LINEAR PROGRAMMING PROBLEM

Definition and Examples

Linear Program

Objective Function

maximize $\quad 2x_1 \quad +3x_2 \quad -x_3 \quad +x_4$

subject to

$$
\begin{array}{rrrrcr}
x_1 & -x_2 & & & \leq & 10 \\
2x_1 & +x_2 & -x_3 & & \geq & -5 \\
& -x_2 & & +x_4 & = & 4
\end{array}
$$

Constraints

Linear Program (General Form)

Objective
Function

$$\{\leq, \geq, =\}$$

$$
\begin{array}{llll}
\max & c_1 x_1 & +c_2 x_2 & +\cdots+ & c_n x_n \\
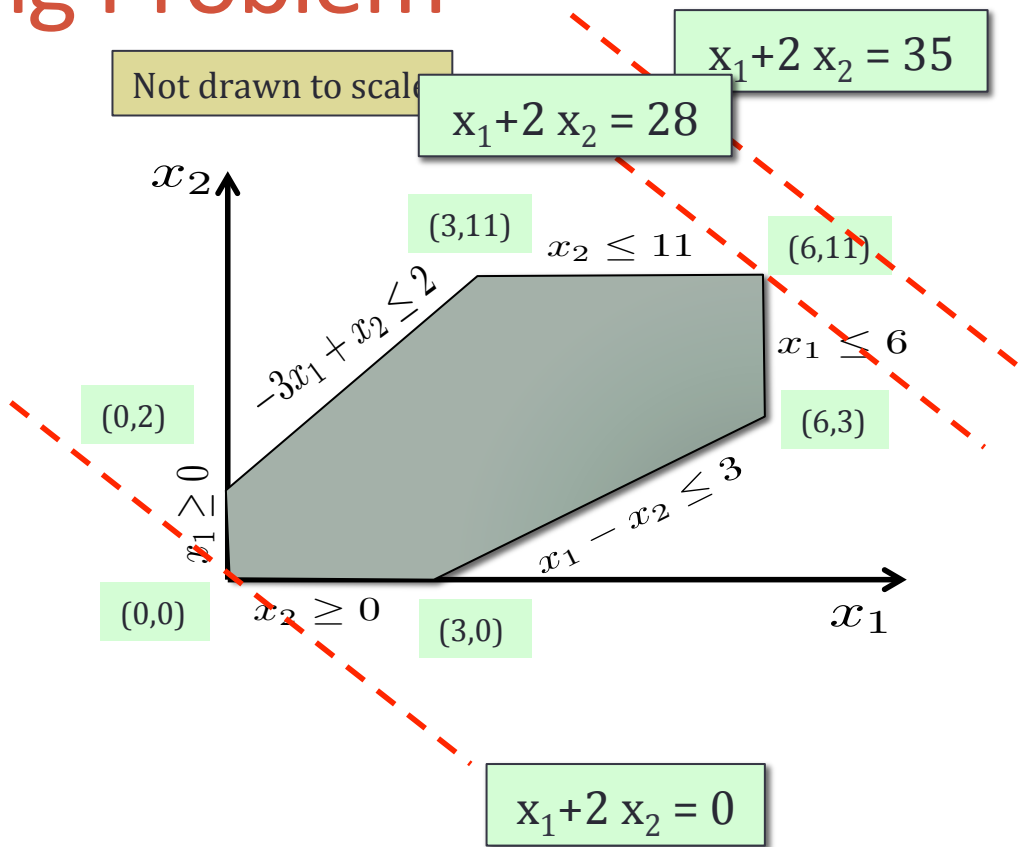\text{s.t.} & a_{11} x_1 & +a_{12} x_2 & +\cdots+ & a_{1n} x_n & \leq & b_1 \\
& & & \ddots & & \vdots \\
& a_{m1} x_1 & +a_{m2} x_2 & +\cdots+ & a_{mn} x_n & \leq & b_m
\end{array}
$$

Constraints

Linear Programming Problem

$$\begin{array}{rcccc}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
& & +x_2 & \leq & 11 \\
& x_1 & -x_2 & \leq & 3 \\
& x_1 & & \leq & 6 \\
& x_1, & x_2 & \geq & 0
\end{array}$$

Solution: $x_1 = 6$, $x_2 = 11$
Optimal Objective Value: 28

Not drawn to scale

$x_1 + 2x_2 = 35$

$x_1 + 2x_2 = 28$

x_2

(3,11) $x_2 \leq 11$ (6,11)

$-3x_1 + x_2 \leq 2$

$x_1 \leq 6$

(0,2)

(6,3)

$x_1 - x_2 \leq 3$

$x_1 \geq 0$

(0,0) $x_2 \geq 0$ (3,0)

x_1

$x_1 + 2x_2 = 0$

Overview

- Solving a Linear Program.
  - Visualizing Linear Programs.
  - What does solving a Linear Program mean?

- Algorithms for Linear Programming.
  - Simplex.
  - Ellipsoidal Methods.
  - Interior Point Methods.