ENTERING/LEAVING VARIABLE CHOICE HEURISTICS

Entering Variable Choice

RECAP

$\boxed{\text{Choose } x_{Ij} \text{ such that } c_{Ij} > 0.}$

$$
\begin{array}{rclllllll}
x_{B1} & = & b_1 & +a_{11}x_{I1} & + \cdots & +a_{1j}x_{Ij} & + \cdots & +a_{1n}x_{In} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & + \cdots & +a_{2j}x_{Ij} & + \cdots & +a_{2n}x_{In} \\
 & \vdots & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & + \cdots & +a_{mj}x_{Ij} & + \cdots & +a_{mn}x_{In} \\
\hline
z & = & c_0 & +c_1x_{I1} & + \cdots & +c_jx_{Ij} & + \cdots & +c_nx_{In}
\end{array}
$$

Selecting the Leaving Variable

RECAP

$$
\begin{array}{ccccccc|l}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +a_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} & \rightarrow x_{Ij} \leq \frac{b_1}{-a_{1j}} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +a_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} & \rightarrow x_{Ij} \leq \infty \\
 & \vdots & & & & & & & \\
x_{Bi} & = & b_i & +a_{i1}x_{I1} & +\cdots & +\mathbf{a_{ij}}x_{Ij} & +\cdots & +a_{in}x_{In} & \boxed{\rightarrow x_{Ij} \leq \frac{b_i}{-a_{ij}}} \\
 & \vdots & & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +a_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} & \rightarrow x_{Ij} \leq \frac{b_m}{-a_{mj}} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +c_jx_{Ij} & +\cdots & +c_nx_{In} &
\end{array}
$$

Minimum bound

Entering Variable Choice

- Often, we can have more than one choice for entering variable.
  - For each entering variable, more than one choice of leaving.

- Question: If multiple choices, then how to choose?

Heuristic #1: Largest Objective Coefficient

$$
\begin{array}{rclllll}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +{\color{red}a_{1j}}x_{Ij} & +\cdots & +a_{1n}x_{In} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +{\color{red}a_{2j}}x_{Ij} & +\cdots & +a_{2n}x_{In} \\
 & \vdots & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +{\color{red}a_{mj}}x_{Ij} & +\cdots & +a_{mn}x_{In} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +{\color{red}c_j}x_{Ij} & +\cdots & +c_nx_{In}
\end{array}
$$

Idea: Larger the value of c_j, the more increase in z per unit increase in x_{Ij}.

Heuristic #2: Greedy Choice

- Explore all possible choices.
- Choose the entering/leaving combination that yield the most increase in z.

Bland's Rule

- Associate unique index with each problem/slack variable.

$$x_j \text{ has index } j.$$

- Amongst all possible entering variables, choose least possible index.

- Amongst all possible leaving choices, choose least index choice.

Example: Bland’s Rule

$$
\begin{array}{rcrrr}
x_3 & = & 1 & +3x_1 & -25x_2 \\
x_4 & = & 11 & +0x_1 & -x_2 \\
x_5 & = & 3 & -x_1 & +x_2 \\
x_6 & = & 3 & -x_1 & +0x_2 \\
\hline
z & = & 0 & +x_1 & +25x_2
\end{array}
$$