DUALITY: BASIC IDEAS

Dual Problem

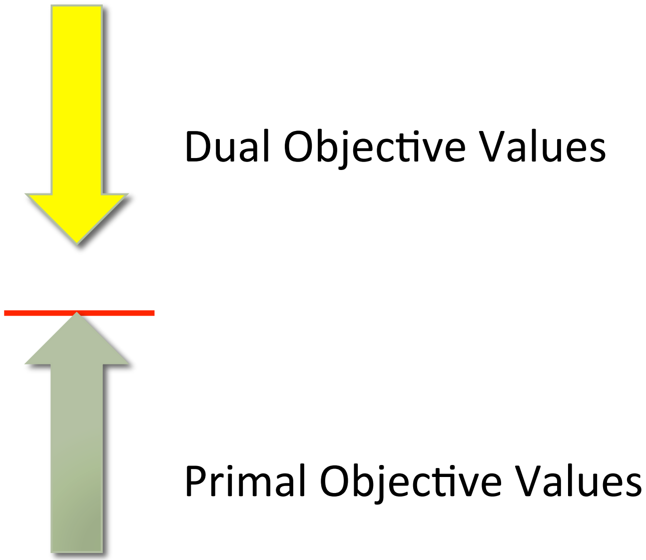
Powerful way of viewing optimization problem

$$
\begin{array}{llll}
\max & f(\vec{x}) & & \\
\text{s.t.} & C(\vec{x}) & \leq & \vec{d}
\end{array}
\xrightarrow{\text{Dualize}}
\boxed{
\begin{array}{llll}
\min & g(\vec{y}) & & \\
\text{s.t.} & P(\vec{y}) & \leq & \vec{q}
\end{array}}
$$

General Opt. Problem
(Primal Problem)

Dual Problem

Dualization Motivation

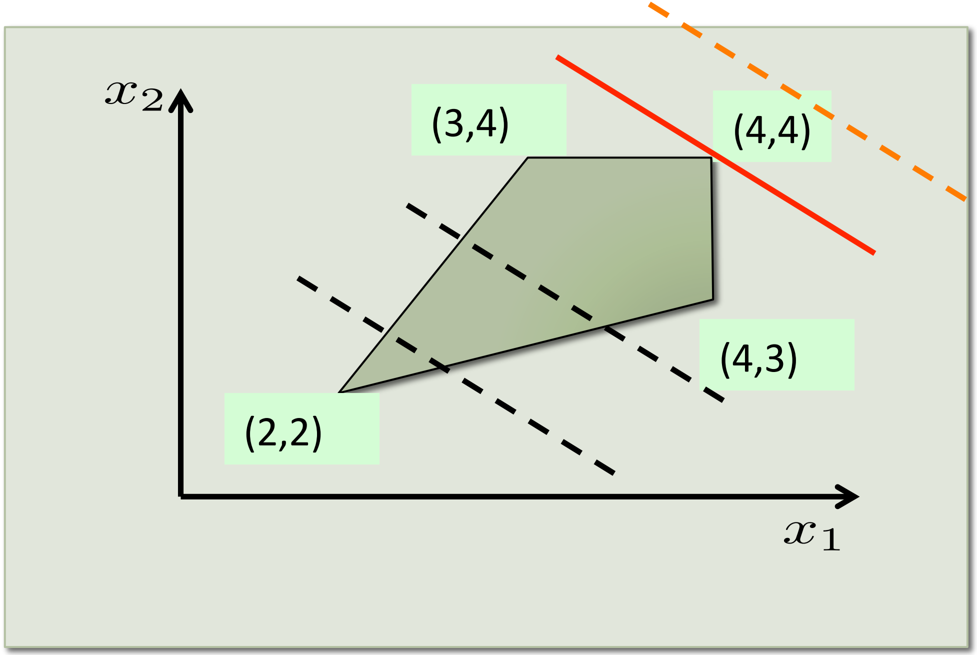


Get upper bounds to primal optimum and obtain best upper bound

Example

$$
\begin{array}{lrcl}
\text{max.} & x_1 + 2x_2 & & \\
\text{s.t.} & -2x_1 + x_2 & \leq & -2 \\
 & x_2 & \leq & 4 \\
 & x_1 - 2x_2 & \leq & -2 \\
 & x_1 & \leq & 4 \\
 & x_1, x_2 & \geq & 0
\end{array}
$$

Optimal Solution: z = 12



Example

$$
\begin{array}{rrcr}
\text{max.} & x_1 + 2x_2 & & \\
\text{s.t.} & -2x_1 + x_2 & \leq & -2 \\
 & x_2 & \leq & 4 \\
 & x_1 - 2x_2 & \leq & -2 \\
 & x_1 & \leq & 4 \\
 & x_1, x_2 & \geq & 0
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{rcrrcr}
-2x_1 + x_2 \leq -2 & \Rightarrow & -4x_1 & +2x_2 & \leq & -4 \\
x_1 \leq 4 & \Rightarrow & 5x_1 & & \leq & 20 \\
\hline
 & & x_1 & +2x_2 & \leq & 16
\end{array}
$$

Example

$$
\begin{array}{rrcr}
\text{max.} & x_1 + 2x_2 & & \\
\text{s.t.} & -2x_1 + x_2 & \leq & -2 \\
 & x_2 & \leq & 4 \\
 & x_1 - 2x_2 & \leq & -2 \\
 & x_1 & \leq & 4 \\
 & x_1, x_2 & \geq & 0
\end{array}
$$

$$
\begin{array}{lcrrcr}
x_1 \leq 4 & \Rightarrow & x_1 & & \leq & 4 \\
x_2 \leq 4 & \Rightarrow & & 2x_2 & \leq & 8 \\
\hline
 & & x_1 & +2x_2 & \leq & 12
\end{array}
$$