

Dual Certificate

$$\begin{array}{llll} \max & \mathbf{c}^\top \mathbf{x} & & \\ & A \mathbf{x} & \leq & \mathbf{b} \\ & \mathbf{x} & \geq & 0 \end{array}$$

Someone tells us \mathbf{x}^* is an optimal solution.

Do we trust them?

- Check if \mathbf{x}^* is feasible?
- **How to check if it is optimal?**

Dual Certificates

- Given x (Claimed to be primal optimal solution) and
- Given y (Claimed to be dual optimal solution).

We can use both to convince ourselves.

1. Check feasibility of x using primal problem
2. Form dual problem and check feasibility of y
3. Check that primal objective value is equal to dual objective value.

Example

$$\begin{array}{llll}
 \text{max.} & x_1 + 2x_2 & & \\
 \text{s.t.} & -2x_1 + x_2 & \leq & -2 \\
 & x_2 & \leq & 4 \\
 & x_1 - 2x_2 & \leq & -2 \\
 & x_1 & \leq & 4 \\
 & x_1, x_2 & \geq & 0
 \end{array}$$

$$\begin{array}{llllll}
 \text{min} & -2y_1 & +4y_2 & -2y_3 & +4y_4 & \\
 & -2y_1 & & +y_3 & +y_4 & \geq 1 \\
 & y_1 & +y_2 & -2y_3 & & \geq 2 \\
 & y_1, & y_2, & y_3, & y_4 & \geq 0
 \end{array}$$

$$x_1 = 4, x_2 = 4, z = 12$$

$$y_1 = 0, y_2 = 2, y_3 = 0, y_4 = 1$$