PIVOTING ALGORITHM

For the optimization phase.

Overview

Input: Dictionary D (Feasible)

Output: Dictionary D’ or STOP with answer.

1. Select Entering Variable.
    1. If no entering variable: dictionary is final. STOP with optimal soln.

2. For entering variable, select leaving variable.
    1. If no leaving variable: dictionary is unbounded. STOP (unbounded)

3. Perform Pivoting (Row Operations)

Dictionary Structure

$$\begin{array}{lll} \mathbf{x_B} & = \mathbf{b} & +A\mathbf{x_I} \\ \hline z & = c_0 & +\mathbf{c}^\intercal\ \mathbf{x_I} \end{array}$$

$$\begin{array}{lllll} x_{B1} & = b_1 & +a_{11}x_{I1} & +\cdots & +a_{1n}x_{In} \\ x_{B2} & = b_2 & +a_{21}x_{I1} & +\cdots & +a_{2n}x_{In} \\ & \vdots & & & \\ x_{Bm} & = b_m & +a_{m1}x_{I1} & +\cdots & +a_{mn}x_{In} \\ \hline z & = c_0 & +c_1x_{I1} & +\cdots & +c_nx_{In} \end{array}$$

Entering Variable Choice

Choose x_{Ij} such that $c_{Ij} > 0$.

$$
\begin{array}{rcllllll}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +a_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +a_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} \\
 & \vdots & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +a_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +c_jx_{Ij} & +\cdots & +c_nx_{In}
\end{array}
$$

Leaving Variable Analysis

$$
\begin{array}{lcllllll|l}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +a_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} & \rightarrow\ x_{Ij} \leq ? \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +a_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} & \rightarrow\ x_{Ij} \leq ? \\
 & \vdots & & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +a_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} & \rightarrow x_{Ij} \leq ? \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +c_jx_{Ij} & +\cdots & +c_nx_{In} &
\end{array}
$$

$$a_{1j} < 0 \ \Rightarrow x_{Ij} \leq -\frac{b_1}{a_{1j}}$$

$$a_{1j} \geq 0 \ \Rightarrow x_{Ij} \leq \infty$$

Leaving Variable Analysis

$$
\begin{array}{lcllllll|l}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +a_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} & \rightarrow\ x_{Ij} \leq \frac{b_1}{-a_{1j}} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +a_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} & \rightarrow\ x_{Ij} \leq \infty \\
 & \vdots & & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +a_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} & \rightarrow x_{Ij} \leq \frac{b_m}{-a_{mj}} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +c_jx_{Ij} & +\cdots & +c_nx_{In} &
\end{array}
$$

$$a_{ij} < 0 \ \Rightarrow x_{Ij} \leq -\frac{b_i}{a_{ij}}$$

$$a_{ij} \geq 0 \ \Rightarrow x_{Ij} \leq \infty$$

Selecting the Leaving Variable

$$
\begin{array}{lcllllll|l}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +a_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} & \rightarrow\ x_{Ij} \leq \frac{b_1}{-a_{1j}} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +a_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} & \rightarrow\ x_{Ij} \leq \infty \\
 & \vdots & & & & & & & \\
x_{Bi} & = & b_i & +a_{i1}x_{I1} & +\cdots & +\mathbf{a_{ij}}x_{Ij} & +\cdots & +a_{in}x_{In} & \rightarrow x_{Ij} \leq \frac{b_i}{-a_{ij}} \\
 & \vdots & & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +a_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} & \rightarrow x_{Ij} \leq \frac{b_m}{-a_{mj}} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +c_jx_{Ij} & +\cdots & +c_nx_{In} &
\end{array}
$$

Minimum bound

Pivoting

$$
\begin{array}{lcl|lll|l}
x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +a_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} & \rightarrow\ x_{Ij} \leq \frac{b_1}{-a_{1j}} \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +a_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} & \rightarrow\ x_{Ij} \leq \infty \\
 & \vdots & & & & & & & \\
x_{Bi} & = & b_i & +a_{i1}x_{I1} & +\cdots & +\mathbf{a_{ij}}x_{Ij} & +\cdots & +a_{in}x_{In} & \rightarrow x_{Ij} \leq \frac{b_i}{-a_{ij}} \\
 & \vdots & & & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +a_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} & \rightarrow x_{Ij} \leq \frac{b_m}{-a_{mj}} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +c_jx_{Ij} & +\cdots & +c_nx_{In} &
\end{array}
$$

$$
x_{Ij} = -\frac{b_i}{a_{ij}} + \frac{a_{i1}}{-a_{ij}}x_{I1} + \cdots + \frac{a_{in}}{-a_{ij}}x_{In} + \frac{-1}{-a_{ij}}x_{Bi}
$$

Pivoting Row Operations

$$x_{Ij} = -\frac{b_i}{a_{ij}} + \frac{a_{i1}}{-a_{ij}} x_{I1} + \cdots + \frac{a_{in}}{-a_{ij}} x_{In} + \frac{-1}{-a_{ij}} x_{Bi}$$

$$x_{B1} = b_1 + a_{11}x_{I1} + \cdots + a_{1j}x_{Ij} + \cdots + a_{1n}x_{In} \Big|$$

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Output: Dictionary D’ or STOP with answer.

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    1. If no entering variable: dictionary is final. STOP with optimal soln.

2. For entering variable, select leaving variable.
    1. If no leaving variable: dictionary is unbounded. STOP (unbounded)

3. Perform Pivoting (Row Operations)