The Law of Large Numbers

Lecturer: John Guttag

Law of Large Numbers

In repeated **independent tests** with the same **actual probability** p of a particular outcome in each test, the chance that the **fraction of times** that outcome occurs differs from p converges to zero as the number of trials goes to infinity.

Gambler's Fallacy

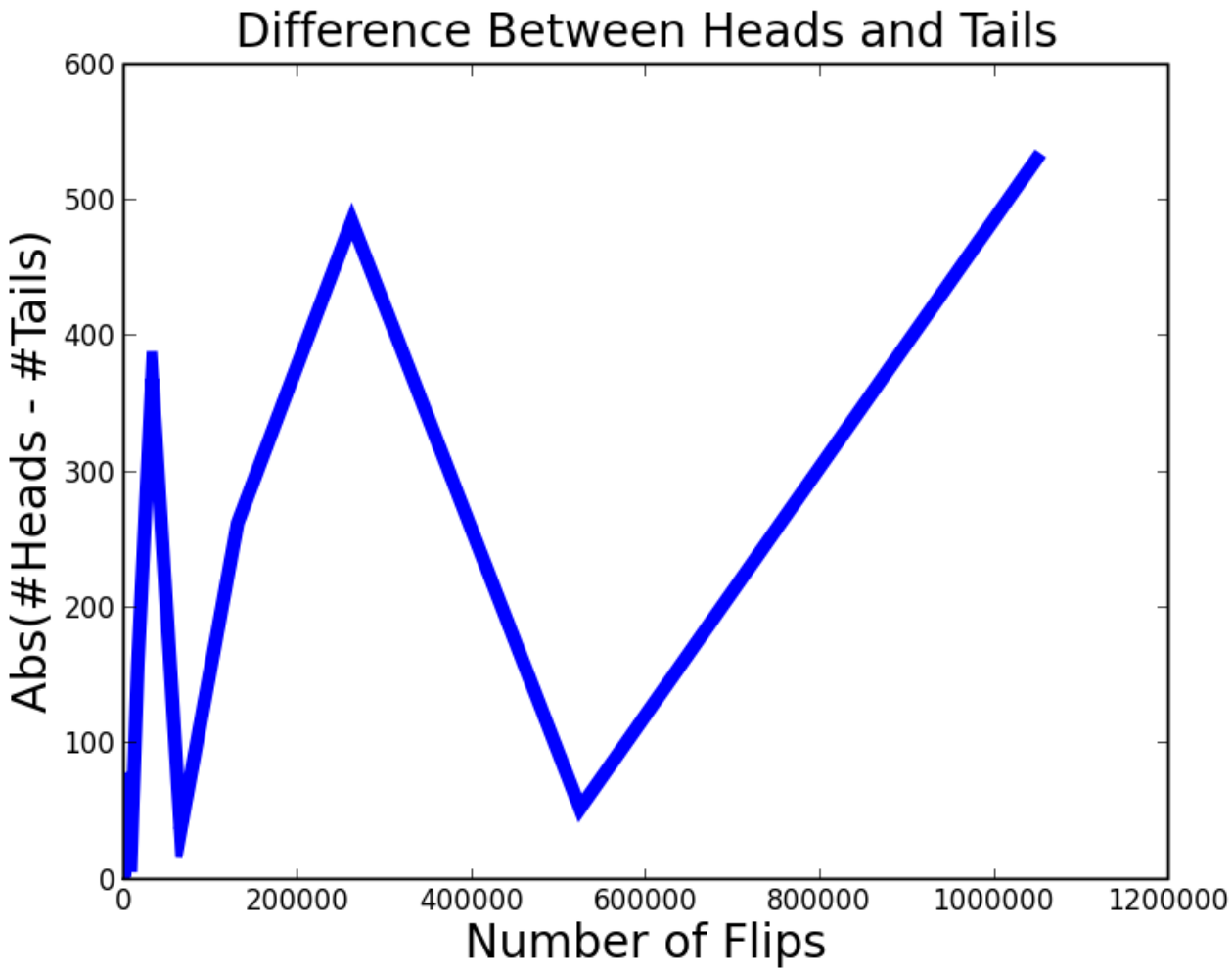
If deviations from expected behavior occur, these deviations are likely to be evened out by opposite deviations in the future.

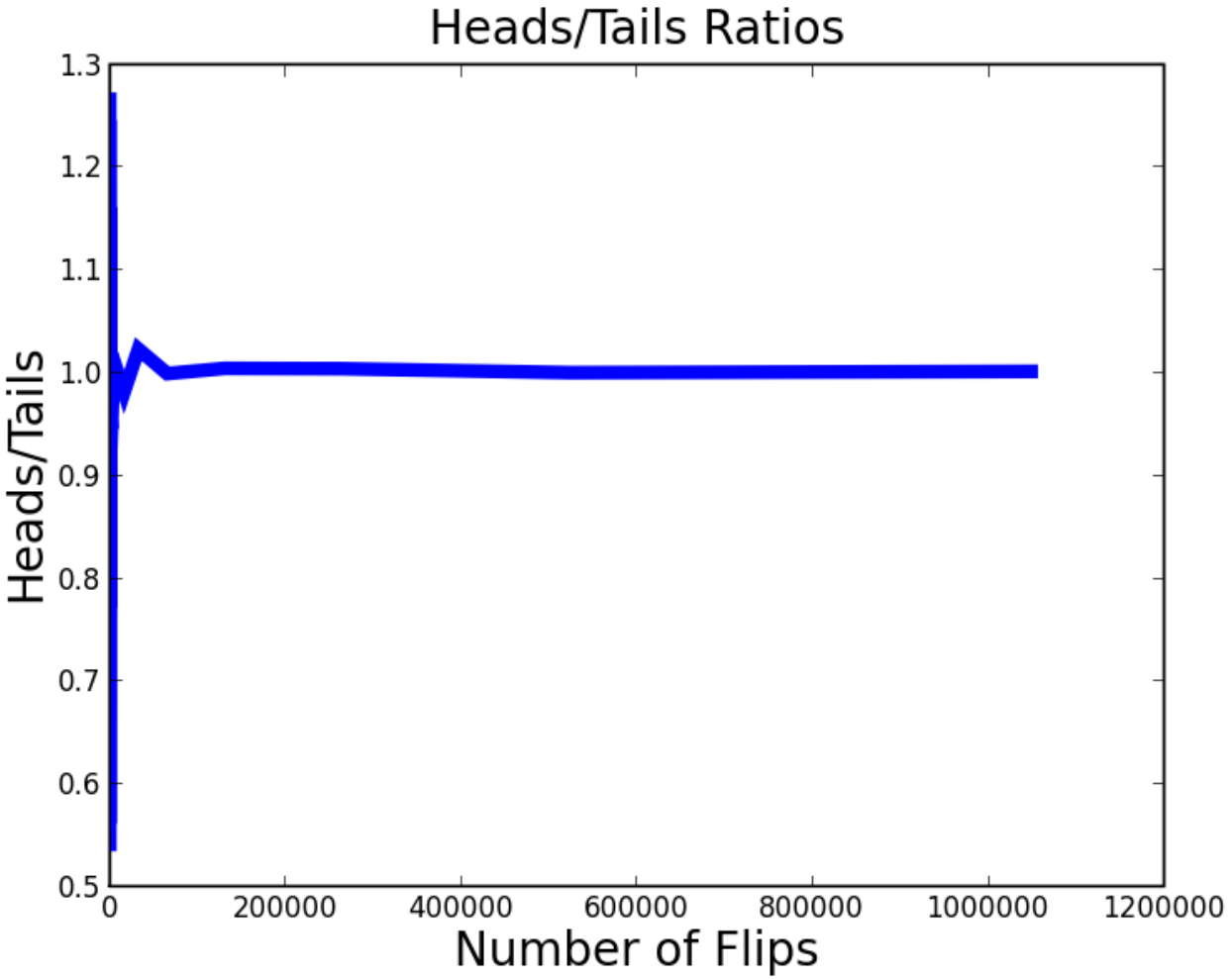
```
def flipPlot(minExp, maxExp):
    """Assumes minExp and maxExp positive
               integers; minExp < maxExp
       Plots results of 2**minExp to
              2**maxExp coin flips"""
    ratios = []
    diffs = []
    xAxis = []
    for exp in range(minExp, maxExp + 1):
        xAxis.append(2**exp)
. . .
```

```
for numFlips in xAxis:
        numHeads = 0
        for n in range(numFlips):
            if random.random() < 0.5:
                numHeads += 1
        numTails = numFlips - numHeads
        ratios.append(numHeads/float(numTails))
        diffs.append(abs(numHeads - numTails))

. . .
```

```
pylab.title('Difference Between Heads and Tails')
    pylab.xlabel('Number of Flips')
    pylab.ylabel('Abs(#Heads - #Tails)')
    pylab.plot(xAxis, diffs)
    pylab.figure()
    pylab.title('Heads/Tails Ratios')
    pylab.xlabel('Number of Flips')
    pylab.ylabel('Heads/Tails')
    pylab.plot(xAxis, ratios)
```


Difference Between Heads and Tails
Abs(#Heads - #Tails)
Number of Flips
0
100
200
300
400
500
600
0
200000
400000
600000
800000
1000000
1200000


Heads/Tails Ratios
Heads/Tails
Number of Flips
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
200000
400000
600000
800000
1000000
1200000

How Much Is Enough?

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How Much is Enough?

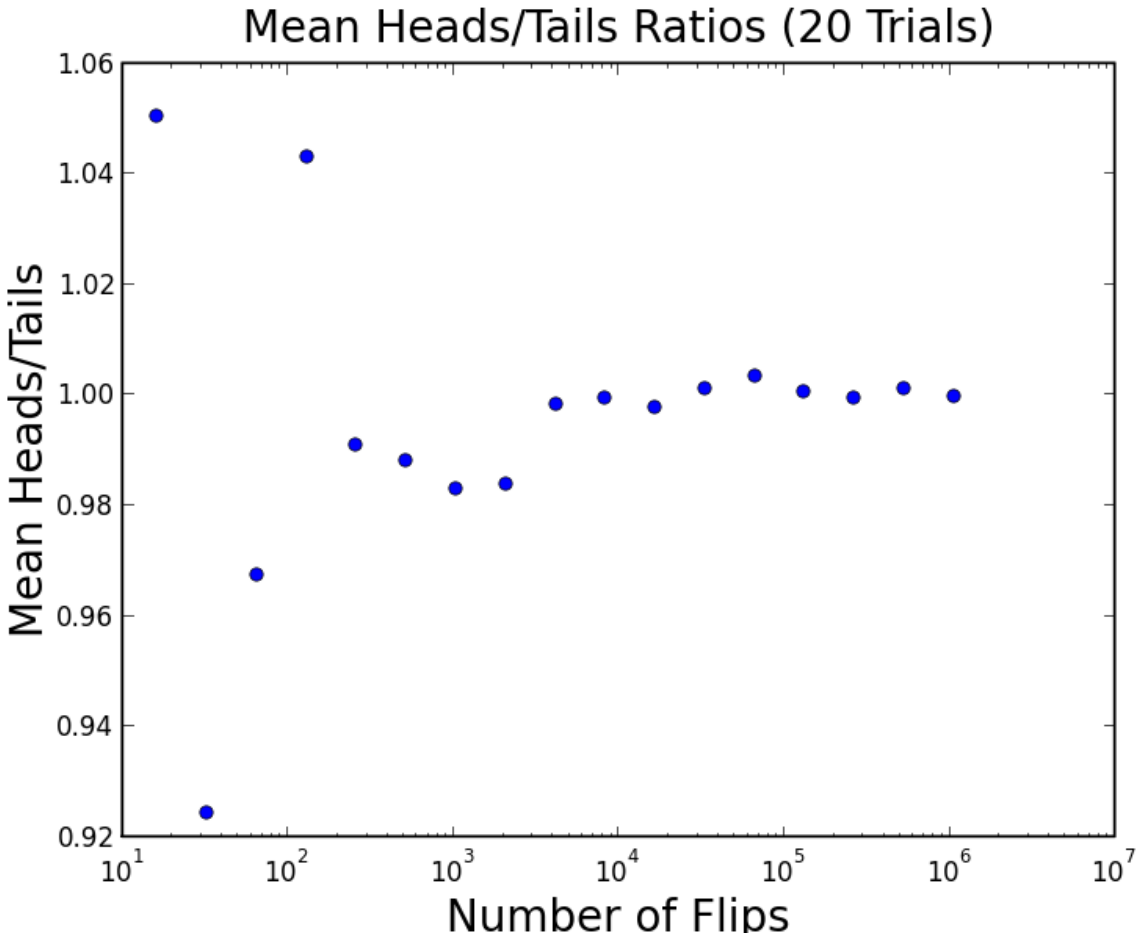
How many samples do we need to look at in order to have a justified confidence that something that is true about the population of samples is also true about the population from which the samples were drawn?

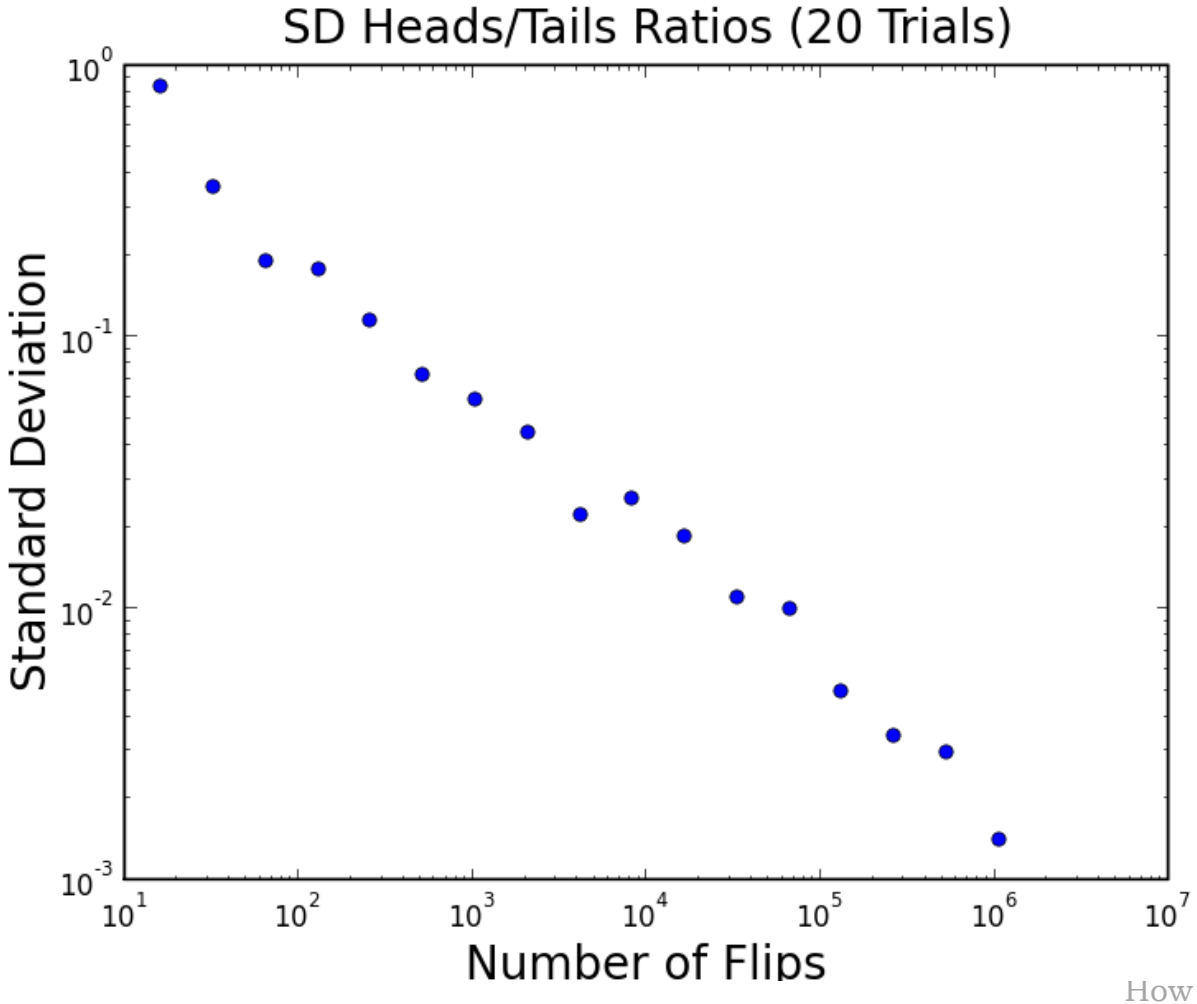
Depends upon the variance in the underlying distribution

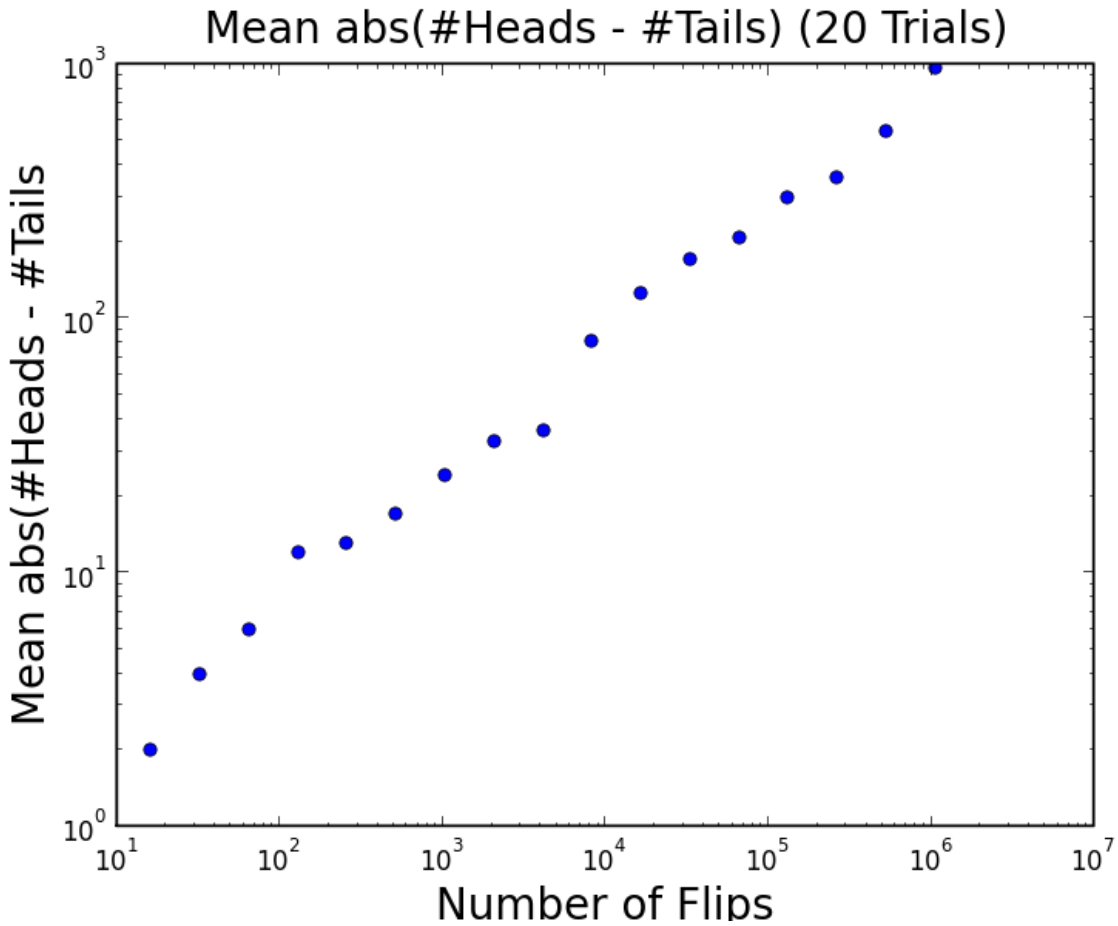
Variance

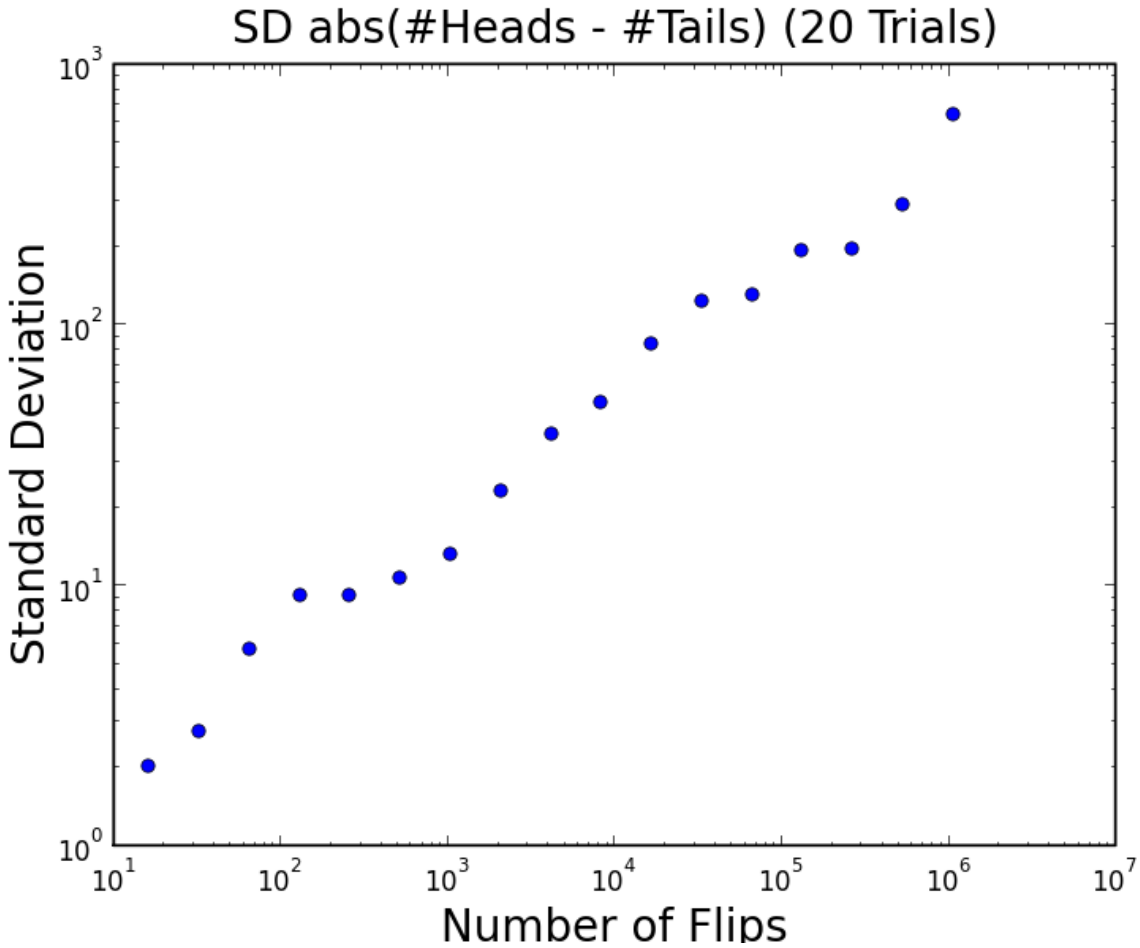
We measure the amount of variance in the outcomes of multiple trials.

```
def stdDev(X):
    mean = sum(X)/float(len(X))
    tot = 0.0
    for x in X:
        tot += (x - mean)**2
    return (tot/len(X))**0.5
```


Mean Heads/Tails Ratios (20 Trials)
Mean Heads/Tails
Number of Flips
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
10^1
10^2
10^3
10^4
10^5
10^6
10^7


SD Heads/Tails Ratios (20 Trials)
Standard Deviation
Number of Flips
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}


Mean abs(#Heads - #Tails) (20 Trials)
Mean abs(#Heads - #Tails
Number of Flips
10^0
10^1
10^2
10^3
10^1
10^2
10^3
10^4
10^5
10^6
10^7


SD abs(#Heads - #Tails) (20 Trials)
Standard Deviation
Number of Flips
10^0
10^1
10^2
10^3
10^1
10^2
10^3
10^4
10^5
10^6
10^7

Standard Deviations and Histograms

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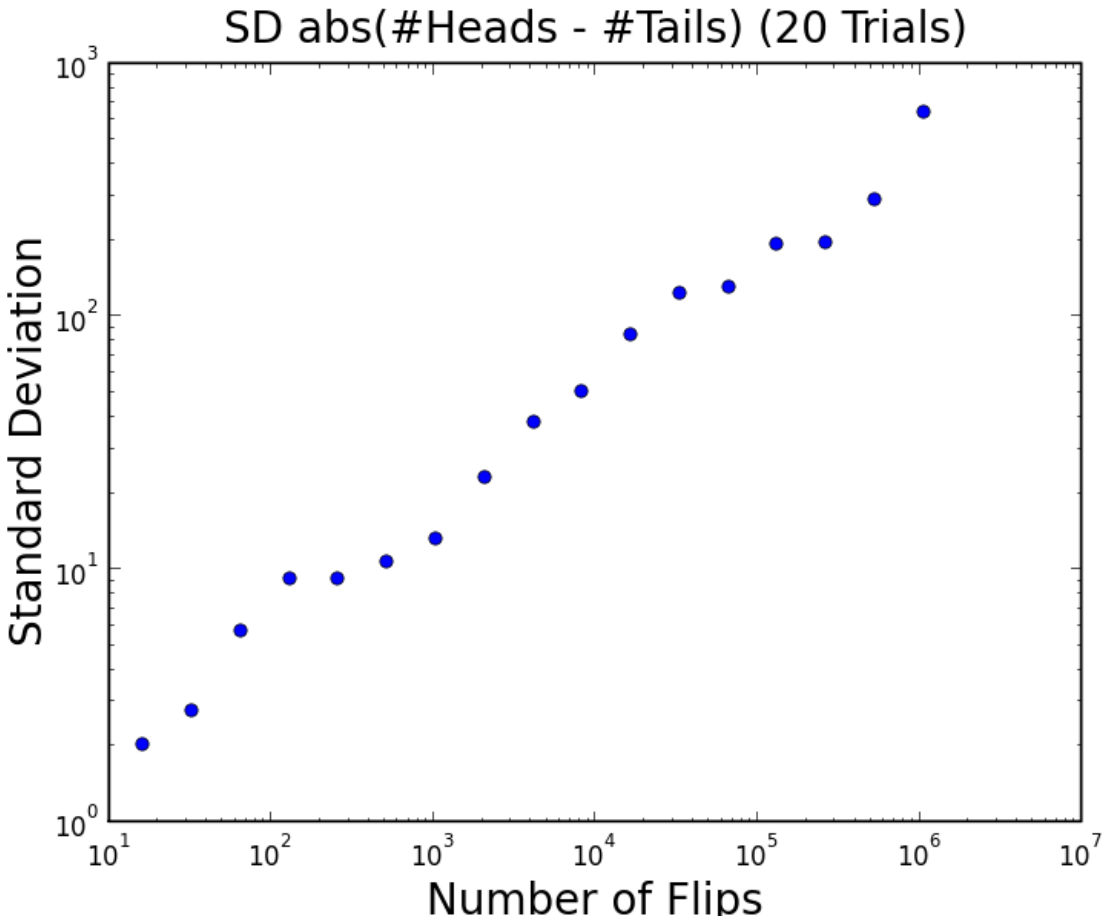
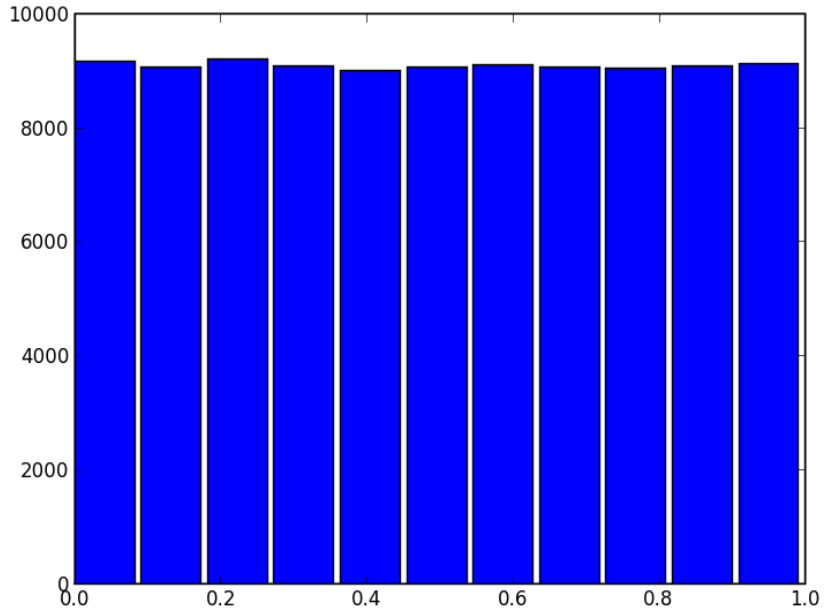

SD abs(#Heads - #Tails) (20 Trials)
Standard Deviation
Number of Flips
10^0
10^1
10^2
10^3
10^1
10^2
10^3
10^4
10^5
10^6
10^7

```
def CV(X):
    mean = sum(X)/float(len(X))
    try:
        return stdDev(X)/mean
    except ZeroDivisionError:
        return float('NaN')
```

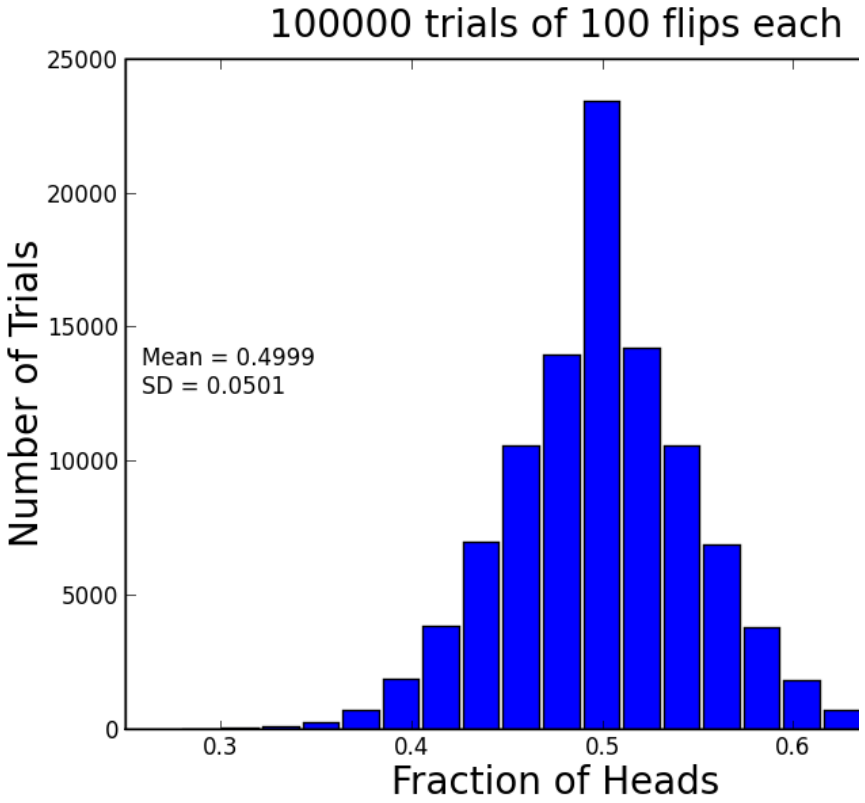
```
def flip(numFlips):

def flipSim(numFlipsPerTrial, numTrials):

def labelPlot(nf, nt, mean, sd):

def makePlots(nf1, nf2, nt):
  """nt = number of trials per experiment
     nf1 = number of flips 1st experiment
     nf2 = number of flips 2nd experiment"""
```

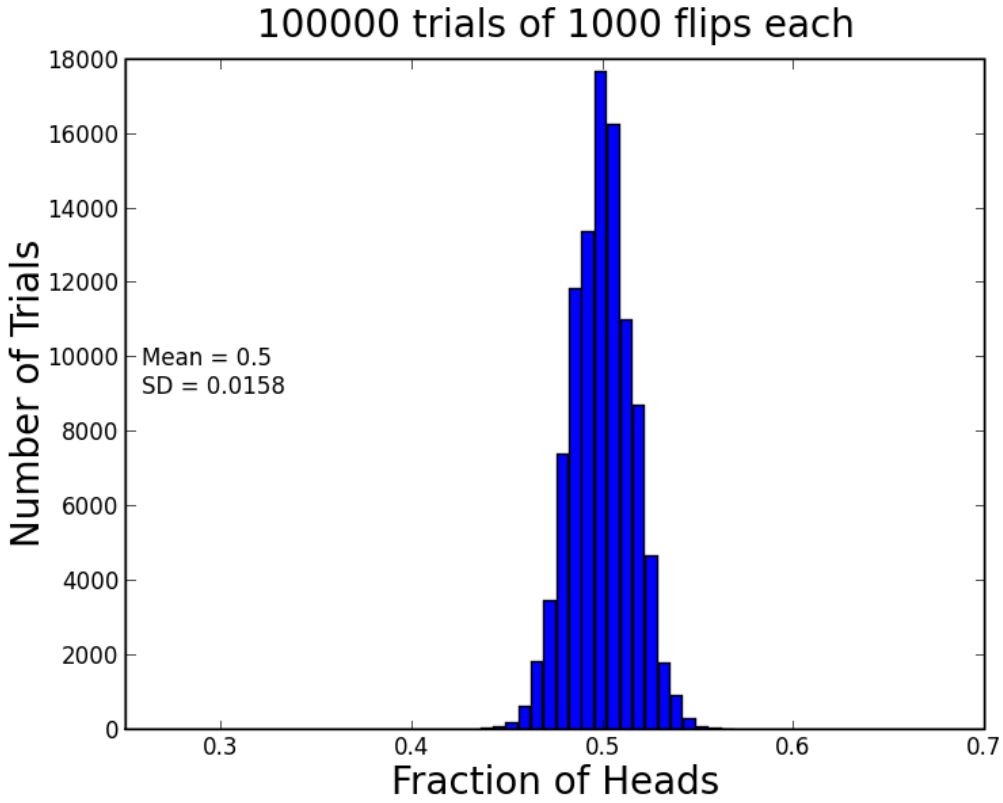
```
def makePlots(numFlips1, numFlips2, numTrials):
    val1, mean1, sd1 = flipSim(numFlips1, numTrials)
    pylab.hist(val1, bins = 20)
    xmin,xmax = pylab.xlim()
    ymin,ymax = pylab.ylim()
    labelPlot(numFlips1, numTrials, mean1, sd1)
    pylab.figure()
    val2, mean2, sd2 = flipSim(numFlips2, numTrials)
    pylab.hist(val2, bins = 20)
    pylab.xlim(xmin, xmax)
    ymin, ymax = pylab.ylim()
    labelPlot(numFlips2, numTrials, mean2, sd2)
```

```
vals = []
for i in range(100000):
    num = random.random()
    vals.append(num)
pylab.hist(vals, bins = 11)
```



```
vals = []
for i in range(100000):
    num = random.random()
    vals.append(num)
pylab.hist(vals, bins = 11)
xmin, xmax = pylab.xlim()
ymin, ymax = pylab.ylim()
print 'x-range =', xmin, '-', xmax
print 'y-range =', ymin, '-', ymax
pylab.figure
pylab.hist(vals, bins = 11)
#pylab.xlim(-1.0, 2.0)
```


100000 trials of 100 flips each
Number of Trials
Fraction of Heads
Mean = 0.4999
SD = 0.0501
25000
20000
15000
10000
5000
0
0.3
0.4
0.5
0.6


100000 trials of 1000 flips each
Number of Trials
Fraction of Heads
Mean = 0.5
SD = 0.0158
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0.3
0.4
0.5
0.6
0.7