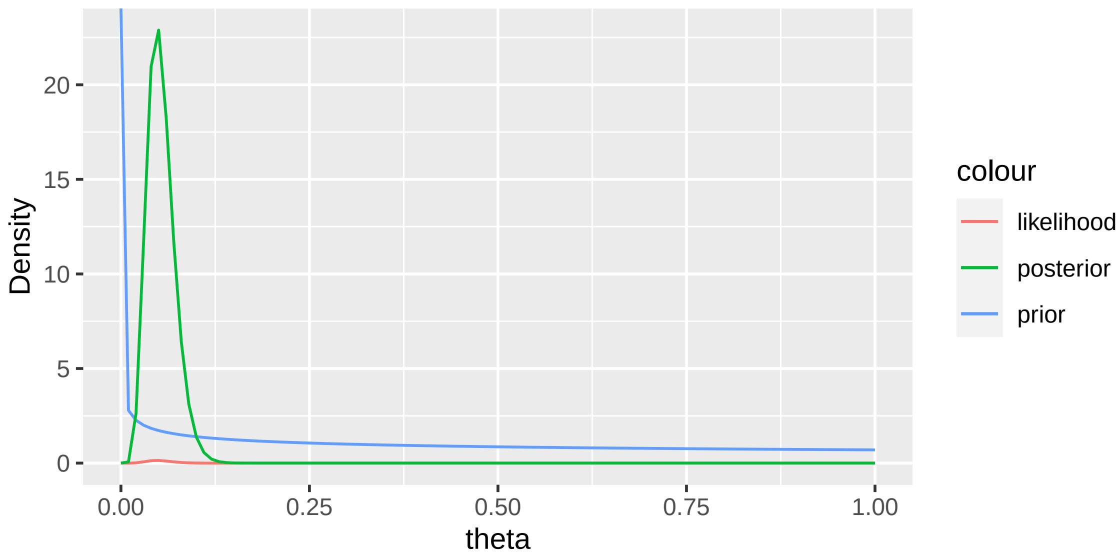
Bayes



Here is how the data looks like:

	No. of cases	No. of participants
BNT162b2	8	21720
Placebo	162	21728
Total	170	43448

Let $W = X \mid X+Y$ represents the the number of Covid-19 cases within vaccination group out of the total number of Covid-19 cases $X+Y$, and it follows the distribution of $\text{Binom}(n, \theta = \frac{n_1\pi_1}{n_1\pi_1+n_2\pi_2})$. Since $n_1 \approx n_2$, we approximate $\theta = \frac{\pi_1}{\pi_1+\pi_2}$.

Therefore, we have $W \sim \text{Binom}(n, theta = \frac{\pi_1}{\pi_1+\pi_2})$, where we assume $g(\theta) \sim \text{Beta}(\alpha = 0.7000102, \beta = 1)$. According to Beta Binomial, we have posterior $h(\pi \mid x) \sim \text{Beta}(\alpha + x = 8.7000102, n - x + \beta = 163)$.

Back to the relationship between efficacy and θ, we have:

$$\theta = \frac{\pi_1}{\pi_1+\pi_2} = \frac{\frac{\pi_1}{\pi_2}}{\frac{\pi_1}{\pi_2}+\frac{\pi_2}{\pi_2}} = \frac{1-\psi}{1-\psi+1} = \frac{1-\psi}{2-\psi} \quad (1)$$

We also get $\psi = \frac{1-2\theta}{1-\theta}$ from equation (1).

We know the $\hat{\psi} = \frac{1-2\hat{\theta}}{1-\hat{\theta}}$, and therefore we get the estimated efficacy:

```
theta_hat <- 8/(8+162)
efficacy <- (1-2*theta_hat)/(1-theta_hat)
efficacy
```

```
## [1] 0.9506173
```

Let ψ_1 and ψ_2 be the unknown 2.5th percentile and 97.5th percentile, respectively. Then a 95% credible interval for ψ is:

$$P(\psi_1 \leq \psi \leq \psi_2) = 0.95$$
$$P(\psi_1 \leq \frac{1-2\theta}{1-\theta} \leq \psi_2) = 0.95$$
$$P(\frac{\psi_2 - 1}{\psi_2 - 2} \leq theta \leq \frac{\psi_1 - 1}{\psi_1 - 2}) = 0.95$$

We have the 95% credible interval for θ:

```
ci <- hdi(qbeta, credMass=0.95, shape1=8.7000102, shape2=163)
ci[1]
```

```
##      lower
## 0.02055199
```

```
ci[2]
```

```
##      upper
## 0.08389353
```

We derive two equations:

$$\frac{\psi_1 - 1}{\psi_1 - 2} = 0.08389353$$
$$\frac{\psi_2 - 1}{\psi_2 - 2} = 0.02055199$$

```
q1 <- uniroot(function(x){(x-1)/(x-2)-0.08389353}, lower=0, upper=1)$root
q2 <- uniroot(function(x){(x-1)/(x-2)-0.02055199}, lower=0, upper=1)$root
```

We solved these two equations and get $\psi_1 = 0.9084002$ and $\psi_2 = 0.9790169$. Thus, The 95% credible interval for ψ is $[0.9084002, 0.9790169]$.

Now, we want to test whether the efficacy is above 0.3.

$$\begin{aligned} P(\psi \geq 0.3 \mid W) &= P(\frac{1-2\theta}{1-\theta} > 0.3 \mid W) \\ &= P(1 - 2\theta > 0.3(1-\theta) \mid W) \\ &= P(1.7\theta < 0.7 \mid W) \\ &= P(\theta < 0.4116 \mid W) \end{aligned}$$

```
pbeta(0.4116, 8.7000102, 163)
```

```
## [1] 1
```

Since the p-value is much greater than the conventional significance level, we conclude that efficacy is above 0.3.