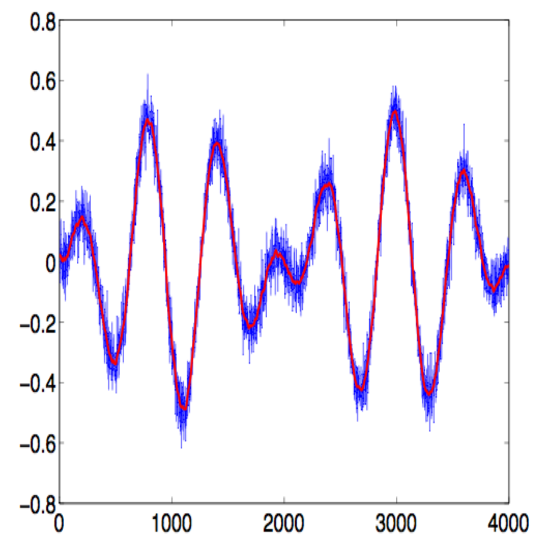
De-Noising Application



- Given n corrupted, by noise, data points

- Find n new data points that are:

  1.) Similar to the corrupted data

  2.) But smoother (i.e. the difference between neighboring data points should be small

1.) 2.)

$$\text{minimize} \quad \|\boldsymbol{x} - \boldsymbol{x}_{\text{cor}}\|^2 + \mu \sum_{k=1}^{n-1} (x_{k+1} - x_k)^2$$

if $\mu \to 0$

$x \to x_{cor}$

if $\mu \to \infty$

$x \to$ constant

De-Noising Application

- Re-cast problem as Least-Squares

$$\text{minimize} \quad ||A\boldsymbol{x} - \boldsymbol{b}||^2$$

$$(\overbrace{mx_1 + b}^{Y(x_1)} - y_1)^2 + (\overbrace{mx_1 + b}^{Y(x_2)} - y_1)^2 + \cdots + (mx_n + b - y_n)^2$$

$$(\underbrace{\begin{bmatrix} x_1 & 1 \end{bmatrix}}_{a_1^T} \begin{bmatrix} m \\ b \end{bmatrix} - y_1)^2 + (\underbrace{\begin{bmatrix} x_2 & 1 \end{bmatrix}}_{a_2^T} \begin{bmatrix} m \\ b \end{bmatrix} - y_2)^2 \cdots$$

$$(\boldsymbol{a}_1^T x - \underset{\uparrow\, y_1}{b_1})^2 + (\boldsymbol{a}_1^T x - \underset{\uparrow\, y_2}{b_1})^2 \cdots + (\boldsymbol{a}_n^T x - b_n)^2$$

$$A = \begin{bmatrix} \boldsymbol{a_1}^T \\ \boldsymbol{a_2}^T \\ \vdots \\ \boldsymbol{a_n}^T \end{bmatrix}$$

$$\text{minimize} \quad \|\boldsymbol{x} - \boldsymbol{x}_{\text{cor}}\|^2$$

$$(x_1 - x_{cor}(1))^2 + (x_2 - x_{cor}(2))^2 + \cdots$$

$$\Big(\underbrace{[1\,0\,0 \cdots 0]}_{a_1^T}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} - \underbrace{x_{cor}(1)}_{b_1}\Big)^2 + \Big(\underbrace{[0\,1\,0\,0 \cdots 0]}_{a_2^T}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} - \underbrace{x_{cor}(2)}_{b_2}\Big)^2 + \cdots$$

$$A = \begin{bmatrix} I^{n\times n} \\ ? \end{bmatrix} \qquad b = \begin{bmatrix} x_{cor} \\ ? \end{bmatrix}$$

$$\text{minimize} \quad \mu \sum_{k=1}^{n-1} (x_{k+1} - x_k)^2$$

inside

$$\underbrace{\left(\sqrt{\mu}\,(x_2 - x_1)\right)^2} + \left(\sqrt{\mu}\,(x_3 - x_2)\right)^2 + \cdots \left(\sqrt{\mu}\,(x_n - x_{n-1})\right)^2$$

$$\left(\underbrace{\sqrt{\mu}\,[-1\ 1\ 0\ 0 \cdots 0]}_{a_1^T} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} - \underset{b_1}{0}\right)^2 + \left(\underbrace{\sqrt{\mu}\,[0\ -1\ 1\ 0 \cdots 0]}_{a_2^T} x - \underset{b_2}{0}\right)^2$$