1.

```r
#1
mosaic::binom.test(x=23+15+6+8,n=13+18+23+15+6+8,ci.method="Wald",alternative = "two.sided")
mosaic::binom.test(x=23+15+6+8,n=13+18+23+15+6+8,ci.method="Score",alternative = "two.sided")
mosaic::binom.test(x=23+15+6+8,n=13+18+23+15+6+8,ci.method="Plus4",alternative = "two.sided")

mosaic::prop.test(x=23+15+6+8,n=13+18+23+15+6+8,correct=F,alternative = "two.sided") #turn off continuity correction
mosaic::prop.test(x=23+15+6+8,n=13+18+23+15+6+8,alternative = "two.sided") #turn on continuity correction
```

```
        Exact binomial test (Wald CI)

data:  23 + 15 + 6 + 8 out of 83
number of successes = 52, number of trials = 83, p-value = 0.02753
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.5224389 0.7305731
sample estimates:
probability of success
              0.626506


        Exact binomial test (Score CI without continuity correction)

data:  23 + 15 + 6 + 8 out of 83
number of successes = 52, number of trials = 83, p-value = 0.02753
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.5190169 0.7228031
sample estimates:
probability of success
              0.626506


        Exact binomial test (Plus 4 CI)

data:  23 + 15 + 6 + 8 out of 83
number of successes = 52, number of trials = 83, p-value = 0.02753
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.5187312 0.7226481
sample estimates:
probability of success
              0.626506
```

```
        1-sample proportions test without continuity correction

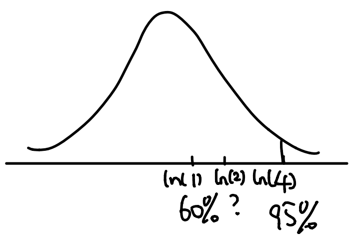
data:  + out of +23 + 15 + 6 out of 13 + 18 + 23 + 15 + 68 out of 8
X-squared = 5.3133, df = 1, p-value = 0.02116
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.5190169 0.7228031
sample estimates:
       p
0.626506


        1-sample proportions test with continuity correction

data:  + out of +23 + 15 + 6 out of 13 + 18 + 23 + 15 + 68 out of 8
X-squared = 4.8193, df = 1, p-value = 0.02814
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.5129510 0.7282365
sample estimates:
       p
0.626506
```

I find the binom.test "score" matches with prop.test without continuity.

2. We know $Y = \ln(X) \sim \text{Norm}(\mu, \sigma)$, then we can calculate μ and σ



$$\begin{cases} \dfrac{\ln(1)-\mu}{sd} = 0.253 \\ \dfrac{\ln(4)-\mu}{sd} = 1.645 \end{cases}$$

$\ln(1) - \mu = 0.253sd \Rightarrow \mu = \ln(1) - 0.253sd$

$\ln(4) - \mu = 1.645sd \Rightarrow$

$\ln(4) - \ln(1) + 0.253sd = 1.645\ sd$

$\ln(4) - \ln(1) = 1.392\ sd$

$sd = 0.996$

$\mu = \ln(1) - 0.253sd = -0.25$

pnorm $(q = \ln(2), \mu = -0.25, Sd = 0.996) = 0.828$

```
> pnorm(log(2),-0.25,0.996)
[1] 0.828164
> mosaic::binom.test(x=95,n=150,p=0.828,alternative="less")


data:  95 out of 150
number of successes = 95, number of trials = 150, p-value = 9.804e-09
alternative hypothesis: true probability of success is less than 0.828
95 percent confidence interval:
 0.0000000 0.6989511
sample estimates:
probability of success
             0.6333333
```

Let π be percentage rats die from dose = 2.

$H_0: \pi \geq 0.828$ $H_1: \pi < 0.828$

We can see p-value is < 0.05 so we reject H_0. So we conclude that manufacturers have overestimated the effect of the poison.

3. We know $\frac{|x-n\pi_0|}{\sqrt{n\pi_0(1-\pi_0)}} \le z_{\frac{\alpha}{2}}$ and square it on both sides.

$$\frac{(x-n\pi_0)^2}{n\pi_0(1-\pi_0)} \le z_{\frac{\alpha}{2}}^2$$

$$-4x^2(n^2+z_{\frac{\alpha}{2}}^2 n)$$

$$\frac{x^2-2xn\pi_0+n^2\pi_0^2}{n\pi_0(1-\pi_0)} \le z_{\frac{\alpha}{2}}^2$$

$$x^2-2xn\pi_0+n^2\pi_0^2 \le z_{\frac{\alpha}{2}}^2 n\pi_0(1-\pi_0)$$

$$x^2-2xn\pi_0+n^2\pi_0^2 \le z_{\frac{\alpha}{2}}^2 n\pi_0 - z_{\frac{\alpha}{2}}^2 n\pi_0^2$$

$$(n^2+z_{\frac{\alpha}{2}}^2 n)\pi_0^2 + (-2xn - z_{\frac{\alpha}{2}}^2 n)\pi_0 + x^2 \le 0$$

$$\pi_0 = \frac{2xn + z_{\frac{\alpha}{2}}^2 n \pm \sqrt{\cancel{4x^2n^2} + z_{\frac{\alpha}{2}}^4 n^2 + 4xn^2 z_{\frac{\alpha}{2}}^2 - \cancel{4x^2n^2} - 4x^2 z_{\frac{\alpha}{2}}^2 n}}{2(n^2+z_{\frac{\alpha}{2}}^2 n)}$$

$$= \frac{2n^2\hat{\pi} + z_{\frac{\alpha}{2}}^2 n \pm \sqrt{z_{\frac{\alpha}{2}}^2 \cdot 4n^4 \left(\frac{z_{\frac{\alpha}{2}}^2}{4n^2} + \frac{x}{n^2} - \frac{x^2}{n^3}\right)}}{2n^2\left(1+\frac{z_{\frac{\alpha}{2}}^2}{n}\right)}$$

$$= \frac{2n^2\hat{\pi} + 2n^2\cdot\frac{z_{\frac{\alpha}{2}}^2}{2n} \pm z_{\frac{\alpha}{2}}\cdot 2n^2\sqrt{\frac{z_{\frac{\alpha}{2}}^2}{4n^2}+\frac{n\hat{\pi}}{n^2}-\frac{n^2\hat{\pi}^2}{n^3}}}{2n^2\left(1+\frac{z_{\frac{\alpha}{2}}^2}{n}\right)}$$

$$= \frac{\hat{\pi} + \frac{z_{\frac{\alpha}{2}}^2}{2n} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}+\frac{z_{\frac{\alpha}{2}}^2}{4n^2}}}{1+\frac{z_{\frac{\alpha}{2}}^2}{n}}$$

4.

```
```{r}
#4
library(dplyr)
data <- read.csv("gss16.csv")
data <- data %>% filter(wrkstat=="Working fulltime" | wrkstat=="Working parttime") %>%
  mutate(snp_insta=case_when(snapchat=="Yes" | instagrm=="Yes" ~ "Yes",
    snapchat=="No" & instagrm=="No" ~ "No"))
mosaic::binom.test(x=nrow(filter(data,snp_insta=="Yes")),n=nrow(filter(data,snp_insta=="Yes")) +
                     nrow(filter(data,snp_insta=="No")),ci.method="Plus4")
```
```

```
	Exact binomial test (Plus 4 CI)

data:  nrow(filter(data, snp_insta == "Yes")) out of 905L
number of successes = 354, number of trials = 905, p-value = 6.129e-11
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.3599077 0.4233706
sample estimates:
probability of success
             0.3911602
```

I choose to use "Plus4" because it's the best compromise in those ci.method. And we can see that 0.5 is not contained in the confidence interval. So we can conclude that there are less than 50% of people use snapchat or instagram at work