1. (a)

````
```{r}
#1(a)
set.seed(5172013)
x <- c(0.10, 0.33, 0.67, 1.00, 1.40, 1.70, 1.97, 2.2, 2.5)
pnorm(x)-0.5

result <- rep(NA,9)
for(i in 1:9) {
  xi <- x[i]
  t <- runif(10000,0,1)
  J=function(t) {
    return(xi*exp(-(xi*t)^2/2)/sqrt(2*pi))
  }
  eva_uni <- J(t)
  result[i] <- mean(eva_uni)
}
result
```
````

```
[1] 0.03982784 0.12930002 0.24857110 0.34134475 0.41924334 0.45543454 0.47558081 0.48609655 0.49379033
[1] 0.03982797 0.12928128 0.24834516 0.34212371 0.41846701 0.45760454 0.47718328 0.48162806 0.49608007
```

(b)

````
```{r}
#1(b)
unifSample <- c(runif(10000,0,1)[1:5000],1-runif(5000,0,1))
result_2 <- rep(NA,9)
for(i in 1:9) {
  xi <- x[i]
  J=function(t) {
    return(xi*exp(-(xi*t)^2/2)/sqrt(2*pi))
  }
  t <- unifSample
  eva_uni <- J(t)
  result_2[i] <- mean(eva_uni)
}
result_2
```
````

```
[1] 0.03982743 0.12928562 0.24846105 0.34102928 0.41857793 0.45451773 0.47451304 0.48498963 0.49278080
```

(c)

````
```{r}
#1(c)
N <- 1000
xi <- 1.97
result <- rep(NA,1000)
result_2 <- rep(NA,1000)
for(i in 1:1000) {
  J=function(t) {
    return(xi*exp(-(xi*t)^2/2)/sqrt(2*pi))
  }
  t1 <- runif(1000,0,1)
  eva_uni <- J(t1)
  result[i] <- mean(eva_uni)
  t2 <- c(t1[1:500],1-t1[1:500])
  eva <- J(t2)
  result_2[i] <- mean(eva)
}
sd(result)
sd(result_2)
```
````

```
[1] 0.006933658
[1] 0.0004294857
```

We can see the sd of the second set is much smaller than the first set.

2. (a) $f(x) = \dfrac{e^x}{(1+e^x)^2}$

For $x>0$, $f_1(x) = \dfrac{e^x}{(1+e^x)^2}$

For $x<0$, $f_2(x) = \dfrac{e^{-x}}{(1+e^{-x})^2} = \dfrac{\frac{1}{e^x}}{(1+\frac{1}{e^x})^2} = \dfrac{\frac{1}{e^x}}{1+\frac{2}{e^x}+\frac{1}{e^{2x}}} = \dfrac{e^x}{e^{2x}+2e^x+1} = \dfrac{e^x}{(1+e^x)^2}$

Thus $f_1(x) = f_2(x)$, so $f(x)$ is symmetric about 0.

$F(x) = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \dfrac{e^x}{(1+e^x)^2}\,dx \qquad u = 1+e^x$

$= \int \dfrac{1}{u^2}\,du = \dfrac{u^{-2+1}}{-2+1} = \dfrac{(1+e^x)^{-1}}{-1} = \dfrac{e^x}{1+e^x}$

(b) $F(x) = \dfrac{e^x}{1+e^x} = u$

$e^x = u + ue^x$

$(e^x - ue^x) = u$

$e^x(1-u) = u$

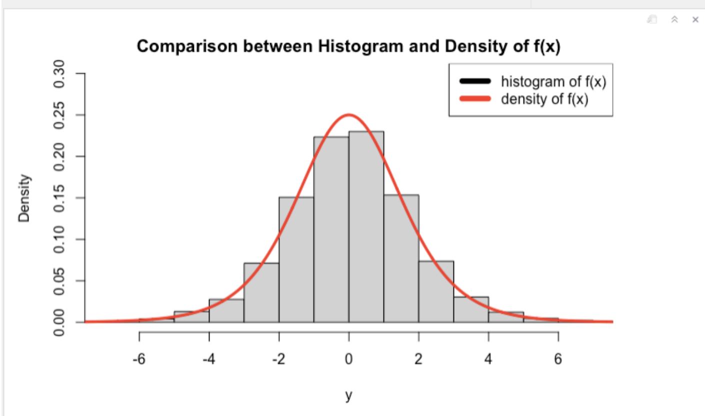
$e^x = \dfrac{u}{1-u}$

$x = \log\left(\dfrac{u}{1-u}\right) \quad$ for $0<u<1$

(c)

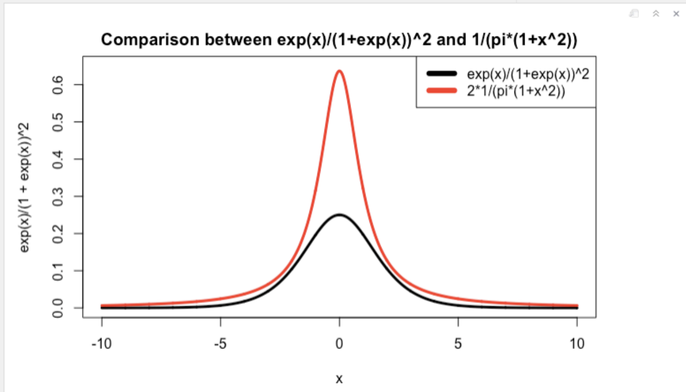
```{r}
#2(c)
fx <- function(x) {
  return(exp(x)/(1+exp(x))^2)
}

x <-  runif(10000)
y <- log(x/(1-x))
hist(y,freq=FALSE,xlim=c(-7,7),ylim=c(0,0.3),main="Comparison between Histogram and Density of f(x)")
x1 <- seq(-10,10,0.01)
lines(x=x1,y=exp(x1)/(1+exp(x1))^2,col="red",lwd=3)
legend("topright",c("histogram of f(x)","density of f(x)"),col=c("black","red"),lwd=6)
```



3. (a)

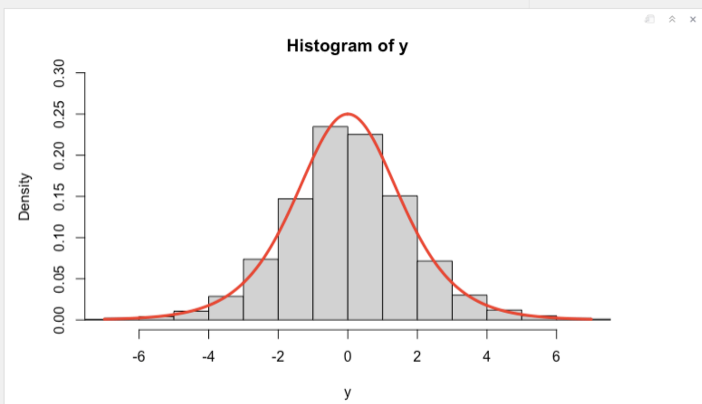
````
```{r}
#3(a)
fx <- function(x) {
  return(exp(x)/(1+exp(x))^2)
}
fc <- function(x) {
  return(1/(pi*(1+x^2)))
}
x=seq(-10,10,0.01)
plot(x=x,y=exp(x)/(1+exp(x))^2,xlim=c(-10,10),ylim=c(0,0.65),type="l",lwd=3,main="Comparison between exp(x)/(1+exp(x))^2 and 1/(pi*(1+x^2))")
lines(x=seq(-10,10,0.01),y=2*1/(pi*(1+x^2)),type="l",lwd=3,col="red")
legend("topright",c("exp(x)/(1+exp(x))^2","2*1/(pi*(1+x^2))"),col=c("black","red"),lwd=6)
```
````



(b) (c)

````
```{r}
#3(b)(c)
fx <- function(x) {
  return(exp(x)/(1+exp(x))^2)
}
cauchy <- function(x) {
  return(1/(pi*(1+x^2)))
}
N <- 20000
p = rcauchy(N)
u <- runif(N)
y <- p[which(u<fx(p)/(2*cauchy(p)))]
hist(y,freq=FALSE,xlim=c(-7,7),ylim=c(0,0.3))
x<-seq(-7,7,0.01)
lines(x=x,y=exp(x)/(1+exp(x))^2,type="l",lwd=3,col="red")

```
````



4.

```r
#4
M <- matrix(nrow=10000,ncol=40)
lamda_1 <- rep(NA,10000)
lamda_2 <- rep(NA,10000)

for(i in 1:10000) {
  M[i,] <- rexp(40,2.7)
}
for(i in 1:10000) {
  x <- M[i,]
  lamda_1[i] <- 40/sum(x)
  lamda_2[i] <- sqrt(40-1)/sqrt(sum((x-mean(x))^2))
}
#bias
bias_1 <- mean(lamda_1) - 2.7
bias_2 <- mean(lamda_2) - 2.7
mcerror_bias_1 <- sd(lamda_1-2.7)/sqrt(10000)
mcerror_bias_2 <- sd(lamda_2-2.7)/sqrt(10000)
print("Bias for Lamda_1 and Lamda_2 and the confidence interval")
bias_1
bias_2
c(bias_1 - 1.96*mcerror_bias_1, bias_1 + 1.96*mcerror_bias_1)
c(bias_2 - 1.96*mcerror_bias_2, bias_2 + 1.96*mcerror_bias_2)

#variance
var_1 <- var(lamda_1)
var_2 <- var(lamda_2)
mcerror_var_1 <- sd((lamda_1-mean(lamda_1))^2)/sqrt(10000)
mcerror_var_2 <- sd((lamda_2-mean(lamda_2))^2)/sqrt(10000)
print("Variance for Lamda_1 and Lamda_2 and the confidence interval")
var_1
var_2
c(var_1 - 1.96*mcerror_var_1, var_1 + 1.96*mcerror_var_1)
c(var_2 - 1.96*mcerror_var_2, var_2 + 1.96*mcerror_var_2)

#mse
mse_1 <- mean((2.7-lamda_1)^2)
mse_2 <- mean((2.7-lamda_2)^2)
mcerror_mse_1 <- sd((lamda_1-2.7)^2)/sqrt(10000)
mcerror_mse_2 <- sd((lamda_2-2.7)^2)/sqrt(10000)
print("MSE for Lamda_1 and Lamda_2 and the confidence interval")
mse_1
mse_2
c(mse_1 - 1.96*mcerror_mse_1, mse_1 + 1.96*mcerror_mse_1)
c(mse_2 - 1.96*mcerror_mse_2, mse_2 + 1.96*mcerror_mse_2)
```

```
[1] "Bias for Lamda_1 and Lamda_2 and the confidence interval"
[1] 0.06644513
[1] 0.1888543
[1] 0.05774702 0.07514325
[1] 0.1767543 0.2009544
[1] "Variance for Lamda_1 and Lamda_2 and the confidence interval"
[1] 0.1969418
[1] 0.3811187
[1] 0.1903286 0.2035551
[1] 0.3685748 0.3936625
[1] "MSE for Lamda_1 and Lamda_2 and the confidence interval"
[1] 0.2013371
[1] 0.4167465
[1] 0.1941888 0.2084854
[1] 0.4017871 0.4317059
```