PS6: Assignment on ILP solvers

Help

The **due date** for this homework is **Tue 16 Dec 2014 2:59 PM CST**.

- [ ] **In accordance with the Coursera Honor Code, I (Kevin Zhu) certify that the answers here are my own work.**

Question 1

This assignment takes you through the process of "binarizing" an ILP to convert it into a 0-1 ILP.

Consider the ILP shown below:

$$\begin{array}{llll} \max & x_1 + x_2 & & \\ \text{s.t.} & x_1 - 3x_2 & \leq & 10 \\ & 2x_1 + 3x_2 & \leq & 15 \\ 0 \leq & x_1 & \leq & 7 \\ 0 \leq & x_2 & \leq & 13 \\ & x_1, x_2 & \mathbb{Z} & \end{array}$$

We represent x_1 by a three bit binary number $b_{1,3}b_{1,2}b_{1,1}$ ($b_{1,3}$ is the most significant bit and $b_{1,1}$ is the least significant bit). Which of the following expressions replaces x_1 in the original ILP?

- $x_1 : 8b_{1,1} + 4b_{1,2} + 2b_{1,1}$
- $x_1 : 4b_{1,3} + 2b_{1,2} + b_{1,1}$
- $x_1 : b_{1,1} + b_{1,2} + b_{1,3}$
- $x_1 : 1000b_{1,3} + 100b_{1,2} + 10b_{1,1}$

Question 2

Continuing with problem 1, what is the minimum number of bits needed to represent x_2 in the binarized problem? Hint: you may have to solve an LP to find out.

- 3
- 4

- 2
- 5

Question 3

In our lecture we considered pure ILPs where all the variables in the problem are integers. In this problem, we consider mixed integer programs (MIPs), wherein a subset of the decision variables are integer variables and the remaining variables are considered real-valued.

Which of the following modifications to branch-and-bound procedure covered will help is solve mixed integer programs? Select all the correct answers. Assume that the problem is maximizing the objective function.

- [ ] Branching is considered when the LP relaxation yields a fractional solution for some decision variable.
- [ ] It is possible to branch on any of the decision variables in the problem.
- [ ] A node can be converted into a leaf whenever its solution satisfies the integrality constraints for the integral decision variables.
- [ ] We cannot prune a node simply because its LP relaxation yields an optimum that is less than or equal to the best objective.
- [ ] We can branch on any real-valued variable x_j whose LP relaxation optimal value is s_j, the branch constraints will be $x_j \leq s_j$ and $x_j \geq s_j$
- [ ] Branching is considered when the LP relaxation yields a fractional solution for an integer decision variable.

Question 4

We apply branch-and-bound on a 0-1 (binary) ILP. Select all the true facts below. Assume that the problem is a maximization problem with n decision variables and m constraints.

- [ ] The maximum depth of the tree is given by the number of decision variables n.
- [ ] We should explore the branch $x_i \geq 1$ first and the $x_i \leq 0$ since it is guaranteed to yield a larger value.
- [ ] For a branch variable x_i, the branch constraints are always $x_i \leq 0$ and $x_i \geq 1$.
- [ ] The total number of nodes in the final branch-and-bound tree will be less than 2^{n+1}

Question 5

Consider the following final dictionary encountered while solving an ILP given below (we assume all problem and slack variables are integers).

$$\begin{array}{c|cccc}
x_2 & \frac{8}{3} & +\frac{2}{3}x_6 & -\frac{10}{3}x_3 \\
x_1 & \frac{5}{3} & -\frac{5}{3}x_6 & +\frac{1}{3}x_3 \\
x_4 & 6 & +\frac{14}{3}x_6 & -\frac{4}{3}x_3 \\
x_5 & \frac{4}{3} & +\frac{5}{6}x_6 & -\frac{1}{3}x_3 \\
\hline
z & -\frac{17}{3} & -\frac{7}{3}x_6 & -\frac{1}{3}x_3
\end{array}$$

Select all the valid cutting plane constraints from the list below.

- [ ] $\frac{2}{3}x_6 - \frac{2}{3}x_3 \geq \frac{2}{3}$ corr. to x_2
- [ ] $\frac{2}{3}x_6 + \frac{2}{3}x_3 \geq \frac{2}{3}$ corresponding to x_1
- [ ] $\frac{1}{3}x_6 + \frac{1}{3}x_3 \geq 0$ corresponding to x_4
- [ ] $\frac{5}{3}x_6 - \frac{1}{3}x_3 \geq \frac{5}{3}$ corr. to x_1
- [ ] $\frac{5}{6}x_6 - \frac{1}{3}x_3 \geq \frac{4}{3}$ corr. to x_5
- [ ] $\frac{2}{3}x_6 - \frac{1}{3}x_3 \geq \frac{2}{3}$ corr. to x_1.
- [ ] $\frac{1}{6}x_6 + \frac{1}{3}x_3 \geq \frac{1}{3}$ corresponding to x_5
- [ ] $\frac{1}{3}x_6 + \frac{1}{3}x_3 \geq \frac{2}{3}$ corresponding to x_2

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