SHADOW COSTS

sensitivity analysis

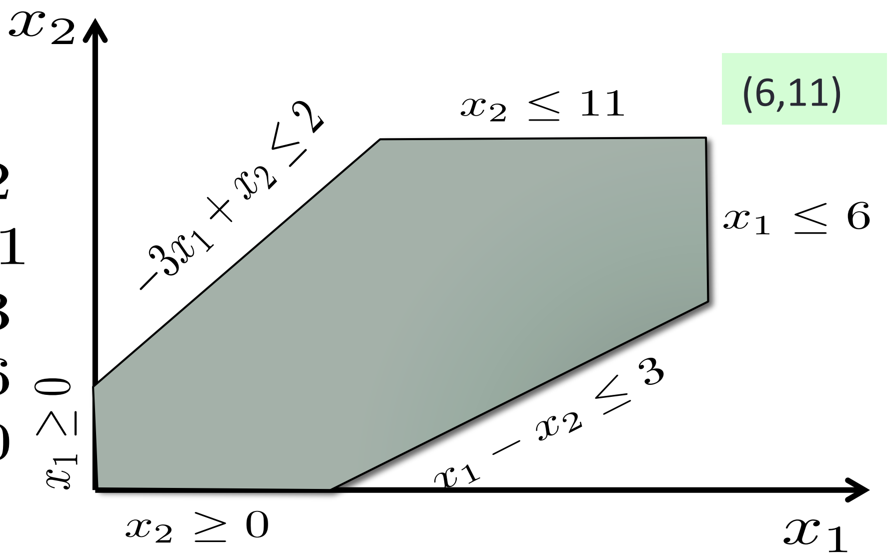
Shadow Cost of Constraints

$$
\begin{array}{lccccccc}
\max\ z = & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & & \\
& a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 & \leftarrow\ y_1 \\
& a_{21}x_1 & +a_{22}x_2 & +\cdots+ & a_{2n}x_n & \leq & b_2 & \leftarrow\ y_2 \\
& \vdots & & \ddots & & \vdots & & \\
& a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m & \leftarrow\ y_m \\
& x_1, & x_2, & \cdots & x_n & \geq & 0 &
\end{array}
$$

How does a “small” change in b_i affect the total optimal value?

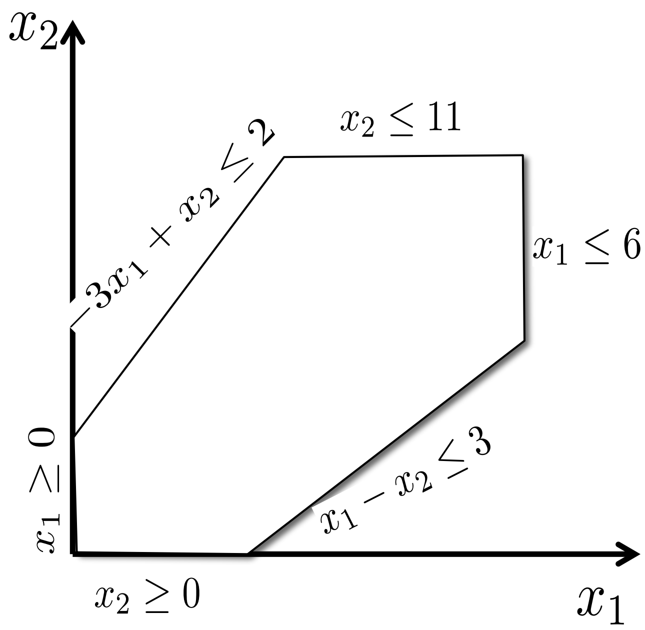
Linear Programming Problem

$$
\begin{array}{llllll}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
$$



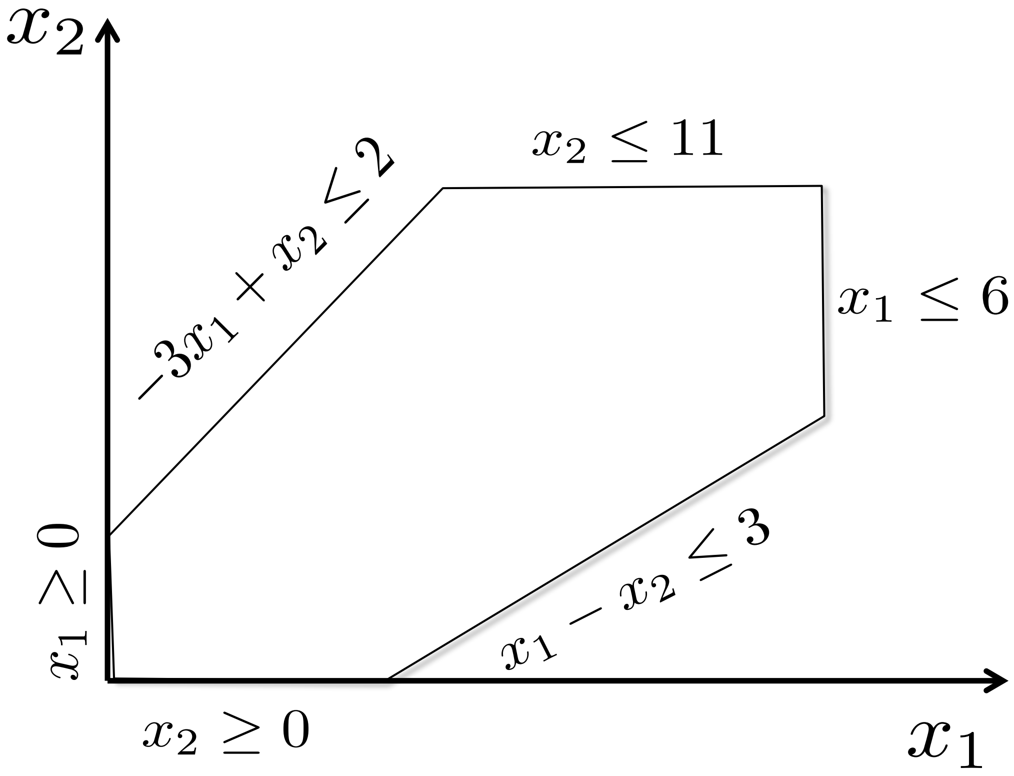
Dual Optimum

$$
\begin{array}{llllll}
\text{max.} & x_1 & +2x_2 & & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 & y_1 : 0 \\
 & & +x_2 & \leq & 11 & y_2 : 2 \\
 & x_1 & -x_2 & \leq & 3 & y_3 : 0 \\
 & x_1 & & \leq & 6 & y_4 : 1 \\
 & x_1, & x_2 & \geq & 0 & {\color{red} y_5, y_6} : 0
\end{array}
$$



Geometric View

$$
\begin{array}{rrrcr}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
\qquad
\boxed{\begin{array}{rcl}
y_1 & : & 0 \\
y_2 & : & 2 \\
y_3 & : & 0 \\
y_4 & : & 1 \\
\color{red}{y_5, y_6} & : & 0
\end{array}}
$$



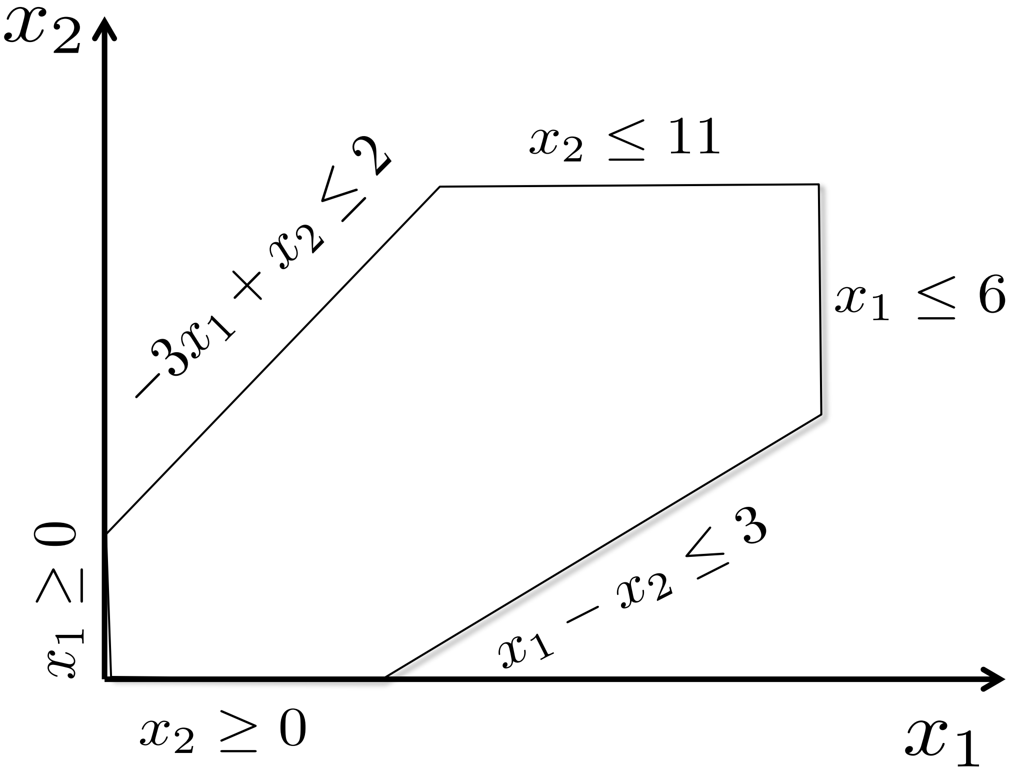
Sensitivity Analysis

$$\begin{array}{rcccccccl}
\max\ z = & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & & & \\
& a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 & \leftarrow & y_1 \\
& a_{21}x_1 & +a_{22}x_2 & +\cdots+ & a_{2n}x_n & \leq & b_2 & \leftarrow & y_2 \\
& \vdots & & \ddots & & \vdots & & & \\
& a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m & \leftarrow & y_m \\
& x_1, & x_2, & \cdots & x_n & \geq & 0 & &
\end{array}$$

1. x^* and y^* be optimal primal/dual from final dictionary
2. dictionary is assumed non-degenerate.

For a *infinitesimally* small change d in b_j (I.e, b_j changes to b_j +d) the objective changes by y_j * d

$$
\begin{array}{rrrcr}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
\qquad
\boxed{\begin{array}{rcl}
y_1 & : & 0 \\
y_2 & : & 2 \\
y_3 & : & 0 \\
y_4 & : & 1 \\
{\color{red} y_5, y_6} & : & 0
\end{array}}
$$



Diet Problem Dual

$$\begin{array}{lrcl} \max & -\mathbf{c}^\intercal \mathbf{x} & & \\ & F^\intercal \mathbf{x} & \leq & \mathbf{u} \\ & -F^\intercal \mathbf{x} & \leq & -\ell \\ & \mathbf{x} & \geq & \mathbf{0} \end{array}$$

$$\begin{array}{lrcl} \min & \mathbf{u}^\intercal \mathbf{y_u} - \ell^\intercal \mathbf{y}_\ell & & \\ & F(\mathbf{y_u} - \mathbf{y}_\ell) & \geq & -\mathbf{c} \\ & \mathbf{y_u},\ \mathbf{y}_\ell & \geq & \mathbf{0} \end{array}$$

DIET PROBLEM: DUAL

What does the dual mean?

Food	Calories	Total_Fat	Protein	Vit_A	Vit_C	Calcium	Price
Peppers	20	0.1	0.7	467.7	66.1	6.7	0.8
Potatoes, Baked	171.5	0.2	3.7	0	15.6	22.7	0.5
Tofu	88.2	5.5	9.4	98.6	0.1	121.8	1.1
Couscous	100.8	0.1	3.4	0	0	7.2	1
White Rice	102.7	0.2	2.1	0	0	7.9	0.4
Macaroni,Ckd	98.7	0.5	3.3	0	0	4.9	0.2
Peanut Butter	188.5	16	7.7	0	0	13.1	0.6

Nutrient	Min	Max
Calories	2000	2250
Total_Fat	0	65
Protein	50	100
Vit A	5000	50000
Vit C	50	20000
Calcium	800	1600

Optimal Solutions

Primal Solution

Food	Opt. Amt.
Peppers	9.55
Potatoes, Baked	0.95
Tofu	5.39
Couscous	0.00
White Rice	0.00
Macaroni,Ckd	11.86
Peanut Butter	0.00

Dual Solution

Nutrient	Dual (yU)	Dual (yL)
Calories	0.000	0.002
Total_Fat	0.000	0.000
Protein	0.021	0.000
Vit A	0.000	0.002
Vit C	0.000	0.000
Calcium	0.000	0.008