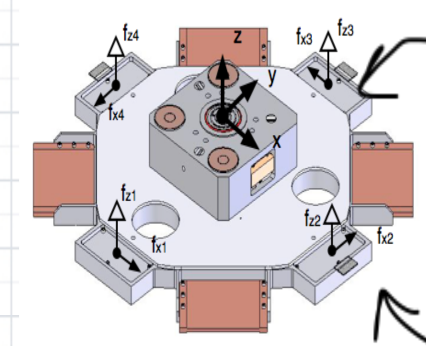
Minimum Power Application

Stage



motor

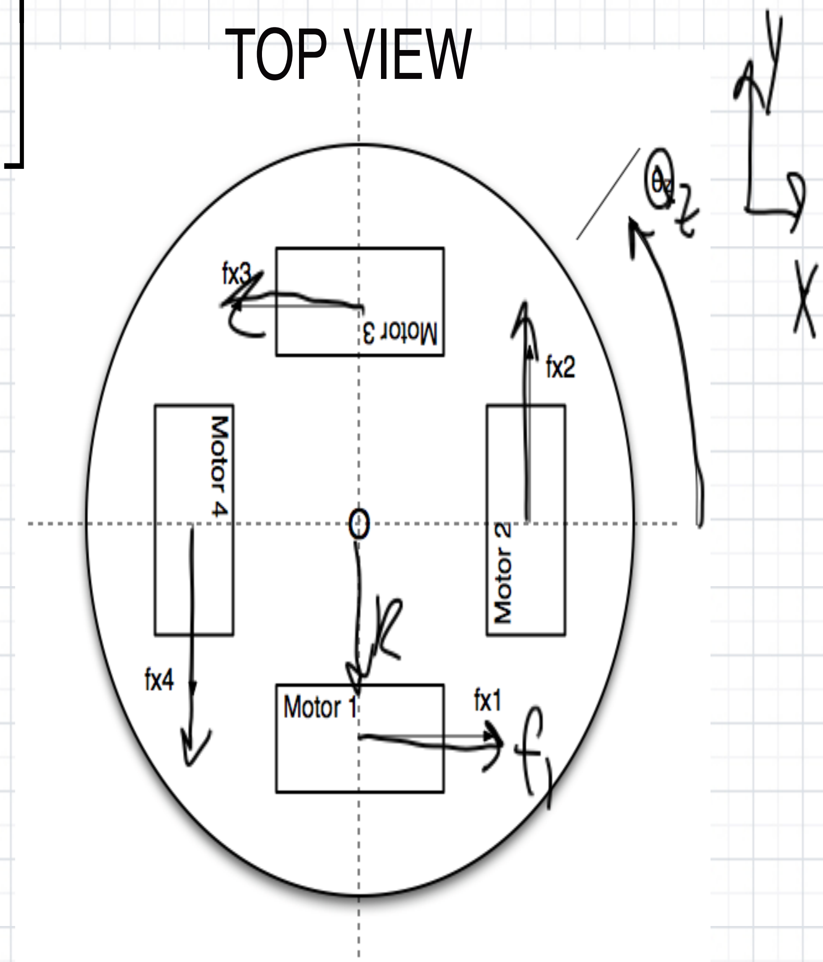
$$\begin{bmatrix} \boxed{F_x} \\ \boxed{F_y} \\ \boxed{\tau_z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ R & R & R & R \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

Global forces

$$\boxed{\underset{3\times1}{\boldsymbol{F}} = A\underset{4\times1}{\boldsymbol{f}}}$$

$F_x = f_1 - f_3$

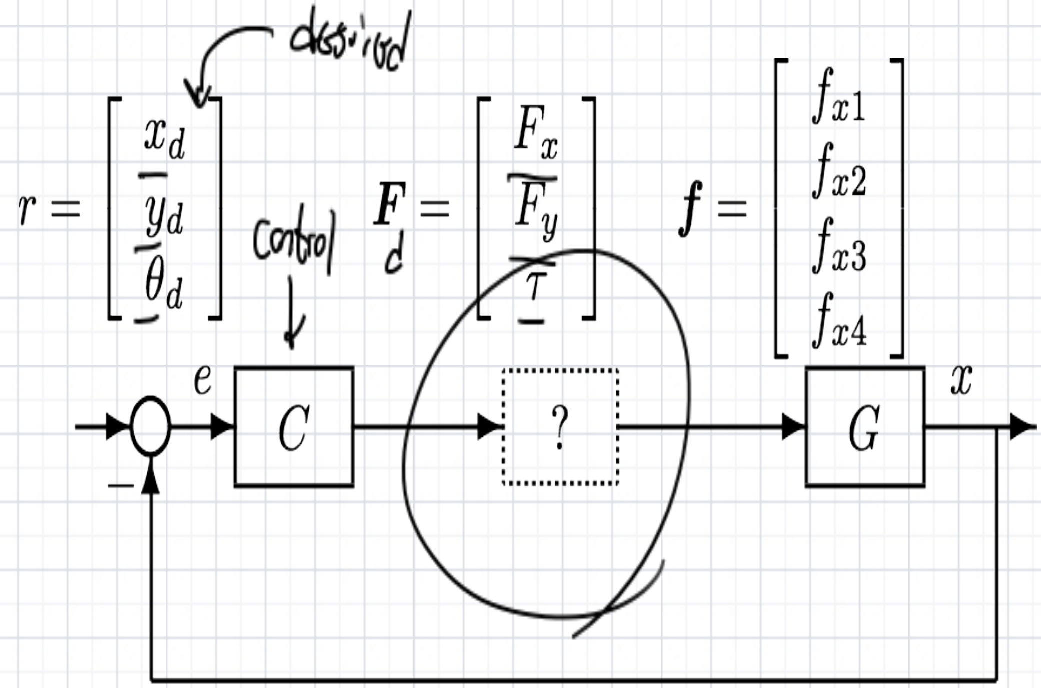
TOP VIEW



* Assuming θ is small

(small angle approximation)

Control Loop



3 eqns $\qquad$ 4 unknowns

$F \rightarrow f$

☆ Assume that force ≈ current (electromagnetic)

Current^2 ≈ Power $\quad \Rightarrow$ force^2 ≈ Power

$$\min f_1^2 + f_2^2 + f_3^2 + f_4^2 = \|f\|_2^2 = \min \text{Power}$$

$$\text{s.t.} \quad F = \underline{A} f$$

Least-Norm Solution

$\min \|f\|_2^2 \;?\; \min$ Power

s.t $F_d = Af$

Given

motor 1 $\quad f_1 = \frac{Fx}{2} + \frac{\tau_z}{4R}$

$$f_2 = \frac{Fy}{2} + \frac{\tau_z}{4R}$$

motor 3 $\quad f_3 = -\frac{Fx}{2} + \frac{\tau_z}{4R}$

$$f_4 = -\frac{Fy}{2} + \frac{\tau_z}{4R}$$

$$f^* = A^+ F_d = \begin{bmatrix} f_1^* \\ f_2^* \\ f_3^* \\ f_3^* \end{bmatrix} ?$$

Null Space of A

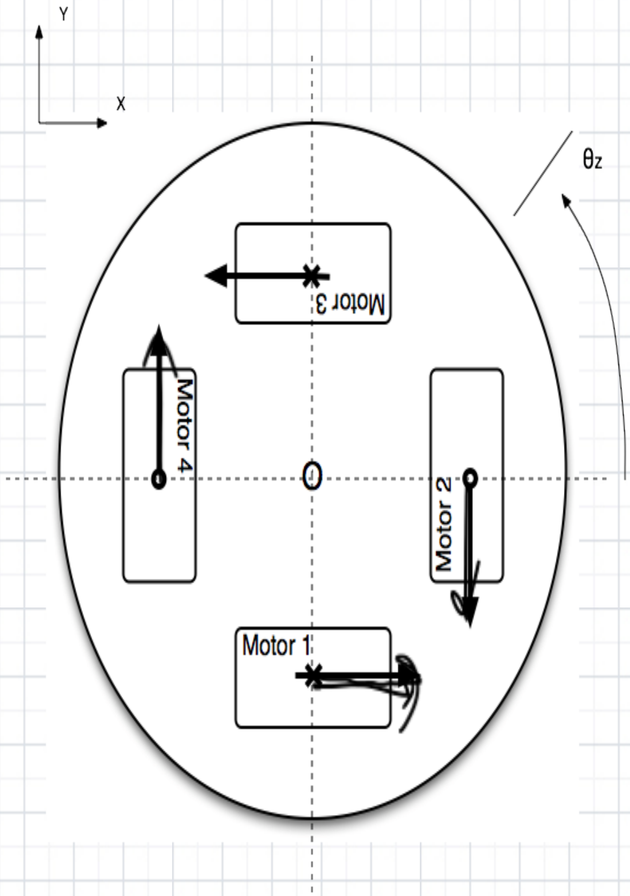
* Why are there an infinite set of solutions!

if $X_n \in N(A)$

$\Rightarrow \quad A X_n = 0 = \text{zero}!$

$$F = Af = A(\underbrace{f + f_N}_{\hat{f} \neq f}) \quad \text{where } f_N \in N(A)$$

$$= Af + \cancel{A f_N}^{\,0}$$



$f_1 = c$

$f_2 = -c$

$f_3 = c$

$f_4 = -c$

$Af = 0$

where $f \neq 0$

$$F_d = Af = A\Big(\underbrace{f + c\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix}}_{\hat{f}}\Big) = Af$$

$$\hat{f} \neq f$$