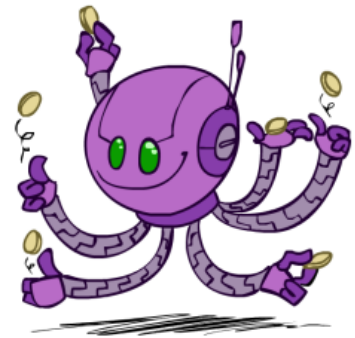
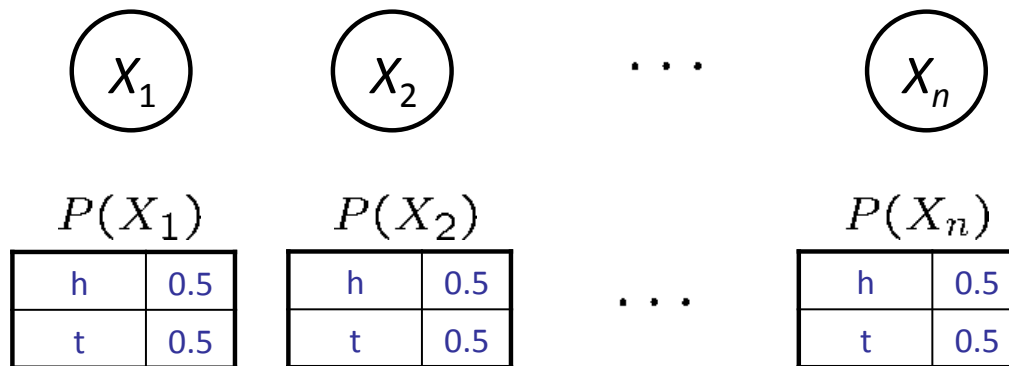
- Assume conditional independences: $P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$

→ Consequence: $P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$

- Not every BN can represent every joint distribution

- The topology enforces certain conditional independencies

Example: Coin Flips

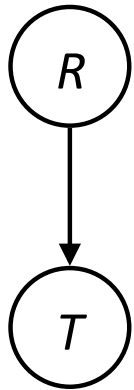


$$P(h, h, t, h) =$$

Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.

DEMO

Example: Traffic


$$P(R)$$

+r	1/4
-r	3/4

$$P(T|R)$$

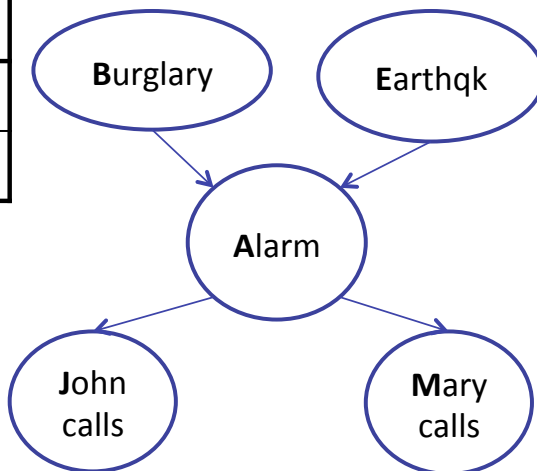
+r	+t	3/4
+r	-t	1/4
-r	+t	1/2
-r	-t	1/2

$$P(+r, -t) =$$



Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999



E	P(E)
+e	0.002
-e	0.998



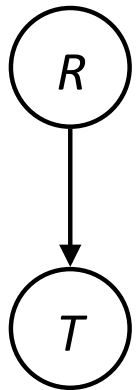
A	J	P(J A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

B	E	A	P(A B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

Example: Traffic

- Causal direction


$$P(R)$$

+r	1/4
-r	3/4

$$P(T|R)$$

+r	+t	3/4
	-t	1/4
-r	+t	1/2
	-t	1/2

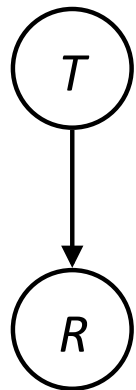
$$P(T, R)$$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16



Example: Reverse Traffic

- Reverse causality?


$$P(T)$$

+t	9/16
-t	7/16

$$P(R|T)$$

+t	+r	1/3
	-r	2/3
-t	+r	1/7
	-r	6/7

A cartoon illustration showing a yellow bird-like character driving a red convertible car in the rain. A teal car is following behind it. The scene is set against a background of rain clouds and falling rain.

$$P(T, R)$$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16

Causality?

- When Bayes' nets reflect the true causal patterns:

- Often simpler (nodes have fewer parents)
- Often easier to think about
- Often easier to elicit from experts

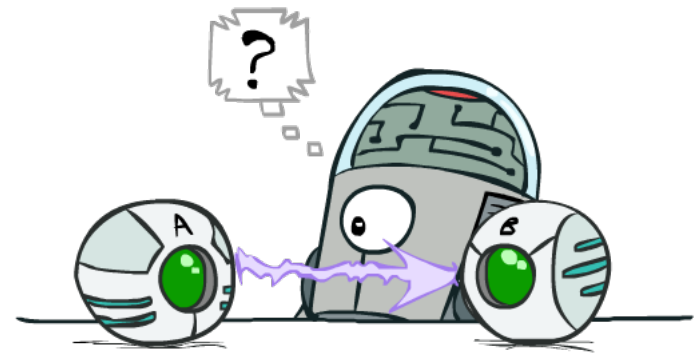
- BNs need not actually be causal

- Sometimes no causal net exists over the domain (especially if variables are missing)
- E.g. consider the variables *Traffic* and *Drips*
- End up with arrows that reflect correlation, not causation

- What do the arrows really mean?

- Topology may happen to encode causal structure
- **Topology really encodes conditional independence**

$$P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$$



Bayes' Nets

- So far: how a Bayes' net encodes a joint distribution
- Next: how to answer queries about that distribution
 - Key idea: conditional independence
 - Today: assembled BNs using an intuitive notion of conditional independence as causality
 - Next: formalize these ideas
 - Main goal: answer queries about conditional independence and influence
- After that: how to answer numerical queries (inference)

