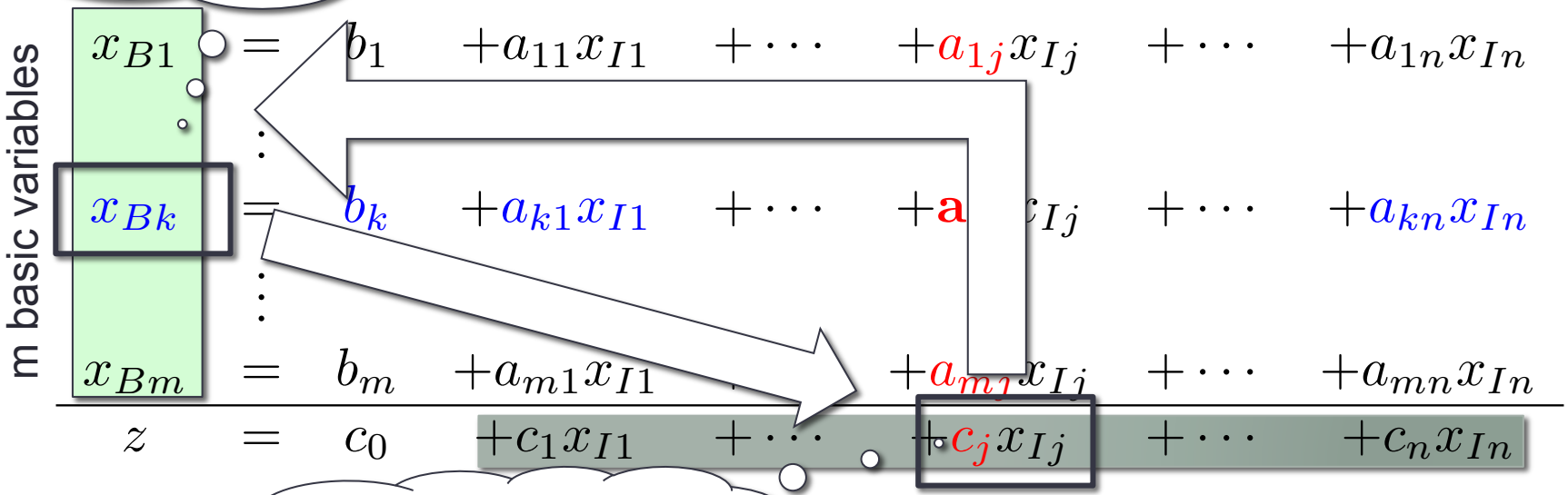
ACT III: PIVOTING

Going from one dictionary to the next.

Pivoting (from 2K feet)

$$\begin{array}{rcl} \mathbf{x}_B & = & \mathbf{b} - \mathbf{A}\mathbf{x}_I \\ \hline z & = & c_0 + \mathbf{c}^\mathsf{T}\,\mathbf{x}_I \end{array}$$

2. Choose Leaving Variable



1. Choose Entering Variable

Pivoting Steps

1. Choose an entering variable.

2. For the choice of entering variable, find a leaving variable.

3. Perform substitutions and obtain next dictionary.

Example

Chvátal, Chapter 2

$$
\begin{array}{lrcl}
\text{max.} & 5x_1 + 4x_2 + 3x_3 & & \\
\text{s.t.} & 2x_1 + 3x_2 + x_3 & \leq & 5 \\
& 4x_1 + x_2 + 2x_3 & \leq & 11 \\
& 3x_1 + 4x_2 + 2x_3 & \leq & 8 \\
& x_1, x_2, x_3 & \geq & 0
\end{array}
$$

Already in standard form.

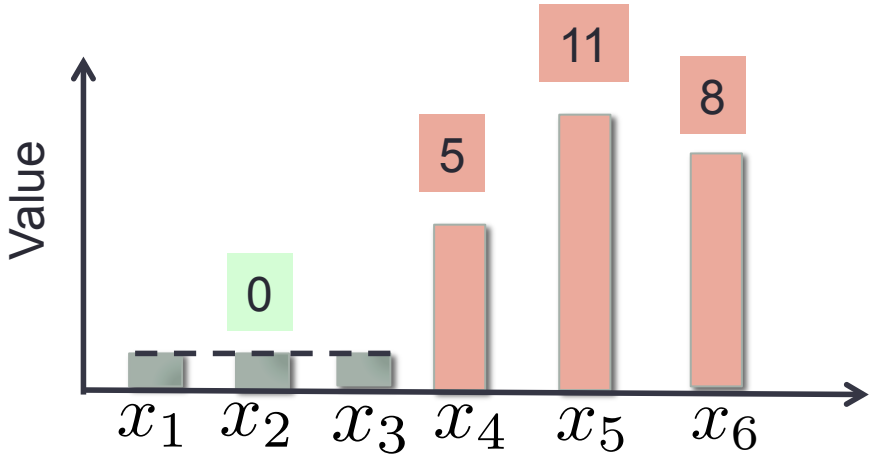
Example (add slack)

$$
\begin{array}{lrl}
\text{max.} & 5x_1 + 4x_2 + 3x_3 & \\
\text{s.t.} & 2x_1 + 3x_2 + x_3 & \leq 5 \\
& 4x_1 + x_2 + 2x_3 & \leq 11 \\
& 3x_1 + 4x_2 + 2x_3 & \leq 8 \\
& x_1, x_2, x_3 & \geq 0
\end{array}
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$

Choosing an entering variable

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$



Leaving Variable

$$
\begin{aligned}
x_4 &= 5 - 2x_1 - 3x_2 - x_3 \\
x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
\hline
z &= 0 + 5x_1 + 4x_2 + 3x_3
\end{aligned}
$$

Leaving Variable Analysis

$$
\begin{array}{rcrlll}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$

$$
\begin{array}{rcl}
x_4 = 5 - 2x_1 - 3x_2 - x_3 & \Rightarrow & x_1 \leq \frac{5}{2} \\
x_5 = 11 - 4x_1 - x_2 - 2x_3 & \Rightarrow & x_1 \leq \frac{11}{4} \\
x_6 = 8 - 3x_1 - 4x_2 - 2x_3 & \Rightarrow & x_1 \leq \frac{8}{3} \\
\hline
z = 0 + 5x_1 + 4x_2 + 3x_3 & &
\end{array}
$$