LP Equivalent: ℓ_∞ - Norm Approximation

- ℓ_∞ - Norm: $\|\boldsymbol{y}\|_\infty = \max_i |y_i| = \max_i \max(-y_i, y_i)$

$\|r\|_\infty$ $\quad |y_i|$

- Fitting/Approximation Problem:

$$\text{minimize} \quad \|A\boldsymbol{x} - \boldsymbol{b}\|_\infty$$

- LP Equivalent:

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & A\boldsymbol{x} - \boldsymbol{b} \le t\mathbf{1} \\ & -(A\boldsymbol{x} - \boldsymbol{b}) \ge t\mathbf{1} \end{array}$$

LP Equivalent: ℓ_∞ - Norm Approximation

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & Ax - b \preceq t\mathbf{1} = \begin{bmatrix} t \\ t \\ t \\ \vdots \end{bmatrix} \\ & -(Ax - b) \succeq t\mathbf{1} \end{array}$$

$\min \|Ax-b\|_\infty$

$-Ax + b$

$\le$

$\hat{x} = \begin{bmatrix} x \\ t \end{bmatrix}$

$\min \underbrace{\begin{bmatrix} \vec{0} & 1 \end{bmatrix}^T}_{\hat{c}^T} \underbrace{\begin{bmatrix} x \\ t \end{bmatrix}}_{\hat{x}}$

$Ax - \mathbb{1}t \le b$

$\mathbb{1}$ is false

$-Ax - \mathbb{1}t \le -b$

$-Ax + b \le t\mathbb{1}$

$$\underbrace{\begin{bmatrix} A & -\mathbb{1} \\ A & -\mathbb{1} \end{bmatrix}}_{\hat{A}} \begin{bmatrix} x \\ t \end{bmatrix} \le \underbrace{\begin{bmatrix} b \\ -b \end{bmatrix}}_{\hat{b}}$$

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- Matrix Form:

$$
\begin{array}{ll}
\text{minimize} & \begin{bmatrix} \mathbf{0} & 1 \end{bmatrix}^T \begin{bmatrix} \boldsymbol{x} \\ t \end{bmatrix} \\
\text{subject to} & \begin{bmatrix} A & -\mathbf{1} \\ -A & -\mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ t \end{bmatrix} \leq \begin{bmatrix} \boldsymbol{b} \\ -\boldsymbol{b} \end{bmatrix}
\end{array}
$$