

DICTIONARIES AND VERTICES

Main Message

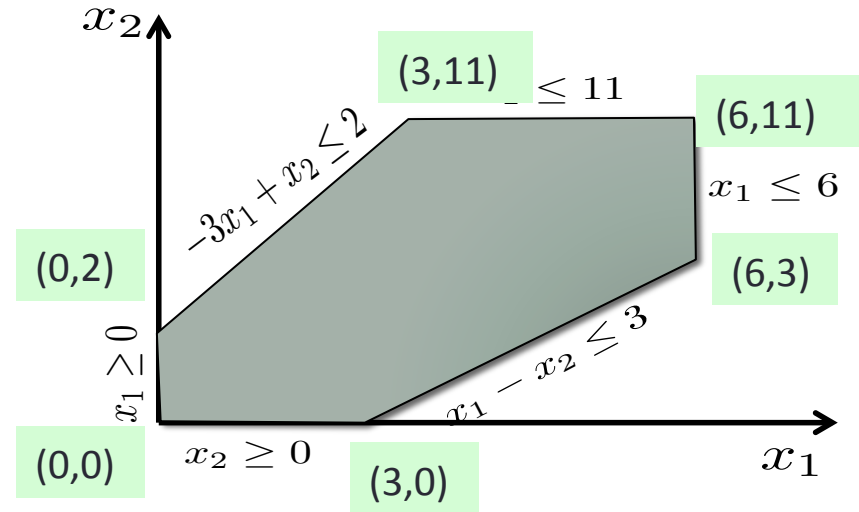
Dictionaries of Simplex = Vertices of the feasible region.

Linear Programming Problem

From Two Weeks Ago.

$$\begin{array}{llllll} \text{max.} & x_1 & +2x_2 & & & \\ \text{s.t.} & -3x_1 & +x_2 & \leq & 2 & \\ & & +x_2 & \leq & 11 & \\ & x_1 & -x_2 & \leq & 3 & \\ & x_1 & & \leq & 6 & \\ & x_1, & x_2 & \geq & 0 & \end{array}$$

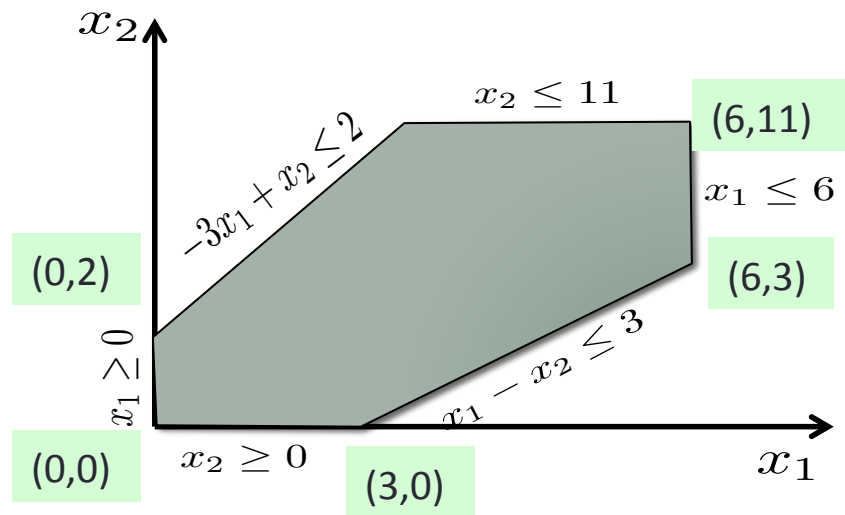
Note: Not drawn to scale



Goal: Solve LP using Simplex and visualize!

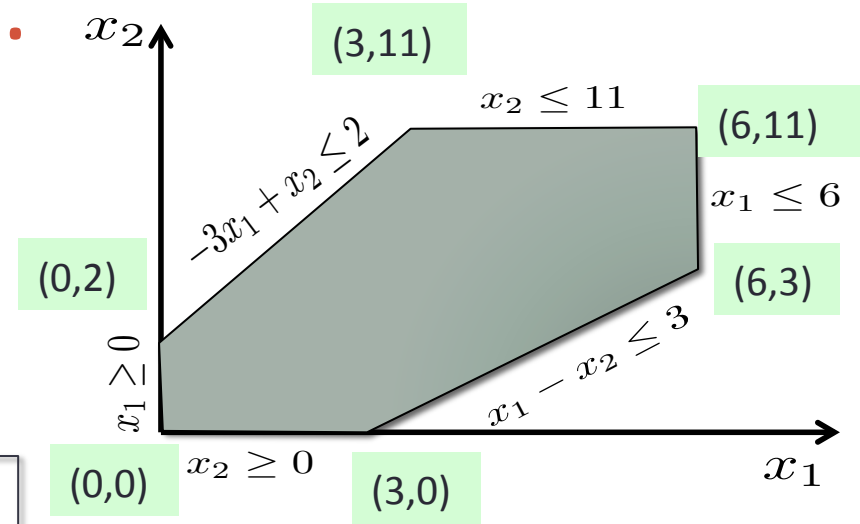
Linear Programming Problem

$$\begin{array}{llllll}
 \text{max.} & x_1 & +2x_2 & & & \\
 \text{s.t.} & -3x_1 & +x_2 & \leq & 2 & \\
 & & +x_2 & \leq & 11 & \\
 & x_1 & -x_2 & \leq & 3 & \\
 & x_1 & & \leq & 6 & \\
 & x_1, & x_2 & \geq & 0 &
 \end{array}$$



Dictionary Vertex Corr.

$$\begin{array}{llllll}
 \max & x_1 & +2x_2 & & & \\
 \text{s.t.} & -3x_1 & +x_2 & \leq & 2 & \leftarrow x_3 \\
 & & x_2 & \leq & 11 & \leftarrow x_4 \\
 & x_1 & -x_2 & \leq & 3 & \leftarrow x_5 \\
 & x_1 & & \leq & 6 & \leftarrow x_6 \\
 & x_1, & x_2 & \geq & 0 &
 \end{array}$$



$$x_1 =$$

$$x_2 =$$

$$x_5 =$$

$$x_6 =$$

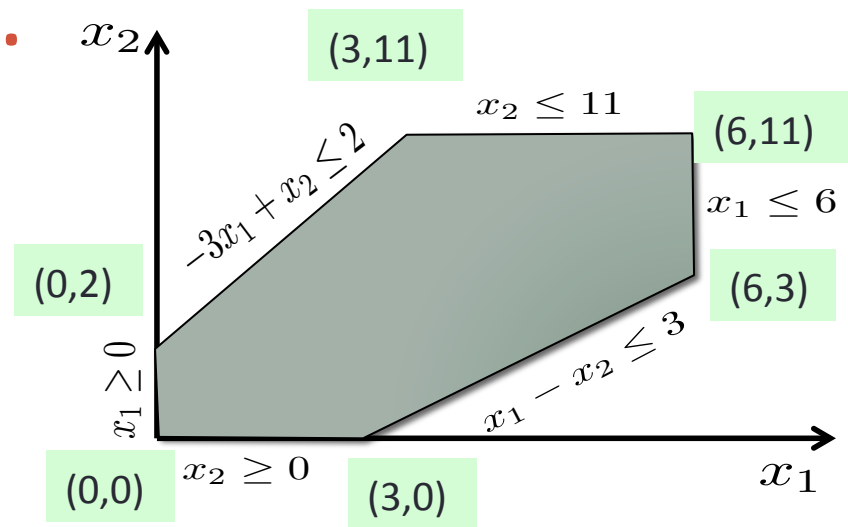
$$z =$$

$$x_4 \quad x_3$$

Dictionary Vertex Corr.

$$\begin{array}{llllll}
 \max & x_1 & +2x_2 & & & \\
 \text{s.t.} & -3x_1 & +x_2 & \leq & 2 & \leftarrow x_3 \\
 & & x_2 & \leq & 11 & \leftarrow x_4 \\
 & x_1 & -x_2 & \leq & 3 & \leftarrow x_5 \\
 & x_1 & & \leq & 6 & \leftarrow x_6 \\
 & x_1, & x_2 & \geq & 0 &
 \end{array}$$

x_1	\dots		
x_3	\dots		
x_4	\dots		
x_6	\dots		
z	$?$	$?x_2$	$?x_5$



Example #3

$$\max x_1 + x_2 - x_3$$

$$x_1 \leq 1 \leftarrow x_4$$

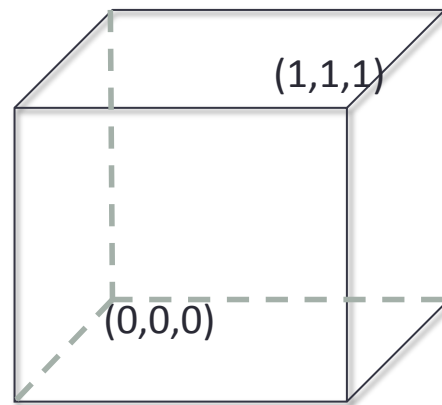
$$x_2 \leq 1 \leftarrow x_5$$

$$x_3 \leq 1 \leftarrow x_6$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$x_3 \geq 0$$



Linear Programming Problem (Standard Form)

$$\begin{aligned} \max \quad & \mathbf{c}^\top \mathbf{x} \\ & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned}$$

Feasible Dictionary

$$x_{B1} = b_1 + a_{11}x_{I1} + \cdots + a_{1j}x_{Ij} + \cdots + a_{1n}x_{In}$$

$$\vdots$$

$$x_{Bm} = b_m + a_{m1}x_{I1} + \cdots + a_{mj}x_{Ij} + \cdots + a_{mn}x_{In}$$

$$z = c_0 + c_1x_{I1} + \cdots + c_jx_{Ij} + \cdots + c_nx_{In}$$

- (1) Solution associated will make at least n constraints active.
- (2) Rank of active constraints is n.

Summary

- **Vertex (definition).**
 - A feasible point that makes at least n inequalities active.
 - The rank of active inequalities equals n .
- Feasible Dictionaries in Simplex:
 - Solution associated must be a vertex of the feasible region.
- What does pivoting do?