PROOFS OF THEOREMS

Dual Dictionary

$$\begin{array}{c|cc} \mathbf{x_B} & \mathbf{b} & +A\mathbf{x_I} \\ \hline z & z_0 & +\mathbf{c}^\intercal \mathbf{x_I} \end{array}$$

Primal Problem Dictionary

$\mathbf{x}$	$\mathbf{x_s}$
$\mathbf{y_s}$	$\mathbf{y}$

$$\begin{array}{c|cc} \mathbf{x_I}^c & -\mathbf{c} & -A^\intercal \mathbf{x_B}^c \\ \hline d & -z_0 & -\mathbf{b}^\intercal \mathbf{x_B}^c \end{array}$$

Dual Problem Dictionary

Theorem

Let **x** be a primal solution and **y** be a dual solution such that

$$\mathbf{c}^\intercal \mathbf{x} = \mathbf{b}^\intercal \mathbf{y}$$

It follows that **x** is primal optimal and **y** is dual optimal.

Fundamental Result of Simplex

The solution associated with any (feasible) final dictionary of the primal problem is optimal.

$$\begin{array}{c|cc} \mathbf{x_B} & \mathbf{b} & +A\mathbf{x_I} \\ \hline z & z_0 & +\mathbf{c}^\intercal \mathbf{x_I} \end{array}$$

Feasible + Final Primal Dict.

$$\begin{array}{c|cc} \mathbf{x_I}^c & -\mathbf{c} & -A^\intercal \mathbf{x_B}^c \\ \hline d & -z_0 & -\mathbf{b}^\intercal \mathbf{x_B}^c \end{array}$$

Feasible + Final Dual Dict.

Primal Objective = Dual Objective Value

Strong Duality

- Let $\mathbf{x}^*$ be a primal optimal solution for an LP.
  - The dual problem has an optimal solution, and
  - Any dual optimal solution $\mathbf{y}^*$ satisfies

$$\mathbf{b}^\intercal \ \mathbf{y}^* = \ \mathbf{c}^\intercal \mathbf{x}^*$$

Proof of Strong Duality Theorem