

BRANCH-AND-BOUND METHOD

Heuristics

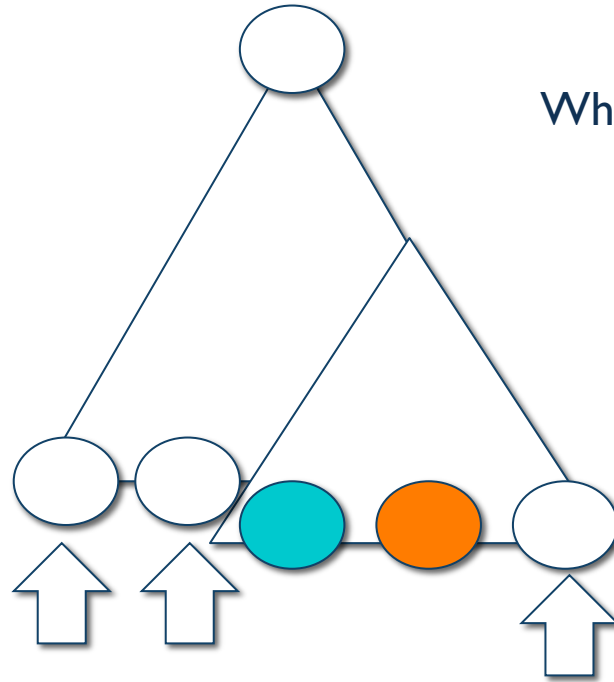
BnB choices to be made

- Which unexplored regular leaf node should I expand?
- How to choose the branching variables?
- Other tricks:
 - Randomized rounding to find an integer solution?
 - Carrying out BnB at the dictionary level.

BRANCH-AND-BOUND

Choice of unexplored node to expand

General Situation

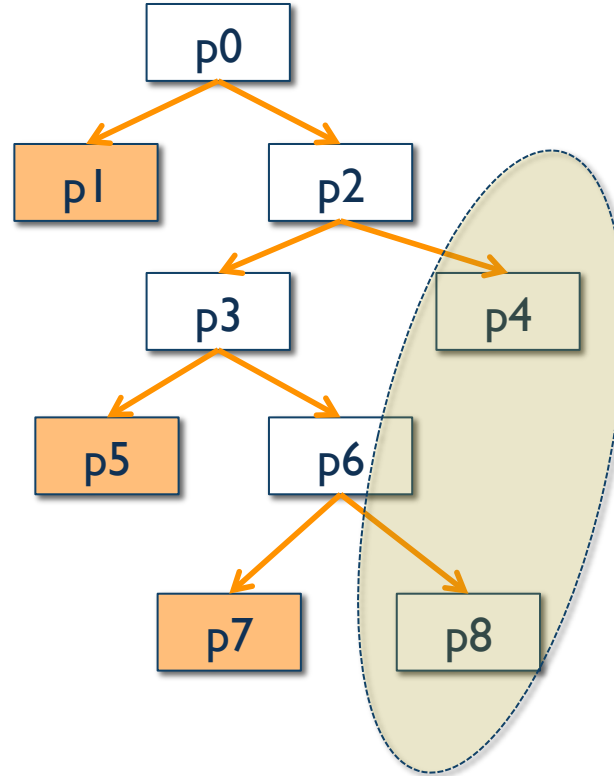


Which node should I examine first?

Unexplored “frontier”

Branch-and-bound

Q: Which one should we examine first?



Unexplored
Nodes

Goal

- Minimize the number of nodes explored in the tree.
- Which node should I expand first:
 - Yield integral solutions.
 - Improve the value of bestObjective.

Unexplored Node Selection Heuristic

- Deepest node first.
 - Select the node that has largest depth in the tree to explore first.
- Best LP relaxation solution.
 - Select the node whose LP relaxation has the best answer.
- Breadth First
 - Search breadth-first.

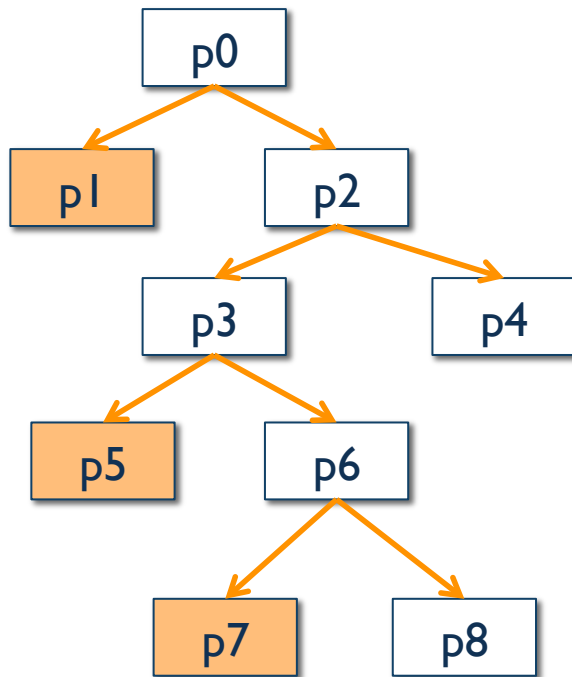
In Practice...

- ILPs come from some problem domain:
 - eg., We are converting a minimum cost vertex cover computation to ILP.
- Selection heuristics are best found by experimenting with a large set of problems from the domain.
- There is no general rule, unfortunately.

BRANCH-AND-BOUND

Choosing the branch variable

BnB Tree



p3.ampl

x1.val = 1
x2.val = 0
x3.val = 4.57142857142857
x4.val = 0
x5.val = 0.714285714285714
x6.val = 0.285714285714286
Optimal Value: -3.857143

p8.ampl

x1.val = 1
x2.val = 0
x3.val = 6
x4.val = 0
x5.val = 1
x6.val = 0.5
Optimal Value: -4.750000

p6.ampl

x1.val = 1
x2.val = 0
x3.val = 5
x4.val = 0.333333333333333
x5.val = 0.666666666666667
x6.val = 0.333333333333333
Optimal Value: -4.066667

Original Problem

```
var x1 integer >= 0, <= 10;
var x2 integer >= 0, <= 10;
var x3 integer >= 0, <= 10;
var x4 integer >= 0, <= 10;
var x5 integer >= 0, <= 10;
var x6 integer >= 0, <= 10;
maximize obj: -x1 - 2* x2 -0.5 * x3 - 0.2* x4 - x5 +0.5* x6;
c1: x1 + 2 * x2 >= 1;
c2: x1 + x2 + 3* x6 >= 1;
c3: x1 + x2 + x6 >= 1;
c4: x3 - 3* x4 >= 1;
c5: x3 - 2* x4 -5* x5 >= 1;
c6: x4 + 3* x5 -4 *x6 >= 1;
c7: x2 + x5 + x6 >= 1;
solve;
display x1, x2,x3, x4, x5, x6;
end;
```

BnB Final Tree

$x1.val = 0$

$x2.val = 1$

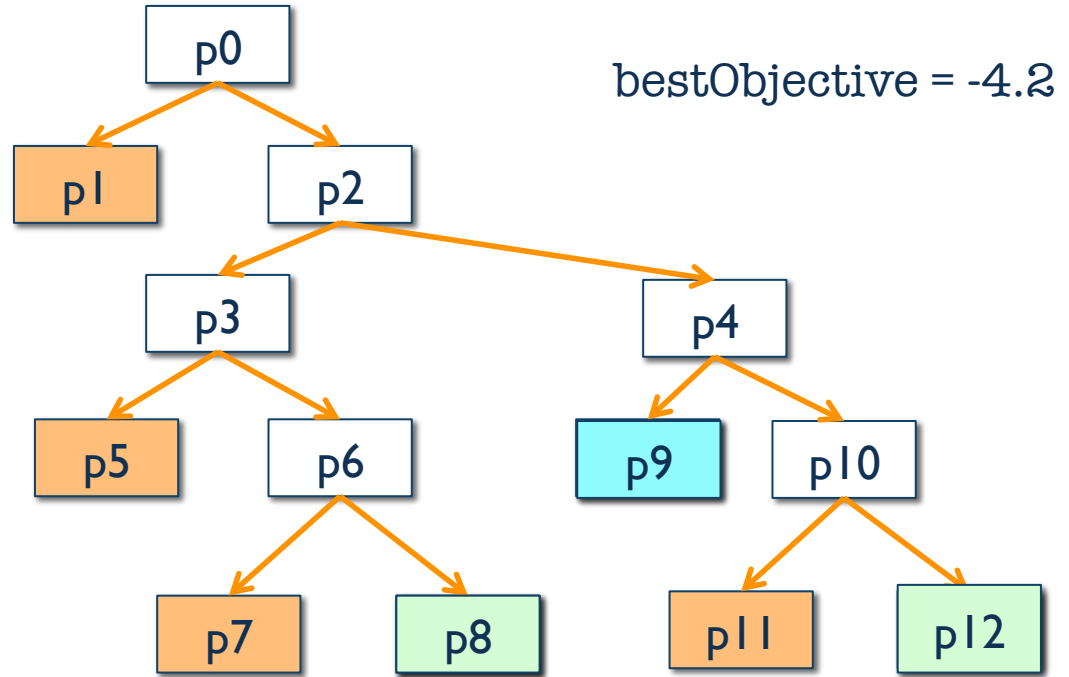
$x3.val = 4$

$x4.val = 1$

$x5.val = 0$

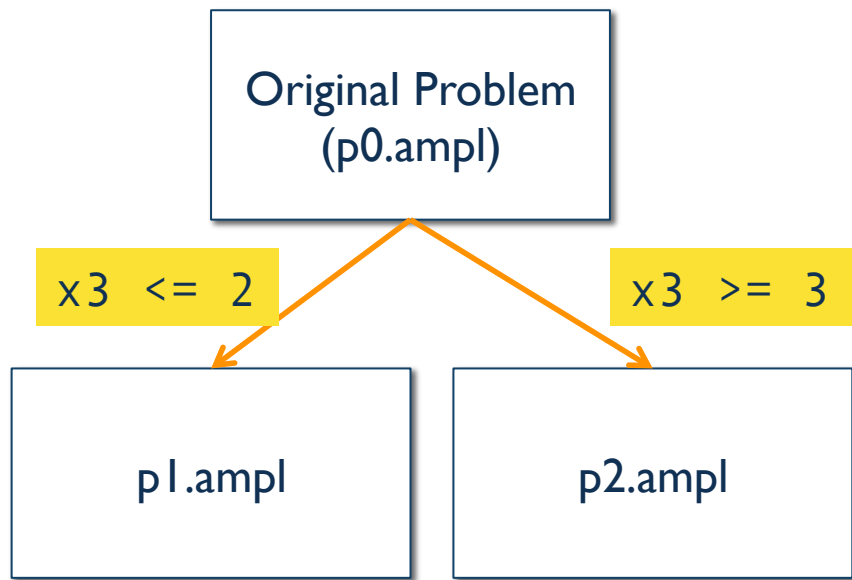
$x6.val = 0$

Optimal Value: -4.200000



Initial Node

bestObjective := - Infinity



$x1.val = 0.3333333333333333$

$x2.val = 0.6666666666666667$

$x3.val = 2.666666666666667$

$x4.val = 0$

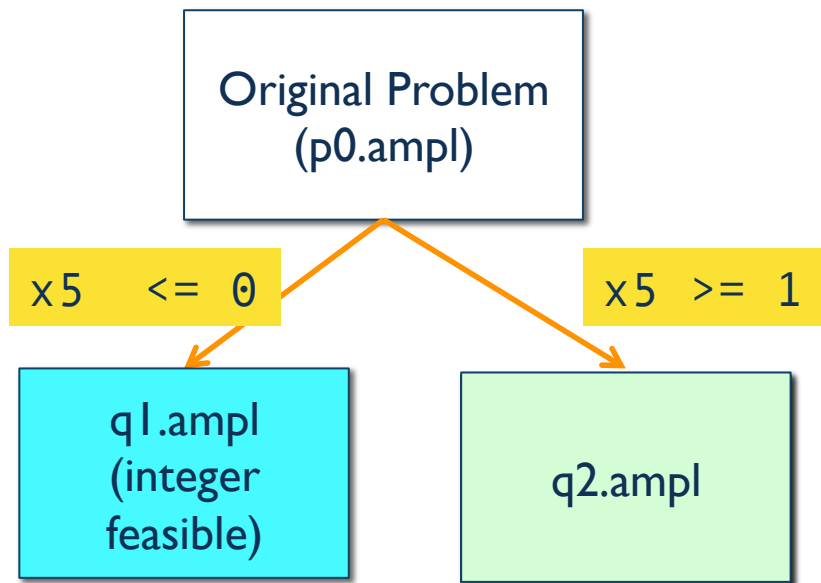
$x5.val = 0.3333333333333333$

$x6.val = 0$

Optimal Value: -3.333333

What if we chose a different branch variable..?

bestObjective := - Infinity



bestObjective := -4.2

x1.val = 0
x2.val = 0.5
x3.val = 6
x4.val = 0
x5.val = 1
x6.val = 0.5
Optimal Value: -4.750000

How to select a branch variable?

- The choice of a branch variable is very important.
- There is no single “best heuristic”.
- Often, some expertise with the problem domain informs the heuristic.