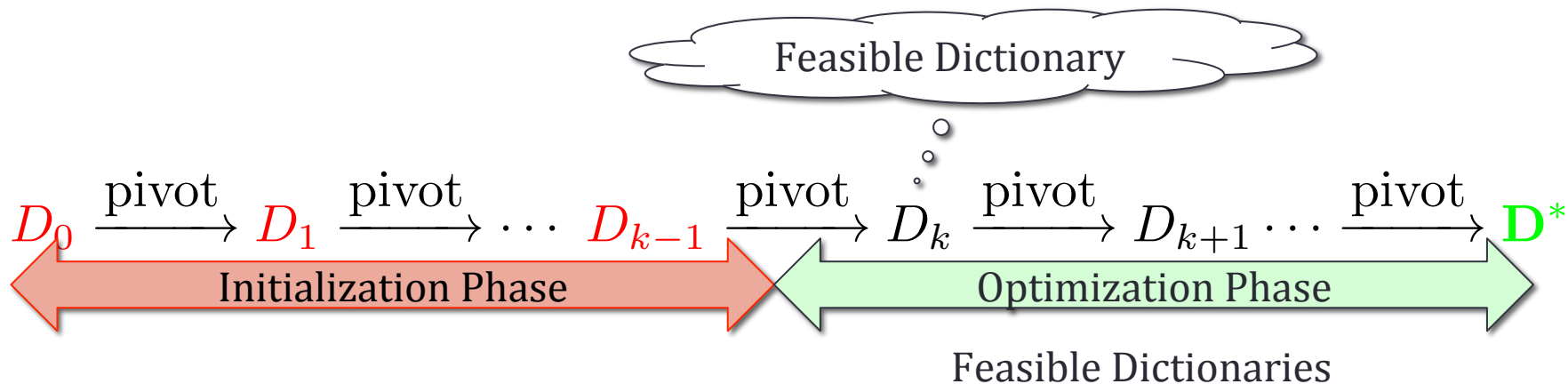


SIMPLEX METHOD: INITIALIZATION PHASE

How to find a feasible point using Simplex.

Simplex Algorithm



Initial Dictionary

$$\begin{array}{ll}\max & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0\end{array}$$



$$\begin{array}{ll}\max & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} + \mathbf{x}_s = \mathbf{b} \\ & \mathbf{x}, \mathbf{x}_s \geq 0\end{array}$$

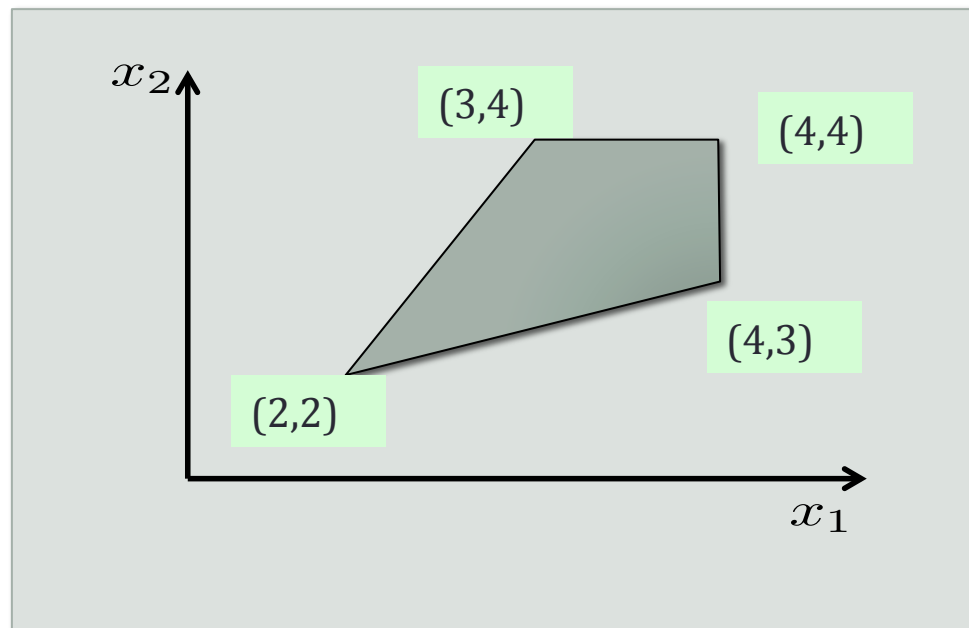


Feasible?

$$\begin{array}{rcl} \mathbf{x}_s & = & \mathbf{b} - A\mathbf{x} \\ \hline z & = & 0 + \mathbf{c}^T \mathbf{x} \end{array}$$

Example

$$\begin{array}{llll} \text{max.} & x_1 + 2x_2 & & \\ \text{s.t.} & -2x_1 + x_2 & \leq & -2 \\ & x_2 & \leq & 4 \\ & x_1 - 2x_2 & \leq & -2 \\ & x_1 & \leq & 4 \\ & x_1, x_2 & \geq & 0 \end{array}$$



Step 1: Adding Slack

$$\begin{array}{llll} \text{max.} & x_1 + 2x_2 & & \\ \text{s.t.} & -2x_1 + x_2 + x_3 & = & -2 \\ & x_2 + x_4 & = & 4 \\ & x_1 - 2x_2 + x_5 & = & -2 \\ & x_1 + x_6 & = & 4 \\ & x_1, x_2, x_3, \dots, x_6 & \geq & 0 \end{array}$$

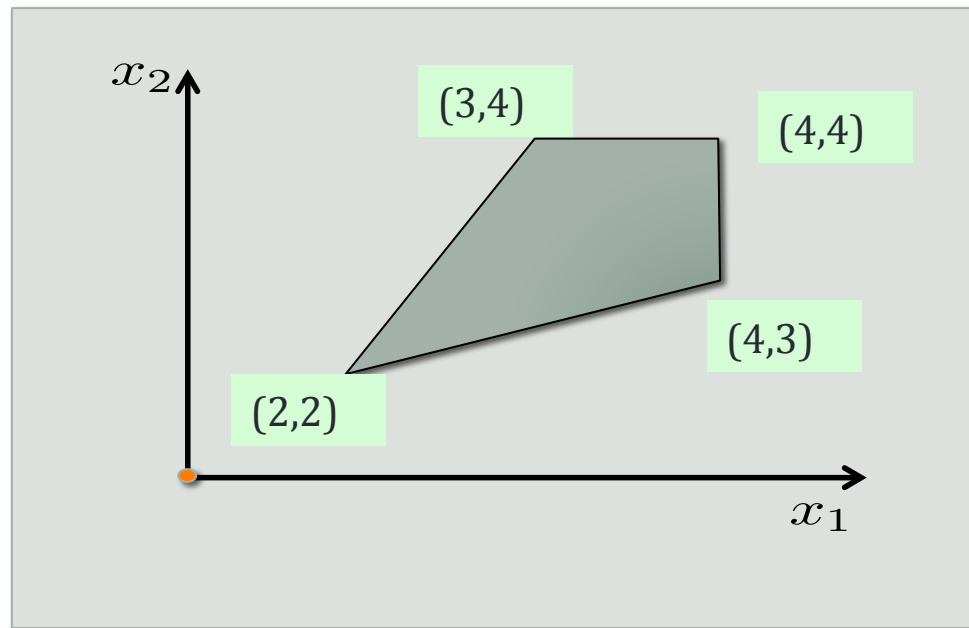
Step 2: Initial Dictionary

max.	$x_1 + 2x_2$	
s.t.	$-2x_1 + x_2 + x_3$	$= -2$
	$x_2 + x_4$	$= 4$
	$x_1 - 2x_2 + x_5$	$= -2$
	$x_1 + x_6$	$= 4$
	$x_1, x_2, x_3, \dots, x_6$	≥ 0

x_3	$=$	-2	$+2x_1$	$-x_2$
x_4	$=$	4	$+0x_1$	$-x_2$
x_5	$=$	-2	$-x_1$	$+2x_2$
x_6	$=$	4	$-x_1$	$+0x_2$
<hr/>				
z	$=$	0	$+x_1$	$+2x_2$

Infeasible Initial Dictionary

x_3	$=$	-2	$+2x_1$	$-x_2$
x_4	$=$	4	$+0x_1$	$-x_2$
x_5	$=$	-2	$-x_1$	$+2x_2$
x_6	$=$	4	$-x_1$	$+0x_2$
<hr/>				
z	$=$	0	$+x_1$	$+2x_2$



Initialization Phase Simplex : Overview

- **Goal:** Get to a dictionary with feasible solution.
 - Alternatively, conclude problem infeasibility.
- **Strategy:**
 - Modify the problem
 - Perform Simplex on Modified Problem.