Problem Set 1A: Basics of Linear Programming

Help

The **due date** for this homework is **Mon 10 Nov 2014 3:00 PM CST**.

This assignment is worth a total of 17 points

To be able to solve this assignment, please review Week #0 materials.

The questions in this assignment concern the linear programming problem below.

$$\begin{array}{lllllll} \max & 2x_1 & +3x_2 & & & \\ \text{s.t.} & x_1 & & & \leq & 5 \\ & x_1 & -x_2 & +x_3 & \leq & 10 \\ & & x_2 & & \leq & 15 \\ & & & x_3 & \leq & 22 \\ & x_1 & & & \geq & 0 \\ & & x_2 & & \geq & 0 \\ & & & x_3 & \geq & 0 \end{array}$$

Answer the questions below for this fixed linear programming problem.

☐ **In accordance with the Coursera Honor Code, I (Kevin Zhu) certify that the answers here are my own work.**

Question 1

How many decision variables does the LP have?

Question 2

Which of the following statements are true about the **feasible region** of the LP (recalled below)?

$$
\begin{array}{llllll}
\max & 2x_1 & +3x_2 & & & \\
\text{s.t.} & x_1 & & & \leq & 5 \\
& x_1 & -x_2 & +x_3 & \leq & 10 \\
& & x_2 & & \leq & 15 \\
& & & x_3 & \leq & 22 \\
& x_1 & & & \geq & 0 \\
& & x_2 & & \geq & 0 \\
& & & x_3 & \geq & 0
\end{array}
$$

Select all options that are correct and make sure that incorrect options are not selected.

- [ ] The feasible region is a three dimensional paraboloid (a bowl shaped figure).
- [ ] The point $(x_1 : 0, x_2 : 0, x_3 : 0)$ belongs to the feasible region of this LP.
- [ ] The linear programming problem is infeasible, so the feasible region is empty.
- [ ] The feasible region contains a point where the variable x_1 achieves a strictly negative value.
- [ ] The feasible region is a three dimensional *polyhedron* .

Question 3

Which of the following options holds for the point $(x_1 : 5, x_2 : 15, x_3 : 22)$? Choose the correct answer.

For your convenience the problem is recalled below:

$$
\begin{array}{llllll}
\max & 2x_1 & +3x_2 & & & \\
\text{s.t.} & x_1 & & & \leq & 5 \\
& x_1 & -x_2 & +x_3 & \leq & 10 \\
& & x_2 & & \leq & 15 \\
& & & x_3 & \leq & 22 \\
& x_1 & & & \geq & 0 \\
& & x_2 & & \geq & 0 \\
& & & x_3 & \geq & 0
\end{array}
$$

- ○ This point is **not** an optimal solution of the LP.
- ○ It satisfies more than a two-thirds majority of the constraints in the LP. Therefore, it can be treated as belonging to the feasible region.
- ○ It is one of the optimal solutions to the LP.

Question 4

Exactly one of the solutions given below is claimed to be an optimal solution to this problem.

Assuming this, choose the correct optimal solution.

For your convenience the problem is recalled below:

$$\begin{array}{llllll}
\max & 2x_1 & +3x_2 & & & \\
\text{s.t.} & x_1 & & & \leq & 5 \\
& x_1 & -x_2 & +x_3 & \leq & 10 \\
& & x_2 & & \leq & 15 \\
& & & x_3 & \leq & 22 \\
& x_1 & & & \geq & 0 \\
& & x_2 & & \geq & 0 \\
& & & x_3 & \geq & 0
\end{array}$$

- $(x_1 : 5, x_2 : 15, x_3 : 20)$
- $(x_1 : 5, x_2 : 10, x_3 : 0)$
- $(x_1 : 0, x_2 : 0, x_3 : 0)$
- $(x_1 : 200, x_2 : 300, x_3 : 0)$

Question 5

Assume that optimal solution obtained from a correct LP solver, sets x_1, x_2, x_3 to integer values yielding an objective value of 55.

Suppose we turn the problem into an ILP by further constraining the decision variables to integers, what is the optimal objective value of the ILP?

Question 6

Suppose we add a fresh inequality (different from all the previously existing ones) to the (old) LP (**LP-A**) above to obtain a *new* LP (**LP-B**), which of the following possibilities are true?

Assume that the fresh inequality involves x_1, x_2 and x_3 and in particular, does not introduce any new decision variables.

Please ensure that all the correct answers are selected and the incorrect answers remain unselected.

- [ ] **LP-B** (new LP) will always be feasible, no matter what inequality is added.
- [ ] The feasible region of **LP-B** (new LP) can become unbounded for some choice of the new inequality.
- [ ] **LP-B** (new LP) can become infeasible for some choice of the new inequality.

- [ ] **In accordance with the Coursera Honor Code, I (Kevin Zhu) certify that the answers here are my own work.**

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