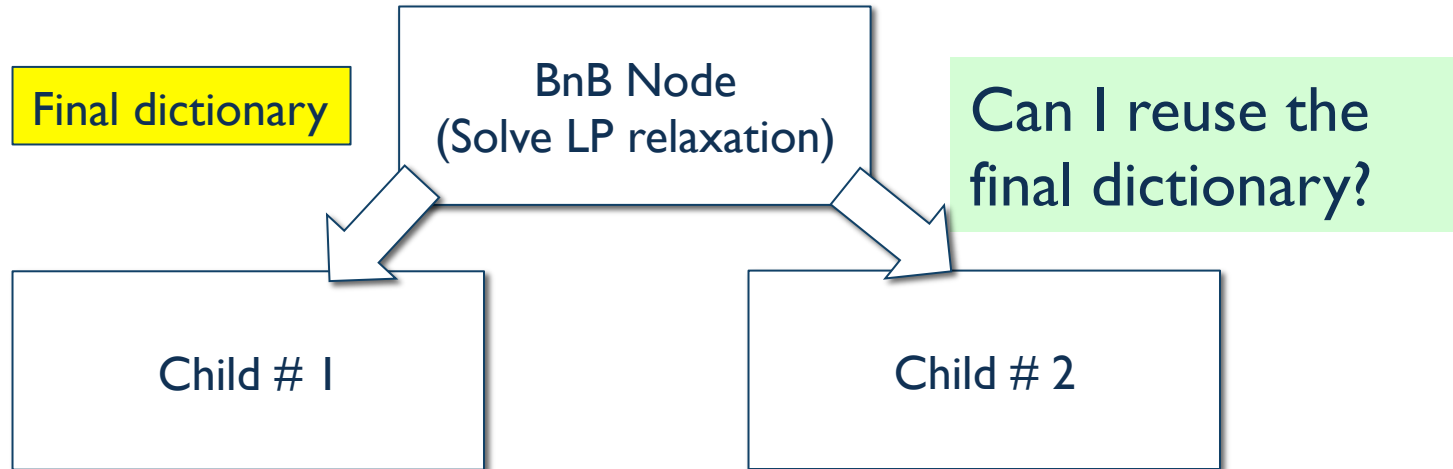


BRANCH-AND-BOUND

At the dictionary level

Thus far

- Use LP solver as a black box.
- This lecture:
 - Consider how to fold branch-and-bound in a dictionary setup.



View from the dictionary

\mathbf{x}_B	\mathbf{b}	\dots
z	c_0	$+\mathbf{c}_N^T \mathbf{x}_N$

Final dictionary

BnB Node
(Solve LP relaxation)

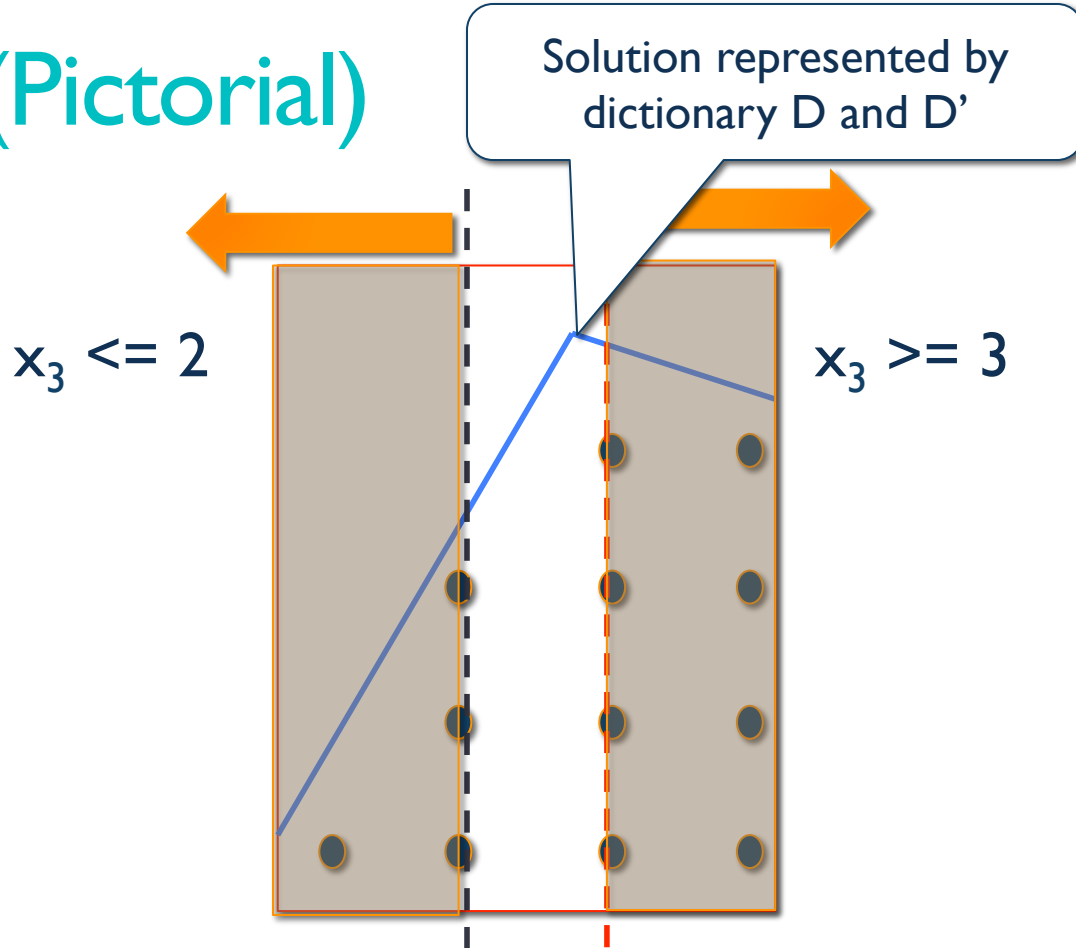
Claim: Dictionary D'
is always infeasible.

Child # 1

$$x_j \leq \lfloor s_j \rfloor$$

\mathbf{x}_B	\mathbf{b}	\dots
w	\dots	
z	c_0	$+\mathbf{c}_N^T \mathbf{x}_N$

Proof (Pictorial)



Example (ILP)

```
var x1 >= 0, <= 10;
```

```
var x2 >= 0, <= 10;
```

```
var x3 >= 0, <= 10;
```

```
maximize obj: x1 + x2 - 5 * x3 ;
```

```
c1: -2 * x1 + 7 * x2 <= 1;
```

```
c2: x1 - 2 * x2 + 5 * x3 <= 3;
```

```
c3: x1 + x2 - 3 * x3 <= 7;
```

```
solve;
```

```
display x1, x2, x3;
```

```
end;
```

$$x_4 : 10 - x_1$$

$$x_5 : 10 - x_2$$

$$x_6 : 10 - x_3$$

$$x_7 : 1 + 2x_1 - 7x_2$$

$$x_8 : 3 - x_1 + 2x_2 - 5x_3$$

$$x_9 : 7 - x_1 - x_2 + 3x_3$$

Slack variables are also integers.

Final Dictionary

x_4	4.3333333333	$+0.333333x_8$	$+0.666667x_9$	$-0.333333x_3$
x_5	8.6666666667	$-0.333333x_8$	$+0.333333x_9$	$-2.666667x_3$
x_6	10			$-x_3$
x_7	3	$-3x_8$	$+x_9$	$-18x_3$
x_1	5.6666666667	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
x_2	1.3333333333	$+0.333333x_8$	$-0.333333x_9$	$+2.666667x_3$
z	7.0	$+0x_8$	$-x_9$	$-2x_3$

$$x_1 \geq 6 \quad \longrightarrow \quad x_{10} : -6 + x_1$$

Dictionary After Branch

$$x_{10} : -6 + x_1$$

x_4	4.3333333333	$+0.333333x_8$	$+0.666667x_9$	$-0.333333x_3$
x_5	8.6666666667	$-0.333333x_8$	$+0.333333x_9$	$-2.666667x_3$
x_6	10			$-x_3$
x_7	3	$-3x_8$	$+x_9$	$-18x_3$
x_1	5.6666666667	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
x_2	1.3333333333	$+0.333333x_8$	$-0.333333x_9$	$+2.666667x_3$
x_{10}	-0.33333	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
z	7.0	$+0x_8$	$-x_9$	$-2x_3$

Dictionary becomes primal infeasible.

Back to initialization phase
Simplex?

Dictionary After Branch

x_4	4.3333333333	$+0.333333x_8$	$+0.666667x_9$	$-0.333333x_3$
x_5	8.6666666667	$-0.333333x_8$	$+0.333333x_9$	$-2.666667x_3$
x_6	10			$-x_3$
x_7	3	$-3x_8$	$+x_9$	$-18x_3$
x_1	5.6666666667	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
x_2	1.3333333333	$+0.333333x_8$	$-0.333333x_9$	$+2.666667x_3$
x_{10}	-0.33333	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
z	7.0	$+0x_8$	$-x_9$	$-2x_3$

Dictionary becomes primal infeasible.

Dual Feasible Dictionary!!
But not dual final.

How to update after branch?

Parent Node
(Final Dictionary)

add branch
constraint

Child Node:

1. Add new row.
2. Primal Infeasible but Dual Feasible

Consider dual complement
dictionary.
(Feasible + but non-final)

Opt. Phase on
dual dictionaries

Final dual dictionary
(also final primal)

LP relaxation
solved for child
node!

General Form Simplex

Parent Node
(Final Dictionary)

add branch
constraint

Child Node:

1. Add new row.
2. Primal Infeasible but Dual Feasible

Consider dual complement
dictionary.
(Feasible + but non-final)

Opt. Phase on
dual dictionaries

Final dual dictionary
(also final primal)

LP relaxation
solved for child
node!

General Form Dictionary

- Give special treatment to bounds on variables.

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$$

- Modify the Simplex algorithm in two ways:
 - Dictionaries now have special ways to track bounds.
 - Pivoting is modified.

Branch-and-Bound with General Form

Parent Node
(Final Dictionary)



Child Node:

1. ~~Add new row.~~
2. Primal Infeasible but Dual Feasible

Simply update
variable bound.