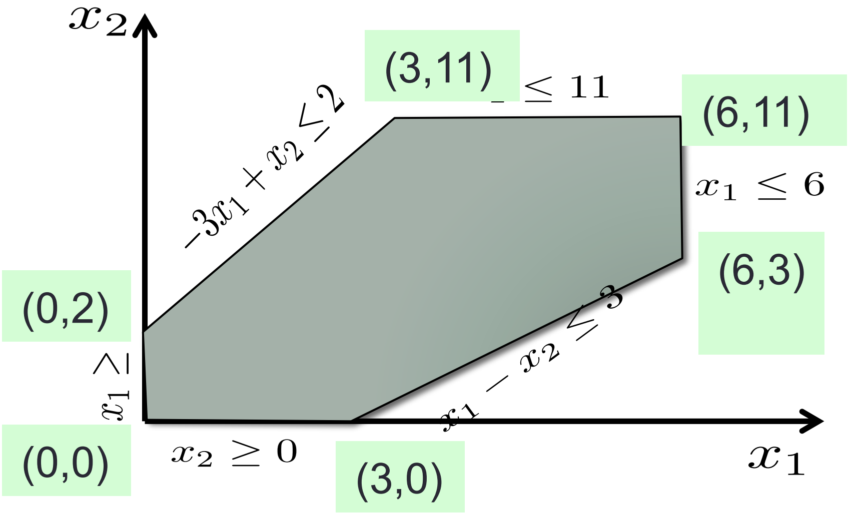
INTERIOR POINT METHODS: BASIC INTRODUCTION

Linear Programming Problem

$$
\begin{array}{llllll}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
$$

Note: Not drawn to scale



Karush-Kuhn-Tucker Conditions

- Very important for many optimization problems.

$$
\begin{array}{llcl}
\max & \mathbf{c}^\intercal \mathbf{x} & & \\
 & A\,\mathbf{x} + \mathbf{x_s} & = & \mathbf{b} \\
 & \mathbf{x}, \mathbf{x_s} & \geq & \mathbf{0}
\end{array}
\qquad\qquad
\begin{array}{llcl}
\min & \mathbf{b}^\intercal \mathbf{y} & & \\
 & A^\intercal\,\mathbf{y} - \mathbf{y_s} & = & \mathbf{c} \\
 & \mathbf{y}, \mathbf{y_s} & \geq & \mathbf{0}
\end{array}
$$

Primal

Dual

Necessary and Sufficient Conditions for optimal solution

$$(\mathbf{x}, \mathbf{x_s}, \mathbf{y}, \mathbf{y_s})$$

KKT conditions for Linear Programs

The primal-dual solution $(\mathbf{x}, \mathbf{x_s}, \mathbf{y}, \mathbf{y_s})$ is optimal iff it satisfies the following conditions:

$$\begin{array}{rcl} A\,\mathbf{x} + \mathbf{x_s} & = & \mathbf{b} \\ \mathbf{x}, \mathbf{x_s} & \geq & \mathbf{0} \end{array}$$

(x,x_s) is primal feasible

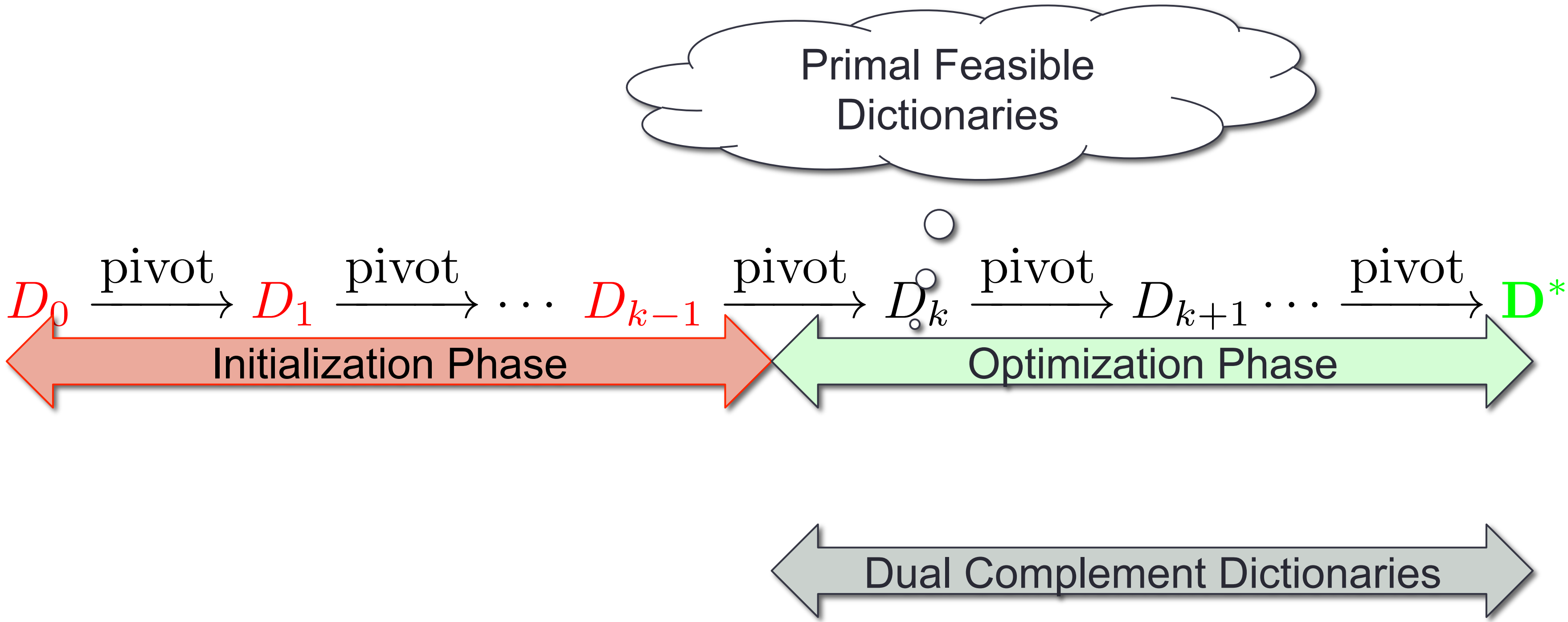
$$\begin{array}{rcl} A^\intercal\,\mathbf{y} - \mathbf{y_s} & = & \mathbf{c} \\ \mathbf{y}, \mathbf{y_s} & \geq & \mathbf{0} \end{array}$$

(y,y_s) is dual feasible

$$\begin{array}{r} x_j y_{s,j} = 0 \\ y_j x_{s,j} = 0 \end{array}$$

Product of complementary pairs is zero.

Simplex Method: Overview



Simplex Method

- Sequence of primal dual solutions (dictionaries)

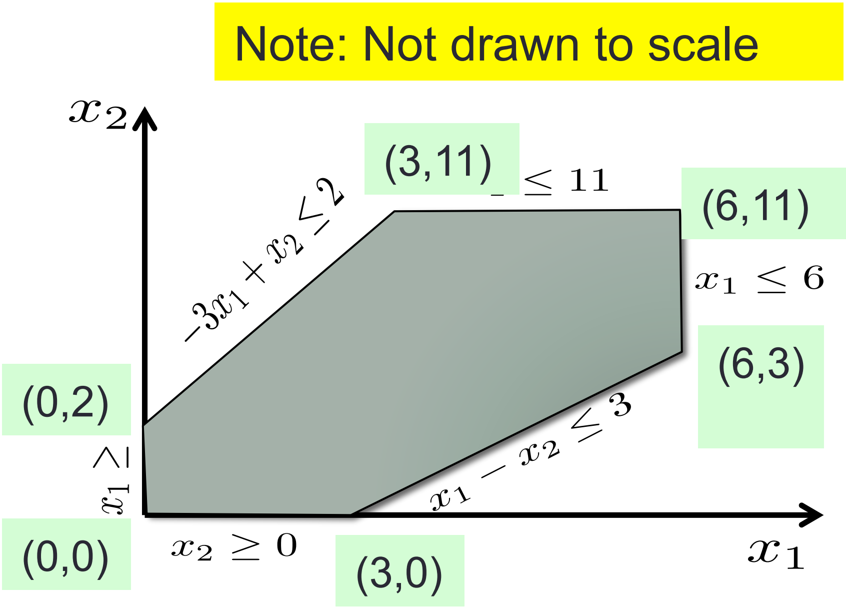
$$\boxed{(\mathbf{x}_0, \mathbf{y}_0) \;\rightarrow\; (\mathbf{x}_1, \mathbf{y}_1) \rightarrow \;\cdots\; \rightarrow \; (\mathbf{x}^*, \mathbf{y}^*)}$$

- Maintain Primal Feasibility.
- Maintain Complementarity Conditions.
- Solutions are vertices of primal/dual feasible regions.
- Dual Feasibility achieved only at the very end.

Interior Point Methods

A class of methods.

- Central Path methods
- Affine Scaling Method
- Active Set
- …



Converges to solution vs. Find the precise answer.

Interior Point Methods

- We will consider a simple central path method.
- Our presentation sequence:
  - Newton’s Method for Solving Equations.
  - Relaxed (mu) complementarity conditions.
  - Central Path.
  - Computing the Newton Step.
  - Adjusting Step Size.
  - Some experiments.