

ACT III: PIVOTING

Going from one dictionary to the next.

Pivoting (from 2K feet)

2. Choose Leaving Variable

$$\begin{array}{rcl} \mathbf{x}_B & = & \mathbf{b} - \mathbf{A}\mathbf{x}_I \\ \hline z & = & c_0 + \mathbf{c}^T \mathbf{x}_I \end{array}$$

m basic variables

$$\begin{array}{lcl} x_{B1} & = & b_1 + a_{11}x_{I1} + \cdots + a_{1j}x_{Ij} + \cdots + a_{1n}x_{In} \\ \vdots & & \vdots \\ x_{Bk} & = & b_k + a_{k1}x_{I1} + \cdots + a_{kj}x_{Ij} + \cdots + a_{kn}x_{In} \\ \vdots & & \vdots \\ x_{Bm} & = & b_m + a_{m1}x_{I1} + \cdots + a_{mj}x_{Ij} + \cdots + a_{mn}x_{In} \end{array}$$

$$z = c_0 + c_1x_{I1} + \cdots + c_jx_{Ij} + \cdots + c_nx_{In}$$

Diagram illustrating the pivoting process. A green box highlights the column of basic variables $x_{B1}, \dots, x_{Bk}, \dots, x_{Bm}$. A blue box highlights the row of the leaving variable x_{Bk} . A red box highlights the coefficient a_{kj} in the row x_{Bk} and column x_{Ij} . A red box also highlights the coefficient c_j in the objective function row z and column x_{Ij} . Arrows indicate the pivot operation: a purple arrow from a_{1j} to a_{kj} , a blue arrow from a_{kj} to a_{mj} , and a white arrow from c_j to a_{kj} .

1. Choose Entering Variable

n non-basic variables

Pivoting Steps

1. Choose an entering variable.
2. For the choice of entering variable, find a leaving variable.
3. Perform substitutions and obtain next dictionary.

Example

Chvátal, Chapter 2

$$\begin{array}{llll} \text{max.} & 5x_1 + 4x_2 + 3x_3 & & \\ \text{s.t.} & 2x_1 + 3x_2 + x_3 & \leq & 5 \\ & 4x_1 + x_2 + 2x_3 & \leq & 11 \\ & 3x_1 + 4x_2 + 2x_3 & \leq & 8 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

Already in standard form.

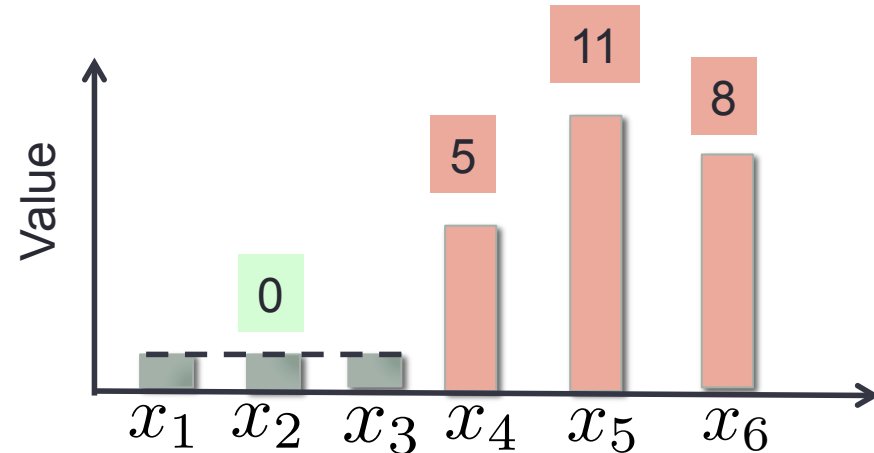
Example (add slack)

$$\begin{array}{llll} \text{max.} & 5x_1 + 4x_2 + 3x_3 & & \\ \text{s.t.} & 2x_1 + 3x_2 + x_3 & \leq & 5 \\ & 4x_1 + x_2 + 2x_3 & \leq & 11 \\ & 3x_1 + 4x_2 + 2x_3 & \leq & 8 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

$$\begin{array}{rcccc} x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\ x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\ x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\ \hline z & = & 0 & +5x_1 & +4x_2 & +3x_3 \end{array}$$

Choosing an entering variable

$$\begin{array}{rclclcl} x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\ x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\ x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\ \hline z & = & 0 & +5x_1 & +4x_2 & +3x_3 \end{array}$$



Leaving Variable

x_4	=	5	-2 x_1	-3 x_2	- x_3
x_5	=	11	-4 x_1	- x_2	-2 x_3
x_6	=	8	-3 x_1	-4 x_2	-2 x_3
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z	=	0	+5 x_1	+4 x_2	+3 x_3

Leaving Variable Analysis

$$\begin{array}{rclcl} x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\ x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\ x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\ \hline z & = & 0 & +5x_1 & +4x_2 & +3x_3 \end{array}$$

$$\begin{array}{rclcl} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & \Rightarrow & x_1 \leq \frac{5}{2} \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & \Rightarrow & x_1 \leq \frac{11}{4} \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & \Rightarrow & x_1 \leq \frac{8}{3} \end{array}$$

$$z = 0 + 5x_1 + 4x_2 + 3x_3$$