Pivoting

Initial Aux. Dictionary $\xrightarrow{\text{Pivoting Steps*}}$ Final Aux. Dictionary

Special Rule:

Whenever x_0 is one possible leaving variable, preferentially choose x_0 as the leaving variable.

Example

$$
\begin{array}{rcrrrr}
x_0 & = & 2 & +x_3 & -2x_1 & +x_2 \\
x_4 & = & 6 & +x_3 & -2x_1 & +0x_2 \\
x_5 & = & 0 & +x_3 & -3x_1 & +3x_2 \\
x_6 & = & 6 & +x_3 & -3x_1 & +x_2 \\
\hline
w & = & -2 & -x_3 & +2x_1 & -x_2
\end{array}
$$

x1 enters + x5 leaves

$$
\begin{array}{rcrrrr}
x_1 & = & 0 & +\frac{1}{3}x_3 & -\frac{1}{3}x_5 & +x_2 \\
x_0 & = & 2 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 & -x_2 \\
x_4 & = & 6 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 & -2x_2 \\
x_6 & = & 6 & +0x_3 & -x_5 & -2x_2 \\
\hline
w & = & -2 & -\frac{1}{3}x_3 & -\frac{2}{3}x_5 & +x_2
\end{array}
$$

Example (Cont)

$$\begin{array}{rcrrrr}
x_1 & = & 0 & +\frac{1}{3}x_3 & -\frac{1}{3}x_5 & +x_2 \\
x_0 & = & 2 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 & -x_2 \\
x_4 & = & 6 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 & -2x_2 \\
x_6 & = & 6 & +0x_3 & -x_5 & -2x_2 \\
\hline
w & = & -2 & -\frac{1}{3}x_3 & -\frac{2}{3}x_5 & +x_2
\end{array}$$

x2 enters and
x0 leaves

$$\begin{array}{rcrrrr}
x_1 & = & 2 & +\frac{2}{3}x_3 & +\frac{1}{3}x_5 & -x_0 \\
x_2 & = & 2 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 & -x_0 \\
x_4 & = & 2 & -\frac{1}{3}x_3 & -\frac{2}{3}x_5 & +2x_0 \\
x_6 & = & 2 & -\frac{2}{3}x_3 & +\frac{4}{3}x_5 & +2x_0 \\
\hline
w & = & 0 & +0x_3 & +0x_5 & -x_0
\end{array}$$

Finding Initial Dictionary (Orig. Problem)

$$\begin{array}{rcrrrr} x_1 & = & 2 & +\frac{2}{3}x_3 & +\frac{1}{3}x_5 & -x_0 \\ x_2 & = & 2 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 & -x_0 \\ x_4 & = & 2 & -\frac{1}{3}x_3 & -\frac{2}{3}x_5 & +2x_0 \\ x_6 & = & 2 & -\frac{2}{3}x_3 & +\frac{4}{3}x_5 & +2x_0 \\ \hline w & = & 0 & +0x_3 & +0x_5 & -x_0 \end{array}$$

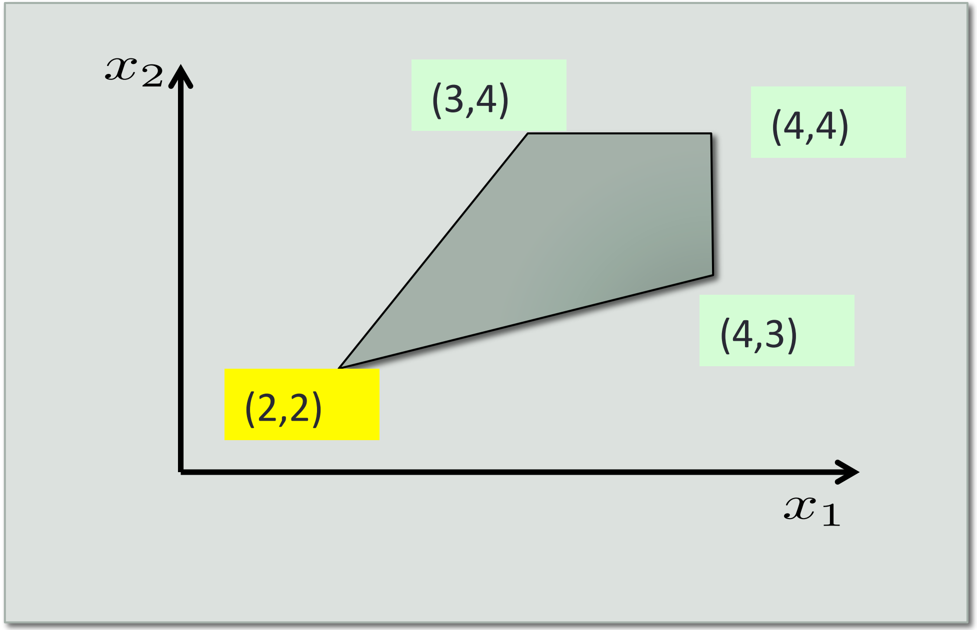
$$\begin{array}{lrcl} \text{max.} & x_1 + 2x_2 & & \\ \text{s.t.} & -2x_1 + x_2 & \leq & -2 \\ & x_2 & \leq & 4 \\ & x_1 - 2x_2 & \leq & -2 \\ & x_1 & \leq & 4 \\ & x_1, x_2 & \geq & 0 \end{array}$$

$$\begin{array}{rcrrr} x_1 & = & 2 & +\frac{2}{3}x_3 & +\frac{1}{3}x_5 \\ x_2 & = & 2 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 \\ x_4 & = & 2 & -\frac{1}{3}x_3 & -\frac{2}{3}x_5 \\ x_6 & = & 2 & -\frac{2}{3}x_3 & +\frac{4}{3}x_5 \\ \hline z & = & & & \end{array}$$

$$\begin{aligned} z &= x_1 + 2x_2 \\ &= (2 + \tfrac{2}{3}x_3 + \tfrac{1}{3}x_5) + 2(2 + \tfrac{1}{3}x_3 + \tfrac{2}{3}x_5) \\ &= 6 + \tfrac{4}{3}x_3 + \tfrac{5}{3}x_5 \end{aligned}$$

Initial Dictionary

$$
\begin{array}{rcrcc}
x_1 & = & 2 & +\frac{2}{3}x_3 & +\frac{1}{3}x_5 \\
x_2 & = & 2 & +\frac{1}{3}x_3 & +\frac{2}{3}x_5 \\
x_4 & = & 2 & -\frac{1}{3}x_3 & -\frac{2}{3}x_5 \\
x_6 & = & 2 & -\frac{2}{3}x_3 & +\frac{4}{3}x_5 \\
\hline
z & = & 6 & +\frac{4}{3}x_3 & +\frac{5}{3}x_5
\end{array}
$$



Initialization Phase Simplex

$$\begin{array}{lrcl} \max & -x_0 & & \\ \text{s.t.} & A\mathbf{x} + \mathbf{x_s} - x_0\mathbf{1} & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s}, x_0 & \geq & 0 \end{array}$$

If opt. value = 0 then form initial feasible dictionary for original problem.

Initial Aux. Dictionary

Pivoting Steps*

Final Aux. Dictionary

If opt. value < 0 then problem infeasible.

Non-Standard Pivots

- At the first step:
  - x_0 is entering.
  - Variable with least b_j is leaving.

- During initialization phase:
  - Whenever x_0 can be a leaving variable: we preferentially choose it.

- **Fact:** If x_0 is a leaving variable

  then next dictionary has to be final.

Simplex Algorithm