

REVISED SIMPLEX: EXAMPLE

Example Problem

$$\begin{array}{rcll}
 \max & x_1 & -x_2 & +x_3 & -x_4 & & & & \\
 & 2x_1 & -3x_2 & +7x_3 & -15x_4 & +x_5 & & & = & 10 \\
 & & x_2 & -4x_3 & +6x_4 & & +x_6 & & = & 12 \\
 & -x_1 & & +x_3 & -2x_4 & & & +x_7 & = & 4 \\
 & & x_2 & +x_3 & & & & & +x_8 & = & 16 \\
 & & & & & & x_1, \dots, x_8 & \geq & 0 & &
 \end{array}$$

$$A = \begin{bmatrix} 2 & -3 & 7 & -15 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 12 \\ 4 \\ 16 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Example Problem

$$A = \begin{bmatrix} 2 & -3 & 7 & -15 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 12 \\ 4 \\ 16 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$B = \{5, 6, 7, 8\}$$

Constructing Objective Coefficient

$$A_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_I = \begin{bmatrix} 2 & -3 & 7 & -15 \\ 0 & 1 & -4 & 6 \\ -1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{c}_B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{c}_I = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \pi = \mathbf{c}_B^T A_B^{-1} = [0 \ 0 \ 0 \ 0]$$
$$\hat{\mathbf{c}} = \mathbf{c}_I^T - \pi A_I = [1 \ -1 \ 1 \ -1]$$

Constructing dictionary

$$B = \{5, 6, 7, 8\}$$

- Choose x_3 as entering variable ($j = 3$).
- Leaving variable analysis requires $\hat{\mathbf{b}}$ and $\hat{\mathbf{a}}_j$

$$\hat{\mathbf{b}} = A_B^{-1} \mathbf{b} = \begin{bmatrix} 10 \\ 12 \\ 4 \\ 16 \end{bmatrix} \quad \hat{\mathbf{a}}_j = -A_B^{-1} A_j = \begin{bmatrix} -7 \\ 4 \\ -1 \\ -1 \end{bmatrix}$$

Result

$$A = \begin{bmatrix} 2 & -3 & 7 & -15 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 12 \\ 4 \\ 16 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$B = \{5, 6, 7, 8\}$$

Entering Variable is x_3 Leaving Variable is x_5

$$B = \{3, 6, 7, 8\}$$

Second Dictionary

$$B = \{3, 6, 7, 8\}$$

$$A = \begin{bmatrix} 2 & -3 & 7 & -15 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 12 \\ 4 \\ 16 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Calculations for Pivoting

Ab =

7	0	0	0
-4	1	0	0
1	0	1	0
1	0	0	1

pl = [0.2857 0 0 0]

chat = [0.4286 -0.1429 -0.2857 **3.2857**]

Entering Index = 4

Ai =

2	-3	1	-15
0	1	0	6
-1	0	0	-2
0	1	0	0

bhat =

[1.4286
17.7143
2.5714
14.5714]

Aj =

[2.1429
2.5714
-0.1429
-2.1429]

Leaving Index = 8

Third Dictionary

$$A = \begin{bmatrix} 2 & -3 & 7 & -15 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 12 \\ 4 \\ 16 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \{3, 6, 7, 8\}$$

Variable x_4 enters and x_8 leaves

$$B = \{3, 6, 7, 4\}$$

Calculations for Pivot

Ab =

7	0	0	-15
-4	1	0	6
1	0	1	-2
1	0	0	0

pl = [0.0667 0 0 1.5333]

cHat = [0.8667 -2.3333 -0.0667 -1.5333]

Entering Index: 1

Ai =

2	-3	1	0
0	1	0	0
-1	0	0	0
0	1	0	1

bHat =

16.0000

35.2000

1.6000

6.8000

Aj =

0

-0.8000

1.2667

0.1333

Leaving Index: 6

Third Dictionary

$$A = \begin{bmatrix} 2 & -3 & 7 & -15 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 12 \\ 4 \\ 16 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \{3, 6, 7, 4\}$$

Variable x_1 enters and x_6 leaves

$$B = \{3, 1, 7, 4\}$$

Pivot Calculations

$A_b =$

7	2	0	-15
-4	0	0	6
1	-1	1	-2
1	0	0	0

$p_{II} =$

0.5000	1.0833	0	2.8333
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$ch_{at} =$

$A_i =$

0	-3	1	0
1	1	0	0
0	0	0	0
0	1	0	1

-1.0833	-3.4167	-0.5000	-2.8333
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Final Dictionary. Objective Value: 47.333