Dictionaries

Dictionary: Solution associated + Feasibility

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$

$$
\begin{array}{rcrr}
\mathbf{x}_B & = & \mathbf{b} & -\mathbf{A}\mathbf{x}_I \\
\hline
z & = & c_0 & +\mathbf{c}^\intercal \mathbf{x}_I
\end{array}
$$

Solution Associated with Dictionary

- Non-basic variables have value 0.
- Basic variables: read off from dictionary.

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$

Another Dictionary

$$
\begin{array}{rcrrrr}
x_4 & = & -28 & +10x_1 & +3x_5 & +5x_3 \\
x_2 & = & 11 & -4x_1 & -x_5 & -2x_3 \\
x_6 & = & -32 & +9x_1 & +4x_5 & +62x_3 \\
\hline
z & = & 44 & -11x_1 & -4x_5 & -5x_3
\end{array}
$$

Feasible vs. Infeasible Dictionary

$$
\begin{aligned}
x_4 &= 5 - 2x_1 - 3x_2 - x_3 \\
x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
\hline
z &= 0 + 5x_1 + 4x_2 + 3x_3
\end{aligned}
$$

Feasible

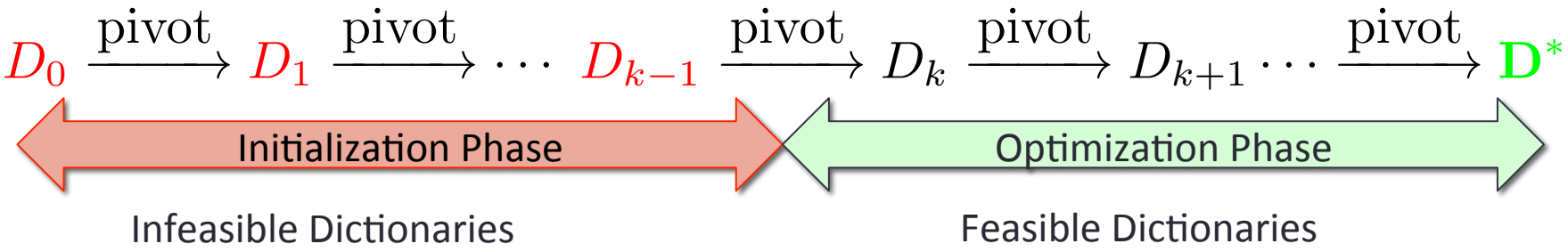
Infeasible

$$
\begin{aligned}
x_4 &= -28 + 10x_1 + 3x_5 + 5x_3 \\
x_2 &= 11 - 4x_1 - x_5 - 2x_3 \\
x_6 &= -32 + 9x_1 + 4x_5 + 62x_3 \\
\hline
z &= 44 - 11x_1 - 4x_5 - 5x_3
\end{aligned}
$$

Why Dictionaries?

- Data structure for Linear Programs.
  - Organize the data in the problem
- Represents candidate solutions to the problem.

Simplex Algorithm



Summary (1)

Transform Problem with Slack Variables

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}^\intercal \mathbf{x} \\ \text{subj.to.} \quad & \mathbf{A}\,\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}^\intercal \mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} + \mathbf{x}_{\text{slack}} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \\ & \mathbf{x}_{\text{slack}} \geq \mathbf{0} \end{aligned}$$

Summary (2)

Dictionary: Solution associated + Feasibility

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$

$$
\begin{array}{rcrr}
\mathbf{x}_B & = & \mathbf{b} & -\mathbf{A}\mathbf{x}_I \\
\hline
z & = & c_0 & +\mathbf{c}^\intercal \mathbf{x}_I
\end{array}
$$

ACT III: PIVOTING

Going from one dictionary to the next.