GOMORY-CHVATAL CUTTING PLANE

Part 1: Setting up the problem.

ILP in Standard Form

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ \text{s.t.} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \\ & \mathbf{x} \in \mathbb{Z} \end{aligned}$$

A, b, c are all assumed to be integers.

Conversion to standard from

- Recap from LP formulations lectures:
  - Equality constraints into two inequalities.
  - Rewrite in case $x_i \geq 0$ is missing:

$$x_i \mapsto x_i^+ - x_i^-$$

  - Convert ≥ to ≤ by negating both sides.

- How do we make sure A, b, c are integers?

Reasonable assumption: original problem coefficients are rationals.

Integer Coefficients.

$$
\begin{array}{llllll}
\max & 2x_1 & +0.3x_2 & -0.1x_3 & & \\
\text{s.t.} & 0.1x_1 & -2x_2 & -x_3 & \leq & 0.25 \\
& 0.5x_1 & -2.6x_2 & +1.3x_3 & \leq & 0.15 \\
& x_1, & x_2, & x_3 & \geq & 0 \\
& x_1, & x_2, & x_3 & \in & \mathbb{Z}
\end{array}
$$

Scale constraints + Objective

$$
\begin{array}{lllllll}
\max & 2x_1 & +0.3x_2 & -0.1x_3 & & & \times 10 \\
\text{s.t.} & 0.1x_1 & -2x_2 & -x_3 & \leq & 0.25 & \times 20 \\
& 0.5x_1 & -2.6x_2 & +1.3x_3 & \leq & 0.15 & \times 100 \\
& x_1, & x_2, & x_3 & \geq & 0 & \\
& x_1, & x_2, & x_4 & \in & \mathbb{Z} &
\end{array}
$$

Conversion to Integer Coefficients.

$$\begin{array}{llllcl}
\max & 2x_1 & +0.3x_2 & -0.1x_3 & & \\
\text{s.t.} & 0.1x_1 & -2x_2 & -x_3 & \leq & 0.25 \\
 & 0.5x_1 & -2.6x_2 & +1.3x_3 & \leq & 0.15 \\
 & x_1, & x_2, & x_3 & \geq & 0 \\
 & x_1, & x_2, & x_4 & \in & \mathbb{Z}
\end{array}$$

$$\begin{array}{llllcl}
\max & 20x_1 & +3x_2 & -x_3 & & \\
\text{s.t.} & 2x_1 & -40x_2 & -20x_3 & \leq & 5 \\
 & 50x_1 & -260x_2 & +130x_3 & \leq & 15 \\
 & x_1, & x_2, & x_3 & \geq & 0 \\
 & x_1, & x_2, & x_4 & \in & \mathbb{Z}
\end{array}$$

Divide result by 10

Adding Slack Variables

$$
\begin{array}{llcl}
\max & \mathbf{c}^\intercal \mathbf{x} & & \\
\text{s.t.} & A\mathbf{x} & \leq & \mathbf{b} \\
 & \mathbf{x} & \geq & \mathbf{0} \\
 & \mathbf{x} & \in & \mathbb{Z}
\end{array}
$$

$$
\boxed{
\begin{array}{llcl}
\max & \mathbf{c}^\intercal \mathbf{x} & & \\
\text{s.t.} & A\mathbf{x} + \mathbf{x_s} & = & \mathbf{b} \\
 & \mathbf{x}, \mathbf{x_s} & \geq & \mathbf{0} \\
 & \mathbf{x}, \mathbf{x_s} & \in & \mathbb{Z}
\end{array}
}
$$

Summary

- We assume problem is in LP standard form.
  - Assume coefficients are rational.
- ILP standard form: coefficients (A,b,c) are integers.
  - Scale the original rational problem.
  - Make sure that result is divided by scale factor for objective.
- Advantage of ILP standard form:
  - Slack variables are naturally integers.
  - No need for separate treatment of decision and slack variables.