

EXAMPLE OF INFEASIBLE PROBLEM.

Example

$$\begin{array}{llllll} \max & x_1 & +2x_2 & -x_3 & & \\ \text{s.t.} & x_1 & -x_2 & & \leq & 5 \\ & & x_2 & +x_3 & \leq & 14 \\ & -x_1 & & +x_3 & \leq & -6 \\ & & & -x_3 & \leq & -7 \end{array}$$

Initial Dictionary

max	x_1	$+2x_2$	$-x_3$		
s.t.	x_1	$-x_2$		\leq	5
		x_2	$+x_3$	\leq	14
	$-x_1$		$+x_3$	\leq	-6
			$-x_3$	\leq	-7

$$x_4 = 5 - x_1 + x_2$$

$$x_5 = 14 - x_2 - x_3$$

$$x_6 = -6 + x_1 - x_3$$

$$x_7 = -7 + x_3$$

$$z = 0 + x_1 + 2x_2 - x_3$$

Auxiliary Problem

$$\begin{array}{llllll} \max & -x_0 & & & & \\ \text{s.t.} & x_1 & -x_2 & & \leq & 5 & +x_0 \\ & & x_2 & +x_3 & \leq & 14 & +x_0 \\ & -x_1 & & +x_3 & \leq & -6 & +x_0 \\ & & & -x_3 & \leq & -7 & +x_0 \end{array}$$

Aux. Problem (Initial Dictionary)

$$\begin{array}{rcllcl} x_4 & = & 5 & +x_0 & -x_1 & +x_2 \\ x_5 & = & 14 & +x_0 & & -x_2 & -x_3 \\ x_6 & = & -6 & +x_0 & +x_1 & & -x_3 \\ x_7 & = & -7 & +x_0 & & & +x_3 \\ \hline w & = & 0 & -x_0 & & & \end{array}$$

x_0 enters and x_7 leaves!

Dictionary D1

$$\begin{array}{rcllcl}
 x_4 & = & 5 & +x_0 & -x_1 & +x_2 \\
 x_5 & = & 14 & +x_0 & & -x_2 & -x_3 \\
 x_6 & = & -6 & +x_0 & +x_1 & & -x_3 \\
 x_7 & = & -7 & +x_0 & & & +x_3 \\
 \hline
 w & = & 0 & -x_0 & & &
 \end{array}$$

$$\begin{array}{rcllcl}
 x_0 & = & 7 & +x_7 & & -x_3 \\
 x_4 & = & 12 & +x_7 & -x_1 & +x_2 & -x_3 \\
 x_5 & = & 21 & +x_7 & & -x_2 & -2x_3 \\
 x_6 & = & 1 & +x_7 & +x_1 & & -2x_3 \\
 \hline
 w & = & -7 & -x_7 & & & +x_3
 \end{array}$$

x_3 enters and x_6 leaves

Dictionary D2

$$\begin{array}{rcccccc}
 x_3 & = & \frac{1}{2} & +\frac{1}{2}x_7 & +\frac{1}{2}x_1 & -\frac{1}{2}x_6 \\
 x_0 & = & \frac{13}{2} & +\frac{1}{2}x_7 & -\frac{1}{2}x_1 & +\frac{1}{2}x_6 \\
 x_4 & = & \frac{23}{2} & +\frac{1}{2}x_7 & -\frac{3}{2}x_1 & +x_2 & +\frac{1}{2}x_6 \\
 x_5 & = & 20 & & -x_1 & -x_2 & +x_6 \\
 \hline
 w & = & -\frac{13}{2} & -\frac{1}{2}x_7 & +\frac{1}{2}x_1 & & -\frac{1}{2}x_6
 \end{array}$$

x_1 enters and x_4 leaves

Dictionary D3

$$\begin{array}{rclclcl}
 x_1 & = & \frac{23}{3} & + \frac{1}{3}x_7 & - \frac{2}{3}x_4 & + \frac{2}{3}x_2 & + \frac{1}{3}x_6 \\
 x_0 & = & \frac{8}{3} & + \frac{1}{3}x_7 & + \frac{1}{3}x_4 & - \frac{1}{3}x_2 & + \frac{1}{3}x_6 \\
 x_3 & = & \frac{13}{3} & + \frac{2}{3}x_7 & - \frac{1}{3}x_4 & + \frac{1}{3}x_2 & - \frac{1}{3}x_6 \\
 x_5 & = & \frac{37}{3} & - \frac{1}{3}x_7 & + \frac{2}{3}x_4 & - \frac{5}{3}x_2 & + \frac{2}{3}x_6 \\
 \hline
 w & = & -\frac{8}{3} & - \frac{1}{3}x_7 & - \frac{1}{3}x_4 & + \frac{1}{3}x_2 & - \frac{1}{3}x_6
 \end{array}$$

x_2 enters and x_5 leaves

Dictionary D4

$$\begin{array}{rcccccc} x_1 & = & \frac{63}{5} & + \frac{1}{5}x_7 & - \frac{2}{5}x_4 & - \frac{2}{5}x_5 & + \frac{3}{5}x_6 \\ x_0 & = & \frac{1}{5} & + \frac{2}{5}x_7 & + \frac{2}{5}x_4 & + \frac{2}{5}x_5 & + \frac{2}{5}x_6 \\ x_3 & = & \frac{34}{5} & + \frac{3}{5}x_7 & - \frac{2}{5}x_4 & - \frac{2}{5}x_5 & - \frac{2}{5}x_6 \\ x_2 & = & \frac{37}{5} & - \frac{1}{5}x_7 & + \frac{2}{5}x_4 & - \frac{3}{5}x_5 & + \frac{2}{5}x_6 \\ \hline w & = & -\frac{1}{5} & - \frac{2}{5}x_7 & - \frac{2}{5}x_4 & - \frac{2}{5}x_5 & - \frac{2}{5}x_6 \end{array}$$

Aux. Problem

$$\begin{array}{llllll} \max & -x_0 & & & & \\ \text{s.t.} & x_1 & -x_2 & & \leq & 5 \quad +x_0 \\ & & x_2 & +x_3 & \leq & 14 \quad +x_0 \\ & -x_1 & & +x_3 & \leq & -6 \quad +x_0 \\ & & & -x_3 & \leq & -7 \quad +x_0 \end{array}$$

Least possible value of x_0 is $\frac{1}{5}$.

Infeasibility: Conclusion

$$\begin{array}{llllll} \max & x_1 & +2x_2 & -x_3 & & \\ \text{s.t.} & x_1 & -x_2 & & \leq & 5 \\ & & x_2 & +x_3 & \leq & 14 \\ & -x_1 & & +x_3 & \leq & -6 \\ & & & -x_3 & \leq & -7 \end{array}$$