Sum of Piecewise- Linear Functions

Min

$$\underbrace{\max_{i=1,2,\cdots,n}(\boldsymbol{a_i}^T\boldsymbol{x}+b_i)}_{f(x)} + \underbrace{\max_{i=1,2,\cdots,m}(\boldsymbol{c_i}^T\boldsymbol{x}+d_i)}_{g(x)}$$

- Equivalent LP

    - Need to add 2 auxiliary variables t_1, t_2

minimize $t_1 + t_2$

subject to $\boldsymbol{a_i}^T\boldsymbol{x}+b_i \leq t_1$

               $\boldsymbol{c_i}^T\boldsymbol{x}+d_i \leq t_2$

$\begin{cases} a_1^T x + b_1 \leq t_1 \\ \quad\vdots \\ a_n^T x + b_n \leq t_1 \\ c_1^T x + d_1 \leq t_2 \\ \quad\vdots \\ c_m^T x + d_m \leq t_2 \end{cases}$

Sum of Piecewise- Linear Functions

- Matrix Form:

$$\text{minimize} \quad t_1 + t_2$$
$$\text{subject to} \quad a_i^T x + b_i \leq t_1$$
$$c_i^T x + d_i \leq t_2$$

$$\hat{A}\hat{x} \leq \hat{b}$$

$$\hat{x} = \begin{bmatrix} x \\ t_1 \\ t_2 \end{bmatrix}$$

$$\text{min} \quad \begin{bmatrix} \vec{0} & 1 & 1 \end{bmatrix}^T \begin{bmatrix} x \\ t_1 \\ t_2 \end{bmatrix}$$

$$A = \begin{bmatrix} a_1^T \\ \vdots \\ a_n^T \end{bmatrix} \; ; \; C = \begin{bmatrix} c_1^T \\ \vdots \\ c_m^T \end{bmatrix}$$

$$\begin{bmatrix} A & -1 & 0 \\ C & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ t_1 \\ t_2 \end{bmatrix} \leq \begin{bmatrix} -b \\ -d \end{bmatrix}$$

$$b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \; ; \; d = \begin{bmatrix} d_1 \\ \vdots \\ d_m \end{bmatrix}$$