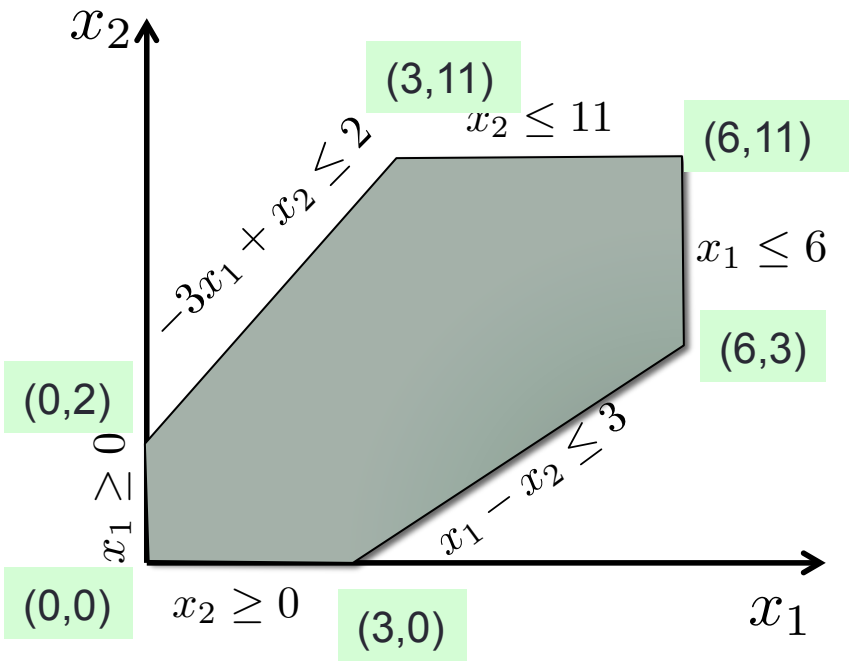
SIMPLEX: EXAMPLE

A Complete Example with Visualization.

Linear Programming Problem

Note: Not drawn to scale

$$
\begin{array}{lrrcr}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
$$



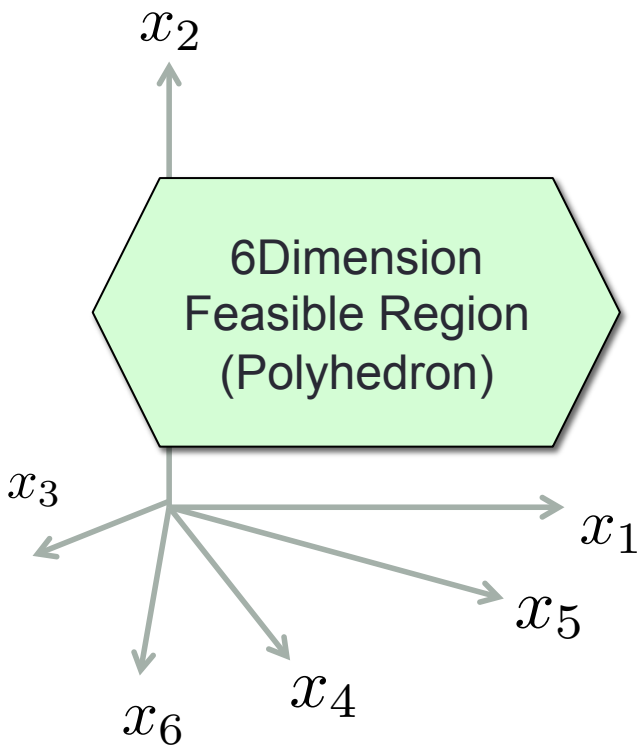
Goal: Solve LP using Simplex and visualize!

Step 1: Add Slack Variables

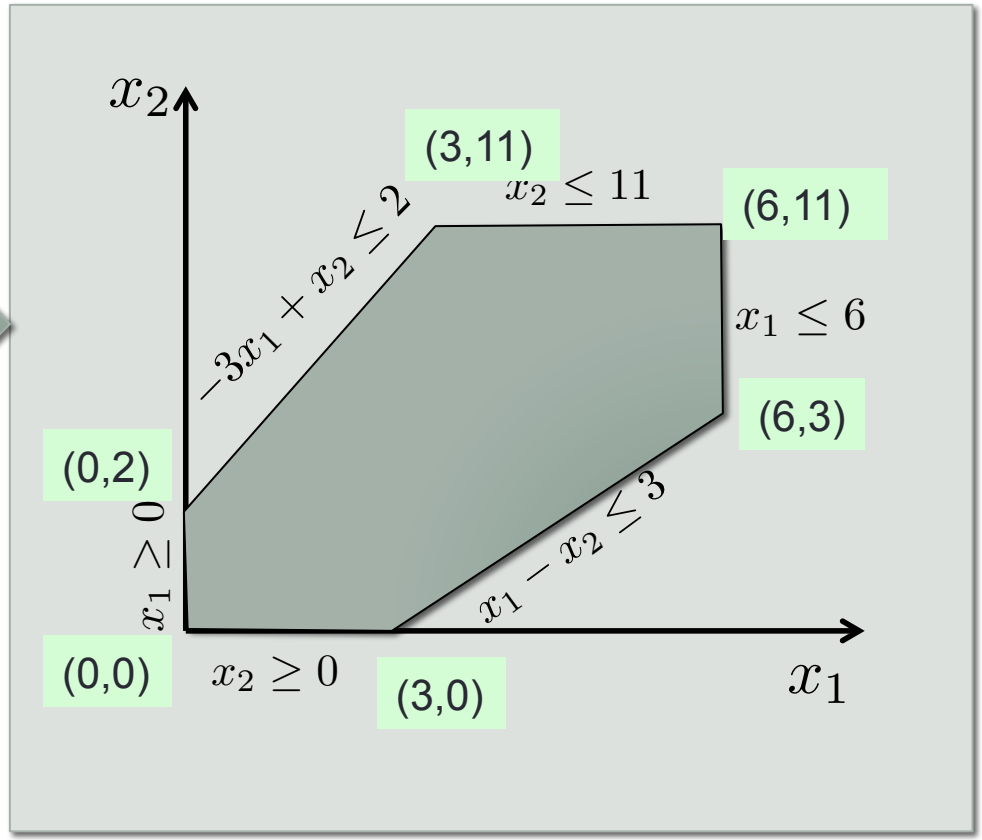
max.	x_1	$+2x_2$		
s.t.	$-3x_1$	$+x_2$	$\leq$	2
		$+x_2$	$\leq$	11
	x_1	$-x_2$	$\leq$	3
	x_1		$\leq$	6
	$x_1,$	x_2	$\geq$	0

$$
\begin{array}{lrrrrr}
\text{max.} & x_1 & +2x_2 & & & \\
\text{s.t.} & -3x_1 & +x_2 & +x_3 & = & 2 \\
 & & +x_2 & +x_4 & = & 11 \\
 & x_1 & -x_2 & +x_5 & = & 3 \\
 & x_1 & & +x_6 & = & 6 \\
 & & x_1, x_2, x_3, \ldots, x_6 & & \geq & 0
\end{array}
$$

Visualizing with Slack

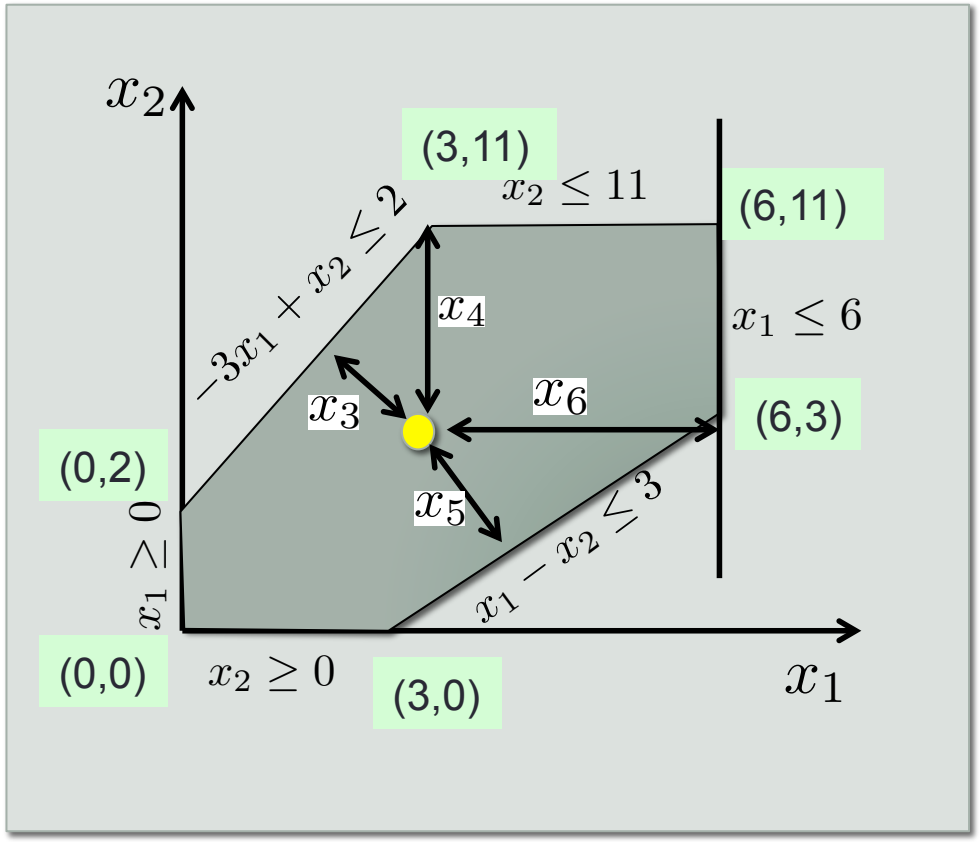


Shadow



Schematic Drawing

Alternative Visualization of Slack



Initial Dictionary

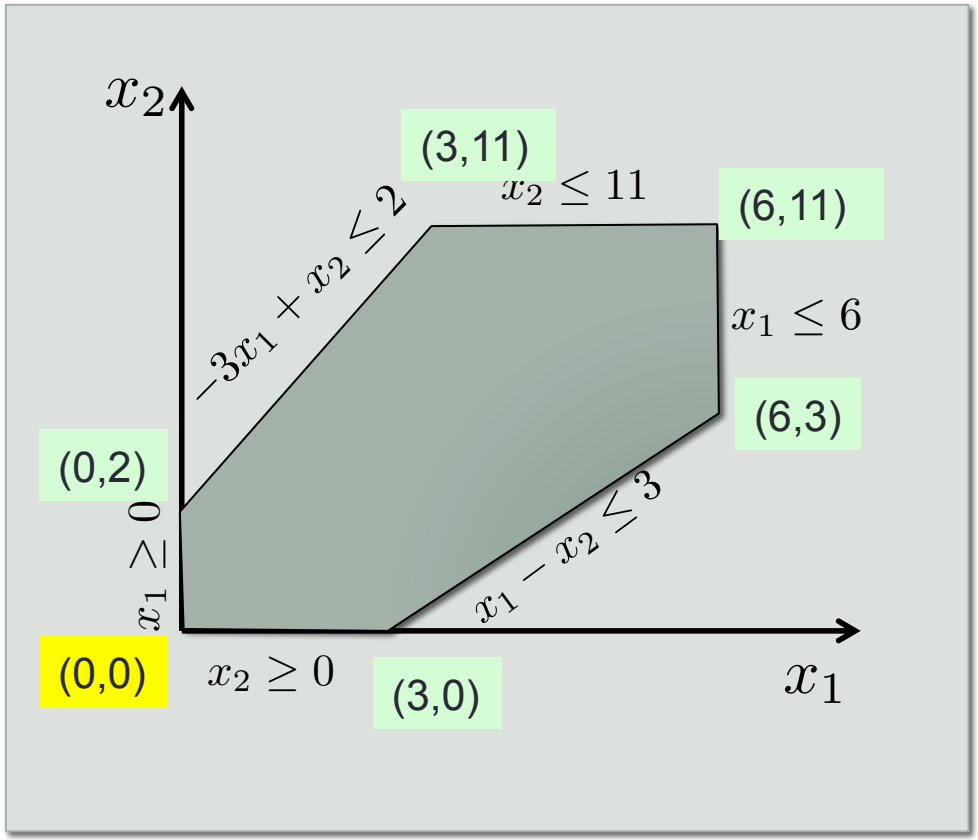
$$
\begin{array}{lrrrcr}
\text{max.} & x_1 & +2x_2 & & & \\
\text{s.t.} & -3x_1 & +x_2 & +x_3 & = & 2 \\
 & & +x_2 & +x_4 & = & 11 \\
 & x_1 & -x_2 & +x_5 & = & 3 \\
 & x_1 & & +x_6 & = & 6 \\
 & & x_1, x_2, x_3, \ldots, x_6 & & \geq & 0
\end{array}
$$

Dict. D1

$$
\begin{array}{rcrrr}
x_3 & = & 2 & +3x_1 & -x_2 \\
x_4 & = & 11 & +0x_1 & -x_2 \\
x_5 & = & 3 & -x_1 & +x_2 \\
x_6 & = & 6 & -x_1 & +0x_2 \\
\hline
z & = & 0 & +x_1 & +2x_2
\end{array}
$$

Solution Associated (Dict. D1)

x_3	$=$	2	$+3x_1$	$-x_2$
x_4	$=$	11	$+0x_1$	$-x_2$
x_5	$=$	3	$-x_1$	$+x_2$
x_6	$=$	6	$-x_1$	$+0x_2$
z	$=$	0	$+x_1$	$+2x_2$



Entering/Leaving Variable Analysis

$$
\begin{array}{rcrrr}
x_3 & = & 2 & +3x_1 & -x_2 \\
x_4 & = & 11 & +0x_1 & -x_2 \\
x_5 & = & 3 & -x_1 & +x_2 \\
x_6 & = & 6 & -x_1 & +0x_2 \\
\hline
z & = & 0 & +x_1 & +2x_2
\end{array}
$$

x_1 enters and x_5 leaves.

x_2 enters and x_3 leaves.

Pivoting

$$
\begin{array}{rcrrr}
x_3 &=& 2 & +3x_1 & -x_2 \\
x_4 &=& 11 & +0x_1 & -x_2 \\
x_5 &=& 3 & -x_1 & +x_2 \\
x_6 &=& 6 & -x_1 & +0x_2 \\
\hline
z &=& 0 & +x_1 & +2x_2
\end{array}
$$

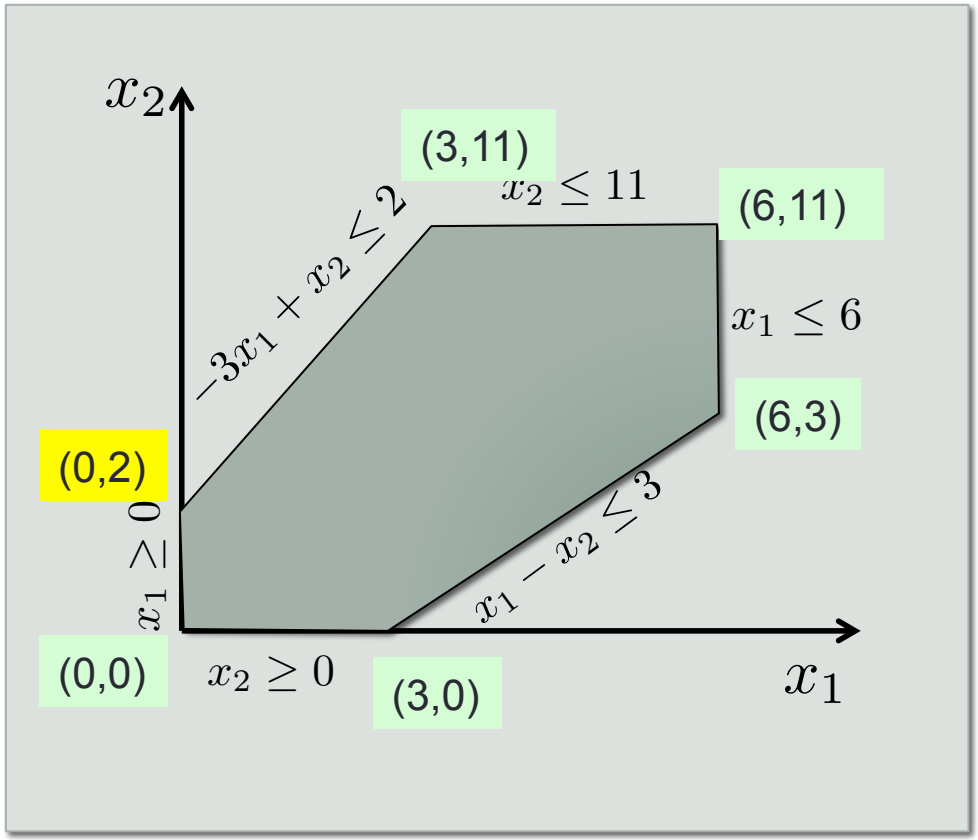
x_2 enters and x_3 leaves.

Dict. D2

$$
\begin{array}{rcrrr}
x_2 &=& 2 & +3x_1 & -x_3 \\
x_4 &=& 9 & -3x_1 & +x_3 \\
x_5 &=& 5 & +2x_1 & -x_3 \\
x_6 &=& 6 & -x_1 & +0x_3 \\
\hline
z &=& 4 & +7x_1 & -2x_3
\end{array}
$$

Solution Associated (D2)

x_2	$=$	2	$+3x_1$	$-x_3$
x_4	$=$	9	$-3x_1$	$+x_3$
x_5	$=$	5	$+2x_1$	$-x_3$
x_6	$=$	6	$-x_1$	$+0x_3$
z	$=$	4	$+7x_1$	$-2x_3$



Entering/Leaving Variable Analysis

$$
\begin{array}{rcrrr}
x_2 & = & 2 & +3x_1 & -x_3 \\
x_4 & = & 9 & -3x_1 & +x_3 \\
x_5 & = & 5 & +2x_1 & -x_3 \\
x_6 & = & 6 & -x_1 & +0x_3 \\
\hline
z & = & 4 & +7x_1 & -2x_3
\end{array}
$$

x_1 enters and x_4 leaves.

Pivoting

$$
\begin{array}{rcrrr}
x_2 &=& 2 & +3x_1 & -x_3 \\
x_4 &=& 9 & -3x_1 & +x_3 \\
x_5 &=& 5 & +2x_1 & -x_3 \\
x_6 &=& 6 & -x_1 & +0x_3 \\
\hline
z &=& 4 & +7x_1 & -2x_3
\end{array}
$$

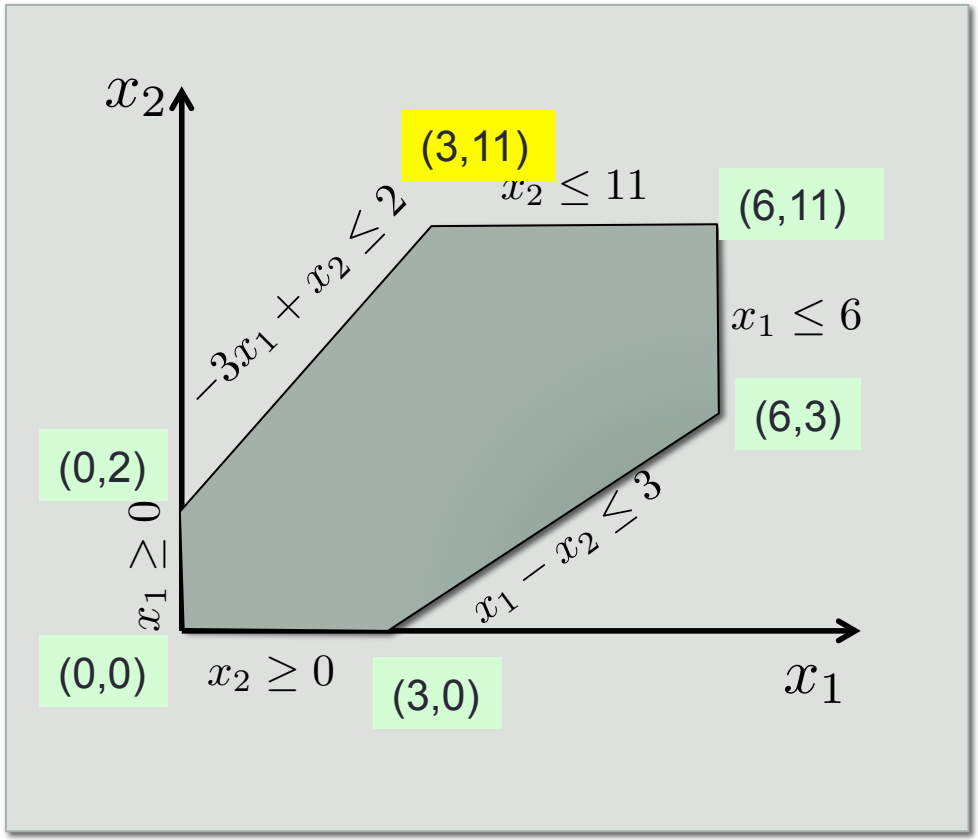
x_1 enters and x_4 leaves.

Dict. D3

$$
\begin{array}{rcrrr}
x_1 &=& 3 & -\frac{1}{3}x_4 & +\frac{1}{3}x_3 \\
x_2 &=& 11 & -x_4 & +0x_3 \\
x_5 &=& 11 & -\frac{2}{3}x_4 & -\frac{1}{3}x_3 \\
x_6 &=& 3 & +\frac{1}{3}x_4 & -\frac{1}{3}x_3 \\
\hline
z &=& 25 & -\frac{7}{3}x_4 & +\frac{1}{3}x_3
\end{array}
$$

Solution Associated with D3

$$
\begin{array}{rcrrr}
x_1 & = & 3 & -\frac{1}{3}x_4 & +\frac{1}{3}x_3 \\
x_2 & = & 11 & -x_4 & +0x_3 \\
x_5 & = & 11 & -\frac{2}{3}x_4 & -\frac{1}{3}x_3 \\
x_6 & = & 3 & +\frac{1}{3}x_4 & -\frac{1}{3}x_3 \\
\hline
z & = & 25 & -\frac{7}{3}x_4 & +\frac{1}{3}x_3
\end{array}
$$



Entering/Leaving Variable Analysis

$$
\begin{array}{rcrrr}
x_1 & = & 3 & -\frac{1}{3}x_4 & +\frac{1}{3}x_3 \\
x_2 & = & 11 & -x_4 & +0x_3 \\
x_5 & = & 11 & -\frac{2}{3}x_4 & -\frac{1}{3}x_3 \\
x_6 & = & 3 & +\frac{1}{3}x_4 & -\frac{1}{3}x_3 \\
\hline
z & = & 25 & -\frac{7}{3}x_4 & +\frac{1}{3}x_3
\end{array}
$$

x_3 enters and x_6 leaves.

Pivoting

x_1	$=$	3	$-\frac{1}{3}x_4$	$+\frac{1}{3}x_3$
x_2	$=$	11	$-x_4$	$+0x_3$
x_5	$=$	11	$-\frac{2}{3}x_4$	$-\frac{1}{3}x_3$
x_6	$=$	3	$+\frac{1}{3}x_4$	$-\frac{1}{3}x_3$
z	$=$	25	$-\frac{7}{3}x_4$	$+\frac{1}{3}x_3$

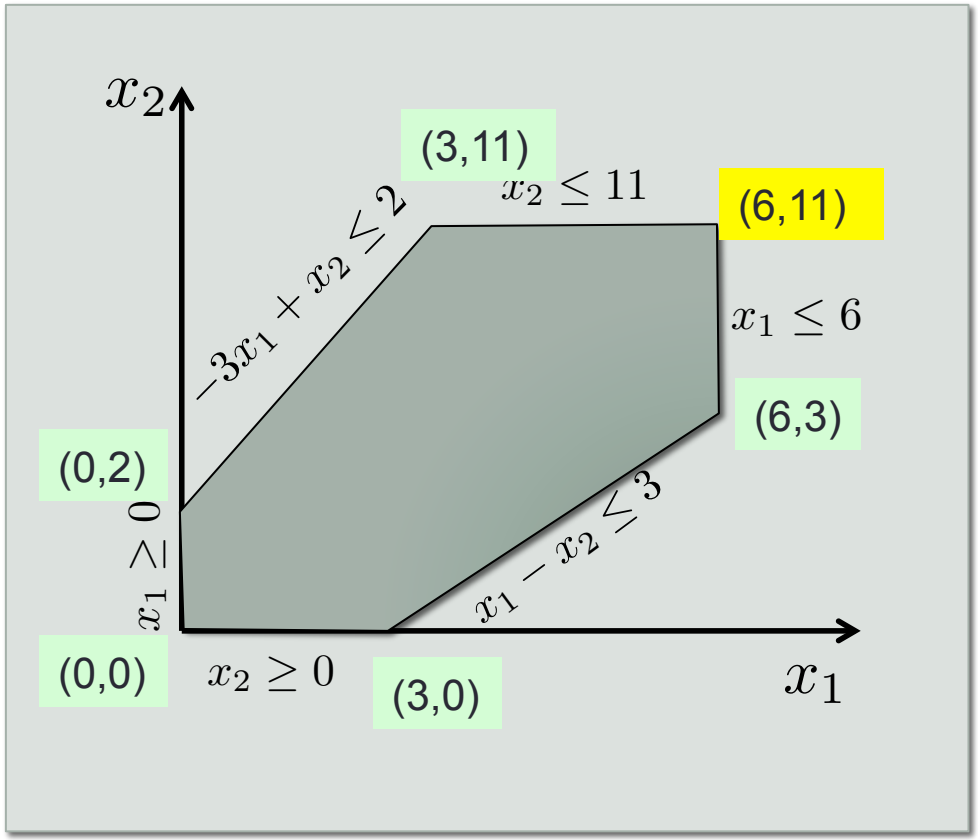
x_3 enters and x_6 leaves.

Dict. D4

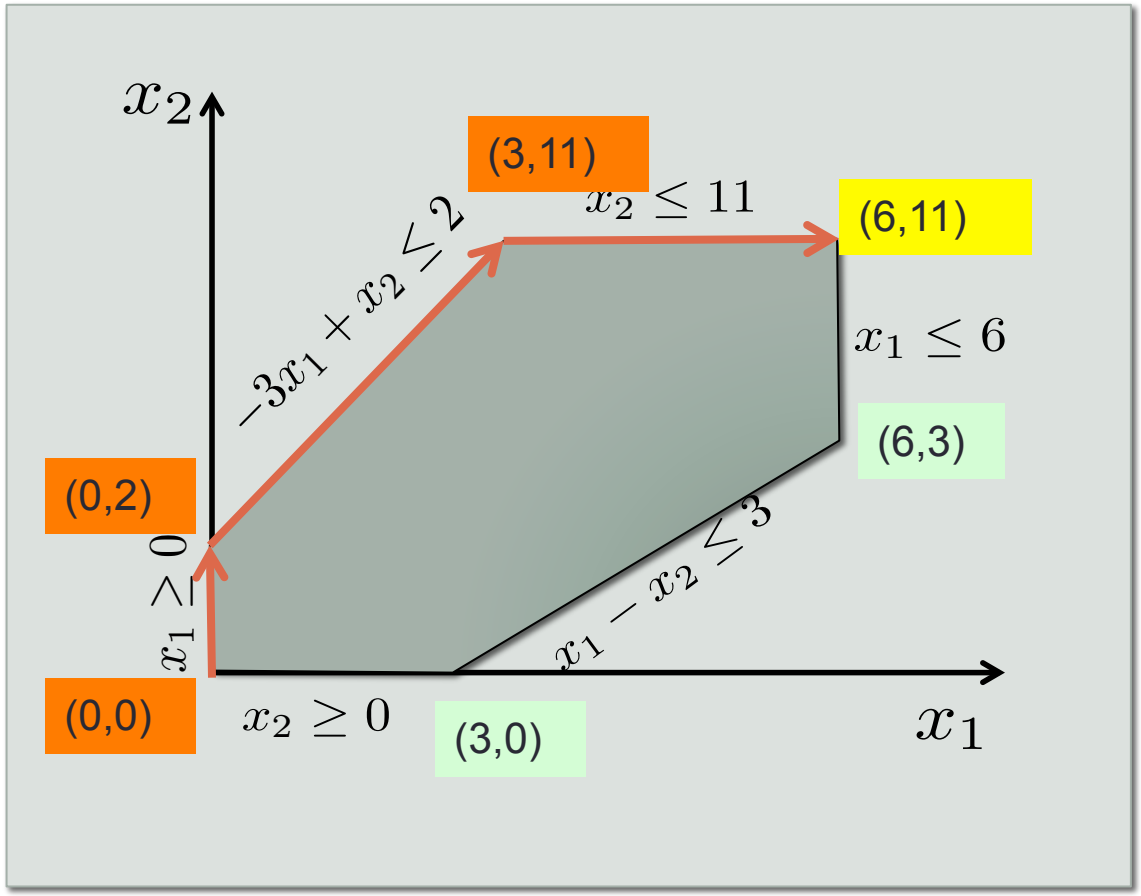
$$
\begin{array}{rcrrr}
x_3 & = & 9 & +x_4 & -3x_6 \\
x_1 & = & 6 & & -x_6 \\
x_2 & = & 11 & -x_4 & +0x_6 \\
x_5 & = & 8 & -x_4 & +x_6 \\
\hline
z & = & 28 & -2x_4 & -x_6
\end{array}
$$

Final Dictionary

$$
\begin{array}{rcrrr}
x_3 & = & 9 & +x_4 & -3x_6 \\
x_1 & = & 6 & & -x_6 \\
x_2 & = & 11 & -x_4 & +0x_6 \\
x_5 & = & 8 & -x_4 & +x_6 \\
\hline
z & = & 28 & -2x_4 & -x_6
\end{array}
$$

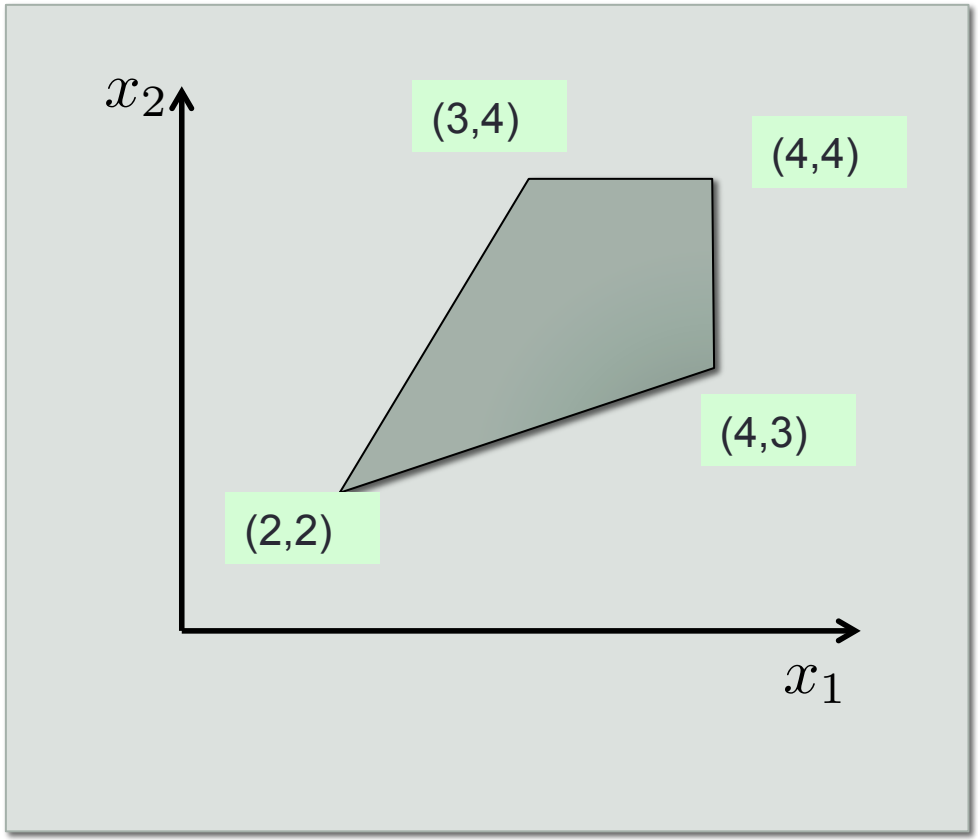


Simplex Dictionaries



Example # 2

$$
\begin{array}{lrcl}
\text{max.} & x_1 + 2x_2 & & \\
\text{s.t.} & -2x_1 + x_2 & \leq & -2 \\
& x_2 & \leq & 4 \\
& x_1 - 2x_2 & \leq & -2 \\
& x_1 & \leq & 4 \\
& x_1, x_2 & \geq & 0
\end{array}
$$



Step 1: Adding Slack

$$
\begin{array}{lrcl}
\text{max.} & x_1 + 2x_2 & & \\
\text{s.t.} & -2x_1 + x_2 + x_3 & = & -2 \\
& x_2 + x_4 & = & 4 \\
& x_1 - 2x_2 + x_5 & = & -2 \\
& x_1 + x_6 & = & 4 \\
& x_1, x_2, x_3, \ldots, x_6 & \geq & 0
\end{array}
$$

Step 2: Initial Dictionary

max.	$x_1 + 2x_2$		
s.t.	$-2x_1 + x_2 + x_3$	$=$	-2
	$x_2 + x_4$	$=$	4
	$x_1 - 2x_2 + x_5$	$=$	-2
	$x_1 + x_6$	$=$	4
	$x_1, x_2, x_3, \ldots, x_6$	$\geq$	0

$$
\begin{aligned}
x_3 &= -2 + 2x_1 - x_2 \\
x_4 &= 4 + 0x_1 - x_2 \\
x_5 &= -2 - x_1 + 2x_2 \\
x_6 &= 4 - x_1 + 0x_2 \\
\hline
z &= 0 + x_1 + 2x_2
\end{aligned}
$$

Infeasible Initial Dictionary

$$
\begin{array}{rcrrr}
x_3 &=& -2 & +2x_1 & -x_2 \\
x_4 &=& 4 & +0x_1 & -x_2 \\
x_5 &=& -2 & -x_1 & +2x_2 \\
x_6 &=& 4 & -x_1 & +0x_2 \\
\hline
z &=& 0 & +x_1 & +2x_2
\end{array}
$$