PRIMAL/DUAL CORRESPONDENCE

Complementary Pairs

Primal vs. Dual

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ & A\,\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned}$$

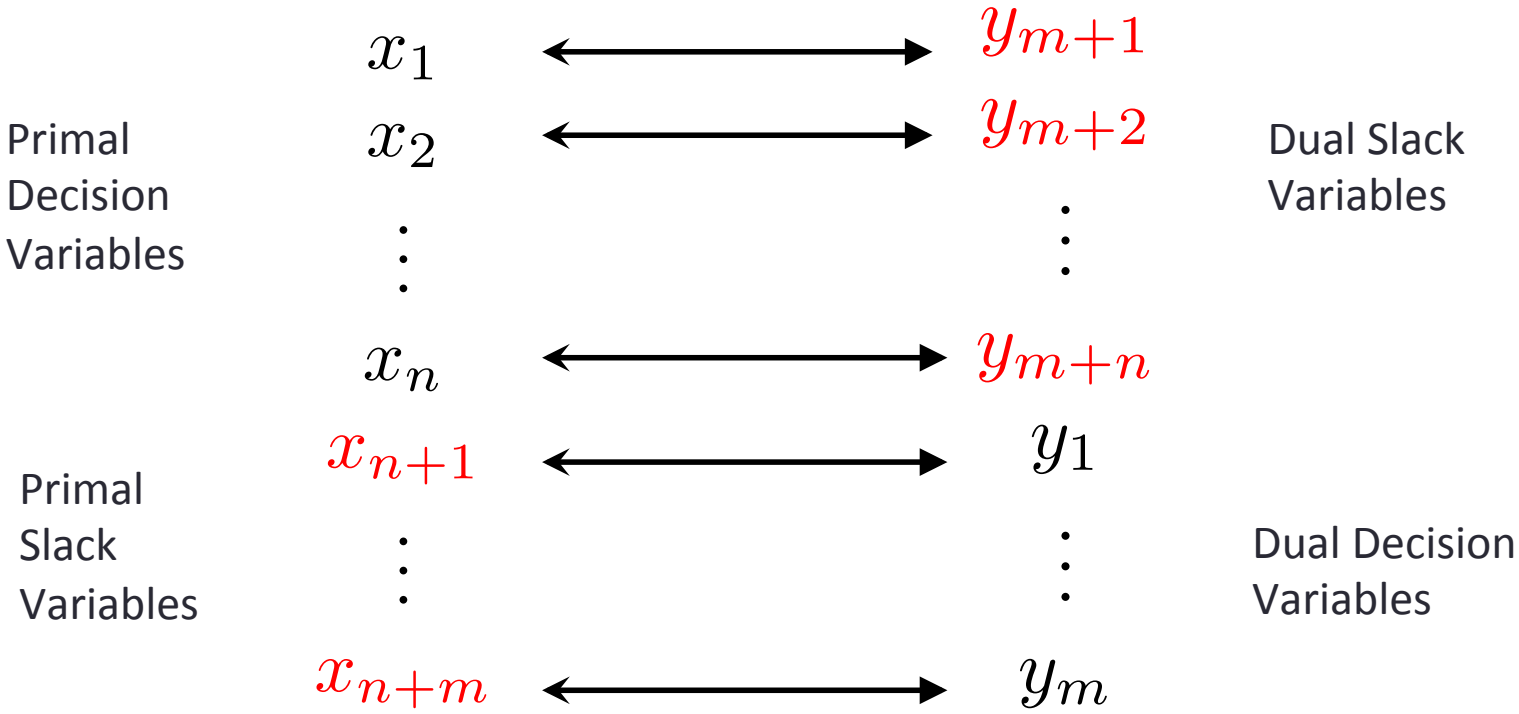
$$\begin{aligned} \min \quad & \mathbf{b}^\intercal \mathbf{y} \\ & A^\intercal\,\mathbf{y} \geq \mathbf{c} \\ & \mathbf{y} \geq 0 \end{aligned}$$

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ & A\,\mathbf{x} + \mathbf{x_s} = \mathbf{b} \\ & \mathbf{x} \geq 0 \\ & \mathbf{x_s} \geq 0 \end{aligned}$$

$$\begin{aligned} \min \quad & \mathbf{b}^\intercal \mathbf{y} \\ & A^\intercal\,\mathbf{y} - \mathbf{y_s} = \mathbf{c} \\ & \mathbf{y} \geq 0 \\ & \mathbf{y_s} \geq 0 \end{aligned}$$

Dual of Dual is the Primal

Complementary Variable Pairs



Complementary Pairs (Example)

$$
\begin{array}{llllcll}
\max & 2x_1 & +3x_2 & -x_3 & & & \\
& x_1 & -x_2 & & \leq & 5 & \leftarrow y_1 \\
& 2x_2 & +x_2 & -x_3 & \leq & -1 & \leftarrow y_2 \\
& x_1 & -x_2 & +x_3 & \leq & 2 & \leftarrow y_3 \\
& x_1 & +x_2 & -x_3 & \leq & 1 & \leftarrow y_4 \\
& -x_1 & & & \leq & 0 & \leftarrow y_5 \\
& & -x_2 & & \leq & 0 & \leftarrow y_6 \\
& & & -x_3 & \leq & 0 & \leftarrow y_7
\end{array}
$$