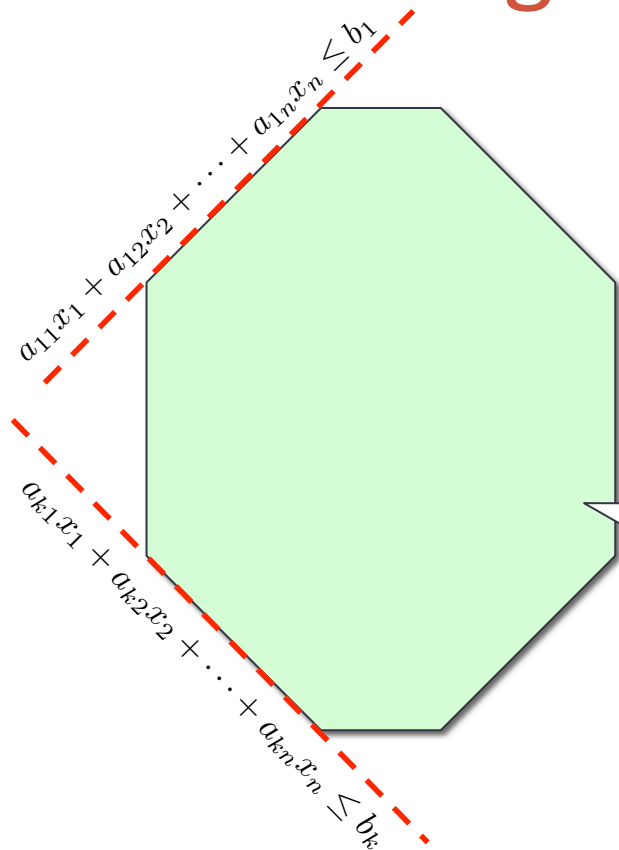


# INTEGER LINEAR PROGRAMMING

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Real vs. Integer Variables

# Feasible Region



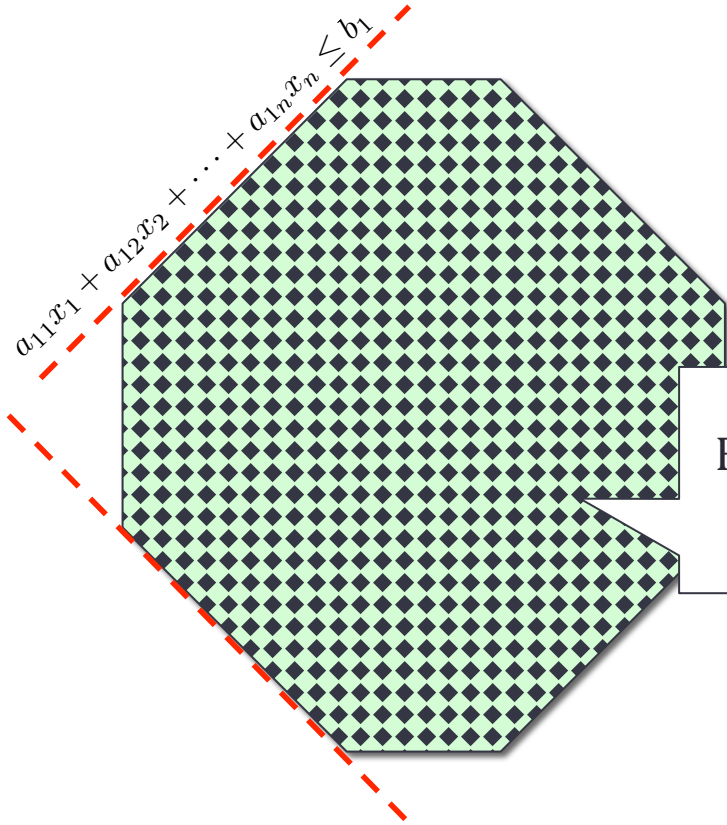
$$\begin{array}{llllll} \max & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & \\ \text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq b_1 \\ & & & & \vdots & \\ & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq b_m \end{array}$$

Feasible Region: Polyhedron  
(n dimensional)

# Linear vs. Integer Linear Programs

$$\begin{array}{llllll} \max & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & \\ \text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq b_1 \\ & & & \ddots & & \vdots \\ & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq b_m \end{array}$$

# Integer Linear Programming



$$\begin{array}{llllll} \max & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & \\ \text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq b_1 \\ & & & & \vdots & \\ & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq b_m \end{array}$$

Feasible Region: Z-Polyhedron  
(n dimensional)

# Linear vs. Integer Linear Programs (Complexity)

Linear Programming  
(Integers)

Nondeterministic  
Polynomial Time

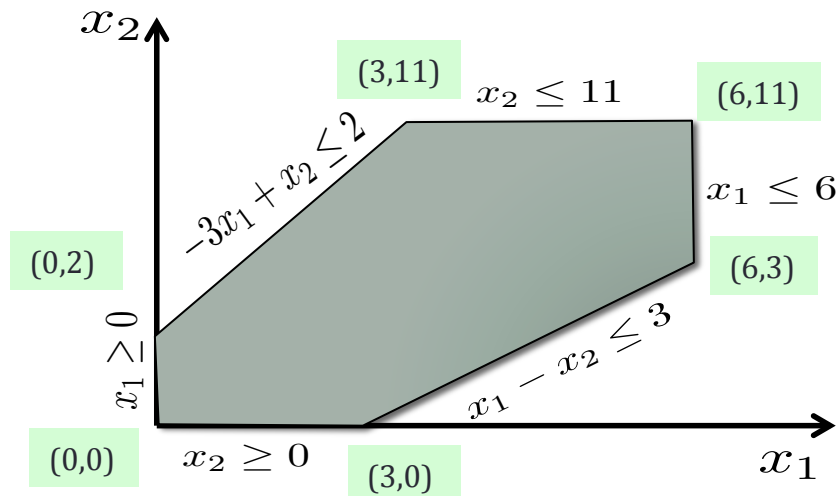
Million Dollar Question: Can Integer Linear Programs be solved  
in polynomial time?  
(  $P \stackrel{?}{=} NP$  )

# Example #1

$$\begin{array}{llllll}
 \text{max.} & x_1 & +2x_2 & & & \\
 \text{s.t.} & -3x_1 & +x_2 & \leq & 2 & \\
 & & +x_2 & \leq & 11 & \\
 & x_1 & -x_2 & \leq & 3 & \\
 & x_1 & & \leq & 6 & \\
 & x_1, & x_2 & \geq & 0 & 
 \end{array}$$

Solution:  $x_1 = 6, x_2 = 11$ pt.  
Objective Value: 28

Not drawn to scale



## Example #2

$$\begin{array}{llllll} \max & & x_2 & & & \\ \text{s.t.} & 3x_1 & +2x_2 & \leq & 6 & \\ & -3x_1 & +2x_2 & \leq & 0 & \\ & x_1, & x_2 & \geq & 0 & \\ & x_1, & x_2 & \in & Z & \end{array}$$

