

Initialization Phase Simplex : Overview

- **Goal:** Get to a dictionary with feasible solution.
 - Alternatively, conclude problem infeasibility.
- **Strategy:**
 - Modify the problem
 - Perform Simplex on Modified Problem.

CONSTRUCTING THE AUXILIARY PROBLEM

AUXILLIARY PROBLEM

$$\begin{array}{llll}
 \text{max.} & x_1 + 2x_2 & & \\
 \text{s.t.} & -2x_1 + x_2 + \textcolor{red}{x}_3 & = & -2 \\
 & x_2 + \textcolor{red}{x}_4 & = & 4 \\
 & x_1 - 2x_2 + \textcolor{red}{x}_5 & = & -2 \\
 & x_1 + \textcolor{red}{x}_6 & = & 4 \\
 & x_1, x_2, \textcolor{red}{x}_3, \dots, \textcolor{red}{x}_6 & \geq & 0
 \end{array}$$

$$\begin{array}{llllll}
 \text{s.t.} & -2x_1 & +x_2 & +x_3 & = & -2 + \textcolor{red}{x}_0 \\
 & & x_2 & +x_4 & = & 4 + \textcolor{red}{x}_0 \\
 & x_1 & -2x_2 & +x_5 & = & -2 + \textcolor{red}{x}_0 \\
 & x_1 & & +x_6 & = & 4 + \textcolor{red}{x}_0 \\
 & & x_1, \dots, x_6, \textcolor{red}{x}_0 & \geq & & 0
 \end{array}$$

Feasibility of Aux. Problem

$$\begin{array}{rclcl} \text{s.t.} & -2x_1 & +x_2 & +x_3 & = & -2 + x_0 \\ & & x_2 & +x_4 & = & 4 + x_0 \\ & x_1 & -2x_2 & +x_5 & = & -2 + x_0 \\ & x_1 & & +x_6 & = & 4 + x_0 \\ & & & & & x_1, \dots, x_6, x_0 \geq 0 \end{array}$$

Claim: Aux. Problem is Feasible.

If Original Problem is Feasible?

$$\begin{array}{llll} \text{max.} & x_1 + 2x_2 & & \\ \text{s.t.} & -2x_1 + x_2 + \textcolor{red}{x}_3 & = & -2 \\ & x_2 + \textcolor{red}{x}_4 & = & 4 \\ & x_1 - 2x_2 + \textcolor{red}{x}_5 & = & -2 \\ & x_1 + \textcolor{red}{x}_6 & = & 4 \\ & x_1, x_2, \textcolor{red}{x}_3, \dots, \textcolor{red}{x}_6 & \geq & 0 \end{array}$$

$$\begin{array}{llllll} \text{s.t.} & -2x_1 & +x_2 & +x_3 & = & -2 + \textcolor{red}{x}_0 \\ & & x_2 & +x_4 & = & 4 + \textcolor{red}{x}_0 \\ & x_1 & -2x_2 & +x_5 & = & -2 + \textcolor{red}{x}_0 \\ & x_1 & & +x_6 & = & 4 + \textcolor{red}{x}_0 \\ & x_1, \dots, x_6, \textcolor{red}{x}_0 & \geq & 0 \end{array}$$

$$\left(\begin{array}{l} x_1 : 2, x_2 : 2, \\ x_3 : 0, x_4 : 2, x_5 : 0, x_6 : 2 \end{array} \right)$$

$$\textcolor{red}{x}_0 = 0$$

If Original Problem is Infeasible?

$$\begin{array}{llll} \text{max.} & x_1 + 2x_2 & & \\ \text{s.t.} & -2x_1 + x_2 + x_3 & = & -2 \\ & x_2 + x_4 & = & 4 \\ & x_1 - 2x_2 + x_5 & = & -2 \\ & x_1 + x_6 & = & 4 \\ & x_1, x_2, x_3, \dots, x_6 & \geq & 0 \end{array}$$

$$\begin{array}{llllll} \text{s.t.} & -2x_1 & +x_2 & +x_3 & = & -2 + x_0 \\ & & x_2 & +x_4 & = & 4 + x_0 \\ & x_1 & -2x_2 & +x_5 & = & -2 + x_0 \\ & x_1 & & +x_6 & = & 4 + x_0 \\ & & & & x_1, \dots, x_6, x_0 & \geq 0 \end{array}$$

No Solution.

Auxiliary Problem

- Add fresh variable x_0
- Aux. Problem is always feasible.

