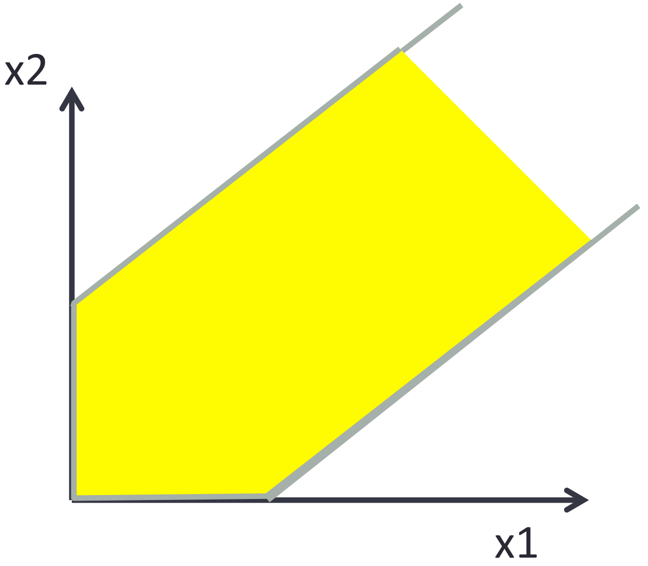
# UNBOUNDED PROBLEMS

How to detect if the problem is unbounded while pivoting.

An example.

# Unbounded Linear Programs

$$\begin{array}{rcccc} \max & x_1 & & & \\ \text{s.t.} & x_1 & -x_2 & \leq & 1 \\ & -x_1 & +x_2 & \leq & 1 \\ & x_1 & , x_2 & \geq 0 & \end{array}$$



# Example

$$
\begin{array}{rrcl}
\text{maximize} & 2x_1 + 3x_2 - 5x_3 & & \\
\text{s.t.} & x_1 - x_2 & \leq & 5 \\
 & -x_1 + \quad\quad x_3 & \leq & 6 \\
 & -2x_1 + \quad\quad x_3 & \leq & 2 \\
 & -x_1 + x_2 & \leq & 4 \\
 & x_1, x_2, x_3 & \geq & 0
\end{array}
$$

# Initial Dictionary

$$\begin{array}{rrrrrl} \text{maximize} & 2x_1 + 3x_2 - 5x_3 & & \\ \text{s.t.} & x_1 - x_2 & \leq & 5 \\ & -x_1 + \quad x_3 & \leq & 6 \\ & -2x_1 + \quad x_3 & \leq & 2 \\ & -x_1 + x_2 & \leq & 4 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

$$\begin{array}{rcl} x_4 & = & 5 - x_1 + x_2 \\ x_5 & = & 6 + x_1 - x_3 \\ x_6 & = & 2 + 2x_1 - x_3 \\ x_7 & = & 4 + x_1 - x_2 \\ \hline z & = & 0 + 2x_1 + 3x_2 - 5x_3 \end{array}$$

# Entering/Leaving Variable Analysis

$$
\begin{array}{rcl}
x_4 & = & 5 - x_1 + x_2 \\
x_5 & = & 6 + x_1 \qquad\quad - x_3 \\
x_6 & = & 2 + 2x_1 \qquad - x_3 \\
x_7 & = & 4 + x_1 - x_2 \\
\hline
z & = & 0 + 2x_1 + 3x_2 - 5x_3
\end{array}
$$

# Second Dictionary

$$
\begin{array}{rcrcrcrcr}
x_2 & = & 4 & + & x_1 & - & x_7 & & \\
x_4 & = & 9 & & & - & x_7 & & \\
x_5 & = & 6 & + & x_1 & & & - & x_3 \\
x_6 & = & 2 & + & 2x_1 & & & - & x_3 \\
\hline
z & = & 12 & + & 5x_1 & - & 3x_7 & - & 5x_3
\end{array}
$$

# Unbounded Dictionary

- No leaving variables.
- Alternatively: all entries in the column corr. to entering variables are non-negative.

$$
\begin{array}{ccccccc}
x_{B1} & = & b_1 & +a_{11}x_{I1} & \cdots & +{\color{red}a_{1j}}x_{Ij} & \cdots \\
x_{B2} & = & b_2 & +a_{21}x_{I1} & \cdots & +{\color{red}a_{2j}}x_{Ij} & \cdots \\
 & \vdots & & & & & \\
x_{Bm} & = & b_m & +a_{m1}x_{I1} & \cdots & +{\color{red}a_{mj}}x_{Ij} & \cdots \\
\hline
z & = & c_0 & +c_1x_{I1} & \cdots & +{\color{red}c_j}x_{Ij} & \cdots
\end{array}
$$

# Unbounded Dictionary

- If we encounter an unbounded dictionary during the optimization phase,
  - Declare that the problem is unbounded and EXIT.