

Matrix Form.

$$\max \quad \mathbf{c}^\top \mathbf{x}$$

$$A \mathbf{x} \leq \mathbf{b}$$

$$\mathbf{x} \geq 0$$

Original Problem

$$A^\top \mathbf{y} - \mathbf{y}_s = \mathbf{c}$$

$$\mathbf{y} \geq 0$$

$$\mathbf{y}_s \geq 0$$

Yields Upper
Bound:

$$\mathbf{b}^\top \mathbf{y}$$

Most Stringent Upper Bound

$$\begin{array}{ll}
 \max & \mathbf{c}^\top \mathbf{x} \\
 & A \mathbf{x} \leq \mathbf{b} \\
 & \mathbf{x} \geq 0
 \end{array}
 \longrightarrow
 \begin{array}{lll}
 A^\top \mathbf{y} & -\mathbf{y}_s & = \mathbf{c} \\
 \mathbf{y} & & \geq 0 \\
 & \mathbf{y}_s & \geq 0
 \end{array}
 \longrightarrow \mathbf{b}^\top \mathbf{y}$$

$$\begin{array}{llll}
 \min & \mathbf{b}^\top \mathbf{y} \\
 & A^\top \mathbf{y} & -\mathbf{y}_s & = \mathbf{c} \\
 & \mathbf{y} & & \geq 0 \\
 & & \mathbf{y}_s & \geq 0
 \end{array}$$

Dual Problem

Dual Problem (cont)

$$\begin{array}{llll} \min & \mathbf{b}^\top \mathbf{y} & & \\ & A^\top \mathbf{y} - \mathbf{y}_s & = & \mathbf{c} \\ & \mathbf{y} & \geq & 0 \\ & \mathbf{y}_s & \geq & 0 \end{array}$$

$$\begin{array}{ll} \max & \mathbf{c}^\top \mathbf{x} \\ & A \mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{array}$$

$$\begin{array}{ll} \min & \mathbf{b}^\top \mathbf{y} \\ & A^\top \mathbf{y} \geq \mathbf{c} \\ & \mathbf{y} \geq 0 \end{array}$$

Dual Problem (Example)

$$\begin{array}{llllll} \max & -2x_1 & +3x_2 & +x_3 & & \\ & x_1 & -x_2 & +x_3 & \leq & 10 \\ & 2x_1 & -3x_2 & -x_3 & \leq & 10 \\ & & +6x_2 & +2x_3 & \leq & 10 \\ & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$