Math381 HW1

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Introduction

Suppose I have a container with weight capacity of 300 kg and volume capacity of 300 liters, and 100 unique objects with various values, weights and volumes. Those values, weights and volumes can be calculated by the equations below:

$$value = floor(50 + 25 * cos(5i))dollars$$

$$weight = floor(30 + 12 * cos(4i + 1))kg$$

$$volume = floor(30 + 12 * cos(2i + 2))liters$$

We are going to use lpsolve to solve three questions under this context.

Question1

Now, I'm going to determine what objects I should put into the container to maximize the total value. Our objects are integer binary variables,

$$x_i(1 <= i <= 100)$$

with 0 representing nonexistence and 1 representing existence. Also, no object can be chosen more than once, since they are all unique objects. Our goal is to maximize the total value in my container with the weight constrain 300 kg and volume constrain 300 liters.

The mathematical formula we use to calculate the total value, total weight and the total volume are:

$$value = \sum_{n=1}^{100} x_i value_i = x_1 value_1 + x_2 value_2 + ... + x_{99} value_{99} + x_{100} value_{100}$$

$$weight = \sum_{n=1}^{100} x_i weight_i = x_1 weight_1 + x_2 weight_2 + ... + x_{99} weight_{99} + x_{100} weight_{100} \leq 300$$

$$volume = \sum_{n=1}^{100} x_i volume_i = x_1 volume_1 + x_2 volume_2 + ... + x_{99} volume_{99} + x_{100} volume_{100} \leq 300$$

The objective function we want to maximize is :

$$max : +57x_1 + 29x_2 + ... + 54x_{99} + 27x_{100};$$

Thus the lp for this question is:
maximun $57\text{x}_1 + 29x_2 + ... + 54x_{99} + 27x_{100}$
subject to

$$+33x_1 + 19x_2 + ... + 34x_{99} + 35x_{100} \leq 300$$

$$+22x_1 + 41x_2 + ... + 35x_{99} + 37x_{100} \leq 300$$

$$x_1, x_2, ..., x_{99}, x_{100} = 0 \text{ or } 1$$

Here is the python code I use to generate the input file for lpsolve:

```python
import math

# This code is used to generate a lp input file for a knapsack problem.

#Gives the objective function.
output = "max: "
for i in range(1,101):
    val = math.floor(50 + 25*math.cos(5*i))
    output = output + "+" + str(val) + "x" + str(i)
output = output + ";"
print(output)

#Gives the weight constraint, which is the total weight is <= 300.
output = "weight = "
for i in range(1,101):
    weight = math.floor(30 + 12 * math.cos(4*i+1))
    output = output + "+" + str(weight) + "x" + str(i)
output = output + ";"
print(output)
print("weight <= 300;")

#Gives the volume constraint, which is the total volume is <= 300.
output = "volume = "
for i in range(1,101):
    volume = math.floor(30 + 12 * math.cos(2*i+2))
    output = output + "+" + str(volume) + "x" + str(i)
output = output + ";"
print(output)
print("volume <= 300;")

# Set the variable constraint.
output = "bin "
for i in range(1,100):
    output = output + "x" + str(i) + ","
output = output + "x100;"
print(output)
```

And the lp input file I generate by using python looks like this:

```
max: +57x1+29x2+...+54x99+27x100;
weight = +33x1+19x2+...+34x99+35x100;
weight <= 300;
volume = +22x1+41x2+...+35x99+37x100;
volume <= 300;
bin x1,x2,...x99,x100;
```

And our final result is:

```
Value of objective function: 920.00000000

Actual values of the variables:
x_10                           1
x_19                           1
x_24                           1
x_29                           1
x_35                           1
x_44                           1
x_49                           1
x_54                           1
x_63                           1
x_73                           1
x_79                           1
x_88                           1
x_98                           1
weight                       298
volume                       297
```

By using lpsolve, we know that we should pick object 10, 19, 24, 29, 35, 44, 49, 54, 63, 73, 79, 88, 98. Also, the total weight is 298 kg and the total volume is 297 liters, which are all under the constraint. Thus, 300 kg and 300 liters are not binding constraints.

Question2

Let's consider if we put a lower bound on the number of objects that we put into the container now. From the previous part, we know that we picked 13 different objects with a total weight of 298 kg and total volume of 297 liters. What if there is a constraint on the number of object we pick? By adjust the code, I generate a new input file which a lower bound of the number of object is added to the file.

We don't need to consider any lower bound less than or equal to 13, since our original answer has 13 objects. Any bound less than or equal to 13 would not affect our answer. Now we can try and see how the answer would change if we put a lower bound as 14.

The new constraint is:

$$\text{number of objects} = \sum_{n=1}^{100} x_i = x_1 + x_2 + ... + x_{99} + x_{100} \geq 14$$

Thus, the new lp for this question is:
maximun $57\text{x}_1 + 29x_2 + ... + 54x_{99} + 27x_{100}$
subject to

$$+33x_1 + 19x_2 + ... + 34x_{99} + 35x_{100} \leq 300$$

$$+22x_1 + 41x_2 + ... + 35x_{99} + 37x_{100} \leq 300$$

$$x_1 + x_2 + ... + x_{99} + x_{100} \geq 14$$

$$x_1, x_2, ..., x_{99}, x_{100} = 0 \text{ or } 1$$

We need to add some code in python to show the new constraint:

```
# Count the number of our objects picked.
output = "count = "
for i in range(1,100):
    output = output + "x" + str(i) + "+"
output = output + "x100;"
print(output)
print("count >= 1;")
```

And the new lp input file is:

```
max: +57x1+29x2+...+54x99+27x100;
weight = +33x1+19x2+...+34x99+35x100;
weight <= 300;
volume = +22x1+41x2+...+35x99+37x100;
volume <= 300;
count = x1+x2+...+x99+x100;
count >= 14;
bin x1,x2,...x99,x100;
```

And our result show as the following:

```
Value of objective function: 907.00000000

Actual values of the variables:
x_4                             1
x_10                            1
x_19                            1
x_24                            1
x_29                            1
x_35                            1
x_38                            1
x_54                            1
x_60                            1
x_63                            1
x_73                            1
x_79                            1
x_82                            1
x_98                            1
weight                        299
volume                        285
count                          14
```

We can see from the result that the objective function decreases from 920 to 907 compare to question 1. We pick object 4, 10, 19, 24, 29, 35, 38, 54, 60, 63, 73, 79, 98 under the constraint, which are 14 objects in total. And the total weight is 299 kg and total volume of 285 liters. Compare with the previous question, we throw object 44, 49, 88 out, and pick object 4, 38, 60, 82 instead.

Objects We Throw Out			
object	value	weight	volume
44	74	35	24
49	74	22	40
88	74	34	24

New Objects We Pick			
object	value	weight	volume
4	60	26	19
38	51	22	19
60	49	22	19
82	49	22	19

We can calculate the average ratio between value and weight and the ratio between value and volume, and compare among objects we throw out and new objects we pick.

For objects we thrown out, the average ratio between value and weight is $\frac{74+74+74}{35+22+34} = 2.43956$. And the ratio between value and volume is $\frac{74+74+74}{24+40+24} = 2.52273$.

For objects we newly picked, the average ratio between value and weight is $\frac{60+51+49+49}{26+22+22+22} = 2.27174$. And the ratio between value and volume is $\frac{60+51+49+49}{19+19+19+19} = 2.75$.

It's hard to see the pattern here, since our question is not only related to the weight but also the volume. However, the rough idea is that we prefer to pick the object contains more value but has smaller weight and volume at the same time. Let's try and see what if we put the lower bound as 15:

```
Value of objective function: 875.00000000

Actual values of the variables:
x_10                            1
x_13                            1
x_19                            1
x_24                            1
x_29                            1
x_32                            1
x_35                            1
x_38                            1
x_54                            1
x_57                            1
x_60                            1
x_73                            1
x_79                            1
x_82                            1
x_98                            1
weight                        299
volume                        299
count                          15
```

We can see from the result that we pick object 10, 13, 19, 24, 29, 32, 35, 38, 54, 57, 60, 73, 79, 82, 97 under the constraint, which are 14 objects in total. And the total weight is 299 kg and total volume of 299 liters. Compare to objects we pick at constraint of 15, now we throw out objects 4, 63, and we get new objects 13, 32, 57.

Objects We Throw Out			
object	value	weight	volume
4	60	26	19
63	66	35	24

New Objects We Pick			
object	value	weight	volume
13	35	18	18
32	25	18	18
57	34	18	18

For objects we thrown out the average ratio between value and weight is $\frac{60+66}{26+35} = 2.065557$. And the ratio between value and volume is $\frac{60+66}{19+24} = 2.93023$.

For objects we newly picked, the average ratio between value and weight is $\frac{35+25+34}{18+18+18} = 1.74074$. And the ratio between value and volume is $\frac{35+25+34}{18+18+18} = 1.74074$.

We observe that from 14 to 15 objects, the ratio between value and weight and between value and volume become smaller, which means that these newly picked objects are not preferable as old ones. However, we choose these object because we need 15 objects at this time.

Let's try and see what if we put the lower bound as 16:

```
[siyuezhu@Siyues-Air osx64 % ./lp_solve hw1_lp.txt
 This problem is infeasible
```

It shows the problem is infeasible, which means the problem cannot be solved with a lower bound of 16. With our lower bound of the number of object we need to take to the knapsack become larger and larger, the weight and volume for each object we take need to be smaller and smaller. One idea is that there has only 15 small objects in the poll that allow us to meet the lower bound of 15. However when we need to take one more object from the poll, no more "small enough" object can be taken out to meed the goal of 16, thus the problem becomes infeasible. Then, any lower bound larger or equal to 16 would make the problem infeasible.

Also, we observe that the more object we pick in total, the smaller the value is. And neither of the constraint of 300 kg nor 300 liters is binding, since all weight and volume are under the bound in all scenario.

Question3

Now, let's consider the same question as part 1 with non-unique objects, which means we can take the same object as many as we want. Define integer variables

$$x_i(1 <= i <= 100)$$

with 0 representing nonexistence and 1 representing existence. Also, we need to constrain that the total weight is less than or equal to 300 kg and the volume is less than or equal to 300 liters. Different from question 2, there is no limit on the number we pick for the total number of object.

The lp for this question is:

Maximun $57\mathrm{x}_1 + 29x_2 + ... + 54x_{99} + 27x_{100}$

subject to

$$+33x_1 + 19x_2 + ... + 34x_{99} + 35x_{100} \leq 300$$
$$+22x_1 + 41x_2 + ... + 35x_{99} + 37x_{100} \leq 300$$
$$x_1, x_2, ..., x_{99}, x_{100} \text{ are all non-negative integer}$$

In the python code, we need to change "bin" to "int", since our objects are not binary, and we can pick the same object as many as we want.

```
# Set the variable constraint.
output = "int "
for i in range(1,100):
    output = output + "x" + str(i) + ","
output = output + "x100;"
print(output)
```

And the new input file we generated is:

```
max: +57x1+29x2+...+54x99+27x100;
weight = +33x1+19x2+...+34x99+35x100;
weight <= 300;
volume = +22x1+41x2+...+35x99+37x100;
volume <= 300;
int x1,x2,...x99,x100;
```

By running the lpsolver, our result is:

```
Value of objective function: 1184.00000000

Actual values of the variables:
x_54                          16
weight                       288
volume                       288
```

The result shows that we take object 54 for 16 times, with total weight 288 kg and total weight 288 liters. And there is not constraint bind in this question. Compare with question 1, we only take one kind of object instead of taking 13 different ones. By observing the weight and the volume of each object, I realize object 54 is the one has the smallest weight with 18 kg and smallest volume with 18 liters and contains the most value compare to other small weight and small volume objects. Also, there are other objects has exactly the same weight and volume and value as object 54, such as object 10, 98. Thus, we can get the same answer if we pick 16 object 10 or 16 object 98.