math381HW6

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1 Introduction

We are going to design a card game and use Monte Carlo method to calculate the probability of winning the game using different strategies. Here is a card game, and the rules are:

• It is a game for 2 people.

• Shuffling half deck of cards and placing them in the middle. Each player takes a card in turn on the top of the deck. Ace and J are considered as 1 and 11. Q, K and joker are all considered as 12.

• After each take, each player rolls two dice and adds the two numbers together. If the sum is equal or greater than the number on the card, then the number on the card can be added to the total score of the player. Otherwise, the player cannot add the number on the card to his/her total score.

• Now, players who did not get points in the previous roll can decide whether they want to gamble. If they want to gamble, then they can roll two dice again, if the sum is equal or greater than the number on the card, then the player can still get the point. If not, then the player would deduct the number on the card from his total score. Any negative number would be considered as 0.

• The game will end after running out of cards.

2 Strategy

Below are some strategies players can follow to win the game. We calculated the winning probability interval for a player using each strategy.

(A) The player would never gamble.

(B) The player would only gamble if the current score is less than the number on the card.

(C) The player would only gamble if the number on the card is less than or equal to 6. Explanation: It is hard to say why we choose 6 as the gamble limit. We just think 6 is the middle number in the range of the card(1-12). The greater the number on the card, the more risky . For example, 10 is larger than 6. If the player loses the gamble for number 10, then he would lose 10 points. However, losing 6 points is not that risky comparing to 10. At the same time, we also want to guarantee the benefit of gambling. The number should not be too small either. Therefore, We choose 6 as our gamble limit.

(D) The player would only gamble if the current score is less than or equal to the other player's score.

Now, we are going to compare each strategy. We are going to make 10 runs of 100,000 simulations for every pair of different strategies and calculate the confidence interval to see the probabilities player 1 would win.

> **Confidence interval**: The probability of one simulation-generated probability is smaller than the true probability is $\frac{1}{2}$. And the probability of ten simulation-generated probabilities are all smaller or all greater than the true probability is $\frac{1}{2^{10}}$. And the sum of the probability of ten simulation-generated probabilities are all smaller or all greater than the true probability is $\frac{2}{2^{10}}$. Thus, the probability that the true probability is in the range (0.43411, 0.43672) is 99.8% according to:

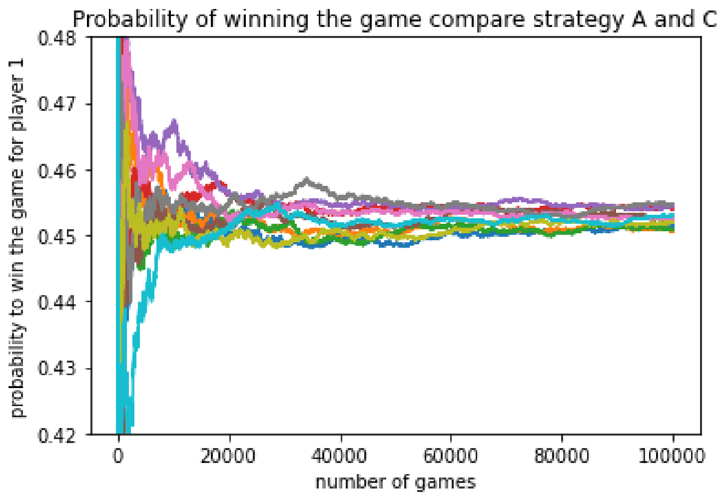
$$1 - \frac{2}{2^{10}} = 99.8\%$$

We list the confidence intervals for using different strategies in the table. Player 1 follows the strategy on the column, and player 2 follows the strategy on the row. And the number of the confidence interval is the probability for player 1 to win the game. The code for calculating the probabilities can be find on github.

Interval of winning probability				
strategy	A	B	C	D
A	(0.4974, 0.0.50048)	(0.51933, 0.52583)	(0.44933, 0.45196)	(0.53831, 0.54438)
B	(0.51933, 0.52583)	(0.49715, 0.50286)	(0.43737, 0.43433)	(0.51633, 0.52071)
C	(0.44933, 0.45196))	(0.43737, 0.43433)	(0.4975, 0.50175)	(0.5838, 0.58436)
D	(0.53831, 0.54438)	(0.51633, 0.52071)	(0.5838, 0.58436)	(0.52196, 0.52572)

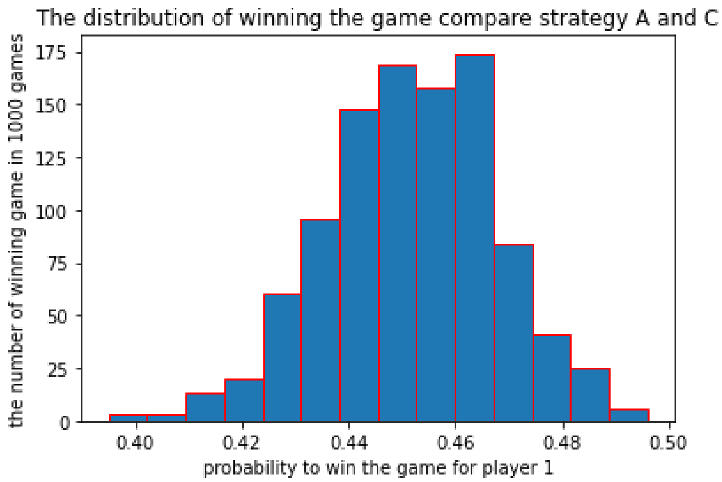
When player 1 and player 2 use the same strategy, they should have the same chance to win the game. Thus, we can see when player 1 and player 2 both use strategy A, B or C, the probability of winning rates for them are close to 50%. However, when both players use strategy D, the result show that player 1 has a higher a higher chance to win, which is around 52% percent. Strategy D says that the player would only gamble if the current score is less than or equal to the other player's score. However, player 1 is the player who drew the card first, thus player 1 has a lower chance to gamble. Thus, the probability to win the game is not 50% to 50% when both players follow strategy D.

Overall, we can observe that strategy C is the best strategy over all other strategies to win the game. Now, we are going to run simulations of 100,000 games 10 times, and compare strategy A and C. We want to see how strategy C can perform on improving the chance of winning the game.



We can see that the graph is convergent. And when the simulation number equals to 100,000, the minimum probability to win the game is 43.4113%, and the maximum number to win the game is 43.672% in 10 runs. Thus, the interval is (0.43411, 0.43672). It is obvious to see that when player 1 follows strategy A and player 2 follows strategy C, player 2 has a higher chance to win. And we can conclude strategy C is very effective on improving the winning probability..

Now, we are going to see distribution of average winning probability with 1000 runs of the above game simulation and 1000 game simulations per run:



We can see the histogram is in a bell shape, which indicates the distribution is a normal distribution. And now we can apply the central limit theorem:

- If we sample many values from an unknown distribution and average them, and do this multiple times, the averages will be approximately normally distributed around the mean of the distribution.

Since strategy C is the best strategy to win for the game, let's a take a step further and focus on strategy C in the next section.

3 Choose the best strategy

Strategy C states that the player would only gamble if the number on the card is less than or equal to 6. Now, let's change 6 to other numbers and see how it would affect the winning probability. We define strategy C_x as as the strategy that base on strategy C, and 6 would change to x. For example, C_5 is the strategy that the player would only gamble if the number on the card is less than or equal to 5.

Interval of winning probability							
strategy	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_4	(0.4983, 0.5038)	(0.48097, 0.48676)	(0.45898, 0.46278)	(0.43766, 0.4463)	(0.44762, 0.45039)	(0.48971, 0.49444)	(0.568, 0.57505)
C_5	(0.48097, 0.48676)	(0.49969, 0.5029)	(0.47442, 0.47942)	(0.45557, 0.45901)	(0.45985, 0.46581)	(0.50425, 0.50863)	(0.58111, 0.58594)
C_6	(0.45898, 0.46278)	(0.47442, 0.47942)	(0.49828, 0.5027)	(0.47753, 0.48201)	(0.48185, 0.48586)	(0.52302, 0.52866)	(0.59852, 0.6032)
C_7	(0.43766, 0.4463)	(0.45557, 0.45901)	(0.47753, 0.48201)	(0.49616, 0.50097)	(0.49922, 0.50457)	(0.53674, 0.54362)	(0.6106, 0.61495)
C_8	(0.44762, 0.45039)	(0.45985, 0.46581)	(0.48185, 0.48586)	(0.49922, 0.50457)	(0.49648, 0.50173)	(0.53585, 0.54117)	(0.60249, 0.60801)
C_9	(0.48971, 0.49444)	(0.50425, 0.50863)	(0.52302, 0.52866)	(0.53674, 0.54362)	(0.53585, 0.54117)	(0.49914, 0.50198)	(0.56576, 0.5693)
C_{10}	(0.568, 0.57505)	(0.58111, 0.58594)	(0.59852, 0.6032)	(0.6106, 0.61495)	(0.60249, 0.60801)	(0.56576, 0.5693)	(0.49547, 0.50349)

From the interval above, we can see that when player 2 always uses strategy C_9 or C_{10}, the winning probability of player 1 is higher than 50%, which means strategy C_9 and C_{10} are not very effective. Over other strategies such as, C_7 is the most efficient one. Now, let's take a look at how C_7 perform when comparing with strategy A, B and D.

Interval of winning probability				
strategy	A	B	C	D
C$_7$	(0.56614, 0.57006)	(0.57672, 0.58358)	(0.51691, 0.52192)	(0.60156, 0.60587)

We can see that C_7 has a higher winning probability over other strategies. Thus, C_7 is the best strategy.

4 Make C_7 even better

Even though C_7 is very effective on improving the winning rate, we are not put the player's current score into consideration. Now let's change C_7 a little bit, which the player can also gamble when the current score is less than or equal to a number N. A good thing about this is that the player can also gamble if it is not too risky, which means he would not lose too much points when he loss the gamble, since the number he gambles is smaller than N. Now let's define C_{MN} as the strategy described as above. N is the number that the player can gamble as long as the number on the card is less than or equal to N.

Interval of winning probability	
strategy	C$_7$
C$_{71}$	(0.51746, 0.52411)
C$_{72}$	(0.51784, 0.52324)
C$_{73}$	(0.51899, 0.52331)
C$_{74}$	(0.51713, 0.52201)

We can see that all of the lower bounds of confidence intervals comparing strategy C_{7x} and C_7 are all larger than 51%. which means our strategy C_{7x} is more efficient than C_7.

All code can be find on github: https://github.com/zhusiyue1999/math381.