Revised Simplex Method

- **Basic Idea:** Do not store the intermediate dictionary.

- Store the set of basic and non-basic variables.

- At each step, **reconstruct** dictionary from data:
  - Original problem data: A,b,c
  - Set of basic (and non-basic) variables: B

Storage Cost:
Original problem data (sparse)
Basis set O(m + n)

Recap: Dictionary Reconstruction

$$\boxed{\begin{array}{lrcl} \max & \mathbf{c}^\intercal\ \mathbf{x} & & \\ & A\mathbf{x} + \mathbf{x_s} & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s} & \geq & 0 \end{array}}$$

$$\Rightarrow \quad \begin{array}{cccc} \mathbf{x}_B & = & \hat{\mathbf{b}} & +\hat{A}\mathbf{x}_I \\ \hline z & = & z_0 & +\hat{c}\ \mathbf{x}_I \end{array}$$

$$B = \{x_{B1}, \ldots, x_{Bm}\}$$

Basic Variables

Recap: Splitting the Matrix

$$A : \begin{bmatrix} \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ A_1 & A_2 & \cdots & A_{B1} & \cdots & A_{Bm} & \cdots & A_m \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \end{bmatrix}$$

$$\boxed{A_I} \qquad\qquad A_B$$

Recap: Rewriting the Equation

$$A\mathbf{x} + \mathbf{x_s} = \mathbf{b}$$

Step 1

$$A_B\mathbf{x_B} + A_I\mathbf{x_I} = b$$

Step 2

$$A_B\ \mathbf{x_B} = \mathbf{b} - A_I\mathbf{x_I}$$

Is A_B always invertible?

Recap: Dictionary Reconstruction

$$A_B \, \mathbf{x_B} = \mathbf{b} - A_I \mathbf{x_I}$$

$$\mathbf{x_B} = A_B^{-1} \mathbf{b} - A_B^{-1} A_I \mathbf{x_I}$$

Is A_B always invertible?

Recap: Result Dictionary

$$\mathbf{c}^\intercal\ \mathbf{x} = \mathbf{c_B}^\intercal \mathbf{x_B} + \mathbf{c_I}^\intercal \mathbf{x_I}$$

$$\begin{array}{ccccc}
\mathbf{x_B} & = & A_B^{-1}\mathbf{b} & & -A_B^{-1} A_I \mathbf{x_I} \\
\hline
\mathbf{c} & = & \mathbf{c_B}^\intercal\ A_B^{-1}\mathbf{b} & & +(-\mathbf{c_B}^\intercal A_B^{-1} A_I + \mathbf{c_I}^\intercal)\ \mathbf{x_I}
\end{array}$$

Invert matrix A_B: $O(m^3)$ (Gauss-Jordan).

Compute $A_B^{-1} \times A_I$ takes $O(m^2 n)$.

Compute $A_B^{-1}\mathbf{b}$ takes $O(m^2)$

Overall complexity: $O(m^2 * (m+n))$.