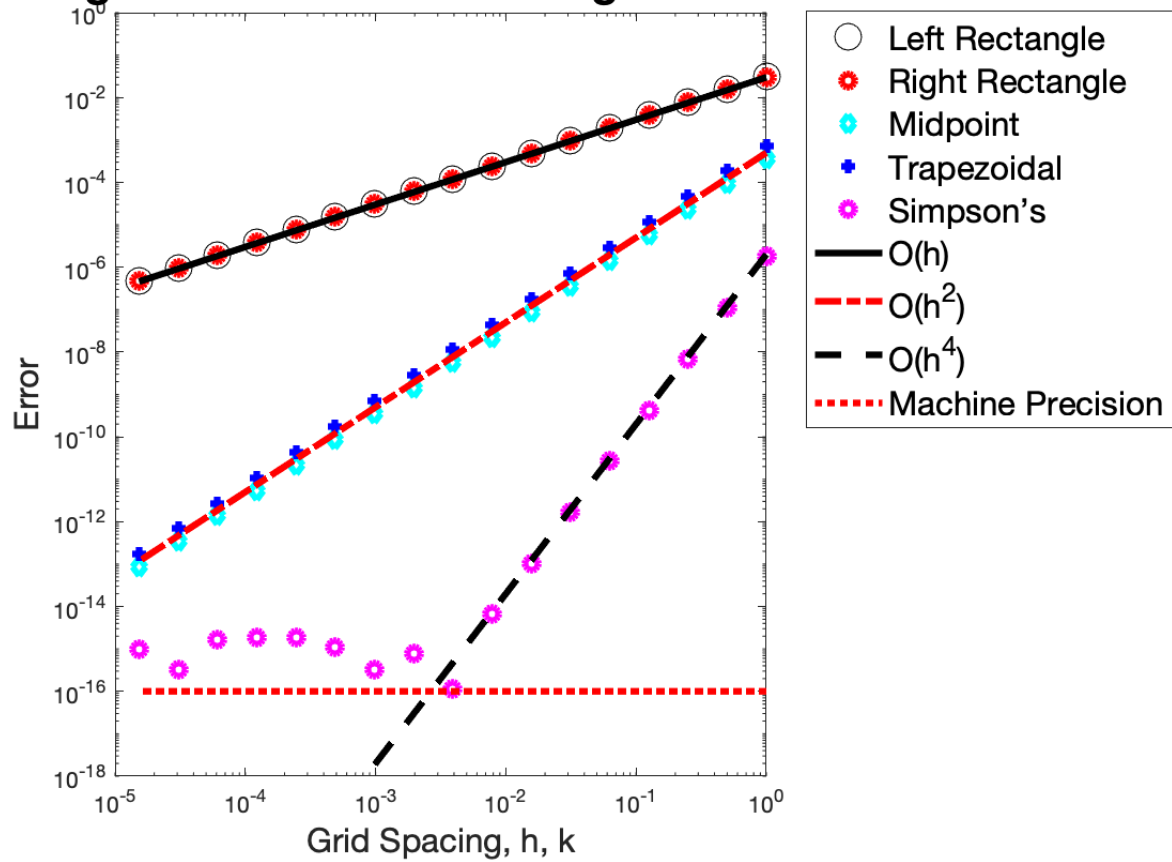Siyue Zhu
AMATH 301 Spring 2020
HW8

Problem 4

vergence of Numerical Integration Schemes

Problem 5

(a)(d)

Code

```matlab
%% Problem 4
clear; close all; clc

f = @(x) exp(-(x-85).^2/50) / sqrt(50*pi);
exact = integral(f, 76, 86);

n = 0:-1:-16;
dx = 2.^n;
x = 76;
error_left = zeros([1 17]);
error_right = zeros([1 17]);
error_midpoint = zeros([1 17]);
error_trap = zeros([1 17]);
error_simpson = zeros([1 17]);

midpoint = zeros([1 17]);
for k = 1:length(dx)
    x = 76:dx(k):86;

    left = dx(k) * sum(f(x(1:end-1)));
    error_left(k) = abs(exact - left);

    right = dx(k) * sum(f(x(2:end)));
    error_right(k) = abs(exact - right);

    mid = f((x(1:end-1) + x(2:end)) / 2);
    midpoint(k) = dx(k) * sum(mid);
    error_midpoint = abs(exact - midpoint);

    trap(k) = dx(k)/2*(f(x(1)) + 2*sum(f(x(2:end-1))) + f(x(end)));
    error_trap = abs(exact - trap);

    simpson(k) = dx(k)/3 * (f(x(1)) + 4*sum(f(x(2:2:end-1))) +
2*sum(f(x(3:2:end-2))) + f(x(end)));
    error_simpson = abs(exact - simpson);
end


loglog(dx, error_left, 'ko','MarkerSize',12)
hold on
loglog(dx, error_right, 'ro', 'LineWidth', 3)
loglog(dx, error_midpoint, 'cd', 'LineWidth', 3)
loglog(dx, error_trap, 'b+', 'LineWidth', 3)
loglog(dx, error_simpson, 'mo', 'LineWidth', 3)

loglog(dx, 0.03*dx, 'k-', 'LineWidth', 3)
loglog(dx, 0.0005*dx.^2, 'r-.', 'LineWidth', 3)
loglog(dx, 0.000002*dx.^4, 'k--', 'LineWidth', 3)

line = 10^-16 + zeros([1 length(dx)]);
loglog(dx, line, 'r:', 'LineWidth', 3)

title('Convergence of Numerical Integration Schemes', 'fontsize', [20])
xlabel('Grid Spacing, h, k', 'fontsize', [15])
```

```
ylabel('Error', 'fontsize', [15])
legend('Left Rectangle', 'Right Rectangle', 'Midpoint', 'Trapezoidal',
'Simpson‚Äôs', 'O(h)', 'O(h^2)', 'O(h^4)', 'Machine Precision', 'fontsize',
[15], 'Location', 'Northeastoutside')
ylim([10^-18 1])
print('HW8_fig1.png','-dpng')
```