PS3: Dualization

Help

The **due date** for this homework is **Mon 24 Nov 2014 3:00 PM CST**.

Dualization Problem Set

The questions in this problem set talk about the video lectures posted for week #3 on dualization. We recommend carefully copying down each problem data onto a sheet of paper and then answer the questions

☑ **In accordance with the Coursera Honor Code, I (Kevin Zhu) certify that the answers here are my own work.** **Thank you!**

Question 1

Consider the primal LP below:

$$
\begin{array}{lrrrcll}
\max & -x_1 & -3x_2 & +x_3 & & & \\
 & -x_1 & -x_2 & & \leq & 5 & \leftarrow \mathsf{y}_1 \\
 & \underline{\mathsf{a}_1} x_1 & & -3x_3 & \leq & 2 & \leftarrow \mathsf{y}_2 \\
 & x_1 & +\underline{\mathsf{a}_2} x_2 & +2x_3 & \leq & 2 & \leftarrow \mathsf{y}_3 \\
 & x_1 & & +\underline{\mathsf{a}_3} x_3 & \leq & 0 & \leftarrow \mathsf{y}_4 \\
 & & x_2 & -x_3 & \leq & 0 & \leftarrow \mathsf{y}_5 \\
 & x_1, & x_2, & x_3 & \geq & 0 &
\end{array}
$$

and the corresponding dual problem:

$$
\begin{array}{lrrrrrcl}
\min & \underline{\mathsf{c}_1} y_1 & +\underline{\mathsf{c}_2} y_2 & +\underline{\mathsf{c}_3} y_3 & & & & \\
\text{s.t.} & -y_1 & -2y_2 & +y_3 & +y_4 & & \geq & \underline{\mathsf{b}_1} \\
 & -y_1 & & +y_3 & & +y_5 & \geq & \underline{\mathsf{b}_2} \\
 & & -3y_2 & +2y_3 & -y_4 & -y_5 & \geq & \underline{\mathsf{b}_3} \\
 & y_1, & y_2, & y_3, & y_4, & y_5 & \geq & 0
\end{array}
$$

Note the missing numbers c_1, c_2, c_3 in the dual.

Write down the values of $c_1,\ c_2,\ c_3$. Your answer should be three numbers separated by a space ( ).

Question 2

Consider the primal LP below:

$$
\begin{array}{lrrrcll}
\max & -x_1 & -3x_2 & +x_3 & & & \\
 & -x_1 & -x_2 & & \leq & 5 & \leftarrow \mathsf{y}_1 \\
 & \underline{\mathsf{a}_1} x_1 & & -3x_3 & \leq & 2 & \leftarrow \mathsf{y}_2 \\
 & x_1 & +\underline{\mathsf{a}_2} x_2 & +2x_3 & \leq & 2 & \leftarrow \mathsf{y}_3 \\
 & x_1 & & +\underline{\mathsf{a}_3} x_3 & \leq & 0 & \leftarrow \mathsf{y}_4 \\
 & & x_2 & -x_3 & \leq & 0 & \leftarrow \mathsf{y}_5 \\
 & x_1, & x_2, & x_3 & \geq & 0 &
\end{array}
$$

and the corresponding dual problem:

$$
\begin{array}{lrrrrrcl}
\min & \underline{\mathsf{c}_1} y_1 & +\underline{\mathsf{c}_2} y_2 & +\underline{\mathsf{c}_3} y_3 & & & & \\
\text{s.t.} & -y_1 & -2y_2 & +y_3 & +y_4 & & \geq & \underline{\mathsf{b}_1} \\
 & -y_1 & & +y_3 & & +y_5 & \geq & \underline{\mathsf{b}_2} \\
 & & -3y_2 & +2y_3 & -y_4 & -y_5 & \geq & \underline{\mathsf{b}_3} \\
 & y_1, & y_2, & y_3, & y_4, & y_5 & \geq & 0
\end{array}
$$

Note the missing numbers b_1, b_2, b_3 in the dual.

Write down the values of $b_1,\ b_2,\ b_3$. Your answer should be three numbers separated by a space ().

Question 3

Consider the primal LP below:

$$
\begin{array}{lrrrcrl}
\max & -x_1 & -3x_2 & +x_3 & & & \\
 & -x_1 & -x_2 & & \leq & 5 & \leftarrow \mathsf{y}_1 \\
 & \underline{\mathsf{a}_1} x_1 & & -3x_3 & \leq & 2 & \leftarrow \mathsf{y}_2 \\
 & x_1 & +\underline{\mathsf{a}_2} x_2 & +2x_3 & \leq & 2 & \leftarrow \mathsf{y}_3 \\
 & x_1 & & +\underline{\mathsf{a}_3} x_3 & \leq & 1 & \leftarrow \mathsf{y}_4 \\
 & & x_2 & -x_3 & \leq & -1 & \leftarrow \mathsf{y}_5 \\
 & x_1, & x_2, & x_3 & \geq & 0 &
\end{array}
$$

and the corresponding dual problem:

$$
\begin{array}{lrrrrrcr}
\min & \underline{\mathsf{c}_1} y_1 & +\underline{\mathsf{c}_2} y_2 & +\underline{\mathsf{c}_3} y_3 & & & & \\
\mathsf{s.t.} & -y_1 & -2y_2 & +y_3 & +y_4 & & \geq & \underline{\mathsf{b}_1} \\
 & -y_1 & & +y_3 & & +y_5 & \geq & \underline{\mathsf{b}_2} \\
 & & -3y_2 & +2y_3 & -y_4 & -y_5 & \geq & \underline{\mathsf{b}_3} \\
 & y_1, & y_2, & y_3, & y_4, & y_5 & \geq & 0
\end{array}
$$

Note the missing numbers a_1, a_2, a_3 in the primal.

Write down the values of $a_1,\ a_2,\ a_3$. Your answer should be three numbers separated by a space ().

Question 4

Consider the dual dictionary:

$$
\begin{array}{c|ccccc}
y_1 & -1 & +2y_2 & -3y_4 & +y_5 & +y_7 \\
y_3 & 2 & -y_2 & +2y_4 & -y_5 & \\
y_6 & -1 & -y_2 & -y_4 & -y_5 & \\
\hline
\xi & -10 & -2y_2 & -3y_4 & -y_5 & +0y_7
\end{array}
$$

The complementary pairs are as follows:

Primal	Dual
x_1	y_5
x_2	y_6
x_3	y_7
x_4	y_1
x_5	y_2
x_6	y_3
x_7	y_4

Answer the following questions about the **complementary primal dictionary** and the **primal, dual** problems that give rise to these dictionary, which are not shown. Select all the correct answers and ensure that incorrect answers are not selected.

It will help you immensely to first write down the primal complement dictionary in advance on a sheet of paper to answer each of the 10 parts below. In doing so, focus on (a) the basic and non-basic variables in this dictionary, (b) the constant column, objective row, and finally (c) the matrix (rows) of the dictionary.

- [ ] The **primal problem** has 3 variables and 4 constraints
- [ ] The complementary **primal dictionary** is degenerate
- [ ] x_4 is an entering variable in the complementary **primal dictionary**
- [ ] If x_2 enters there is no leaving variable, therefore the **primal problem** is unbounded.
- [ ] The **dual problem** has a feasible solution.
- [ ] The **primal problem** has 4 variables and 3 constraints
- [ ] The complementary **primal dictionary** is final
- [ ] The objective value of the complementary **primal dictionary** is -10
- [ ] x_5 is a basic variable in the complementary **primal dictionary**
- [ ] x_6 is an entering variable in the complementary **primal dictionary**

- [x] **In accordance with the Coursera Honor Code, I (Kevin Zhu) certify that the answers here are my own work.** **Thank you!**

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