

SIMPLEX: REVISED SIMPLEX METHOD

Revised Simplex Method

Original LP

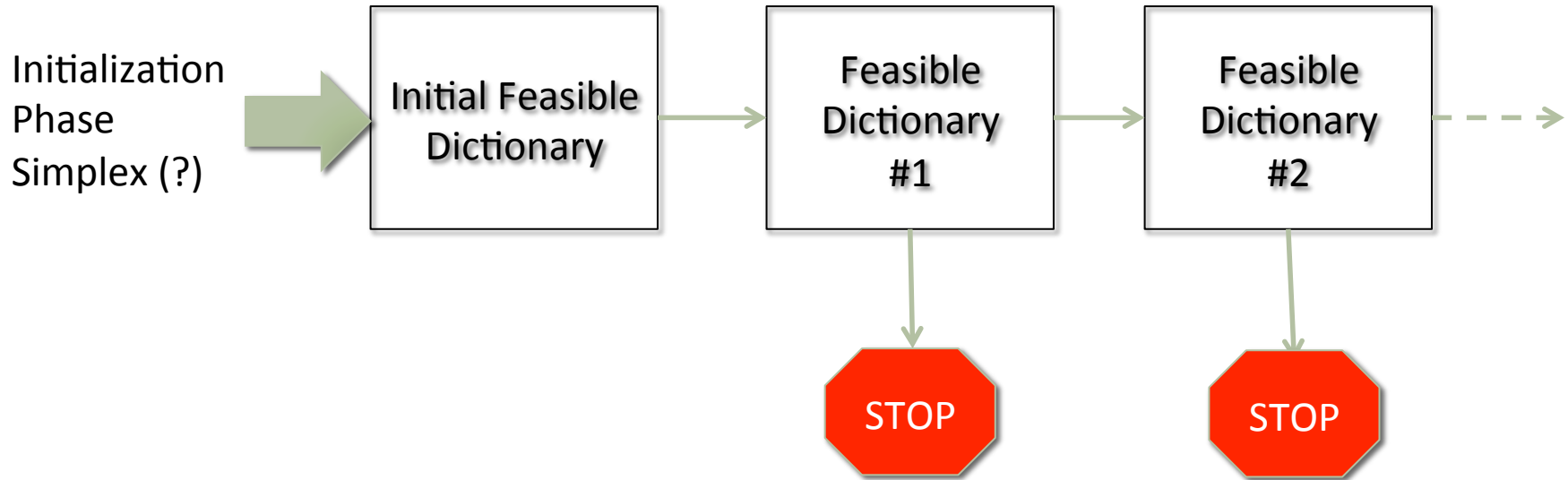
n decision variables
 m rows in A .

$$\begin{array}{ll}\max & \mathbf{c}^\top \mathbf{x} \\ & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0\end{array}$$



$$\begin{array}{ll}\max & \mathbf{c}^\top \mathbf{x} \\ & A\mathbf{x} + \mathbf{x}_s = \mathbf{b} \\ & \mathbf{x}, \mathbf{x}_s \geq 0\end{array}$$

Standard Simplex Method



Complexity of Each Pivoting Operation

$$\begin{array}{rclclclcl}
 x_{B1} & = & b_1 & + a_{11}x_{I1} & + \cdots & + a_{1j}x_{Ij} & + \cdots & + a_{1n}x_{In} \\
 x_{B2} & = & b_2 & + a_{21}x_{I1} & + \cdots & + a_{2j}x_{Ij} & + \cdots & + a_{2n}x_{In} \\
 & & \vdots & & & & & \\
 x_{Bm} & = & b_m & + a_{m1}x_{I1} & + \cdots & + a_{mj}x_{Ij} & + \cdots & + a_{mn}x_{In} \\
 \hline
 z & = & c_0 & + c_1x_{I1} & + \cdots & + c_jx_{Ij} & + \cdots & + c_nx_{In}
 \end{array}$$

Choice of Entering Variable: $O(n)$

Choice of Leaving Variable: $O(m)$

Updating Dictionary: $O(m * n)$

Storage: $O(m*n)$ floating point numbers

Problems with Standard Simplex Algorithm

- Storage Cost is High.

$$\begin{array}{rclclcl}
 x_{B1} & = & b_1 & +a_{11}x_{I1} & +\cdots & +a_{1j}x_{Ij} & +\cdots & +a_{1n}x_{In} \\
 x_{B2} & = & b_2 & +a_{21}x_{I1} & +\cdots & +a_{2j}x_{Ij} & +\cdots & +a_{2n}x_{In} \\
 & & \vdots & & & & & \\
 x_{Bm} & = & b_m & +a_{m1}x_{I1} & +\cdots & +a_{mj}x_{Ij} & +\cdots & +a_{mn}x_{In} \\
 \hline
 z & = & c_0 & +c_1x_{I1} & +\cdots & +c_jx_{Ij} & +\cdots & +c_nx_{In}
 \end{array}$$

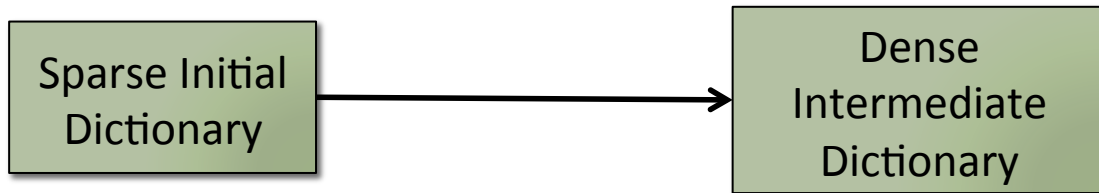
- Sparsity of Original Problem is Lost.
- Accumulation of Floating Point Errors over number of iterations.

Problem # 1 : Storage Cost

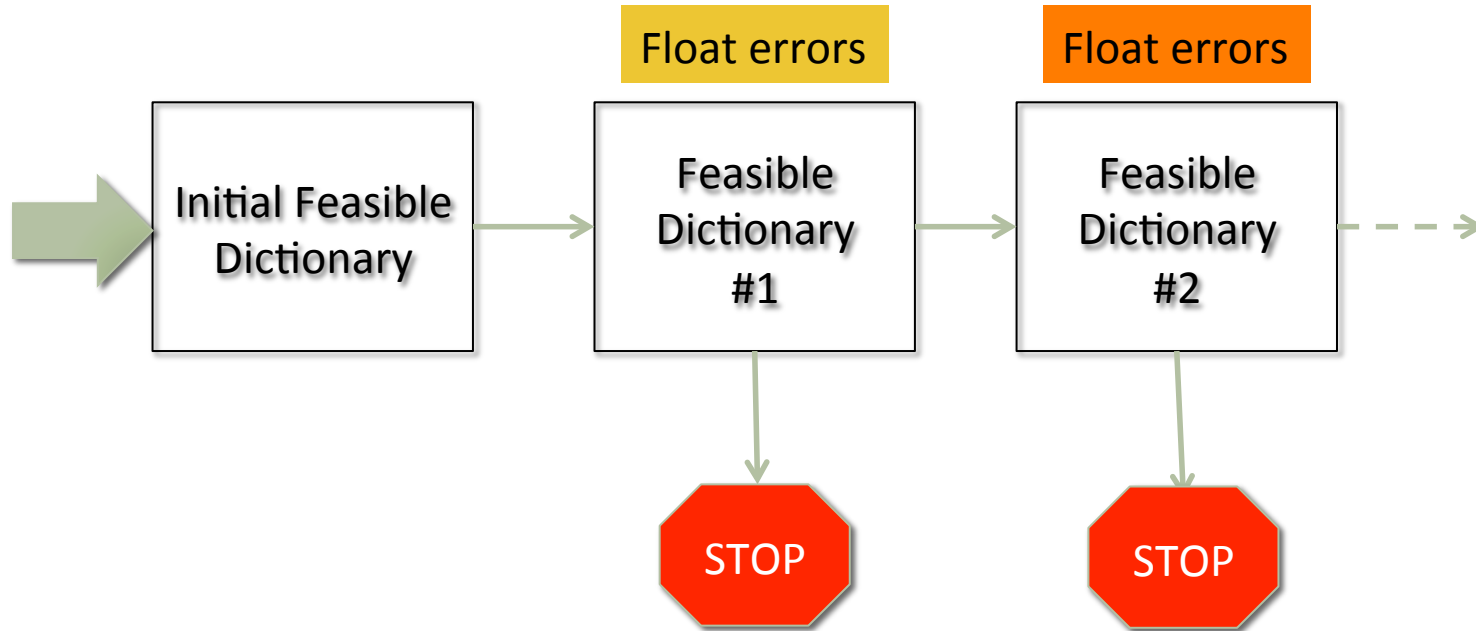
- Typical problem size.
 - $m = 500$ and $n = 10000$
- Cost of storing dictionary: 5×10^6 floating point numbers.
 - Approx. 20 MB

Problem #2: Loss in Sparsity

- Most practical LP instances are sparse.
- Lots of variables in the problem.
- But each inequality involves few variables.
- Fill In Problem:



Problem #3: Floating Point Error Accumulation



Revised Simplex Method

- **Basic Idea:** Do not store the intermediate dictionary.
- Store the set of basic and non-basic variables.
- At each step, **reconstruct** dictionary from data:
 - Original problem data: A, b, c
 - Set of basic (and non-basic) variables: B

Storage Cost:

Original problem data (sparse)

Basis set $O(m + n)$