

Piecewise - Linear Optimization

- Compare ℓ_2 vs ℓ_1 vs ℓ_∞ Minimization

- Histogram of Residuals (intuition)

- Line - Fitting Application

Solutions:

$$\min \|Ax - b\|_p \text{ where } p = \{1, 2, \infty\}$$

- ℓ_2

$$\underline{x^* = A^\dagger b}$$

- closed-form!

- ℓ_∞

$$\begin{array}{ll} \text{minimize} & \begin{bmatrix} 0 & 1 \end{bmatrix}^T \begin{bmatrix} x \\ t \end{bmatrix} = t \\ \text{subject to} & \begin{bmatrix} A & -1 \\ -A & -1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix} \leq \begin{bmatrix} b \\ -b \end{bmatrix} \end{array}$$

$$\hat{x} = \begin{bmatrix} x \\ t \end{bmatrix} \begin{matrix} 80 \\ +1 \\ 81 \end{matrix}$$

- ℓ_1

$$\begin{array}{ll} \text{minimize} & \begin{bmatrix} 0 & 1 \end{bmatrix}^T \begin{bmatrix} x \\ t \end{bmatrix} \\ \text{subject to} & \begin{bmatrix} A & -I \\ -A & -I \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix} \leq \begin{bmatrix} b \\ -b \end{bmatrix} \end{array}$$

$$\hat{x} = \begin{bmatrix} x \\ t \end{bmatrix} \begin{matrix} 80 \\ +200 \\ 280 \end{matrix}$$

Approximation/Fitting Problem

- minimize $\|Ax - b\|_p$

- $A \in \mathbf{R}^{200 \times 80}$ \leftarrow variables

\nwarrow eqns (constraints)

- Generate the elements of (A)
and (b) via randn() function

★ Wank!, these residuals = 0!

