COMPLEXITY OF SIMPLEX

Klee-Minty Cubes.

Simplex Complexity

- Focus on maximum number of pivots required.
- Fortunately, Bland’s rule guarantees eventual termination.
- Upper bound is given by

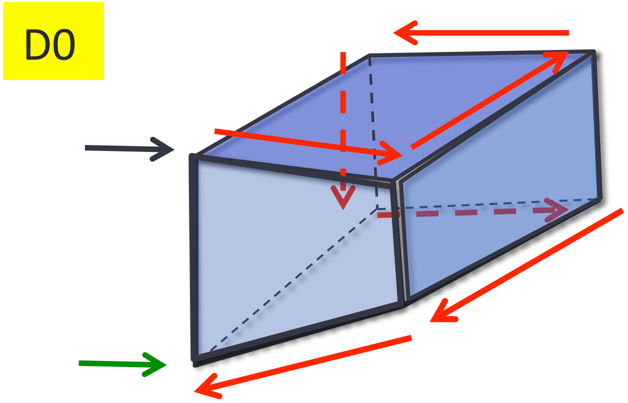
$$\binom{n+m}{m} = \frac{(n+m)!}{m!n!}$$

Klee-Minty Example

$$
\begin{array}{lrcl}
\text{max.} & 10^{n-1}x_1 + 10^{n-2}x_2 + \ldots + 10x_{n-1} + 1x_n & & \\
\text{s.t.} & x_1 & \leq & 1 \\
 & 20x_1 + x_2 & \leq & 100 \\
 & 200x_1 + 20x_2 + x_3 & \leq & 100^2 \\
 & \ldots & & \\
 & 2(\sum_{j=1}^{i-1} 10^{i-j} x_j) + x_i & \leq & 100^{i-1} \\
 & \ldots & & \\
 & x_1, \ldots, x_n & \geq & 0
\end{array}
$$

Klee-Minty Cube

- Feasible region is a “distorted” cube.



Klee-Minty Example

- If we follow the largest coefficient rule, then simplex requires 2^{n-1} iterations to converge.
- Worst-Case Complexity of Simplex (with largest objective coefficient heuristic) is exponential.
- Other pivoting rules have similar worst-case results.
  - Avis and Chvatal's exponential lower bound for Bland's rule on Klee-Minty Cubes.