Cutting plane methods

Complete worked out example

Example (ILP)

```
var x1 integer >= 0, <= 10;
var x2 integer >= 0, <= 10;
var x3  integer  >= 0, <= 10;

maximize obj: x1 + x2 -5* x3 ;

c1: -2* x1 + 7 *x2 <= 1;
c2: x1 - 2 *x2 + 5* x3 <= 3;
c3: x1 + x2 - 3 * x3 <= 7;

solve;
display x1, x2, x3;
end;
```

$$
\begin{aligned}
x_4 &: 10 - x_1 \\
x_5 &: 10 - x_2 \\
x_6 &: 10 - x_3 \\
x_7 &: 1 + 2x_1 - 7x_2 \\
x_8 &: 3 - x_1 + 2x_2 - 5x_3 \\
x_9 &: 7 - x_1 - x_2 + 3x_3
\end{aligned}
$$

Slack variables are also integers.

First Iteration

Initial Dictionary

x_4	10	$-1x_1$		
x_5	10		$-1x_2$	
x_6	10			$-1x_3$
x_7	1	$+2x_1$	$-7x_2$	
x_8	3	$-1x_1$	$+2x_2$	$-5x_3$
x_9	7	$-1x_1$	$-1x_2$	$+3x_3$
z	0	$+1x_1$	$+1x_2$	$-5x_3$

Final Dictionary

x_4	4.33333	$+0.333x_8$	$+0.666667x_9$	$-0.333x_3$
x_5	8.66667	$-0.333x_8$	$+0.333x_9$	$-2.6667x_3$
x_6	10			$-1x_3$
x_7	3	$-3x_8$	$+1x_9$	$-18x_3$
x_1	5.66667	$-0.333x_8$	$-0.6667x_9$	$+0.333x_3$
x_2	1.33333	$+0.333x_8$	$-0.333x_9$	$+2.6667x_3$
z	7		$-1x_9$	$-2x_3$

$$+0.333x_8 + 0.6667x_9 + 0.6667x_3 \geq 0.666667$$

Second Iteration

$$
\begin{array}{r|llll}
x_4 & 4.33333 & +0.333x_8 & +0.6667x_9 & -0.333x_3 \\
x_5 & 8.66667 & -0.333x_8 & +0.333x_9 & -2.6667x_3 \\
x_6 & 10 & & & -1x_3 \\
x_7 & 3 & -3x_8 & +1x_9 & -18x_3 \\
x_1 & 5.66667 & -0.333x_8 & -0.6667x_9 & +0.333x_3 \\
x_2 & 1.33333 & +0.333x_8 & -0.333x_9 & +2.6667x_3 \\
x_{10} & -0.666667 & +0.333x_8 & +0.6667x_9 & +0.6667x_3 \\
\hline
z & 7 & & -1x_9 & -2x_3
\end{array}
$$

Dictionary after C_1 added.

Primal infeasible

Dual Dictionary

Feasible + Non-final

$$
\begin{array}{r|lllllll}
y_8 & -0 & -0.333y_4 & +0.333y_5 & & +3y_7 & +0.333y_1 & -0.333y_2 & -0.333y_{10} \\
y_9 & 1 & -0.6667y_4 & -0.333y_5 & & -1y_7 & +0.6667y_1 & +0.333y_2 & -0.6667y_{10} \\
y_3 & 2 & +0.333y_4 & +2.6667y_5 & +1y_6 & +18y_7 & -0.333y_1 & -2.6667y_2 & -0.6667y_{10} \\
\hline
z & -7 & -4.333y_4 & -8.6667y_5 & -10y_6 & -3y_7 & -5.6667y_1 & -1.333y_2 & +0.6667y_{10}
\end{array}
$$

Naming convention for dual: x_j is complementary to y_j

Second Iteration (cont.)

Final Dual Dictionary

y_{10}	1.28571428571	$-1y_4$	$-0.285714y_5$		$-1.285714y_9$	$+1y_1$	$+0.285714y_2$ $-0.428571y_8$
y_7	0.142857142857		$-0.142857y_5$		$-0.142857y_9$		$+0.142857y_2$ $+0.285714y_8$
y_3	3.71428571429	$+1y_4$	$+0.285714y_5$	$+1y_6$	$-1.714286y_9$	$-1y_1$	$-0.285714y_2$ $+5.428571y_8$
z	-6.57142857143	$-5y_4$	$-8.428571y_5$	$-10y_6$	$-0.428571y_9$	$-5y_1$	$-1.571429y_2$ $-1.142857y_8$

Final Primal Dictionary

x_4	5	$+1x_{10}$		$-1x_3$
x_5	8.42857142857	$+0.285714x_{10}$	$+0.142857x_7$	$-0.285714x_3$
x_6	10			$-1x_3$
x_9	0.428571428571	$+1.285714x_{10}$	$+0.142857x_7$	$+1.714286x_3$
x_1	5	$-1x_{10}$		$+1x_3$
x_2	1.57142857143	$-0.285714x_{10}$	$-0.142857x_7$	$+0.285714x_3$
x_8	1.14285714286	$+0.428571x_{10}$	$-0.285714x_7$	$-5.428571x_3$
z	6.57142857143	$-1.285714x_{10}$	$-0.142857x_7$	$-3.714286x_3$

Cut C_2

$$0.285714x_{10} + 0.142857x_7 + 0.714286x_3 \geq 0.571428571429$$

Third Iteration

Dictionary after C_2 added.

Primal Infeasible

x_4	5	$+1x_{10}$		$-1x_3$
x_5	8.42857142857	$+0.285714x_{10}$	$+0.142857x_7$	$-0.285714x_3$
x_6	10			$-1x_3$
x_9	0.428571428571	$+1.285714x_{10}$	$+0.142857x_7$	$+1.714286x_3$
x_1	5	$-1x_{10}$		$+1x_3$
x_2	1.57142857143	$-0.285714x_{10}$	$-0.142857x_7$	$+0.285714x_3$
x_8	1.14285714286	$+0.428571x_{10}$	$-0.285714x_7$	$-5.428571x_3$
x_{11}	-0.571428571429	$+0.285714x_{10}$	$+0.142857x_7$	$+0.714286x_3$
z	6.57142857143	$-1.285714x_{10}$	$-0.142857x_7$	$-3.714286x_3$

y_{10}	1.28571428571	$-1y_4$	$-0.285714y_5$		$-1.285714y_9$	$+1y_1$	$+0.285714y_2$	$-0.428571y_8$	$-0.285714y_{11}$
y_7	0.142857142857		$-0.142857y_5$		$-0.142857y_9$		$+0.142857y_2$	$+0.285714y_8$	$-0.142857y_{11}$
y_3	3.71428571429	$+1y_4$	$+0.285714y_5$	$+1y_6$	$-1.714286y_9$	$-1y_1$	$-0.285714y_2$	$+5.428571y_8$	$-0.714286y_{11}$
z	-6.57142857143	$-5y_4$	$-8.428571y_5$	$-10y_6$	$-0.428571y_9$	$-5y_1$	$-1.571429y_2$	$-1.142857y_8$	$+0.571429y_{11}$

Dual dictionary: Feasible + Non-final

Third Iteration (Cont).

Dual Final Dictionary

y_{10}	1	$-1y_4$			$-1y_9$	$+1y_1$	$-1y_8$ $+2y_7$
y_{11}	1		$-1y_5$		$-1y_9$	$+1y_2$	$+2y_8$ $-7y_7$
y_3	3	$+1y_4$	$+1y_5$	$+1y_6$	$-1y_9$	$-1y_1$ $-1y_2$	$+4y_8$ $+5y_7$
z	-6	$-5y_4$	$-9y_5$	$-10y_6$	$-1y_9$	$-5y_1$ $-1y_2$	$-4y_7$

Integral Solution

x_4	5	$+1x_{10}$		$-1x_3$
x_5	9		$+1x_{11}$	$-1x_3$
x_6	10			$-1x_3$
x_9	1	$+1x_{10}$	$+1x_{11}$	$+1x_3$
x_1	5	$-1x_{10}$		$+1x_3$
x_2	1		$-1x_{11}$	$+1x_3$
x_8	-0	$+1x_{10}$	$-2x_{11}$	$-4x_3$
x_7	4	$-2x_{10}$	$+7x_{11}$	$-5x_3$
z	6	$-1x_{10}$	$-1x_{11}$	$-3x_3$

Final Primal Dictionary

Final Answer

Final Primal Dictionary

$$
\begin{array}{c|c|ccc}
x_4 & 5 & +1x_{10} & & -1x_3 \\
x_5 & 9 & & +1x_{11} & -1x_3 \\
x_6 & 10 & & & -1x_3 \\
x_9 & 1 & +1x_{10} & +1x_{11} & +1x_3 \\
x_1 & 5 & -1x_{10} & & +1x_3 \\
x_2 & 1 & & -1x_{11} & +1x_3 \\
x_8 & -0 & +1x_{10} & -2x_{11} & -4x_3 \\
x_7 & 4 & -2x_{10} & +7x_{11} & -5x_3 \\
\hline
z & 6 & -1x_{10} & -1x_{11} & -3x_3
\end{array}
$$

Integral Solution

$$x_1 = 5, x_2 = 1, x_3 = 0$$

ANSWER!!