PIVOTING AND VERTICES

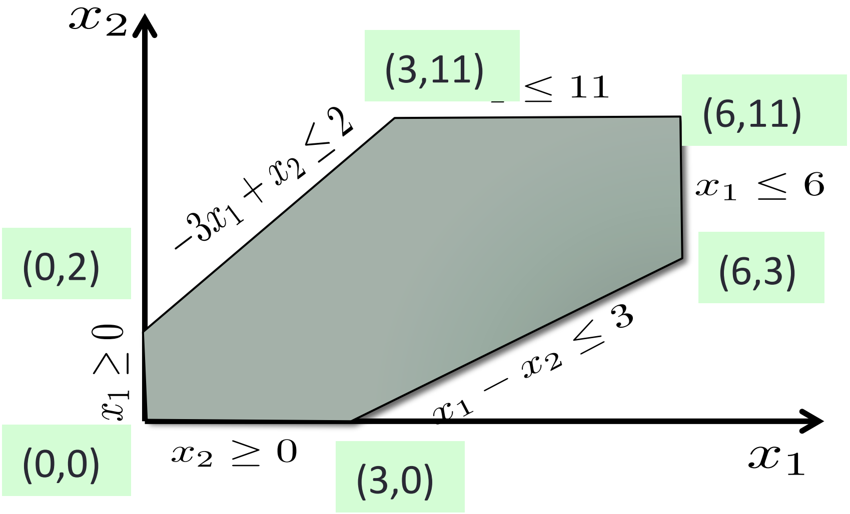
What happens when we pivot?

- Entering variable leaves non-basic set.
  - Leaving variable becomes non-basic.

Adjacent Vertices

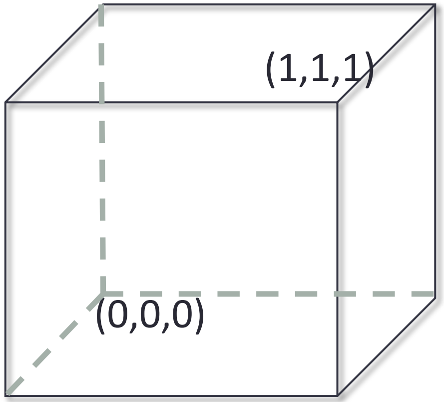
$$
\begin{array}{lrrcr}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
$$

Note: Not drawn to scale



Example #2: Adjacent Vertices

$$
\begin{array}{rrrcl}
x_1 & & & \leq & 1 \\
& x_2 & & \leq & 1 \\
& & x_3 & \leq & 1 \\
x_1 & & & \geq & 0 \\
& x_2 & & \geq & 0 \\
& & x_3 & \geq & 0
\end{array}
$$



Adjacent Vertices

Definition: Two vertices are adjacent if and only if

- At least (n-1) active constraints are common.
- Rank of common active constraints is (n-1).

$$\begin{array}{cccccc} a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 \\ & & \ddots & & \vdots & \\ a_{j1}x_1 & +a_{j2}x_2 & +\cdots+ & a_{jn}x_n & \leq & b_j \\ & & \ddots & & \vdots & \\ a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m \end{array}$$

Active for both vertices

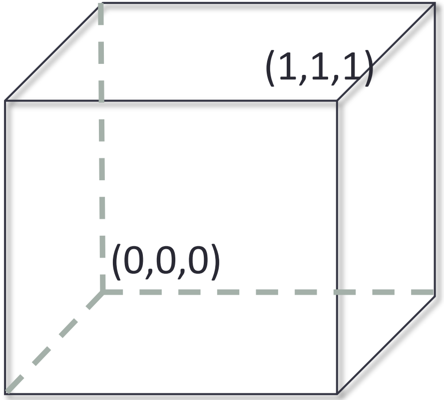
Claim

For non-degenerate/non-final dictionary D_1 if D_2 is obtained on pivot, then the vertices corr. to D_1 and D_2 are adjacent.

$$\begin{array}{c|cc} x_{B1} & \mathbf{b}_1 & \cdots \\ \hline z & c_0 & +c_{N1}x_{N1} \end{array} \qquad \begin{array}{c|cc} x_{B2} & \mathbf{b}_2 & \cdots \\ \hline z & c_2 & +c_{N2}x_{N1} \end{array}$$

Simplex Pivoting Visualization

$$
\begin{array}{rcl}
\max \; x_1 + x_2 - x_3 & & \\
x_1 \qquad\qquad & \leq & 1 \\
x_2 \qquad & \leq & 1 \\
x_3 & \leq & 1 \\
x_1 \qquad\qquad & \geq & 0 \\
x_2 \qquad & \geq & 0 \\
x_3 & \geq & 0
\end{array}
$$



Pivoting Issues

- Does pivoting always move to an adjacent vertex?
  - Yes, if the current dictionary is non-degenerate.

- What happens in the degenerate case?
  - Case-1: Move to an adjacent vertex.
  - Case-2: Remain in the same vertex (?)

- What happens if a dictionary is unbounded?