VISUALIZING LINEAR PROGRAMS

Linear Program (General Form)

Objective Function

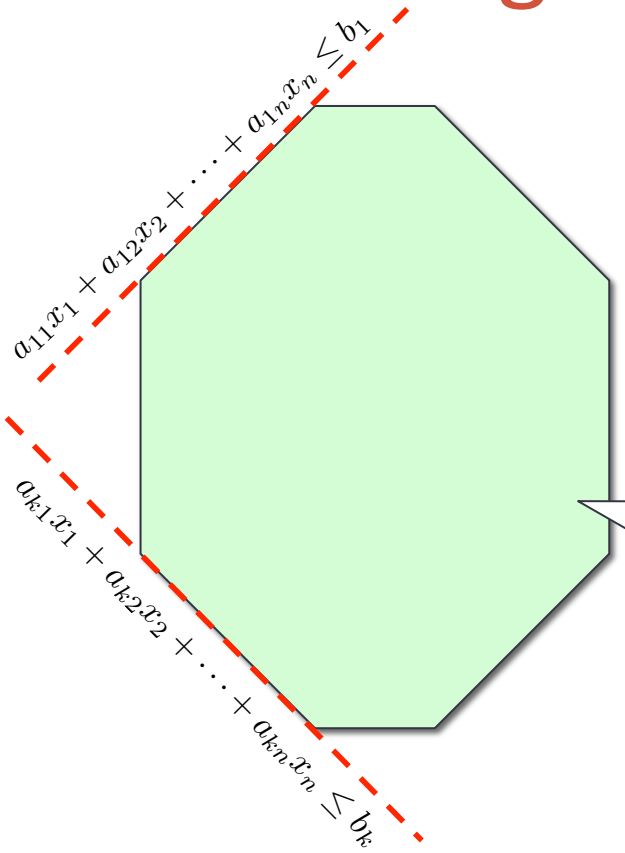
$$\max \quad c_1x_1 \quad +c_2x_2 \quad +\cdots+ \quad c_nx_n$$

$\{\leq, \geq, =\}$

$$\begin{array}{lllll} \text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ \quad a_{1n}x_n & \leq \quad b_1 \\ & & & \ddots & \vdots \\ & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ \quad a_{mn}x_n & \leq \quad b_m \end{array}$$

Constraints

Feasible Region

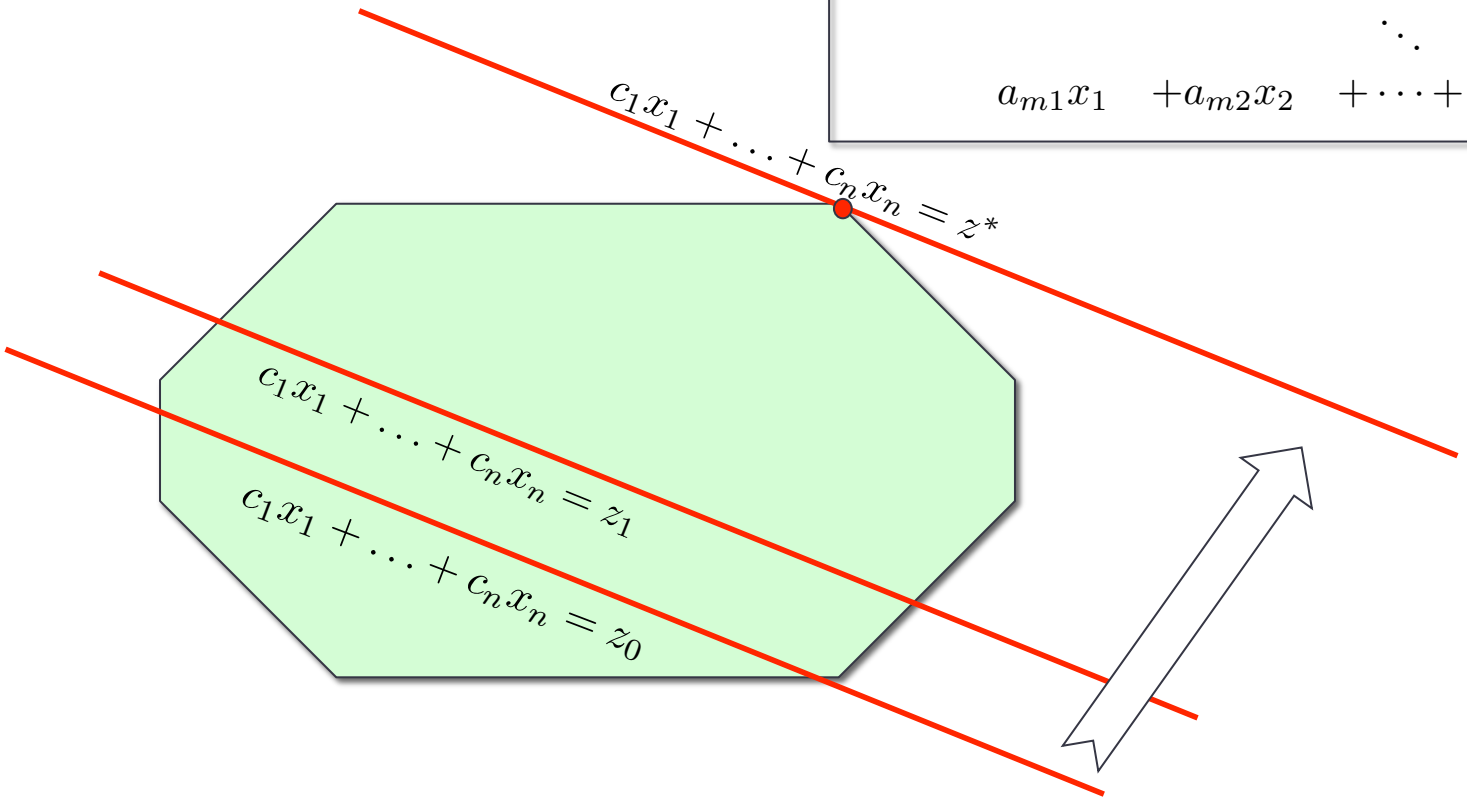


$$
\begin{array}{lccccccc}
\max & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & \\
\text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 \\
 & & & \ddots & & \vdots & \\
 & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m
\end{array}
$$

Feasible Region: Polyhedron
(n dimensional)

Optimization

$$
\begin{array}{lccccccc}
\max & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & \\
\text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 \\
 & & & \ddots & & \vdots & \\
 & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m
\end{array}
$$

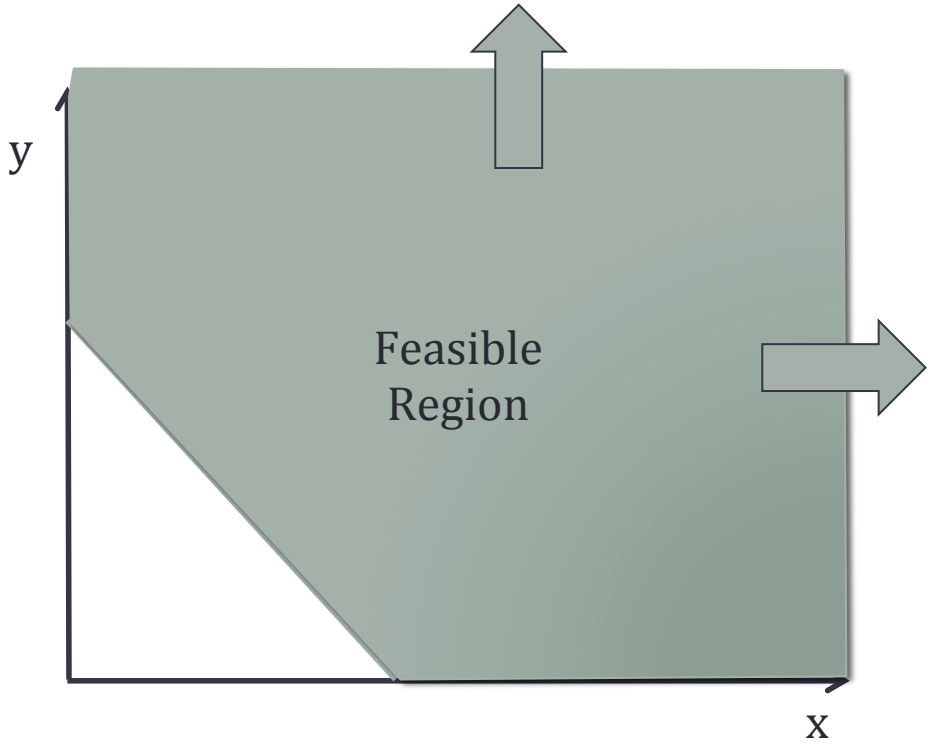


Solving Linear Programs

$$\begin{aligned} \max \quad & \mathbf{c}^\intercal \mathbf{x} \\ & A\,\mathbf{x} \leq \mathbf{b} \end{aligned}$$

- Outcome #1: Optimal Solution(s) exists.
- Outcome #2: Objective Function is unbounded.
- Outcome #3: Feasible Region is empty.

Unbounded Problem (Example)



$$
\begin{array}{lrrrr}
\max & x & & & \\
\text{s.t.} & x & & \geq & 0 \\
 & x & +y & \geq & 1 \\
 & & y & \geq 0 &
\end{array}
$$

Infeasible Problem

- Issue: Constraints contradict each other.

$$
\begin{array}{lllll}
\max & x & & & \\
\text{s.t.} & x & & \geq & 0 \\
\boxed{} & x & +y & \geq & 1 \\
 & & y & \geq 0 & \\
\boxed{} & x & +y & \leq & \frac{1}{2}
\end{array}
$$

Solving Linear Programs

1. Find which of the three cases are applicable.
   - Infeasible?
   - Unbounded?
   - Feasible + Bounded = Optimal?

2. If Optimal, find optimal solution.
   - Note multiple optimal solutions possible.