LOG BARRIER METHOD.

Linear Programming Formulation

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ A\mathbf{x} & \leq \mathbf{b} \\ \mathbf{x} & \geq 0 \end{aligned}$$

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ A\mathbf{x} + \mathbf{x}_s & = \mathbf{b} \\ \mathbf{x}, \mathbf{x_s} & \geq 0 \end{aligned}$$

Primal Standard form with Slack Variables

Log Barrier Trick

Inequality constrained optimization:

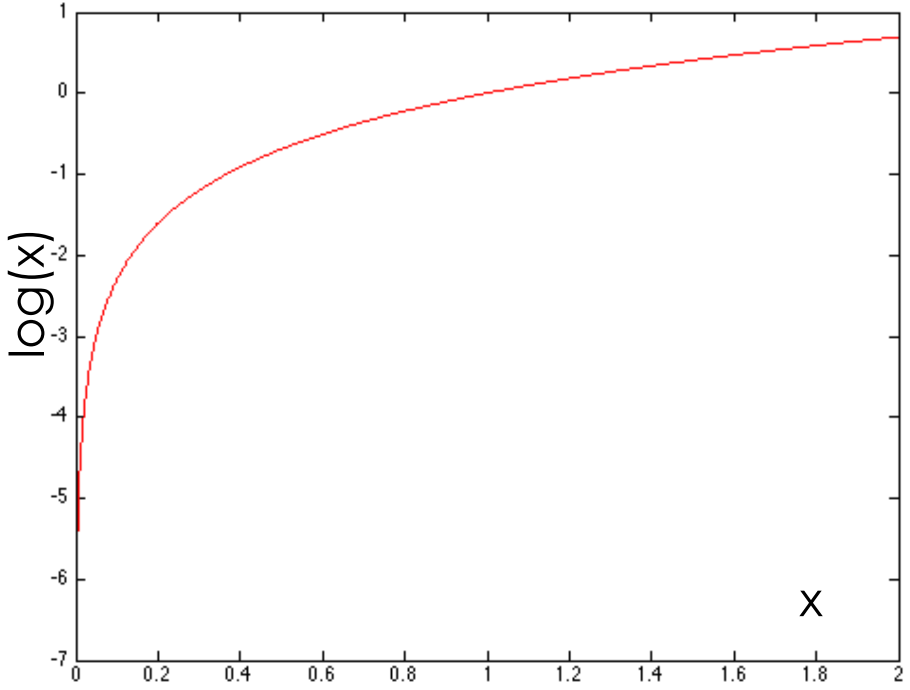
$$\max \; f(x) \text{ s.t. } g(x) \geq 0$$

Log Barrier Transformation of Inequality:

$$\max f(x) + \mu(\log(g(x)))$$

Log Barrier Trick (Log Function)

- log(x) is –Inf if x <= 0
- Adding log(x) to objective forbids x <= 0



Log Barrier Trick

$$\max f(x) + \mu(\log(g(x)))$$

$$\boxed{\text{A } \mu \to 0, \text{ we converge to solution of original problem.}}$$

- Solve log-barrier problem for initial mu (start with g(x) > 0)
- Gradually decrease mu ($\mu \to 0$).
- Stopping criterion: Change in x is below tolerance.

Linear Programming Formulation

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ A\mathbf{x} &\leq \mathbf{b} \\ \mathbf{x} &\geq 0 \end{aligned}$$

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ A\mathbf{x} + \mathbf{x}_s &= \mathbf{b} \\ \mathbf{x}, \mathbf{x_s} &\geq 0 \end{aligned}$$

Primal Standard form with Slack Variables

Log Barrier Formulation

$$\begin{array}{rrcl} \max & \mathbf{c}^\intercal\ \mathbf{x} & & \\ & A\mathbf{x} + \mathbf{x}_s & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s} & \geq & 0 \end{array}$$

$$\max\ \mathbf{c}^\intercal\ \mathbf{x} + \mu \sum_{j=1}^{n} \log(x_j) + \mu \sum_{i=1}^{m} \log(x_{s,i})$$

$$\text{s.t. } A\mathbf{x} + \mathbf{x_s} = \mathbf{b}$$

Equality Constrained Optimization

$$\max \ \mathbf{c}^\intercal \ \mathbf{x} + \mu \textstyle\sum_{j=1}^{n} \log(x_j) + \mu \sum_{i=1}^{m} \log(x_{s,i})$$
$$\text{s.t. } A\mathbf{x} + \mathbf{x_s} = \mathbf{b}$$

$$\boxed{L(\mathbf{x}, \mathbf{y}) = \left(\begin{array}{c} \mathbf{c}^\intercal \mathbf{x} + \mu \ \sum_{j=1}^{n} \log(x_j) + \mu \sum_{i=1}^{m} \log(x_{s,i}) \\ +\mathbf{y}^\intercal \ (A\mathbf{x} + \mathbf{x_s} - \mathbf{b}) \end{array} \right)}$$

$$\frac{\partial L}{\partial x_j} = c_j + \frac{\mu}{x_j} + \mathbf{y}^\intercal A_{:,j}$$