Exercise: Find Leaving Variables

Modified Problem

$$
\begin{array}{rcrrrr}
x_4 & = & 7 & +2x_1 & -3x_2 & +x_3 \\
x_5 & = & 12 & -4x_1 & +x_2 & +2x_3 \\
x_6 & = & 9 & -3x_1 & -4x_2 & +2x_3 \\
\hline
z & = & 10 & +2x_1 & +2x_2 & -3x_3
\end{array}
$$

Note: This dictionary is **not for** the same problem as in slide 2.

Pivoting: Finding new dictionary.

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$

x_1 enters and x_4 leaves

Note: This dictionary is for a different problem than the dictionary in slide 1. The rest of this PPT will keep pivoting the dictionary on this slide.

Pivoting

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -2x_1 & -3x_2 & -x_3 \\
x_5 & = & 11 & -4x_1 & -x_2 & -2x_3 \\
x_6 & = & 8 & -3x_1 & -4x_2 & -2x_3 \\
\hline
z & = & 0 & +5x_1 & +4x_2 & +3x_3
\end{array}
$$

x_1 enters and x_4 leaves

$$
\begin{array}{rcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\
x_5 & = & 1 + 5x_2 + 2x_4 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \\
\hline
z & = & \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{array}
$$

Entering Variable Analysis

$$
\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 \\
\hline
z &= \tfrac{25}{2} - \tfrac{7}{2}x_2 + \tfrac{1}{2}x_3 - \tfrac{5}{2}x_4
\end{aligned}
$$

Leaving Variable Analysis

$$
\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 \\
\hline
z &= \tfrac{25}{2} - \tfrac{7}{2}x_2 + \tfrac{1}{2}x_3 - \tfrac{5}{2}x_4
\end{aligned}
$$

$$
\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 \quad \Rightarrow \quad \underline{x_3 \leq 5} \\
x_5 &= 1 + 5x_2 + 2x_4 \quad \Rightarrow \quad \text{no constraint} \\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 \quad \Rightarrow \quad \underline{x_3 \leq 1}
\end{aligned}
$$

Pivoting

$$
\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 \\
\hline
z &= \tfrac{25}{2} - \tfrac{7}{2}x_2 + \tfrac{1}{2}x_3 - \tfrac{5}{2}x_4
\end{aligned}
$$

x_3 enters and x_6 leaves

$$
\begin{aligned}
x_3 &= 1 + x_2 + 3x_4 - 2x_6 \\
x_1 &= 2 - 2x_2 - 2x_4 + x_6 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
\hline
z &= 13 - 3x_2 - x_4 - x_6
\end{aligned}
$$

Final Dictionary

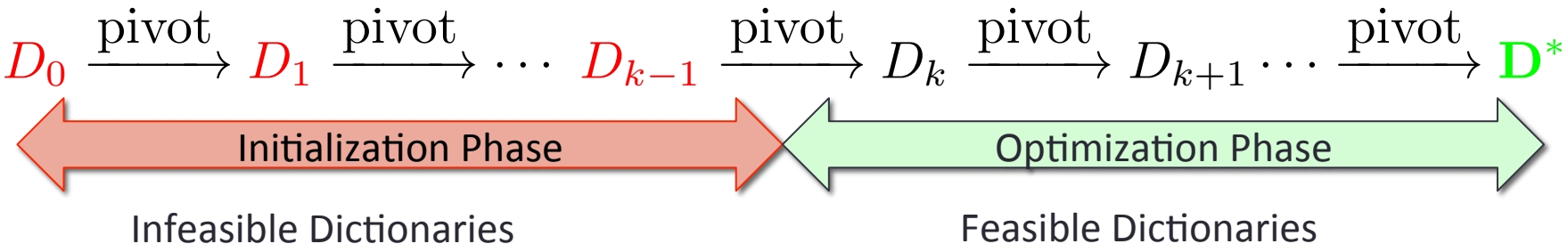
$$
\begin{aligned}
x_3 &= 1 + x_2 + 3x_4 - 2x_6 \\
x_1 &= 2 - 2x_2 - 2x_4 + x_6 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
\hline
z &= 13 - 3x_2 - x_4 - x_6
\end{aligned}
$$

No choice for entering variables.

$$
\begin{aligned}
\mathbf{x}_B &= \mathbf{b} - \mathbf{A}\mathbf{x}_I \\
\hline
z &= c_0 + \mathbf{c}^\intercal \mathbf{x}_I
\end{aligned}
$$

$$\mathbf{c} \leq 0$$

Simplex Algorithm



Summary (1/2)

Choice of leaving variable

$$
\begin{array}{rcllll}
x_{B1} & = & b_1 & -a_{11}x_{I1} & \cdots & -a_{1n}x_{Im} \\
x_{B2} & = & b_2 & -a_{21}x_{I1} & \cdots & -a_{2n}x_{Im} \\
 & \vdots & & & \ddots & \\
x_{Bm} & = & b_n & -a_{m1}x_{I1} & \cdots & -a_{mn}x_{Im} \\
\hline
z & = & c_0 & +c_1x_{I1} & +\cdots & +c_nx_{In}
\end{array}
$$

Choice of entering variable

Summary (2/2)

Pivoting: computing next dictionary

x_3 enters and x_6 leaves

$$
\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 \\
\hline
z &= \tfrac{25}{2} - \tfrac{7}{2}x_2 + \tfrac{1}{2}x_3 - \tfrac{5}{2}x_4
\end{aligned}
$$

$$
\begin{aligned}
x_3 &= 1 + x_2 + 3x_4 - 2x_6 \\
x_1 &= 2 - 2x_2 - 2x_4 + x_6 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
\hline
z &= 13 - 3x_2 - x_4 - x_6
\end{aligned}
$$