PS4: Least-Squares Assignment

Help

The **due date** for this homework is **Mon 1 Dec 2014 3:00 PM CST**.

☑ **In accordance with the Coursera Honor Code, I (Kevin Zhu) certify that the answers here are my own work.** **Thank you!**

Question 1

Which of the following (A, b) pair solve the least squares problem (J = $||Ax - b||_2^2$) with the cost function $J = (x_1 - 2x_2 + x_3 + 1)^2 + (x_2 - 2x_3 + x_4)^2 + (x_3 - 5)^2$

- $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$
- $A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$
- $A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$
- $A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$

Question 2

The following cost function is related to the smoothing (de-noising) problem shown in lecture: $J = ||x - x_{\text{cor}}||^2 + \mu \sum_{k=2}^{n-1} (x_{k-1} - 2x_k + x_{k+1})^2$. In this cost function, the smoothing term (($\mu \sum_{k=2}^{n-1} (x_{k-1} - 2x_k + x_{k+1})^2$) is using a slightly different representation of the derivative than the problem presented in lecture (center-difference vs. backward-difference). Please choose the appropriate D matrix that will correctly represent this version of the smoothing term:

$$D = \sqrt{\mu}\begin{bmatrix} -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 \\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & -1 & 2 & -1 \end{bmatrix}$$

$$D = \mu\begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix}$$

$$D = \sqrt{\mu}\begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix}$$

$$D = \sqrt{\mu}\begin{bmatrix} 2 & -1 & 2 & 0 & \cdots & 0 \\ 0 & 2 & -1 & 2 & \cdots & 0 \\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & 2 & -1 & 2 \end{bmatrix}$$

Question 3

In this problem, we want to best-fit a cubic polynomial $y(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$ given a set of n noisy data points (t_k, y_k). This is similar to the classic line-fitting least-squares problem where the residual is $r_i = y(t_i) - y_i$ but our variables are ($x = [c_0, c_1, c_2, c_3]^T$) rather than the classic ($x = [m, b]^T$). We are still minimizing $||r||^2 = ||Ax - b||^2$. Pick the appropriate \$A\$ and \$b\$ matrices that represent this problem:

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ & & \vdots & \\ 1 & t_{n-1} & t_{n-1}^2 & t_{n-1}^3 \end{bmatrix}; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ & \vdots & \\ 1 & t_n & t_n^2 \end{bmatrix}; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ & & \vdots & \\ 1 & t_n & t_n^2 & t_n^3 \end{bmatrix}; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$A = \begin{bmatrix} t_1^3 & t_1^2 & t_1 & 1 \\ t_2^3 & t_2^2 & t_2 & 1 \\ & & \vdots & \\ t_n^3 & t_n^2 & t_n & 1 \end{bmatrix}; b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Question 4

In this problem, we want to best-fit a quartic polynomial $y(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4$ to data. The data (found here) has 4001 points where the first column of the data represents t_i and the second column of the data represents y_i . Please enter the value you found for c_1 to 2 decimal places

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