General Principle (Attempt 1 of 2)

$$
\begin{array}{lcccccccl}
\max\ z = & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & & \\
& a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 & \leftarrow e_1 \leq b_1 \\
& a_{21}x_1 & +a_{22}x_2 & +\cdots+ & a_{2n}x_n & \leq & b_2 & \leftarrow e_2 \leq b_2 \\
& \vdots & & \ddots & & \vdots & & \\
& a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m & \leftarrow e_m \leq b_m \\
& & & & x_1,\ldots,x_n & \geq & 0 &
\end{array}
$$

$$c_1x_1 + \ldots + c_mx_m \equiv y_1 \times e_1 + y_2 \times e_2 + \cdots + y_m \times e_m$$

$$z^* \leq y_1b_1 + y_2b_2 + \cdots + y_mb_m$$

$$y_1, y_2, \ldots, y_m \geq 0$$

Example

$$
\begin{array}{lllllll}
\max & -2x_1 & +3x_2 & +x_3 & & \\
& x_1 & -x_2 & +x_3 & \leq & 10 \\
& 2x_1 & -3x_2 & -x_3 & \leq & 10 \\
& & +6x_2 & +2x_3 & \leq & 10 \\
& & & x_1, x_2, x_3 & \geq & 0
\end{array}
$$

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& a_{21}x_1 & +a_{22}x_2 & +\cdots+ & a_{2n}x_n & \leq & b_2 & \leftarrow e_2 \leq b_2 \\
& \vdots & & \ddots & & \vdots & & \\
& a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m & \leftarrow e_m \leq b_m \\
& -x_1 & & & & \leq & 0 & \\
& & -x_2 & & & \leq & 0 & \\
& & & \ddots & & & & \\
& & & & -x_n & \leq & 0 &
\end{array}$$

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n \;\equiv\; \sum_{i=1}^{m} y_i(e_i) + y_{m+1}(-x_1) + \cdots + y_{n+m}(-x_n)$$

Example

$$
\begin{array}{lrrrll}
\max & -2x_1 & +3x_2 & +x_3 & & \\
 & x_1 & -x_2 & +x_3 & \leq & 10 \\
 & 2x_1 & -3x_2 & -x_3 & \leq & 10 \\
 & & +6x_2 & +2x_3 & \leq & 10 \\
 & -x_1 & & & \leq & 0 \\
 & & -x_2 & & \leq & 0 \\
 & & & -x_3 & \leq & 0
\end{array}
$$

$$
\boxed{
\begin{array}{lrrrll}
\max & -2x_1 & +3x_2 & +x_3 & & \\
 & x_1 & -x_2 & +x_3 & \leq & 10 \\
 & 2x_1 & -3x_2 & -x_3 & \leq & 10 \\
 & & +6x_2 & +2x_3 & \leq & 10 \\
 & & & x_1, x_2, x_3 & \geq & 0
\end{array}
}
$$

General Principle

$$
\begin{array}{lcccccccc}
\max\ z = & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & & \\
\\
& a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 & \leftarrow \color{red}{\times\ y_1} \\
& a_{21}x_1 & +a_{22}x_2 & +\cdots+ & a_{2n}x_n & \leq & b_2 & \leftarrow \color{red}{\times\ y_2} \\
& \vdots & & \ddots & & \vdots & & \\
& a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m & \leftarrow \color{red}{\times\ y_m} \\
& -x_1 & & & & \leq & 0 & \leftarrow \times\ y_{m+1} \\
& & -x_2 & & & \leq & 0 & \leftarrow \times\ y_{m+2} \\
& & & \ddots & & & & \\
& & & & -x_n & \leq & 0 & \leftarrow \times\ y_{m+n}
\end{array}
$$

Valid Upper Bounds

$$
\begin{array}{ccccccccccc}
a_{11}y_1 & +a_{21}y_2 & +\cdots & +a_{m1}y_m & -y_{m+1} & & & & = & c_1 \\
a_{12}y_2 & +a_{22}y_2 & +\cdots & +a_{m2}y_m & & -y_{m+2} & & & = & c_2 \\
\vdots & & \ddots & & & & \ddots & & & \\
a_{1n}y_1 & +a_{2n}y_2 & +\cdots & +a_{mn}y_m & & & & -y_{m+n} & = & c_n \\
y_1, & y_2, & \cdots & y_m, & y_{m+1}, & y_{m+2}, & \cdots & y_{m+n} & \geq & 0
\end{array}
$$

$$\boxed{b_1y_1 + b_2y_2 + \cdots + b_my_m}$$