DEGENERATE POLYHEDRA

Degenerate Dictionaries

$$
\begin{array}{rclll}
x_1 & = & 3 & -\frac{1}{3}x_4 & +\frac{1}{3}x_3 \\
x_2 & = & 11 & -x_4 & +0x_3 \\
x_5 & = & 11 & -\frac{2}{3}x_4 & -\frac{1}{3}x_3 \\
x_6 & = & 0 & +\frac{1}{3}x_4 & -\frac{1}{3}x_3 \\
\hline
z & = & 25 & -\frac{7}{3}x_4 & +\frac{1}{3}x_3
\end{array}
$$

1. Understand geometry of degeneracy
2. Highly degenerate polyhedra.

Vertex (Definition)

A feasible solution **x** to the constraints is a vertex iff

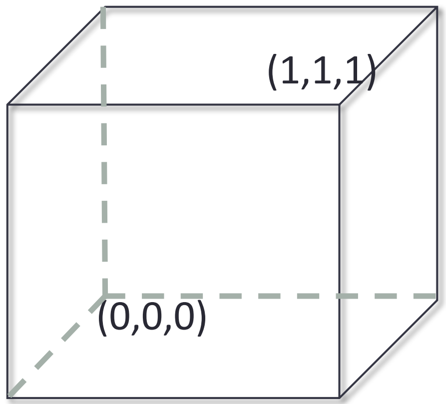
$$\begin{array}{cccccc}
a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 \\
 & & \ddots & & \vdots & \\
a_{j1}x_1 & +a_{j2}x_2 & +\cdots+ & a_{jn}x_n & \leq & b_j \\
 & & \ddots & & \vdots & \\
a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m
\end{array}$$

at least n ineqs.
are active for x.

rank of the active constraints for **x** is n

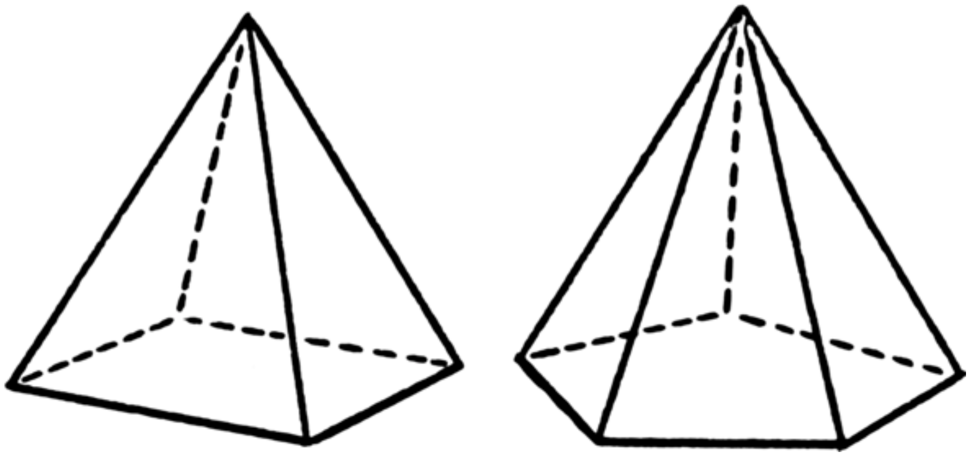
Vertices and Active Constraints

$$\begin{array}{rcl} x_1 & \leq & 1 \\ x_2 & \leq & 1 \\ x_3 & \leq & 1 \\ x_1 & \geq & 0 \\ x_2 & \geq & 0 \\ x_3 & \geq & 0 \end{array}$$

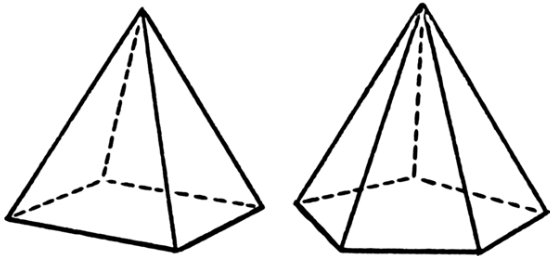


Vertex Issue #2

- Can a vertex activate more than n constraints?



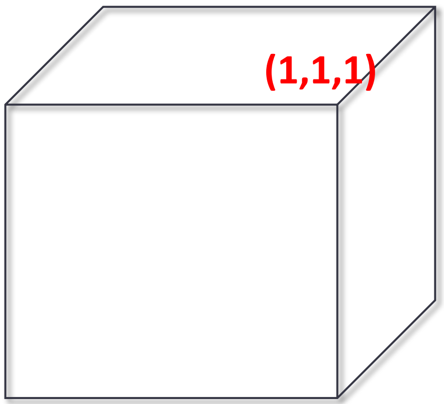
Degenerate Vertex (Definition)

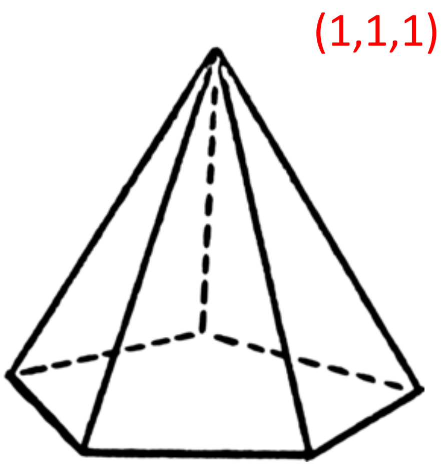


Vertex **x** is degenerate iff

it activates n+k constraints for k > 0

Degenerate vs. Non-degenerate vertex



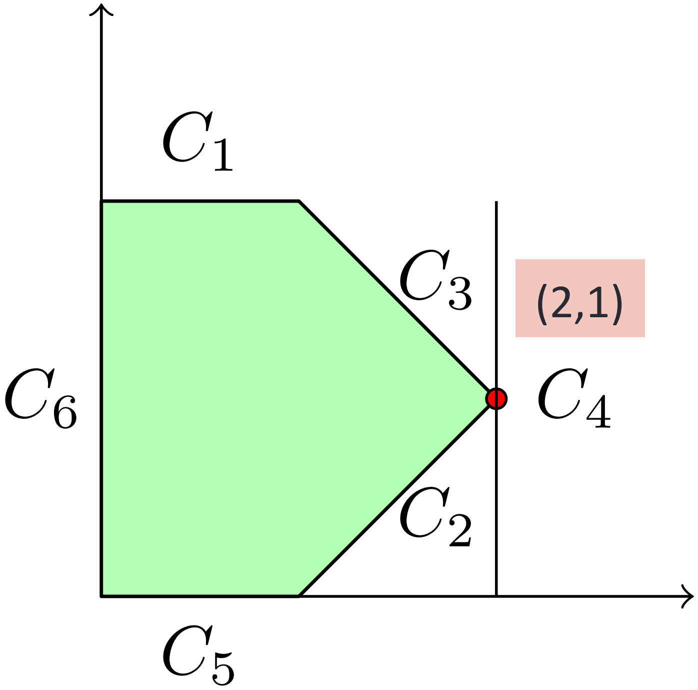


Non-degenerate:
- Activates exactly n constraints.
- Exactly n faces meet at the vertex.
- Unique dictionary associated with vertex.

Degenerate:
- Activates n + k constraints (k > 0)
- More than n faces meet at the vertex.
- Multiple dictionaries associated with vertex.

Degeneracy due redundancy

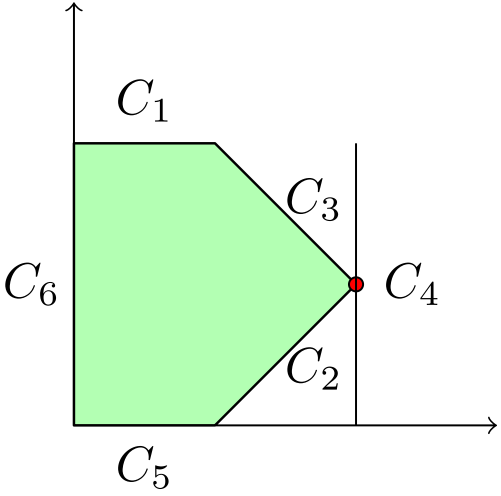
$$
\begin{array}{lrrcl}
\max & x & & & \\
\text{s.t.} & & & & \\
C_1: & & y & \leq & 2 \\
C_2: & x & -y & \leq & 1 \\
C_3: & x & +y & \leq & 3 \\
C_4: & x & & \leq & 2 \\
C_5: & -x & & \leq & 0 \\
C_6: & & -y & \leq & 0
\end{array}
$$



Degeneracy due to redundancy

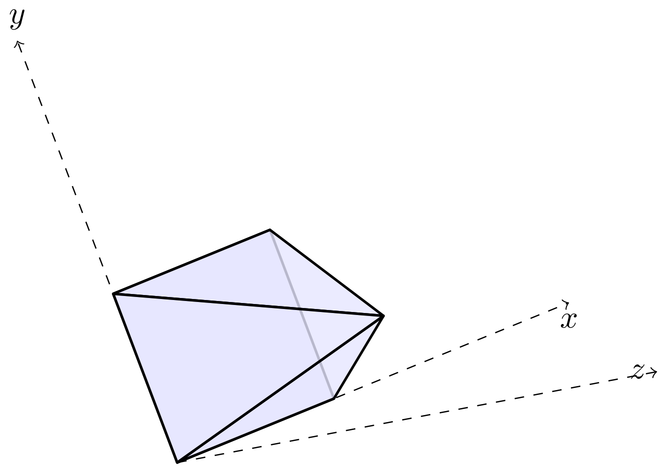
$$\begin{array}{c|cccc} w_1 & 1 & -\frac{1}{2}w_2 & +\frac{1}{2}w_3 \\ x & 2 & -\frac{1}{2}w_2 & -\frac{1}{2}w_3 \\ y & 1 & +\frac{1}{2}w_2 & -\frac{1}{2}w_3 \\ w_4 & 0 & +\frac{1}{2}w_2 & +\frac{1}{2}w_3 \\ \hline z & 2 & -\frac{1}{2}w_2 & -\frac{1}{2}w_3 \end{array}$$

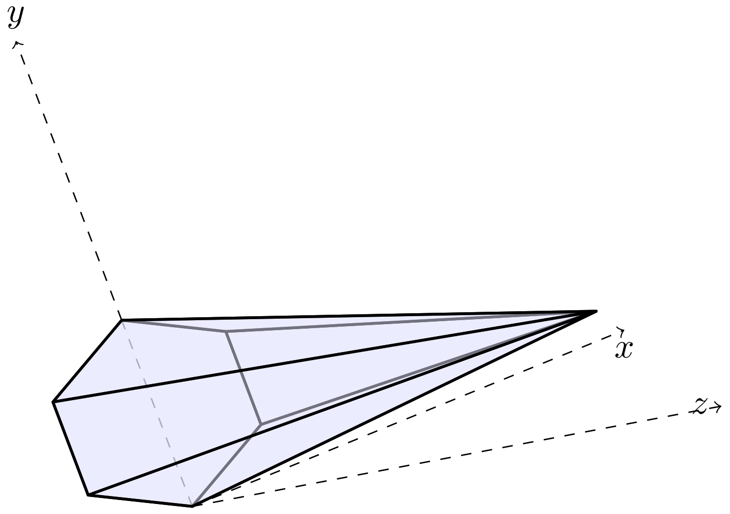
$$\begin{array}{c|cccc} w_1 & 1 & -w_2 & +w_4 \\ x & 2 & & -w_4 \\ y & 1 & +w_2 & -w_4 \\ w_3 & 0 & -w_2 & +2w_4 \\ \hline z & 2 & & -w_4 \end{array}$$



Degeneracy without redundancy

Removing any of the constraints changes feasible region.





Simplex over Degenerate Polyhedra