LINEAR ALGEBRA NOTATION

- Vector / Matrix

- Norms

- Solving a set of linear equations $Ax = b$

Vectors

- lowercase letters (primarily)

- columns

- rows = transpose of columns

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

in

$x \in \mathbb{R}^n$

n of them

each element x_i is a real number

$$x = [x_1 \ x_2 \ \cdots \ x_n]^T$$

How can you add n numbers?

list : $x = [1\ 2\ 3\ \ldots\ 49]^T$

Option 1 : built-in command : sum(x)

Option 2 : Loop

```
Sum = 0;
for ii = 1 to n
    Sum = Sum + x(ii)
end
```

Inner (or dot or scalar) product

$$x^T y = y^T x = \sum_{k=1}^{n} x_k y_k = \langle x, y \rangle = x \cdot y \quad \sim \quad \text{scalar}$$

$$(x^T y)^T = y^T x$$

$$[x_1 \ x_2 \ \cdots \ x_n] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = x_1 y_1 + x_2 y_2 + \cdots x_n y_n$$

$$\text{sum}(x)$$

$$= x^T \mathbb{1} = [x_1 \ x_2 \cdots x_n] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$= \sum_{i=1}^{n} x_i$$

Matrices

- Uppercase letters
  (primarily)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \\ \vdots & & \ddots & \\ a_{m1} & & & a_{mn} \end{bmatrix} ; \; A \in \mathbb{R}^{m \times n}$$

in

Rows

$$A = \begin{bmatrix} \text{---} & a_1^T & \text{---} \\ \text{---} & a_2^T & \text{---} \\ & \vdots & \\ \text{---} & a_n^T & \text{---} \end{bmatrix}$$

Columns

$$A = \begin{bmatrix} | & | & & | \\ \hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_n \\ | & | & & | \end{bmatrix}$$

Vector and Matrix: $Ax = b$

- m simultaneous equation

- n unknows

$$
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & & \\
\vdots & & \ddots & \\
a_{m1} & & & a_{mn}
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ \vdots \\ x_n
\end{bmatrix}
=
\begin{bmatrix}
b_1 \\ b_2 \\ \vdots \\ b_n
\end{bmatrix}
$$

m

Eqn 1: $a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$

$\vdots$

Eqn m: $a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n = b_m$

$Ax = b$: Row and Column Notation

Row

$$\begin{bmatrix} — a_1^T — \\ \\ — a_m^T — \end{bmatrix} x = \boxed{\begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} = \begin{bmatrix} a_1^T x \\ a_2^T x \\ \vdots \\ a_m^T x \end{bmatrix}}$$

Column

$$\begin{bmatrix} | & & | \\ \hat{a}_1 & \cdots & \hat{a}_n \\ | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = b$$

$$\boxed{x_1 \hat{a}_1 + x_2 \hat{a}_2 + \cdots x_n \hat{a}_n = b}$$

Solving $Ax = b$

$A \in \mathbf{R}^{m \times n}$ $b \in \mathbf{R}^{m}$

$x \in \mathbf{R}^{n}$

A $m = n$

☆ A is Full-Rank : Rows + Colums of A are linearly-independent
=> Invertable

$$\boxed{x = A^{-1} b}$$

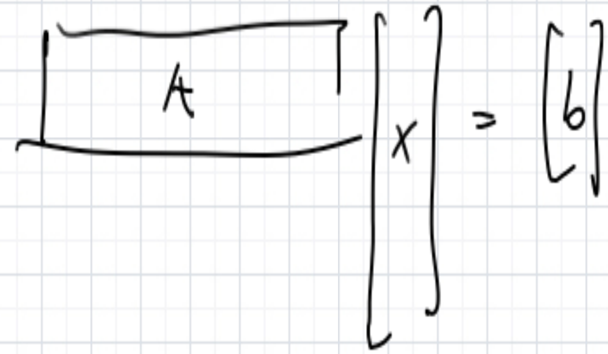
✓

Solving $Ax = b$

- $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^{m}$ are given

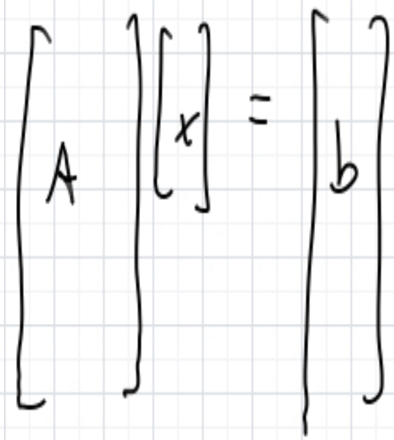
- $x \in \mathbf{R}^{n}$ is unknown

- if A is "fat" ($m < n$) - undetermined system



$\Rightarrow$ many solutions

- if A is "tall" ($m > n$) - overdetermined system



$\Rightarrow$ no solutions

3 Important Norms

- Turn a vector into a constant (compare vectors)

- Euclidean or ℓ_2 - Norm

$\|x\|_2$ or $\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots x_n^2}$

- Sum-absolute-value or ℓ_1 - Norm

$\|x\|^2 = x_1^2 + x_2^2 + \cdots x_n^2$

- Euclidean or ℓ_∞ - Norm

$x^T x = \|x\|^2$

3 Important Norms

- Sum-absolute-value or ℓ_1 - Norm

$$\|x\|_1 = |x_1| + |x_2| + \cdots |x_n|$$

- ~~Euclidean~~ or ℓ_∞ - Norm

Chebyshev - spell?

$$\|x\|_\infty = \max\{|x_1|, |x_2|, |x_3|, \ldots, |x_n|\}$$