LINEAR PROGRAMMING ALGORITHMS

Linear Programming

- Solving systems of Linear Inequalities.
  - Early work by Fourier (Fourier-Motzkin Elimination Algorithm).
  - In symbolic logic, this is called “Linear Arithmetic”.

- World War II: Optimal allocation of resources.
  - Advent of electronic/mechanical calculating machines.
  - L.V. Kantorovich in USSR (1940) and G.B. Dantzig et al. in the USA (1947).

SIMPLEX

- *Simplex:* algorithm for solving LPs.

- First Published by George B. Dantzig

> G.B Dantzig: Maximization of a linear function of variables subject to linear inequalities, 1947.

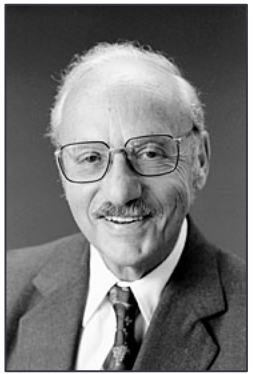
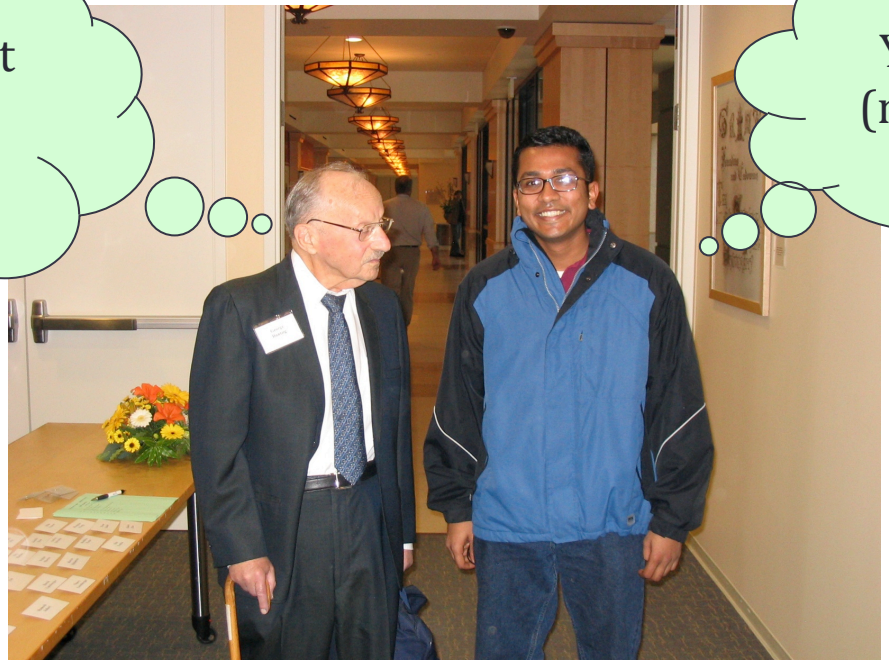


Photo credit:
Stanford University

- Prof. Dantzig contributed numerous seminal ideas to this field.

Prof. Dantzig at his 90th birthday celebrations



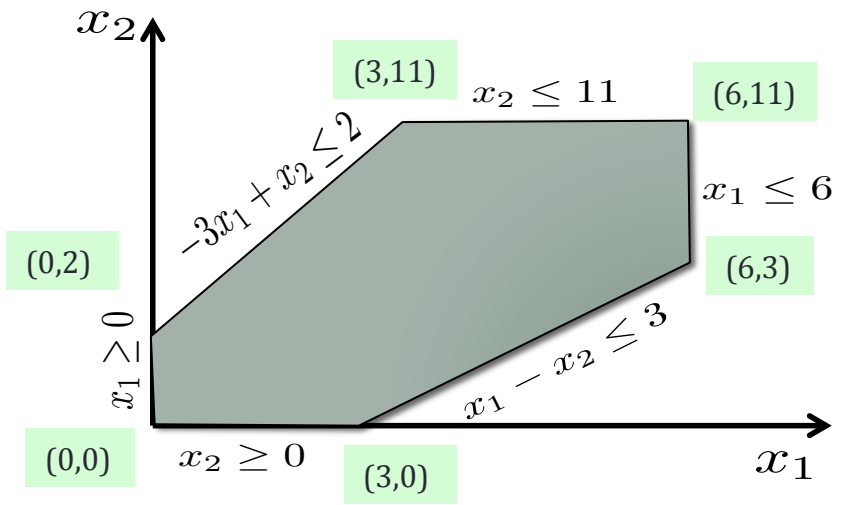
Your instructor (many years ago)

A very special picture for your instructor!!

Visualizing the Simplex Algorithm

$$
\begin{array}{lrrcr}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
$$

Not drawn to scale



Solution: $x_1 = 6$, $x_2 = 11$
Objective Value: 28

Linear Programming Theory

- Duality: John Von Neumann
  - Early work by Lagrange.
  - Connections to game theory.
- Generalized to Karush-Kuhn-Tucker Conditions.
- Complexity of Simplex:
  - Exponential time in the worst case (Klee + Minty).
  - Polynomial time in the “average case”.
  - Much remains to be understood.

Polynomial Time Algorithms

- Leonid Khachiyan's ellipsoidal algorithm [Kachiyan'1980]
  - First polynomial time algorithm.

- Interior Point Methods
  - Ideas go back to Isaac Newton (Newton-Raphson).
  - First algorithms for Linear Programs by Narendra Karmarkar [Karmarkar'1984]
    - Interior point methods are useful for non-linear programming (Cf. Nocedal + Wright textbook).

Applications of Linear Programming Theory

- Too numerous to list exhaustively…

- Major application areas:
  - Operations Research.
    - Optimal allocation of resources.
    - Decision making.
  - Computer Science
    - Algorithms, Machine Learning, Automated Reasoning, Robotics.
  - Engineering
    - Control Theory

In this course…

- We will first study Simplex algorithm.
  - Understand duality of Linear Programs.
- Finally, study interior point methods.