4.58. The Endurance data set contains data from a 1983 paper [KM83] testing the effects of vitamin C on grip strength and muscular endurance. This data set is also available from OzDASL (http://www.statsci.org/data), where the following description is given.

> The effect of a single 600 mg dose of ascorbic acid versus a sugar placebo on the muscular endurance (as measured by repetitive grip strength trials) of fifteen male volunteers (19–23 years old) was evaluated. The study was conducted in a double-blind manner with crossover.
>
> Three initial maximal contractions were performed for each subject, with the greatest value indicating maximal grip strength. Muscular endurance was measured by having the subjects squeeze the dynamometer, hold the contraction for three seconds, and repeat continuously until a value of 50% maximum grip strength was achieved for three consecutive contractions. Endurance time was defined as the number of repetitions required to go from maximum grip strength to the initial 50% value. Subjects were given frequent positive verbal encouragement in an effort to have them complete as many repetitions as possible.

a) Use a paired t-test to compare the two treatments. Is there a significant difference?

b) Now repeat the analysis, but first take the logarithm of each endurance measurement before doing the analysis.

c) Since the difference of logarithms is the logarithm of the quotient, repeat the analysis using the quotient of the two measurements in place of the difference.

d) Let's try one more transformation. This time use the difference between the reciprocals of the endurance measurements.

e) Now analyze these data using the sign test.

f) We have several different analyses, each with a different p-value. How do we decide which is the "right" analysis?

```{r}
endurance <- read_csv("Endurance.csv") %>%
  select(-Order) %>%
  pivot_wider(names_from=Treatment,values_from=Endurance) %>%
  mutate(diff=Vitamin-Placebo)
t.test(~diff, data=endurance, mu=0, alternative="two.sided")

endurance <- endurance %>% mutate(diff_log=log(Vitamin)-log(Placebo))
t.test(~diff_log, data=endurance, mu=0, alternative="two.sided")

endurance <- endurance %>% mutate(diff_quotient=Vitamin/Placebo)
t.test(~diff_quotient, data=endurance, mu=1, alternative="two.sided")

endurance <- endurance %>% mutate(diff_reciprocal=1/Vitamin-1/Placebo)
t.test(~diff_reciprocal, data=endurance, mu=0, alternative="two.sided")

endurance <- endurance %>% mutate(bigger=Vitamin > Placebo)
mosaic::binom.test(~bigger,alternative="two.sided",p=0.5,data=endurance)

```

```
	One Sample t-test

data:  diff
t = -0.78538, df = 14, p-value = 0.4453
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -180.82308   83.88975
sample estimates:
mean of x
-48.46667


	One Sample t-test

data:  diff_log
t = -1.8968, df = 14, p-value = 0.07868
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.56304724  0.03455067
sample estimates:
 mean of x
-0.2642483


	One Sample t-test

data:  diff_quotient
t = -0.95832, df = 14, p-value = 0.3542
alternative hypothesis: true mean is not equal to 1
95 percent confidence interval:
 0.6104985 1.1489252
sample estimates:
mean of x
0.8797119
```

```
	One Sample t-test

data:  diff_reciprocal
t = 2.4111, df = 14, p-value = 0.03022
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.0001377454 0.0023561868
sample estimates:
  mean of x
0.001246966



data:  endurance$bigger  [with success = TRUE]
number of successes = 4, number of trials = 15, p-value = 0.1185
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.07787155 0.55100324
sample estimates:
probability of success
             0.2666667
```

(a) We can see that the p-value $=0.44$, which is greater than 0.05. So it's not significant. So we conclude there is no significant difference between vitamin C and placebo.

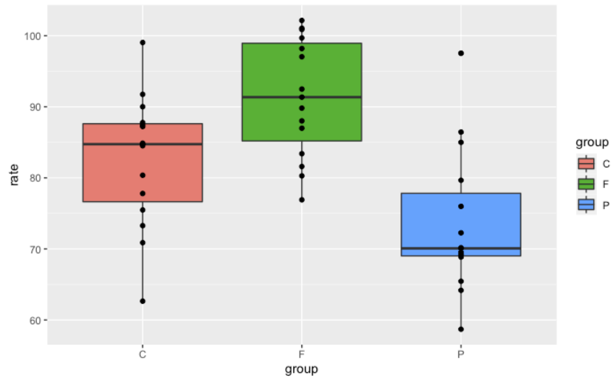
(b) We get p-value $= 0.07868$, and it's greater than 0.05. So we conclude there is no significant difference between vitamin C and placebo.

(c) We get p-value $= 0.3542$, which is greater than 0.05. So we conclude that there is no significant difference between vitamin C and placebo.

(d) We get p-value $= 0.03022$, which is smaller than 0.05. So we reject H_0. We conclude that according to reciprocal of endurance, there is effect of vitamin C and placebo on endurance.

(e) We get p-value $= 0.1185$, which is greater than 0.05. So we conclude there is no significant difference between vitamin C and placebo.

(f) In all 5 tests, the quotient one has the smallest p-value and it's smaller than 0.05. I think we can decide the rightness by the value of p-value. The smaller the p-value is, the more significant the analysis is.



group <fctr>	n <int>	xbar <dbl>	s2 <dbl>	sd <dbl>
C	15	82.52407	85.40670	9.241575
F	15	91.32513	69.57452	8.341134
P	15	73.48307	99.39732	9.969820

3 rows

2.(a) We can see from the boxplot, that people in group F has the highest heart rates and people in group P has the smallest heart rates. So we guess that people seem to have a higher heart rates when their friend in the room, and lower heart rate when their dog is in the room. We can see that the $\bar{x}$ for group F has mean = 91.33, group C has mean = 82.52 and group P has mean = 73.48

(b)

```{r}
library(fastR2)
pet <- PetStress
ggplot(data=PetStress,mapping=aes(x=group,y=rate)) +
  geom_boxplot(mapping=aes(fill=group)) +
  geom_point()

pet %>% group_by(group) %>% summarize(n=n(), xbar=mean(rate), s2=var(rate), sd=sqrt(s2))
pet_PF <- pet %>% filter(group!="C")
pet_CF <- pet %>% filter(group!="P")
pet_CP <- pet %>% filter(group!="F")

mosaic::t.test(rate~group,data=pet_PF,conf.level=0.85)
mosaic::t.test(rate~group,data=pet_CF,conf.level=0.85)
mosaic::t.test(rate~group,data=pet_CP,conf.level=0.85)

```

PF

```
        Welch Two Sample t-test

data:  rate by group
t = 5.316, df = 27.154, p-value = 1.282e-05
alternative hypothesis: true difference in means is not equal to 0
85 percent confidence interval:
 12.86988 22.81425
sample estimates:
mean in group F mean in group P
       91.32513        73.48307
```

CF

```
        Welch Two Sample t-test

data:  rate by group
t = -2.7381, df = 27.711, p-value = 0.01067
alternative hypothesis: true difference in means is not equal to 0
85 percent confidence interval:
 -13.560175  -4.041958
sample estimates:
mean in group C mean in group F
       82.52407        91.32513
```

CP

```
        Welch Two Sample t-test

data:  rate by group
t = 2.5758, df = 27.84, p-value = 0.01561
alternative hypothesis: true difference in means is not equal to 0
85 percent confidence interval:
  3.844821 14.237179
sample estimates:
mean in group C mean in group P
       82.52407        73.48307
```

The first t-test compares group P and F. The P-value = 0 is not in the confidence interval, so we reject the null hypothesis that dogs and friends would provide the same influence to the heart rate.

The second t-test compares group C and F. The p-value = 0.01067 is not in the confidence interval, so we reject the null hypothesis that the heart rate when performing a skillful task alone is the same when performing a skillful task with a good friend in the room.

The second t-test compares group C and P. The p-value = 0.01561 is not in the confidence interval, so we reject the null hypothesis that the heart rate when performing a skillful task alone is the same when performing a skillful task with a dog in the room.

3. (a) $P(U<u) = P(\lambda S < u) = P(S \leq \frac{u}{\lambda})$

$$= \int_0^{\frac{u}{\lambda}} \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x} \, dx$$

$$P(U=u) = \frac{d}{du} P(U \leq u) = \frac{\lambda^n}{\Gamma(n)} \left(\frac{u}{\lambda}\right)^{n-1} e^{-u} \frac{1}{\lambda} = \frac{u^{n-1}}{\Gamma(n)} e^{-u}$$

(b) $q_1 = \text{qgamma}(\frac{\alpha}{2}, n, 1)$

$q_2 = \text{qgamma}(1-\frac{\alpha}{2}, n, 1)$

$q_1 < u < q_2$

$q_1 < \lambda s < q_2$

$\frac{q_1}{s} < \lambda < \frac{q_2}{s}$

$\frac{s}{q_2} < \frac{1}{\lambda} < \frac{s}{q_1}$

So, confidence interval is $[\frac{s}{q_2}, \frac{s}{q_1}]$

(c)

```
```{r}
expci <- function(x, conf.level=0.95){
n=length(x)
xbar=mean(x)
alpha=1-conf.level
q1=qgamma(alpha/2,n,1)
q2=qgamma(1-alpha/2,n,1)
interval=c(s/q2,s/q1)
return(list(conf.int=interval, estimate=s/n)) }

CIsim(n=5,samples=5000,rdist=rexp, args=list(rate=1/10),estimand=10, method=expci)
```
```

```
Interval coverage:
   cover
n   Low Yes High
  5   0   1    0
```

We can see that the confidence intervals work very well in terms of their coverage probability.