Diet Problem

- Variables

- Constraints

- Objective

- Formulate an LP (Inequality Form)

Problem Description

X_{18} - serving of carrots

- Variables

variable x_j (out of n variables)
is the # of servings (units) of food j

- Healthy Diet Requirement (Constraints)

  - For nutrient i (of m nutrients) you should have at least l_i amount and at most u_i amount.

$i = 8$ - Vit A

$l_8 = 5000$

$u_8 = 50000$

  - In 1 serving of food j there is a_{ij} amount of nutrient i

$a_{8,18}$ - amount of VitA in a serving of carrots

- Find the Cheapest Diet (Objective)

1 serving of food j, costs c_j

C_{18} - cost per serving of carrots

Inequality Form

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \end{aligned}$$

- Objective

Total cost $= c_1 x_1 + \cdots c_n x_n$

$$\min \underbrace{\begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}^T}_{c^T} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}}_{x}$$

Inequality Form

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \end{aligned}$$

- Non-Negative Constraint

$x_{18} \not\geq 0$

$x_1 \geq 0$

$x_2 \geq 0$

$\vdots$

$-Ix \leq 0$ ($h \times n$, 0 $n \times 1$)

$$\begin{bmatrix} -1 & & \\ & \ddots & \\ & & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Inequality Form

$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && Ax \leq b \end{aligned}$$

- Nutrient Bounds

$$l_1 \leq a_{11} x_1 + a_{12} x_2 + \cdots a_{1n} x_n \leq u_1$$

$$\vdots$$

$$l_m \leq a_{m1} x_1 + a_{m2} x_2 + \cdots a_{mn} x_n \leq u_m$$

$$\begin{bmatrix} l_1 \\ \vdots \\ l_m \end{bmatrix} = l \leq \hat{A} x \leq u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$$

Inequality Form

$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && Ax \leq b \end{aligned}$$

$-Ix \leq \vec{0}$

$\hat{A}x \geq \ell \iff -\hat{A}x \leq -\ell$

$\hat{A}x \leq u$

$$\begin{bmatrix} \hat{A} \\ -\hat{A} \\ -I \end{bmatrix} x \leq \begin{bmatrix} u \\ -\ell \\ \vec{0} \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$