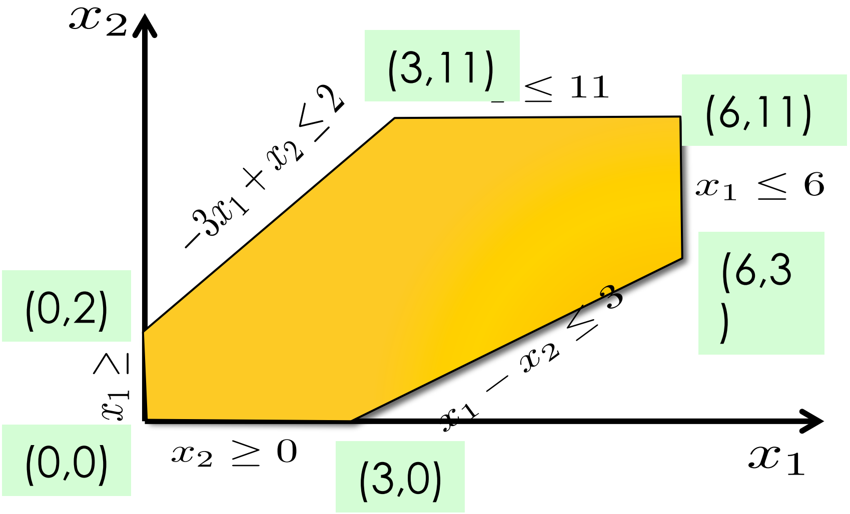
THE CENTRAL PATH

Linear Programming Problem

$$
\begin{array}{lrrcr}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
$$

Note: Not drawn to scale



Overview

$$
\begin{array}{lrcl}
\max & \mathbf{c}^\intercal \ \mathbf{x} & & \\
 & A\mathbf{x} + \mathbf{x}_s & = & \mathbf{b} \\
 & \mathbf{x}, \mathbf{x_s} & \geq & 0
\end{array}
$$

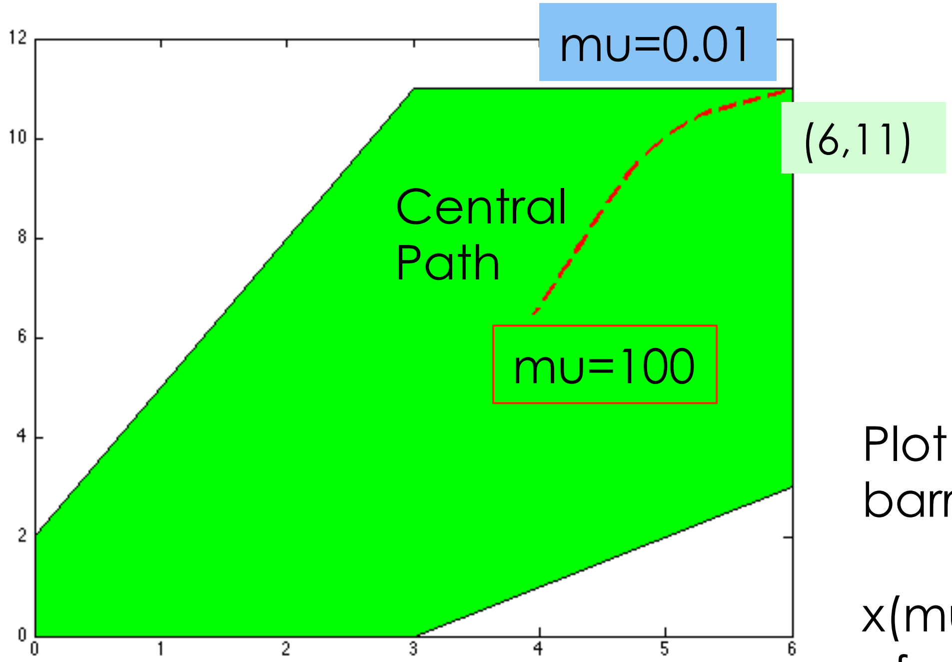
Primal Problem

Log Barrier Trick

$$\max \ \mathbf{c}^\intercal \ \mathbf{x} + \mu \sum_{j=1}^{n} \log(x_j) + \mu \sum_{i=1}^{m} \log(x_{s,i})$$
$$\text{s.t. } A\mathbf{x} + \mathbf{x_s} = \mathbf{b}$$

A $\mu \to 0$, we converge to solution of original problem.

Central Path



Plot optimal solutions for barrier problem:

x(mu) as a function of mu.

Solving Linear Programs

- Start with a large value of mu.
  - Use Newton's method to solve for mu-KKT conditions.

- As we iterate, gradually reduce mu.
  - mu' = 0.1 * mu

- Stop when value of primal infeasibility, dual infeasibility, and mu are small enough.