REVISED SIMPLEX METHOD

Description of Pivoting

Revised Simplex: Partial Reconstruction

1. Solution for Basic Variables

3. Column for entering Variable

$$
\begin{array}{lclccccc}
x_{B1} & = & b_1 & & & +a_{1j} x_{Ij} & & \\
x_{B2} & = & b_2 & & & +a_{2j} x_{Ij} & & \\
 & \vdots & & & & & & \\
x_{Bm} & = & b_m & & & +a_{mj} x_{Ij} & & \\
\hline
z & = & c_0 & +c_1 x_{I1} & + \cdots & +c_j x_{Ij} & + \cdots & +c_n x_{In}
\end{array}
$$

2. Objective row coefficients

Choosing Entering Variable

$$
\begin{array}{ccc}
\mathbf{x}_B & = & \hat{\mathbf{b}} & +\hat{A}\mathbf{x}_I \\
\hline
z & = & z_0 & \boxed{+\hat{c}\ \mathbf{x}_I}
\end{array}
$$

Q: How do we compute the objective coefficients (aka reduced costs)?

Computing Objective Coefficients

- Original Problem Data: *A, b, c*
- Current Basis Set: $B = \{x_{B1}, x_{B2}, \ldots, x_{Bm}\}$

Choosing Entering Variable

$$
\begin{array}{ccccccc}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +{\color{red}a_{1j}}x_{Ij} & +\cdots & +a_{1n}x_{In} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +{\color{red}a_{2j}}x_{Ij} & +\cdots & +a_{2n}x_{In} \\
 & \vdots & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +{\color{red}a_{mj}}x_{Ij} & +\cdots & +a_{mn}x_{In} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +{\color{red}c_j}x_{Ij} & +\cdots & +c_nx_{In}
\end{array}
$$

$$\pi = \mathbf{c_B}^\intercal A_B^{-1}$$

$$\hat{\mathbf{c}} = \mathbf{c}_I^\intercal - \pi A_I$$

Computing the Objective Row

- Compute $\pi = \mathbf{c}_B^{\mathsf{T}} A_B^{-1}$

  by solving the system of equations

$$\boxed{\pi A_B = \mathbf{c}_B^{\mathsf{T}}}$$

- Obtain obj. row coefficients by computing

$$\boxed{\hat{\mathbf{c}} = \mathbf{c}_I^{\mathsf{T}} - \pi A_I}$$

- Objective value is $z = \pi \mathbf{b}$

Entering Variable Analysis

$$z = c_0 + c_1 x_{I1} + \cdots + c_j x_{Ij} + \cdots + c_n x_{In} \Leftarrow \hat{\mathbf{c}}$$

Choose positive coefficient c_j

If no such coefficient, then dictionary is final.

Leaving Variable Analysis

$$
\begin{array}{rclccccc}
x_{B1} & = & b_1 & & & +a_{1j}x_{Ij} & & \\
x_{B2} & = & b_2 & & & +a_{2j}x_{Ij} & & \\
 & \vdots & & & & & & \\
x_{Bm} & = & b_m & & & +a_{mj}x_{Ij} & & \\
\hline
z & = & c_0 & +c_1 x_{I1} & +\cdots & +c_j x_{Ij} & +\cdots & +c_n x_{In}
\end{array}
$$

$$\hat{\mathbf{b}} \text{ and } \hat{\mathbf{a}}_j$$

Leaving Variable Analysis

- Compute $\hat{\mathbf{b}} = A_B^{-1}\mathbf{b}$ by solving the equations

$$A_B\hat{\mathbf{b}} = \mathbf{b}$$

- Compute $\hat{\mathbf{a}}_j = -A_B^{-1}A_j$ by solving the equations

$$A_B\hat{\mathbf{a}}_j = -A_j$$

We have enough data to perform leaving variable analysis.