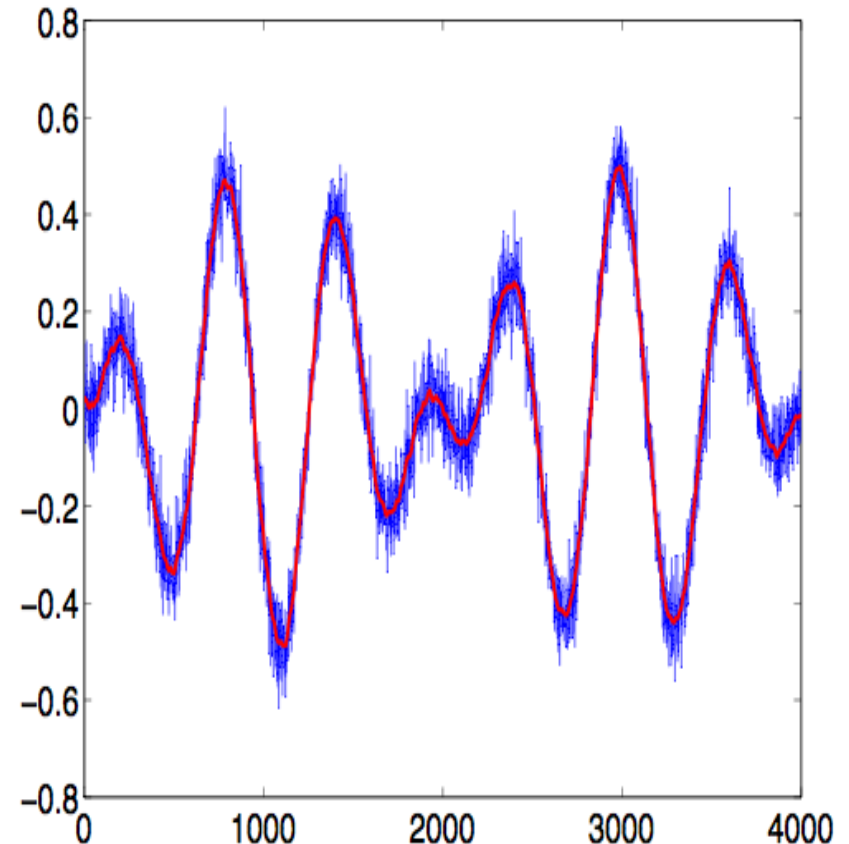# Least-Squares
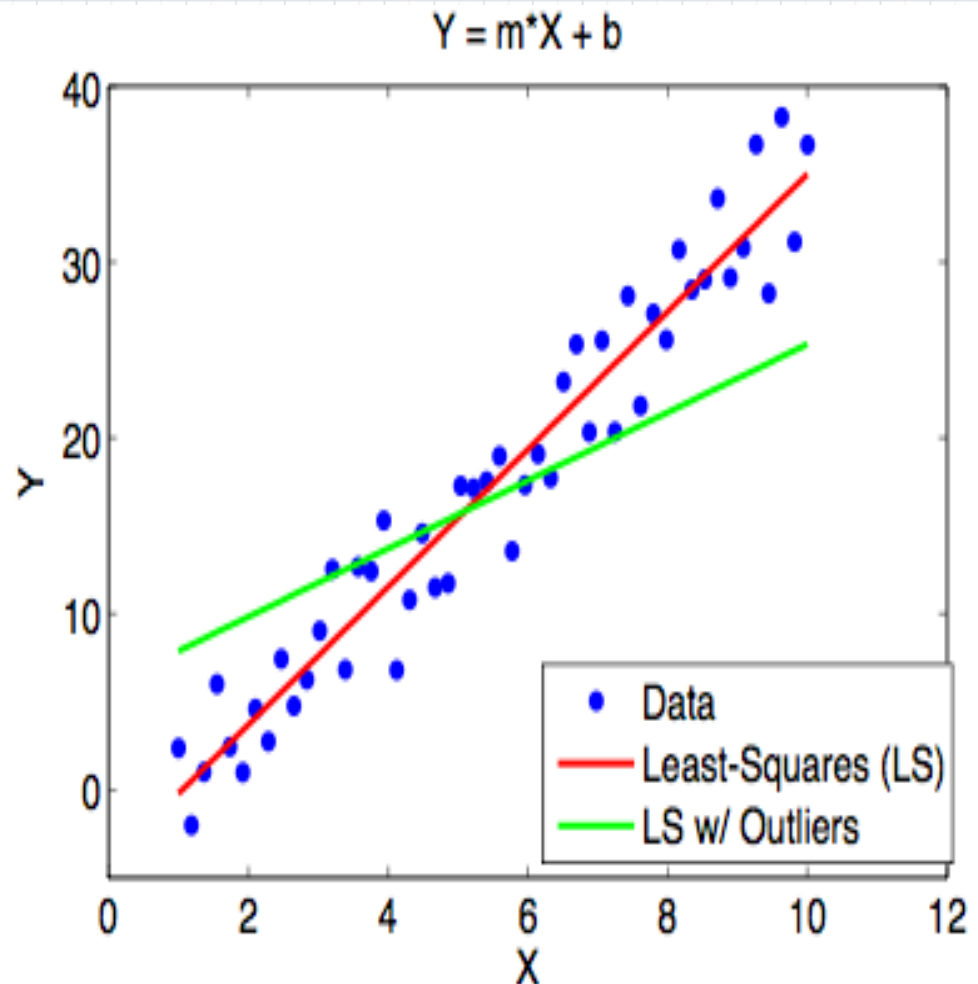
- Problem Description

- Overdetermined Problem

- "Best" Fit

- Matrix Form Solution

- De-Noising Application

Problem Description

- n data points (x_i, y_i)

- Questions: Can you fit a line $(y = mx_i + b)$ through all the data points?

In General, No!

$Y = m^*X + b$

Legend:
- Data
- Least-Squares (LS)
- LS w/ Outliers

Overdetermined Problem

- Too many equations, not enough unknowns

- No Solution! ✓
  (In General)

$$mx_1 + b = y_1$$
$$mx_2 + b = y_2$$
$$\vdots$$

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

A x

$$Ax = b$$

Matrix Form Solution

- Matrix Solution: Solve the Normal Equation: $(A^T A)x^\star = A^T b$

$$x^\star = (A^T A)^{-1} A^T b$$

$A^\dagger$ — psuedo-inverse

- In Matlab

```
>> x_star = pinv(A)*b;
```

```
>> x_star = A\b;
```

left divide