

DUAL DICTIONARIES

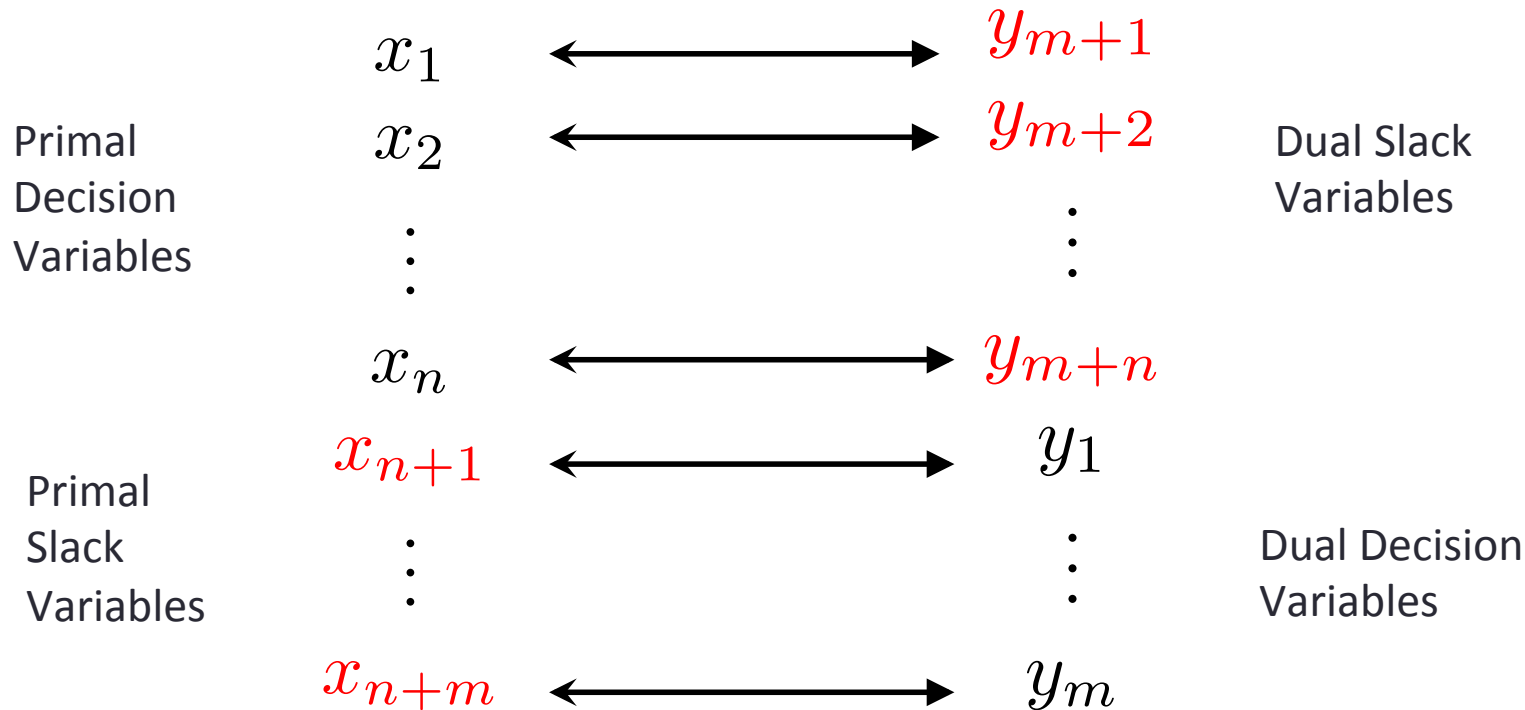
Primal vs. Dual

$$\begin{array}{llll} \max & \mathbf{c}^\top \mathbf{x} & & \\ & A \mathbf{x} & \leq & \mathbf{b} \\ & \mathbf{x} & \geq & 0 \end{array}$$

$$\begin{array}{llll} \min & \mathbf{b}^\top \mathbf{y} & & \\ & A^\top \mathbf{y} & \geq & \mathbf{c} \\ & \mathbf{y} & \geq & 0 \end{array}$$

$$\begin{array}{llll} \max & -\mathbf{b}^\top \mathbf{y} & & \\ \text{s.t.} & -A^\top \mathbf{y} & \leq & -\mathbf{c} \\ & \mathbf{y} & \geq & 0 \end{array}$$

Complementary Variable Pairs



Example

$$\begin{array}{llll}
 \text{max.} & x_1 & +2x_2 & \\
 \text{s.t.} & -3x_1 & +x_2 & \leq 2 \\
 & & +x_2 & \leq 11 \\
 & x_1 & -x_2 & \leq 3 \\
 & x_1 & & \leq 6 \\
 & x_1, & x_2 & \geq 0
 \end{array}$$

$$\begin{array}{llllll}
 \text{max} & -2y_1 & -11y_2 & -3y_3 & -6y_4 & \\
 & 3y_1 & & -y_3 & -y_4 & \leq -1 \\
 & -y_1 & -y_2 & +y_3 & & \leq -2 \\
 & y_1, & y_2, & y_3, & y_4, & \geq 0
 \end{array}$$

x_1	x_2	x_3	x_4	x_5	x_6
y_5	y_6	y_1	y_2	y_3	y_4

Primal vs. Dual Dictionaries

$$\begin{array}{rclcl}
 x_3 & = & 2 & +3x_1 & -x_2 \\
 x_4 & = & 11 & +0x_1 & -x_2 \\
 x_5 & = & 3 & -x_1 & +x_2 \\
 x_6 & = & 6 & -x_1 & +0x_2 \\
 \hline
 z & = & 0 & +x_1 & +2x_2
 \end{array}$$

Primal Problem
Dictionary

x_1	x_2	x_3	x_4	x_5	x_6
y_5	y_6	y_1	y_2	y_3	y_4

Dual Problem
Dictionary

$$\begin{array}{c|cccccc}
 y_5 & -1 & -3y_1 & & +y_3 & +y_4 \\
 y_6 & -2 & +y_1 & +y_2 & -y_3 & \\
 \hline
 d & 0 & -2y_1 & -11y_2 & -3y_3 & -6y_4
 \end{array}$$

Dual Dictionary

$$\begin{array}{c|cc}
 \mathbf{x}_B & \mathbf{b} & +A\mathbf{x}_I \\
 \hline
 z & z_0 & +\mathbf{c}^\top \mathbf{x}_I
 \end{array}$$

Primal Problem
Dictionary

\mathbf{X}	\mathbf{x}_s
\mathbf{y}_s	\mathbf{y}

$$\begin{array}{c|cc}
 \mathbf{x}_I^c & -\mathbf{c} & -A^\top \mathbf{x}_B^c \\
 \hline
 d & -z_0 & -\mathbf{b}^\top \mathbf{x}_B^c
 \end{array}$$

Dual Problem
Dictionary

Example #2

$$\begin{array}{rclcl} x_1 & = & 3 & -\frac{1}{3}x_4 & +\frac{1}{3}x_3 \\ x_2 & = & 11 & -x_4 & +0x_3 \\ x_5 & = & 11 & -\frac{2}{3}x_4 & -\frac{1}{3}x_3 \\ x_6 & = & 3 & +\frac{1}{3}x_4 & -\frac{1}{3}x_3 \\ \hline z & = & 18 & -\frac{7}{3}x_4 & +\frac{1}{3}x_3 \end{array}$$

x_1	x_2	x_3	x_4	x_5	x_6
y_5	y_6	y_1	y_2	y_3	y_4

Example #3

$$x_3 = 9 + x_4 - 3x_6$$

$$x_1 = 6 - x_6$$

$$x_2 = 11 - x_4 + 0x_6$$

$$x_5 = 8 - x_4 + x_6$$

$$z = 21 - 2x_4 - x_6$$

x_1	x_2	x_3	x_4	x_5	x_6
y_5	y_6	y_1	y_2	y_3	y_4

Dual Dictionary

Primal Problem
Dictionary

$$\begin{array}{c|cc} \mathbf{x}_B & \mathbf{b} & + A\mathbf{x}_I \\ \hline z & z_0 & + \mathbf{c}^\top \mathbf{x}_I \end{array}$$

\mathbf{X}	\mathbf{x}_s
\mathbf{y}_s	\mathbf{y}

Dual Problem
Dictionary

$$\begin{array}{c|cc} \mathbf{x}_I^c & -\mathbf{c} & -A^\top \mathbf{x}_B^c \\ \hline d & -z_0 & -\mathbf{b}^\top \mathbf{x}_B^c \end{array}$$

Primal vs. Dual Dictionary

\mathbf{x}_B	\mathbf{b}	$+A\mathbf{x}_I$
z	z_0	$+\mathbf{c}^\top \mathbf{x}_I$

\mathbf{x}_I^c	$-\mathbf{c}$	$-A^\top \mathbf{x}_B^c$
d	$-z_0$	$-\mathbf{b}^\top \mathbf{x}_B^c$

Non-Final



Infeasible

Feasible + Final



Feasible + Final

Pivoting the primal

$$D_i \xrightarrow[\substack{x_j \text{ leaves}}]{\substack{x_i \text{ enters}}} D_{i+1}$$

$$D_i^c \xrightarrow[\substack{x_j^c \text{ enters}}]{\substack{x_i^c \text{ leaves}}} D_{i+1}^c$$

Simplex Optimization Phase

