APPLYING NEWTON METHOD TO SOLVING LPS

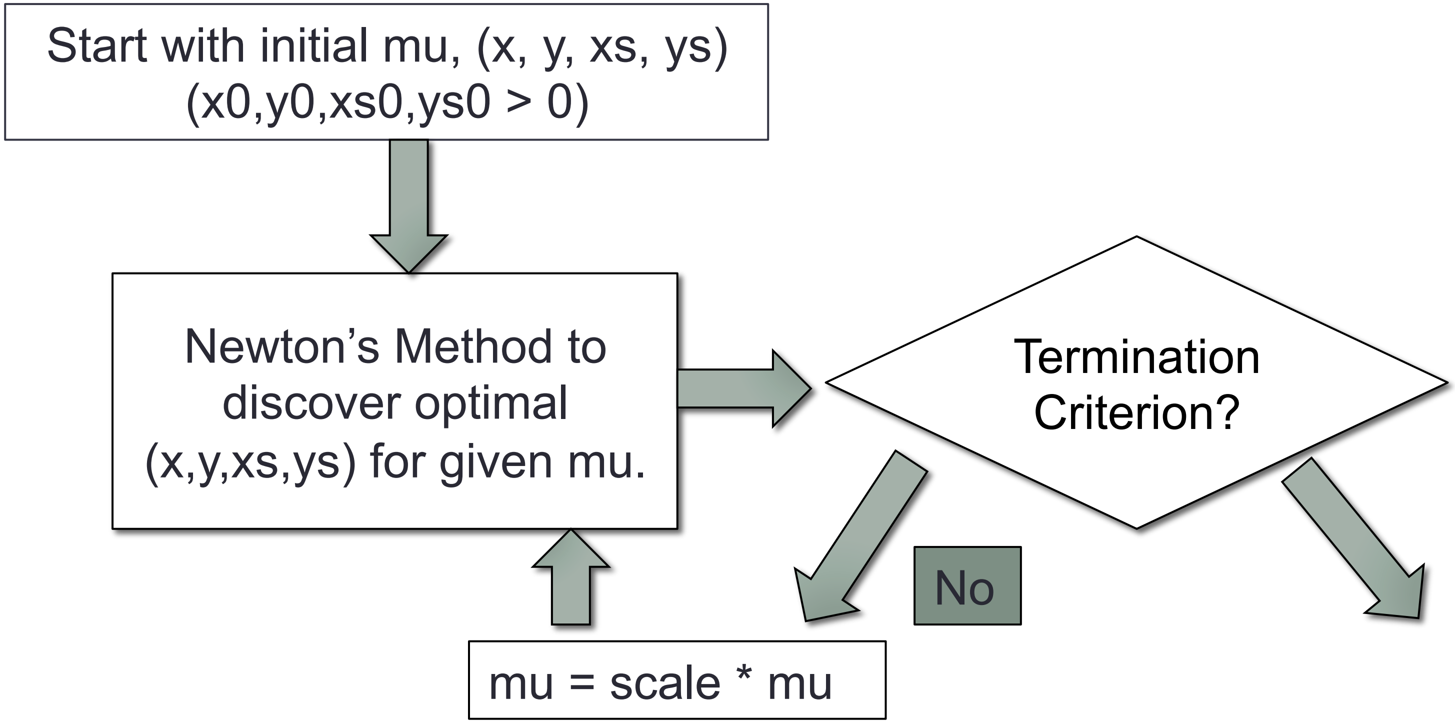
Mu KKT conditions

$$\boxed{\begin{aligned} A\mathbf{x} + \mathbf{x_s} &= \mathbf{b} && \text{Primal} \\ A^\intercal \mathbf{y} - \mathbf{y_s} &= \mathbf{c} && \text{Dual} \\ XY_s\mathbf{e} &= \mu\mathbf{e} && \text{Mu-Complementarity} \\ X_sY\mathbf{e} &= \mu\mathbf{e} && \end{aligned}}$$

$$\begin{aligned} X &= \mathrm{diag}(\mathbf{x}) \\ X_s &= \mathrm{diag}(\mathbf{x_s}) \\ Y &= \mathrm{diag}(\mathbf{y}) \\ Y_s &= \mathrm{diag}(\mathbf{y_s}) \end{aligned}$$

As mu approaches 0,
we obtain
KKT conditions!!

Overall Algorithm



Newton Step

$$A\mathbf{x} + \mathbf{x_s} = \mathbf{b} \qquad \text{Primal}$$

$$A^\intercal \mathbf{y} - \mathbf{y_s} = \mathbf{c} \qquad \text{Dual}$$

$$XY_s\mathbf{e} = \mu\mathbf{e} \qquad \text{Mu-Complementarity}$$

$$X_sY\mathbf{e} = \mu\mathbf{e}$$

Solve for $F(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y}_s) = \mathbf{0}$

$$\boxed{F(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y_s}) = \begin{bmatrix} A\mathbf{x} + \mathbf{x}_s - \mathbf{b} \\ A^\intercal \mathbf{y} = \mathbf{y_s} - \mathbf{c} \\ XY_s\mathbf{e} - \mu\mathbf{e} \\ X_sY\mathbf{e} - \mu\mathbf{e} \end{bmatrix}}$$

Calculating Newton Step -1

$$F(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y_s}) = \begin{bmatrix} A\mathbf{x} + \mathbf{x}_s - \mathbf{b} \\ A^\intercal \mathbf{y} = \mathbf{y_s} - \mathbf{c} \\ XY_s\mathbf{e} - \mu\mathbf{e} \\ X_sY\mathbf{e} - \mu\mathbf{e} \end{bmatrix}$$

$$\nabla F = \begin{bmatrix} A & I_{m\times m} & 0_{m\times m} & 0_{m\times n} \\ 0_{n\times m} & 0_{n\times m} & A^\intercal & -I_{n\times n} \\ Y_s & 0_{n\times m} & 0_{n\times m} & X \\ 0_{m\times n} & Y & X_s & 0_{m\times n} \end{bmatrix}$$

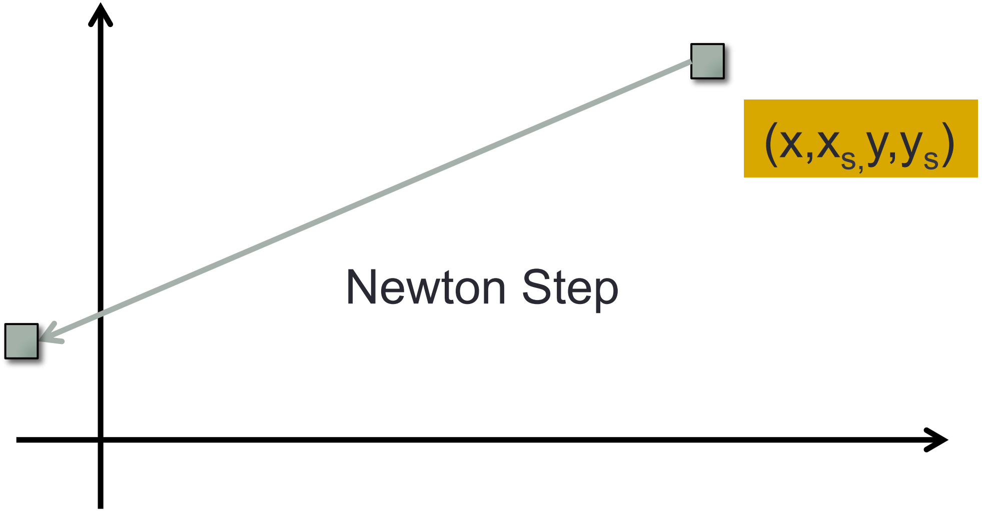
$$\Delta(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y}_s) = -(\nabla F)^{-1} \times F(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y}_s)$$

Calculating Newton Step -2

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} := \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} + \Delta(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y}_s)$$

Warning: This can violate the non-negativity of (x, x_s, y, y_s)

Sizing the Newton Step



Newton step can make some components of x, x_s, y, y_s negative.

Applying the Newton Step

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} := \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} + \Delta(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y}_s)$$

WRONG!!

Use a scale factor $\lambda \in (0,1)$

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} := \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} + \boxed{\lambda} * \Delta(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y}_s)$$

Finding Scale Factor

- Find (*largest*) λsuch that

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} := \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} + \lambda * \Delta(\mathbf{x}, \mathbf{x}_s, \mathbf{y}, \mathbf{y}_s)$$

guarantees that $\begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \\ \mathbf{y} \\ \mathbf{y}_s \end{pmatrix} > 0$

Implementation Detail: We use a smaller value of λ than the largest possible