MU-COMPLEMENTARITY CONDITIONS

Overview

$$\begin{array}{lrcl} \max & \mathbf{c}^\intercal\ \mathbf{x} & & \\ & A\mathbf{x} + \mathbf{x}_s & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s} & \geq & 0 \end{array}$$

Primal Problem

Log Barrier Trick

$$\max\ \mathbf{c}^\intercal\ \mathbf{x} + \mu \sum_{j=1}^{n} \log(x_j) + \mu \sum_{i=1}^{m} \log(x_{s,i})$$
$$\text{s.t. } A\mathbf{x} + \mathbf{x_s} = \mathbf{b}$$

A $\mu \to 0$, we converge to solution of original problem.

Lagrange Multiplier Method

Lagrange Multiplier Method

$$L(\mathbf{x},\mathbf{y}) = \left(\begin{array}{c} \mathbf{c}^\intercal \mathbf{x} + \mu \sum_{j=1}^{n} \log(x_j) + \mu \sum_{i=1}^{m} \log(x_{s,i}) \\ + \mathbf{y}^\intercal \left(A\mathbf{x} + \mathbf{x_s} - \mathbf{b} \right) \end{array} \right)$$

$$\begin{aligned} \nabla_{\mathbf{x}} L(\mathbf{x},\mathbf{y}) &= 0 \\ \nabla_{\mathbf{y}} L(\mathbf{x},\mathbf{y}) &= 0 \end{aligned}$$

First Order Necessary Conditions.

Mu KKT conditions

$$\boxed{\begin{aligned} A\mathbf{x} + \mathbf{x_s} &= \mathbf{b} && \text{Primal} \\ A^\intercal \mathbf{y} - \mathbf{y_s} &= \mathbf{c} && \text{Dual} \\ XY_s\mathbf{e} &= \mu\mathbf{e} && \text{Mu-Complementarity} \\ X_sY\mathbf{e} &= \mu\mathbf{e} && \end{aligned}}$$

$$\begin{aligned} X &= \text{diag}(\mathbf{x}) \\ X_s &= \text{diag}(\mathbf{x_s}) \\ Y &= \text{diag}(\mathbf{y}) \\ Y_s &= \text{diag}(\mathbf{y_s}) \end{aligned}$$

As mu approaches 0,
we obtain
KKT conditions!!