UNBOUNDED POLYHEDRA: RAYS

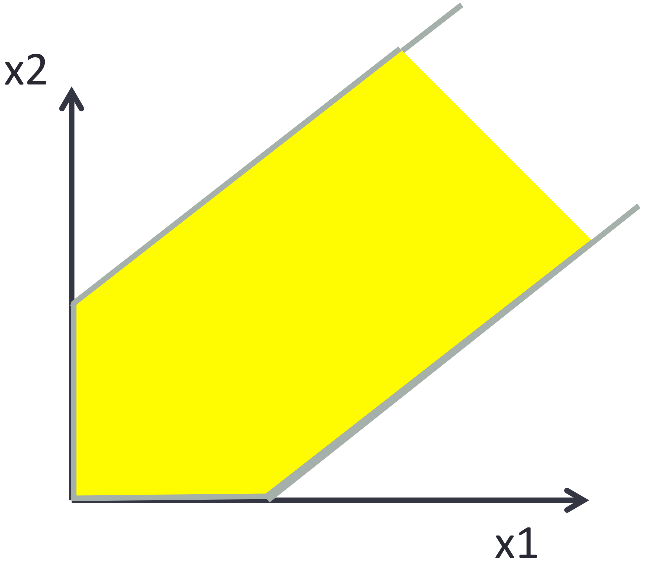
Thus far…

- Feasible Region: Polyhedra
- Vertices:
  - Activate at l
  - Acti
- Vertice
- Simplex ertex.
  - Degene generate

Unbounded Problems.

Unbounded Linear Programs and Rays

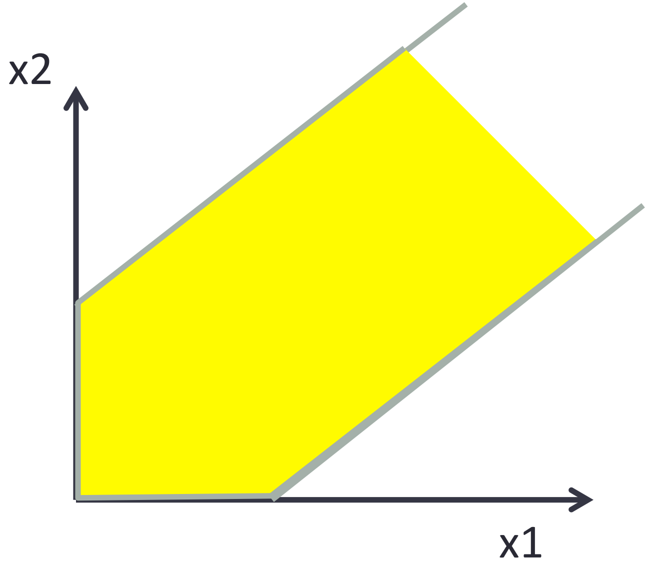
$$\begin{array}{llll}
\max & x_1 & & \\
\text{s.t.} & x_1 & -x_2 & \leq 1 \\
& -x_1 & +x_2 & \leq 1 \\
& x_1 & , x_2 & \geq 0
\end{array}$$



Ray

Vector $\mathbf{r}$ is a ray of polyhedron P iff for every $\mathbf{x} \in P$ and every $\lambda \geq 0$,

$$\mathbf{x} + \lambda \mathbf{r} \in P$$



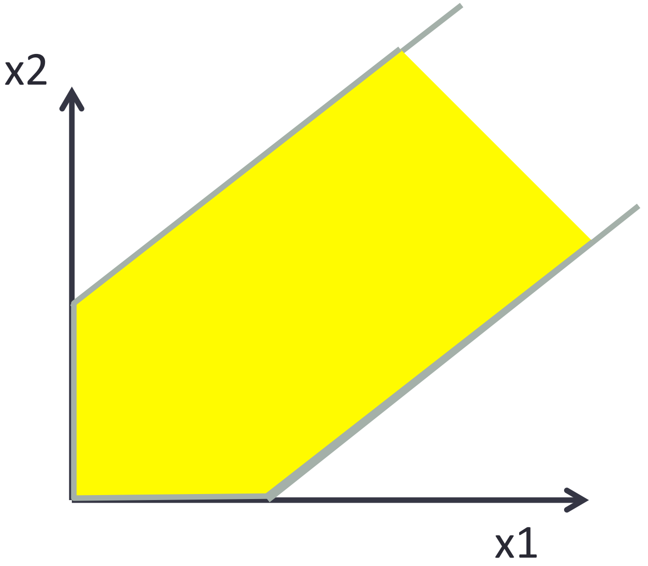
Ray (Fundamental Property)

$$\text{Polyhedron: } A\mathbf{x} \leq \mathbf{b}$$

$$\mathbf{r} \text{ is a ray if and only if } A\mathbf{r} \leq \mathbf{0}$$

Ray (Fundamental Property)

$$
\begin{array}{llllll}
\max & x_1 & & & \\
\text{s.t.} & x_1 & -x_2 & \leq & 1 \\
& -x_1 & +x_2 & \leq & 1 \\
& x_1 & , x_2 & \geq 0 &
\end{array}
$$



Is (1,1) a ray of this polyhedron?

Example

$$
\begin{array}{rrcl}
\text{maximize} & 2x_1 + 3x_2 - 5x_3 & & \\
\text{s.t.} & x_1 - x_2 & \leq & 5 \\
 & -x_1 + \quad\quad x_3 & \leq & 6 \\
 & -2x_1 + \quad\quad x_3 & \leq & 2 \\
 & -x_1 + x_2 & \leq & 4 \\
 & x_1, x_2, x_3 & \geq & 0
\end{array}
$$

Second Dictionary

$$
\begin{array}{rcrcrcrcr}
x_2 & = & 4 & + & x_1 & - & x_7 & & \\
x_4 & = & 9 & & & - & x_7 & & \\
x_5 & = & 6 & + & x_1 & & & - & x_3 \\
x_6 & = & 2 & + & 2x_1 & & & - & x_3 \\
\hline
z & = & 12 & + & 5x_1 & - & 3x_7 & - & 5x_3
\end{array}
$$

Unbounded Dictionary

- No leaving variables.
- Alternatively: all entries in the column corr. to entering variables are non-negative.

$$
\begin{array}{ccccccc}
x_{B1} & = & b_1 & +a_{11}x_{I1} & \cdots & +a_{1j}x_{Ij} & \cdots \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & \cdots & +a_{2j}x_{Ij} & \cdots \\
 & \vdots & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & \cdots & +a_{mj}x_{Ij} & \cdots \\
\hline
z & = & c_0 & +c_1x_{I1} & \cdots & +c_jx_{Ij} & \cdots
\end{array}
$$

Unbounded Dictionary and Ray

$$
\begin{array}{cccccccc}
x_{B1} & = & b_1 & +a_{11}x_{I1} & \cdots & +{\color{red}a_{1j}}x_{Ij} & \cdots \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & \cdots & +{\color{red}a_{2j}}x_{Ij} & \cdots \\
 & \vdots & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & \cdots & +{\color{red}a_{mj}}x_{Ij} & \cdots \\
\hline
z & = & c_0 & +c_1x_{I1} & \cdots & +{\color{red}c_j}x_{Ij} & \cdots
\end{array}
$$