AUXILLIARY PROBLEM

$$
\begin{array}{llcl}
\text{max.} & x_1 + 2x_2 & & \\
\text{s.t.} & -2x_1 + x_2 + x_3 & = & -2 \\
& x_2 + x_4 & = & 4 \\
& x_1 - 2x_2 + x_5 & = & -2 \\
& x_1 + x_6 & = & 4 \\
& x_1, x_2, x_3, \ldots, x_6 & \geq & 0
\end{array}
$$

Same as min. x0

$$
\begin{array}{llllcl}
\max & -x_0 & & & & \\
\text{s.t.} & -2x_1 & +x_2 & +x_3 & = & -2 + x_0 \\
& & x_2 & +x_4 & = & 4 + x_0 \\
& x_1 & -2x_2 & +x_5 & = & -2 + x_0 \\
& x_1 & & +x_6 & = & 4 + x_0 \\
& & & x_1, \ldots, x_6, x_0 & \geq & 0
\end{array}
$$

AUXILLIARY PROBLEM

$$\begin{array}{lrcl} \max & \mathbf{c}^\intercal \mathbf{x} & & \\ \text{s.t.} & A\mathbf{x} + \mathbf{x_s} & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s} & \geq & 0 \end{array} \quad \Rightarrow \quad \begin{array}{lrcl} \max & -x_0 & & \\ \text{s.t.} & A\mathbf{x} + \mathbf{x_s} - x_0\mathbf{1} & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s}, x_0 & \geq & 0 \end{array}$$

1. Aux. problem cannot be unbounded.

2. Aux. problem is always feasible.

Aux. Problem Always Feasible (Proof)

$$\begin{array}{lrcl} \max & \mathbf{c}^\intercal \mathbf{x} & & \\ \text{s.t.} & A\mathbf{x} + \mathbf{x_s} & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s} & \geq & 0 \end{array}$$

$$\Rightarrow$$

$$\begin{array}{lrcl} \max & -x_0 & & \\ \text{s.t.} & A\mathbf{x} + \mathbf{x_s} - x_0\mathbf{1} & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s}, x_0 & \geq & 0 \end{array}$$

$$\mathbf{x} = 0,\ x_0 = -\min(\mathbf{b}, 0), x_s = \mathbf{b} + x_0\mathbf{1}$$

Initialization Phase Simplex

$$\begin{array}{lrcl} \max & -x_0 & & \\ \text{s.t.} & A\mathbf{x} + \mathbf{x_s} - x_0\mathbf{1} & = & \mathbf{b} \\ & \mathbf{x}, \mathbf{x_s}, x_0 & \geq & 0 \end{array}$$

If opt. value = 0 then form initial feasible dictionary for original problem.

Initial Aux. Dictionary

Pivoting Steps*

Final Aux. Dictionary

If opt. value < 0 then problem infeasible.