# BRANCH-AND-BOUND (EXAMPLE)

# Original Problem

```
var x1 integer  >= 0, <= 10;
var x2 integer  >= 0, <= 10;
var x3  integer >= 0, <= 10;
var x4 integer  >= 0, <= 10;
var x5  integer >= 0, <= 10;
var x6  integer >= 0, <= 10;
maximize obj: -x1 - 2* x2 -0.5 * x3 - 0.2* x4 - x5 +0.5* x6;
c1: x1 + 2 * x2 >= 1;
c2: x1 + x2 + 3* x6 >= 1;
c3: x1 + x2 + x6 >= 1;
c4: x3 - 3* x4  >= 1;
c5: x3 - 2* x4 -5* x5 >= 1;
c6: x4 + 3* x5 -4 *x6   >= 1;
c7: x2 + x5 + x6 >= 1;
solve;
display x1, x2,x3, x4, x5, x6;
end;
```

# Original ILP

| integer solution | LP relaxation |
|---|---|
| x1.val = 0 | x1.val = 0.333333333333333 |
| x2.val = 1 | x2.val = 0.666666666666667 |
| x3.val = 4 | x3.val = 2.66666666666667 |
| x4.val = 1 | x4.val = 0 |
| x5.val = 0 | x5.val = 0.333333333333333 |
| x6.val = 0 | x6.val = 0 |
| Optimal Value: -4.200000 | Optimal Value: -3.333333 |

# Initial Node

bestObjective := - Infinity
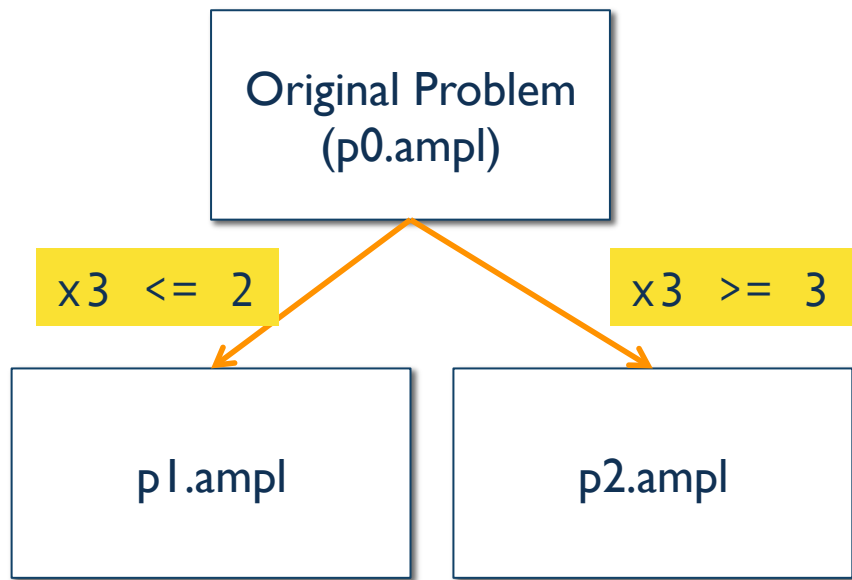


x1.val = 0.333333333333333
x2.val = 0.666666666666667
x3.val = 2.66666666666667
x4.val = 0
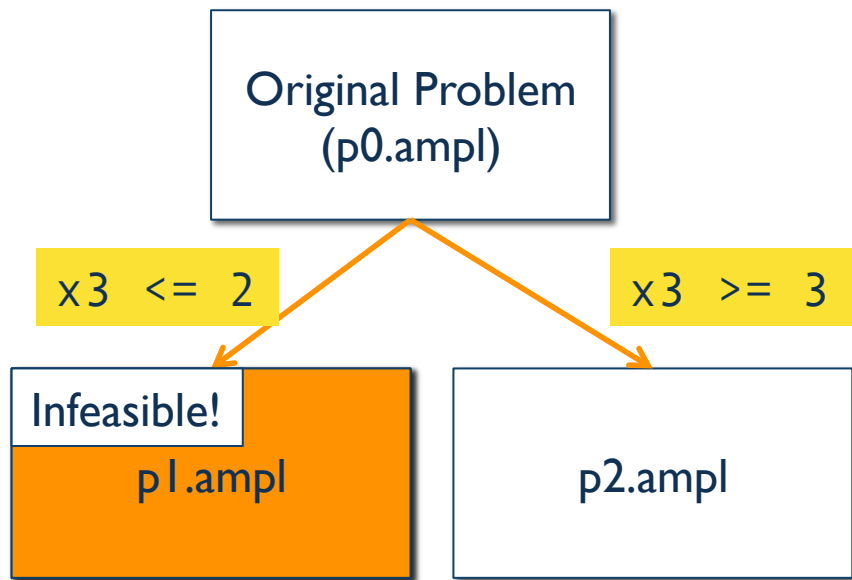x5.val = 0.333333333333333
x6.val = 0
Optimal Value: -3.333333

# Tree #1

bestObjective := - Infinity

Original Problem
(p0.ampl)

x3 <= 2

x3 >= 3

Infeasible!

p1.ampl

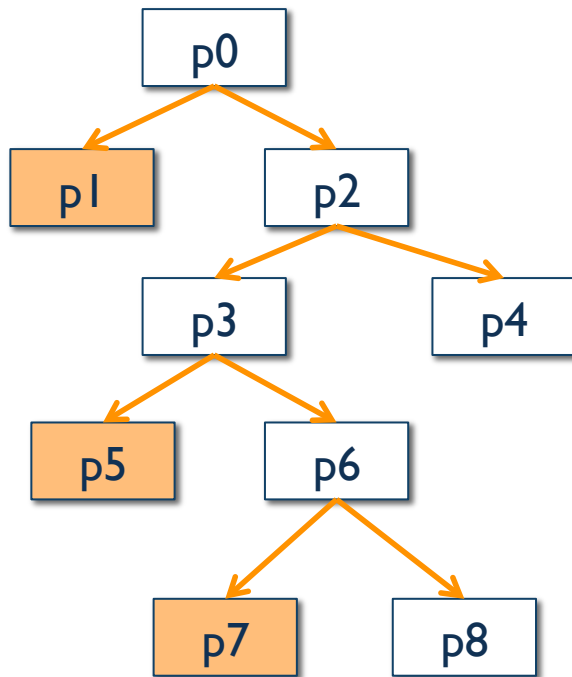p2.ampl

p2.ampl

x1.val = 0.4
x2.val = 0.55
x3.val = 3
x4.val = 0
x5.val = 0.4
x6.val = 0.05

# BnB Tree



p3.ampl

x1.val = 1
x2.val = 0
x3.val = 4.57142857142857
x4.val = 0
x5.val = 0.714285714285714
x6.val = 0.285714285714286
Optimal Value: -3.857143

p8.ampl

x1.val = 1
x2.val = 0
x3.val = 6
x4.val = 0
x5.val = 1
x6.val = 0.5
Optimal Value: -4.750000

p6.ampl

x1.val = 1
x2.val = 0
x3.val = 5
x4.val = 0.333333333333333
x5.val = 0.666666666666667
x6.val = 0.333333333333333
Optimal Value: -4.066667

# BnB Tree



bestObjective = -4.2

p9.ampl
x1.val = 0
x2.val = 1
x3.val = 4
x4.val = 1
x5.val = 0
x6.val = 0
Optimal Value: -4.2

p8.ampl
x1.val = 1
x2.val = 0
x3.val = 6
x4.val = 0
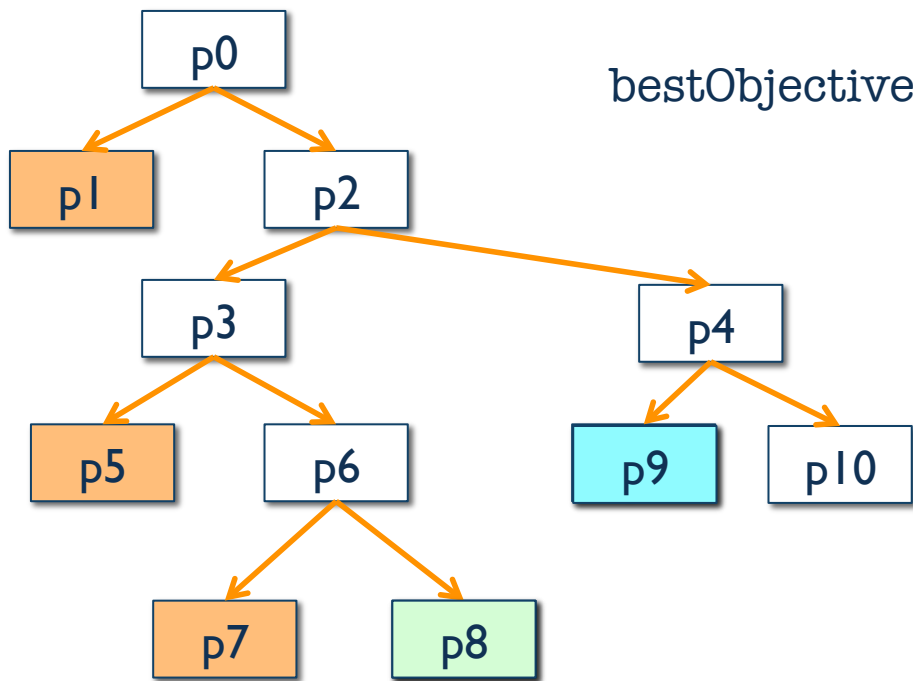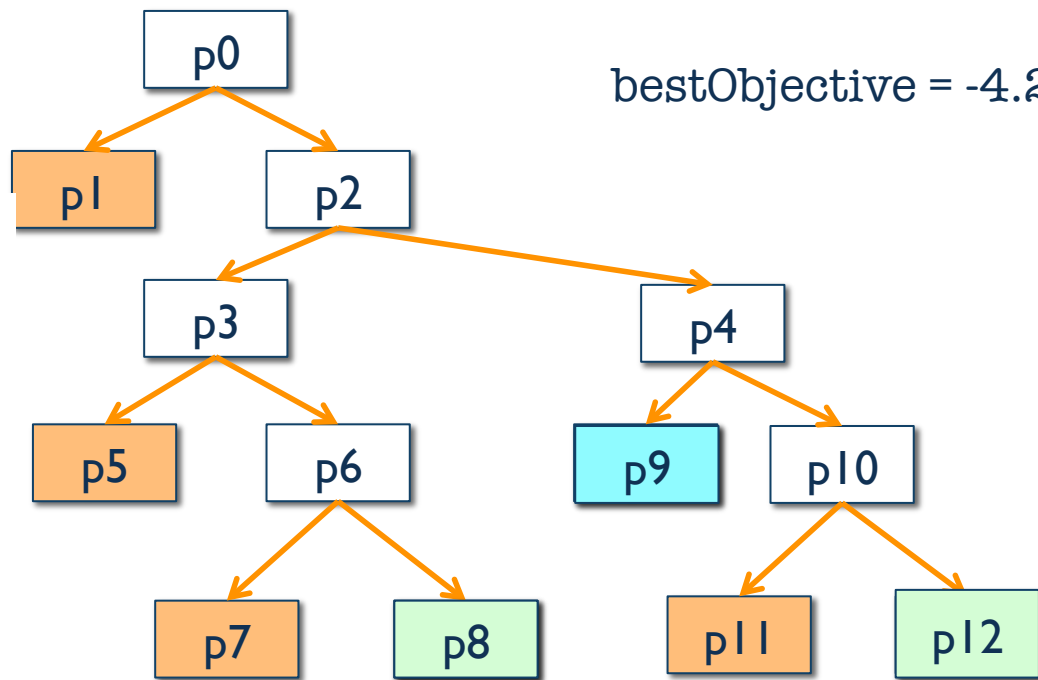x5.val = 1
x6.val = 0.5
Optimal Value: -4.750000

# BnB Tree

bestObjective = -4.2

```
p12.ampl
x1.val = 0
x2.val = 1
x3.val = 6
x4.val = 0
x5.val = 1
x6.val = 0.5
Optimal Value: -5.750000
```

# BnB Final Tree



bestObjective = -4.2

x1.val = 0

x2.val = 1

x3.val = 4

x4.val = 1

x5.val = 0

x6.val = 0

Optimal Value: -4.200000