SOLVING INTEGER LINEAR PROGRAMS

1. Solving the LP relaxation.
2. How to deal with fractional solutions?

Integer Linear Program: Example

$$
\begin{array}{lll}
\max & -x_1 - 2x_2 - 0.5x_3 - 0.2x_4 - x_5 + 0.6x_6 & \\
\text{s.t.} & x_1 + 2x_2 & \geq \quad 1 \\
& x_1 + x_2 + 3x_6 & \geq \quad 1 \\
& x_1 + x_2 + x_6 & \geq \quad 1 \\
& x_3 - 3x_4 & \geq \quad 1 \\
& x_3 - 2x_4 - 5x_5 & \geq \quad 1 \\
& x_4 + 3x_5 - 4x_6 & \geq \quad 1 \\
& x_2 + x_5 + x_6 & \geq \quad 1 \\
0 \leq & x_1, x_2, \cdots, x_6 & \leq \quad 10 \\
& x_1, \ldots, x_6 & \in \quad \mathbb{Z}
\end{array}
$$

Solving ILPs

```
var x1 integer >= 0, <= 10;
var x2 integer >= 0, <= 10;
var x3 integer >= 0, <= 10;
var x4 integer >= 0, <= 10;
var x5 integer >= 0, <= 10;
var x6 integer >= 0, <= 10;

maximize obj: -x1- 2* x2 -0.5*x3- 0.2* x4 - x5 +0.5* x6;
c1: x1 + 2 * x2 >= 1;
c2: x1 + x2 + 3* x6 >= 1;
c3: x1 + x2 + x6 >= 1;
c4: x3 - 3* x4  >= 1;
c5: x3 - 2* x4 -5* x5 >= 1;
c6: x4 + 3* x5 -4 *x6  >= 1;
c7: x2 + x5 + x6 >= 1;
solve;
display x1, x2,x3, x4, x5, x6;
end;
```

Solution

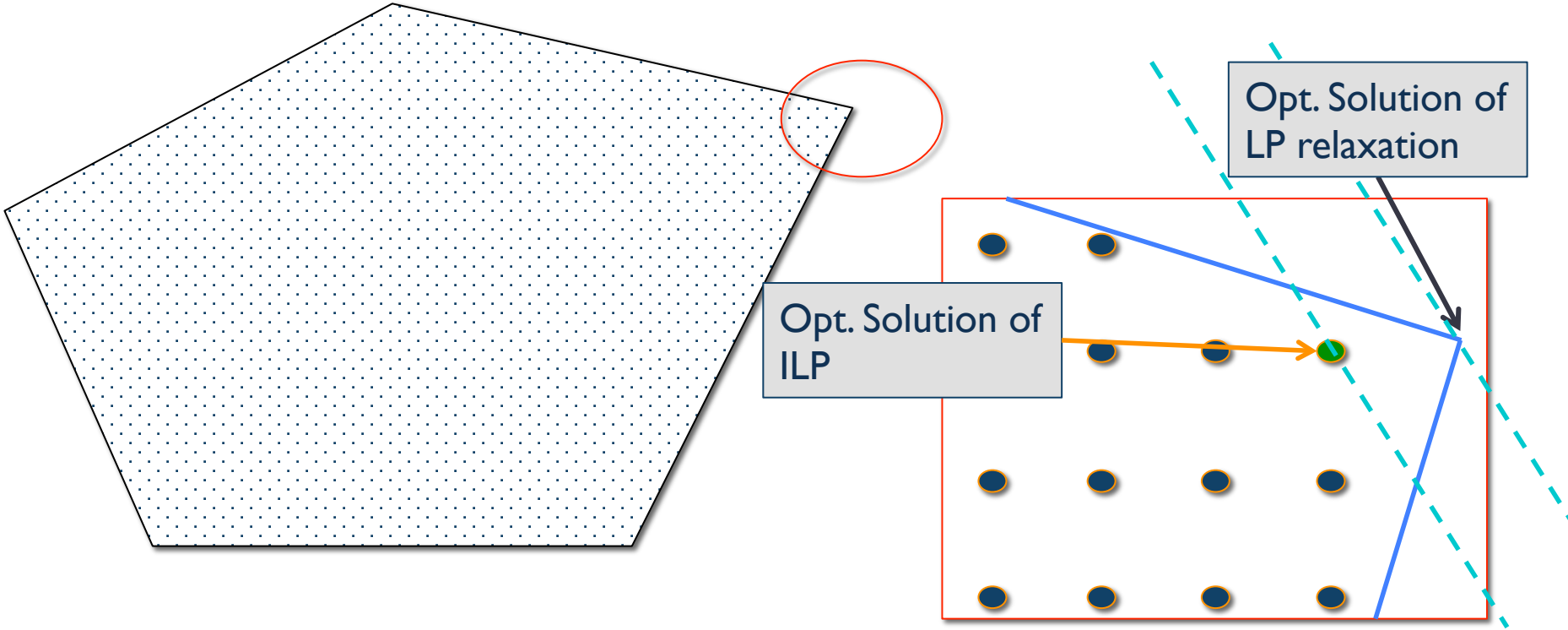
Original ILP

```
x1.val = 0
x2.val = 1
x3.val = 4
x4.val = 1
x5.val = 0
x6.val = 0
Optimal Value: -4.2
```

LP Relaxation

```
x1.val = 0.333333333333333
x2.val = 0.666666666666667
x3.val = 2.66666666666667
x4.val = 0
x5.val = 0.333333333333333
x6.val = 0
Optimal Value: -3.333333
```

Case-1: Both LP and ILP are feasible.



Solving ILPs

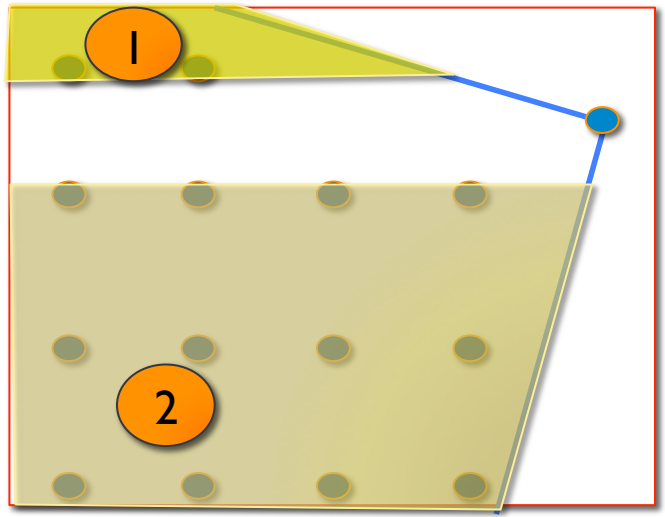
Solve LP relaxation of the ILP.

Case-1: LP relaxation solution satisfies integrality constraint.

Case-2: LP relaxation solution does not satisfy the integrality constraint.

Dealing with Case-2

Branch and Bound



Cutting Plane Method.