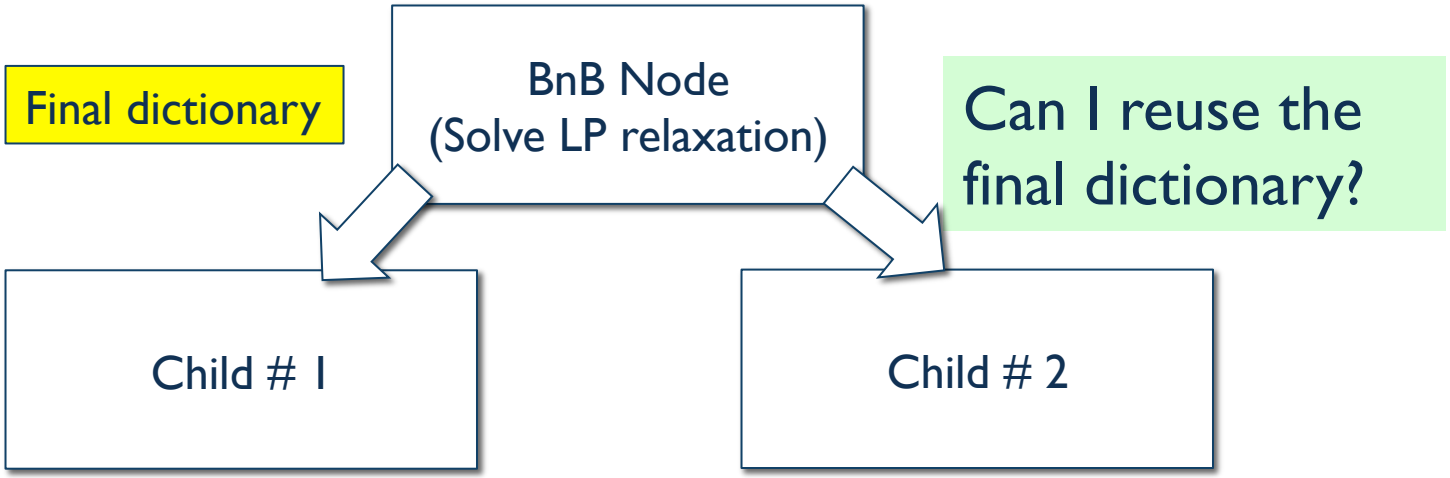
BRANCH-AND-BOUND

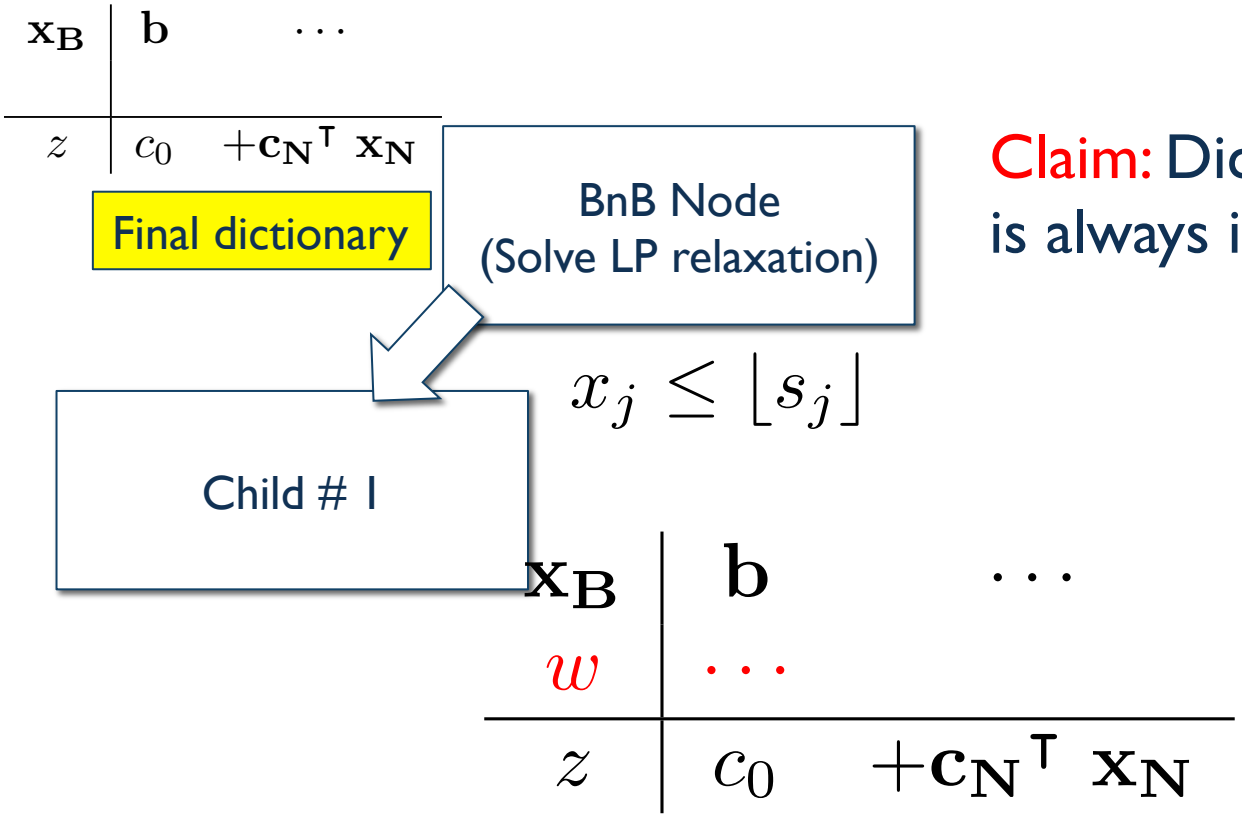
At the dictionary level

Thus far

- Use LP solver as a black box.
- This lecture:
  - Consider how to fold branch-and-bound in a dictionary setup.

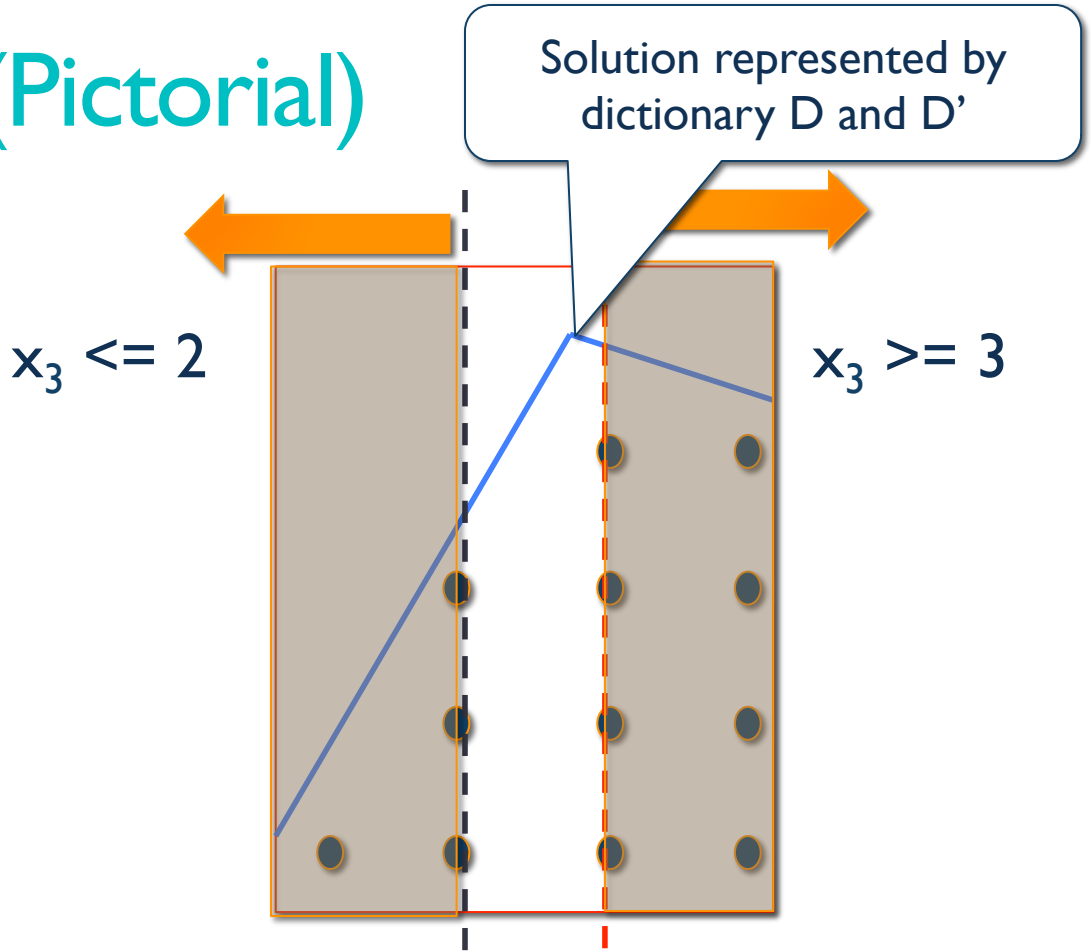


View from the dictionary



Claim: Dictionary D' is always infeasible.

Proof (Pictorial)



Example (ILP)

```
var x1 >= 0, <= 10;
var x2  >= 0, <= 10;
var x3  >= 0, <= 10;

maximize obj: x1 + x2 -5* x3 ;

c1: -2* x1 + 7 *x2 <= 1;
c2: x1 - 2 *x2 + 5* x3 <= 3;
c3: x1 + x2 - 3 * x3 <= 7;

solve;
display x1, x2, x3;
end;
```

$$
\begin{aligned}
x_4 &: 10 - x_1 \\
x_5 &: 10 - x_2 \\
x_6 &: 10 - x_3 \\
x_7 &: 1 + 2x_1 - 7x_2 \\
x_8 &: 3 - x_1 + 2x_2 - 5x_3 \\
x_9 &: 7 - x_1 - x_2 + 3x_3
\end{aligned}
$$

Slack variables are also integers.

Final Dictionary

x_4	4.33333333333	$+0.333333x_8$	$+0.666667x_9$	$-0.333333x_3$
x_5	8.66666666667	$-0.333333x_8$	$+0.333333x_9$	$-2.666667x_3$
x_6	10			$-x_3$
x_7	3	$-3x_8$	$+x_9$	$-18x_3$
x_1	5.66666666667	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
x_2	1.33333333333	$+0.333333x_8$	$-0.333333x_9$	$+2.666667x_3$
z	7.0	$+0x_8$	$-x_9$	$-2x_3$

$x_1 \geq 6$  $x_{10} : \ -6 \ + \ x_1$

Dictionary After Branch

$$x_{10} : \ -6 \ + \ x_1$$

x_4	4.3333333333	$+0.333333x_8$	$+0.666667x_9$	$-0.333333x_3$
x_5	8.66666666667	$-0.333333x_8$	$+0.333333x_9$	$-2.666667x_3$
x_6	10			$-x_3$
x_7	3	$-3x_8$	$+x_9$	$-18x_3$
x_1	5.66666666667	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
x_2	1.33333333333	$+0.333333x_8$	$-0.333333x_9$	$+2.666667x_3$
x_{10}	-0.33333	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
z	7.0	$+0x_8$	$-x_9$	$-2x_3$

Dictionary becomes primal infeasible.

Back to initialization phase Simplex?

Dictionary After Branch

x_4	4.3333333333	$+0.333333x_8$	$+0.666667x_9$	$-0.333333x_3$
x_5	8.66666666667	$-0.333333x_8$	$+0.333333x_9$	$-2.666667x_3$
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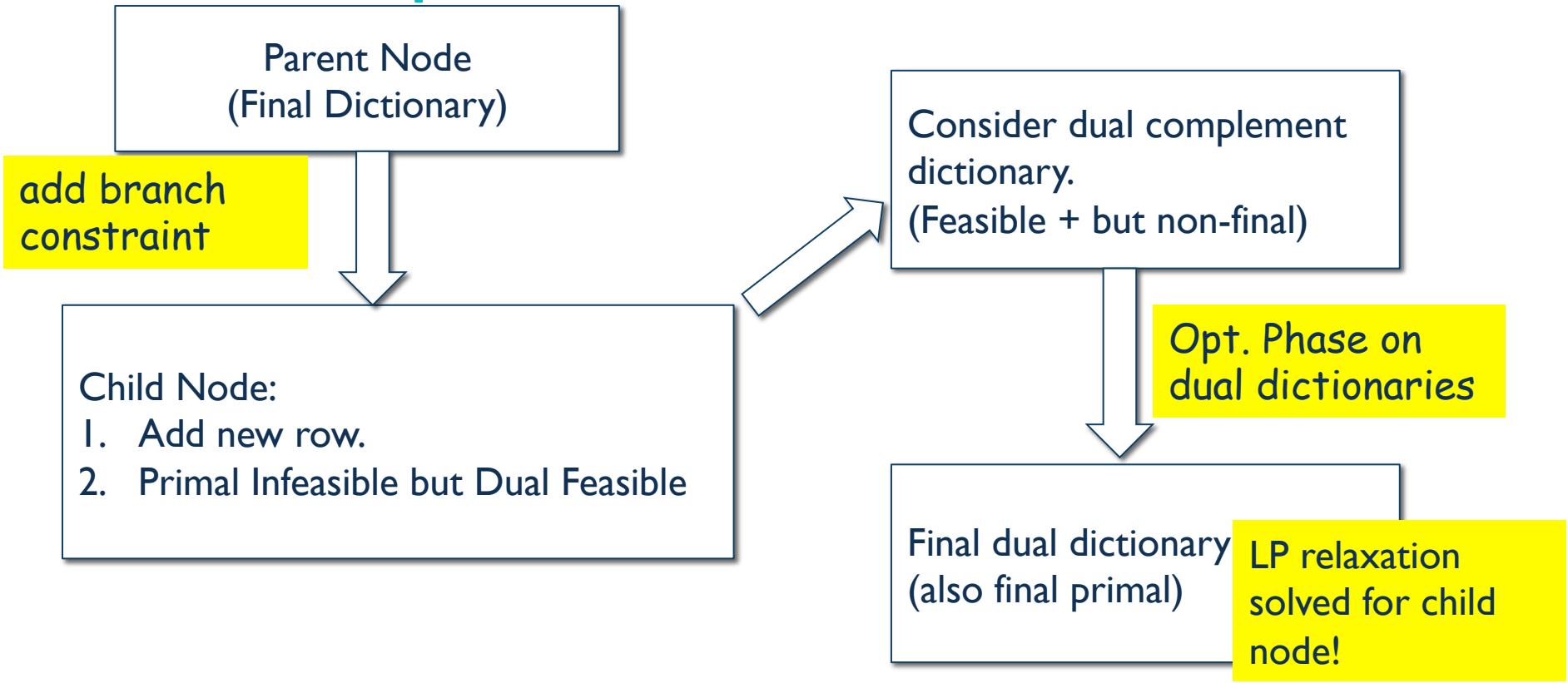
x_1	5.66666666667	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
x_2	1.33333333333	$+0.333333x_8$	$-0.333333x_9$	$+2.666667x_3$
x_{10}	-0.33333	$-0.333333x_8$	$-0.666667x_9$	$+0.333333x_3$
z	7.0	$+0x_8$	$-x_9$	$-2x_3$

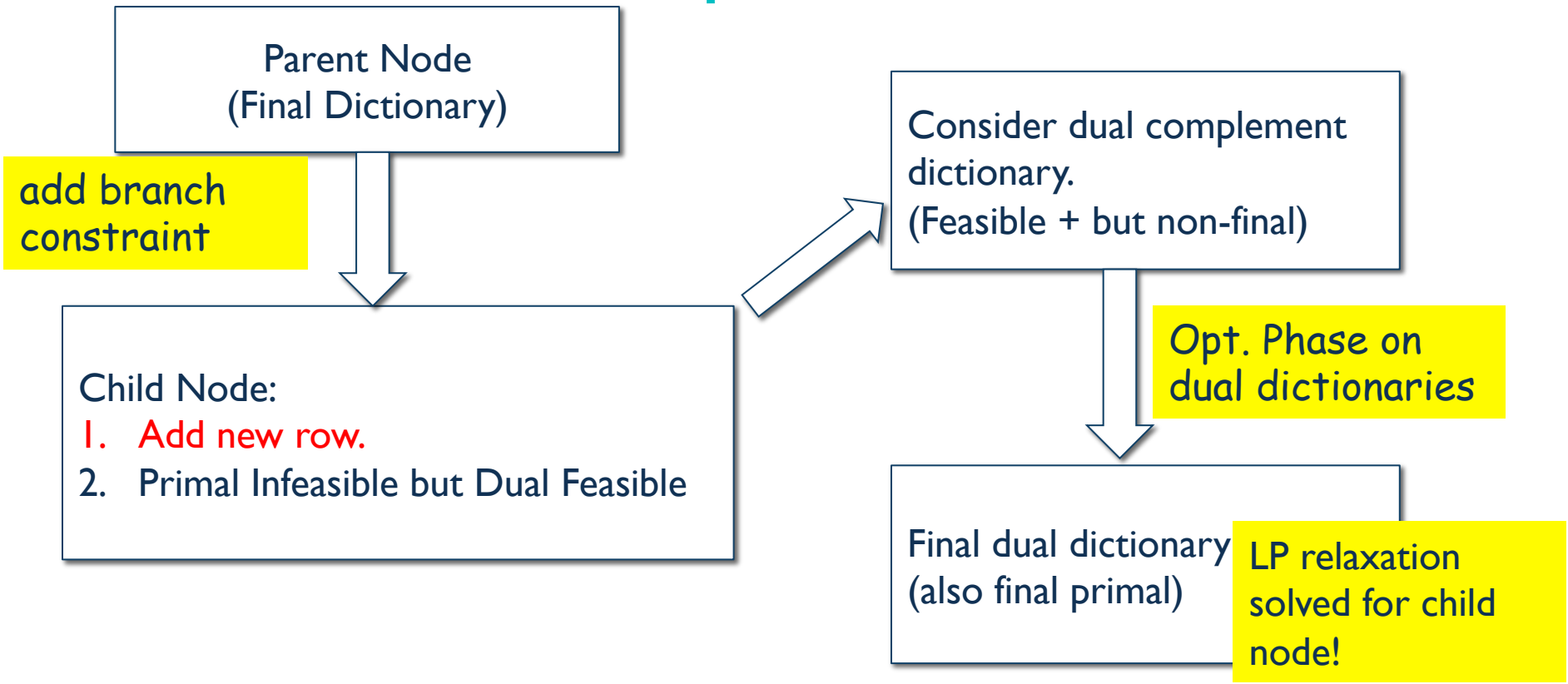
Dictionary becomes primal infeasible.

Dual Feasible Dictionary!!
But not dual final.

How to update after branch?



General Form Simplex



General Form Dictionary

- Give special treatment to bounds on variables.

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$$

- Modify the Simplex algorithm in two ways:
  - Dictionaries now have special ways to track bounds.
  - Pivoting is modified.

Branch-and-Bound with General Form