

Revised Simplex Method

- **Basic Idea:** Do not store the intermediate dictionary.
- Store the set of basic and non-basic variables.
- At each step, **reconstruct** dictionary from data:
 - Original problem data: A, b, c
 - Set of basic (and non-basic) variables: B

Storage Cost:

Original problem data (sparse)

Basis set $O(m + n)$

Recap: Dictionary Reconstruction

$$\begin{array}{ll} \max & \mathbf{c}^\top \mathbf{x} \\ & A\mathbf{x} + \mathbf{x}_s = \mathbf{b} \\ & \mathbf{x}, \mathbf{x}_s \geq 0 \end{array}$$



$$\frac{\mathbf{x}_B}{z} = \frac{\hat{\mathbf{b}} + \hat{A}\mathbf{x}_I}{z_0 + \hat{\mathbf{c}}^\top \mathbf{x}_I}$$

$$B = \{x_{B1}, \dots, x_{Bm}\}$$

Basic Variables

Recap: Splitting the Matrix

$$A : \begin{bmatrix} \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ A_1 & A_2 & \cdots & A_{B1} & \cdots & A_{Bm} & \cdots & A_m \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \end{bmatrix}$$

The diagram illustrates the splitting of matrix A into two parts: A_I and A_B . The matrix A is shown as a block matrix with columns $A_1, A_2, \dots, A_{B1}, \dots, A_{Bm}, \dots, A_m$. The first two columns, A_1 and A_2 , are grouped together by dashed arrows pointing to a dashed box labeled A_I . The columns A_{B1} and A_{Bm} are highlighted in red, and red arrows point from them to a red label A_B .

Recap: Rewriting the Equation

$$A\mathbf{x} + \mathbf{x}_s = \mathbf{b}$$



Step 1

$$A_B\mathbf{x}_B + A_I\mathbf{x}_I = \mathbf{b}$$



Step 2

$$A_B \mathbf{x}_B = \mathbf{b} - A_I\mathbf{x}_I$$

Is A_B always invertible?

Recap: Dictionary Reconstruction

$$A_B \mathbf{x}_B = \mathbf{b} - A_I \mathbf{x}_I$$



$$\mathbf{x}_B = A_B^{-1} \mathbf{b} - A_B^{-1} A_I \mathbf{x}_I$$

Is A_B always invertible?

Recap: Result Dictionary

$$\mathbf{c}^\top \mathbf{x} = \mathbf{c}_B^\top \mathbf{x}_B + \mathbf{c}_I^\top \mathbf{x}_I$$

$$\begin{array}{rcl} \mathbf{x}_B & = & A_B^{-1} \mathbf{b} \qquad \qquad \qquad -A_B^{-1} A_I \mathbf{x}_I \\ \hline \mathbf{c} & = & \mathbf{c}_B^\top A_B^{-1} \mathbf{b} \quad + (-\mathbf{c}_B^\top A_B^{-1} A_I + \mathbf{c}_I^\top) \mathbf{x}_I \end{array}$$

Invert matrix A_B : $O(m^3)$ (Gauss-Jordan).

Compute $A_B^{-1} \times A_I$ takes $O(m^2 n)$.

Compute $A_B^{-1} \mathbf{b}$ takes $O(m^2)$

Overall complexity: $O(m^2 * (m + n))$.