INTERPRETING DUAL VARIABLES: DUAL OF THE DIET PROBLEM

Diet Problem Data

$$F: \left[egin{array}{ccccc} a_{11} & a_{12} & \cdots & a_{1p} \ a_{21} & a_{22} & \cdots & a_{2p} \ dots & \ddots & dots \ a_{r1} & a_{r2} & \cdots & a_{rp} \end{array}
ight]$$

$$: \left(\begin{array}{c} \ell_1 \\ \vdots \\ \ell_p \end{array}\right) \mathbf{u} : \left(\begin{array}{c} u_1 \\ \vdots \\ u_p \end{array}\right)$$

Diet Problem Data

Food	Calories	Total_Fa t		Vit_A	Vit_C	Calcium	Price
Peppers	20	0.1	0.7	467.7	66.1	6.7	0.8
Potatoes, Baked	171.5	0.2	3.7	0	15.6	22.7	0.5
Tofu	88.2	5.5	9.4	98.6	0.1	121.8	1.1
Couscous	100.8	0.1	3.4	0	0	7.2	1
White Rice	102.7	0.2	2.1	0	0	7.9	0.4
Macaroni,Ckd	98.7	0.5	3.3	0	0	4.9	0.2
Peanut Butter	188.5	16	7.7	0	0	13.1	0.6

Nutrient	Min	Max
Calories	2000	2250
Total_Fat	0	65
Protein	50	100
Vit A	5000	50000
Vit C	50	20000
Calcium	800	1600

Diet Problem Setup

$$\begin{array}{cccc} \min & \mathbf{c}^{\mathsf{T}} \mathbf{x} & & \\ \ell & \leq & F^{\mathsf{T}} \mathbf{x} & \leq & \mathbf{u} \\ \mathbf{x} & > & \mathbf{0} & & \end{array}$$

$$\begin{array}{cccc}
 & \mathbf{max} & -\mathbf{c}^{\intercal} \mathbf{x} \\
 & F^{\intercal} \mathbf{x} & \leq & \mathbf{u} \\
 & -F^{\intercal} \mathbf{x} & \leq & -\ell \\
 & \mathbf{x} & \geq & \mathbf{0}
\end{array}$$

Diet Problem Dual

$$\begin{array}{rcl}
 & \mathbf{max} & -\mathbf{c}^{\mathsf{T}} \mathbf{x} \\
 & F^{\mathsf{T}} \mathbf{x} & \leq \mathbf{u} \\
 & -F^{\mathsf{T}} \mathbf{x} & \leq -\ell \\
 & \mathbf{x} & > \mathbf{0}
\end{array}$$

$$egin{array}{lll} \min & \mathbf{u}^{\mathsf{T}}\mathbf{y}_{\mathbf{u}} - \ell^{\mathsf{T}} \; \mathbf{y}_{\ell} \ & F(\mathbf{y}_{\mathbf{u}} - \mathbf{y}_{\ell}) & \geq & -\mathbf{c} \ & \mathbf{y}_{\mathbf{u}}, \; \mathbf{y}_{\ell} & \geq & \mathbf{0} \end{array}$$

What does the dual mean?

		Total Fa					
Food	Calories	Total_Fa t		Vit_A	Vit_C	Calcium	Price
Peppers	20	0.1	0.7	467.7	66.1	6.7	0.8
Potatoes, Baked	171.5	0.2	3.7	0	15.6	22.7	0.5
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Nutrient	Min	Max
Calories	2000	2250
Total_Fat	0	65
Protein	50	100
Vit A	5000	50000
Vit C	50	20000
Calcium	800	1600

Optimal Solutions

Food	Opt. Amt.
Peppers	9.55
Potatoes, Baked	0.95
Tofu	5.39
Couscous	0.00
White Rice	0.00
Macaroni,Ckd	11.86
Peanut Butter	0.00

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Nutrient	Dual (yU)	Dual (yL)
Calories	0.000	0.002
Total_Fat	0.000	0.000
Protein	0.021	0.000
Vit A	0.000	0.002
Vit C	0.000	0.000
Calcium	0.000	0.008