

REVISED SIMPLEX METHOD

Description of Pivoting

Revised Simplex: Partial Reconstruction

1. Solution for
Basic Variables

3. Column for entering Variable

$$\begin{array}{rcl} x_{B1} & = & b_1 + a_{1j}x_{Ij} \\ x_{B2} & = & b_2 + a_{2j}x_{Ij} \\ & \vdots & \\ x_{Bm} & = & b_m + a_{mj}x_{Ij} \\ \hline z & = & c_0 + c_1x_{I1} + \cdots + c_jx_{Ij} + \cdots + c_nx_{In} \end{array}$$

2. Objective row coefficients

Choosing Entering Variable

$$\begin{array}{rcl} \mathbf{x}_B & = & \hat{\mathbf{b}} + \hat{A}\mathbf{x}_I \\ \hline z & = & z_0 + \hat{c} \mathbf{x}_I \end{array}$$

Q: How do we compute the objective coefficients (aka reduced costs)?

Computing Objective Coefficients

- Original Problem Data: A, b, c
- Current Basis Set: $B = \{x_{B1}, x_{B2}, \dots, x_{Bm}\}$

Choosing Entering Variable

$$\begin{array}{rclclcl} x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +\textcolor{red}{a}_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} \\ x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +\textcolor{red}{a}_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} \\ & & \vdots & & & & & \\ x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +\textcolor{red}{a}_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} \\ \hline z & = & c_0 & +c_1x_{I1} & +\cdots & +\textcolor{red}{c}_jx_{Ij} & +\cdots & +c_nx_{In} \end{array}$$

$$\pi = \mathbf{c}_B^\top A_B^{-1}$$

$$\hat{\mathbf{c}} = \mathbf{c}_I^\top - \pi A_I$$

Computing the Objective Row

- Compute $\pi = \mathbf{c}_B^T A_B^{-1}$
by solving the system of equations

$$\boxed{\pi A_B = \mathbf{c}_B^T}$$

- Obtain obj. row coefficients by computing

$$\boxed{\hat{\mathbf{c}} = \mathbf{c}_I^T - \pi A_I}$$

- Objective value is $z = \pi \mathbf{b}$

Entering Variable Analysis

$$z = c_0 + c_1 x_{I1} + \cdots + \color{red}{c_j} x_{Ij} + \cdots + c_n x_{In} \quad \leftarrow \hat{\mathbf{c}}$$

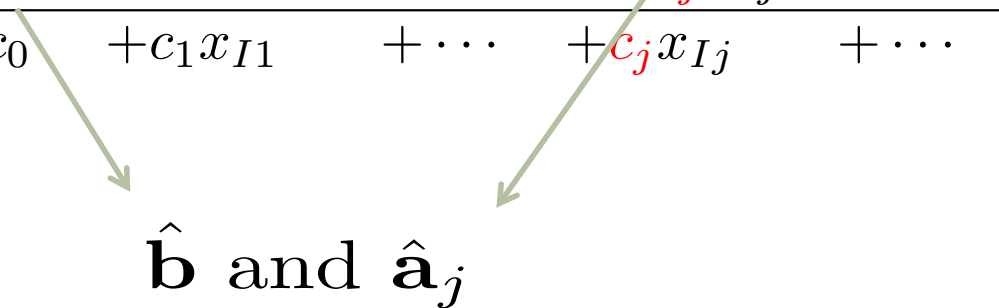
Choose positive coefficient $\color{orange}{c_j}$

If no such coefficient, then dictionary is final.

Leaving Variable Analysis

$$\begin{array}{rcll} x_{B1} & = & b_1 & + a_{1j} x_{Ij} \\ x_{B2} & = & b_2 & + a_{2j} x_{Ij} \\ & \vdots & & \\ x_{Bm} & = & b_m & + a_{mj} x_{Ij} \\ \hline z & = & c_0 & + c_1 x_{I1} + \cdots + c_j x_{Ij} + \cdots + c_n x_{In} \end{array}$$

$\hat{\mathbf{b}}$ and $\hat{\mathbf{a}}_j$



Leaving Variable Analysis

- Compute $\hat{\mathbf{b}} = A_B^{-1} \mathbf{b}$ by solving the equations

$$A_B \hat{\mathbf{b}} = \mathbf{b}$$

- Compute $\hat{\mathbf{a}}_j = -A_B^{-1} A_j$ by solving the equations

$$A_B \hat{\mathbf{a}}_j = -A_j$$

We have enough data to perform leaving variable analysis.