MEC HW1

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Question 1

a) The system is not stable. The eigenvalues of the system are:

$$7.669, -0.3345 + 0.1361i, -0.3343 - 0.1361i.$$

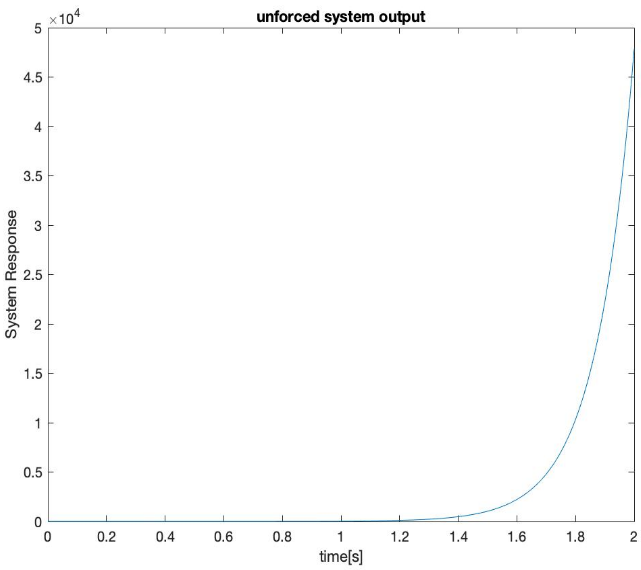
As we can see one of the eigenvalues has a positive real part.
According to the stability test, that would result in an unstable system.

b) The system is controllable

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 5 & 7 \end{bmatrix} \; B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$Wc = [B|AB|AAB] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 7 \end{bmatrix}$$

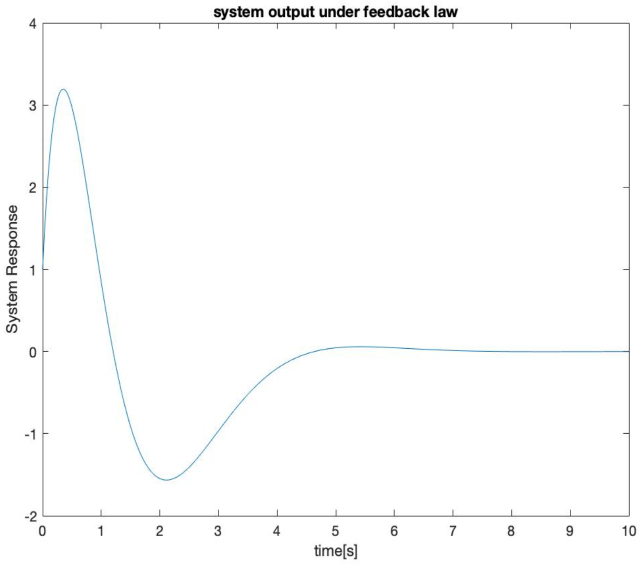
$det(W_c) = -1 \neq 0$ Thus the matrix is invertible and according to the controllability test, the system is controllable.

c)



d) $K = [11, 60, 88]$

e)


system output under feedback law
System Response
time[s]
4
3
2
1
0
-1
-2
0
1
2
3
4
5
6
7
8
9
10

Question 2

a)

$$\dot{X} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{\alpha}{\gamma\alpha-\beta^2*cos^2(x_2)}(F+\frac{\beta}{\alpha}Dsin(x_2)cos(x_2)-\beta x_4^2 sin(x_2)-\mu x_3) \\ \frac{\beta cos(x_2)}{\gamma\alpha-\beta^2*cos^2(x_2)}(F+\frac{\beta}{\alpha}Dsin(x_2)cos(x_2)-\beta x_4^2 sin(x_2)-\mu x_3)+\frac{Dsin(x_2)}{\alpha} \end{bmatrix}$$

b)

$$\text{Solve for } \dot{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

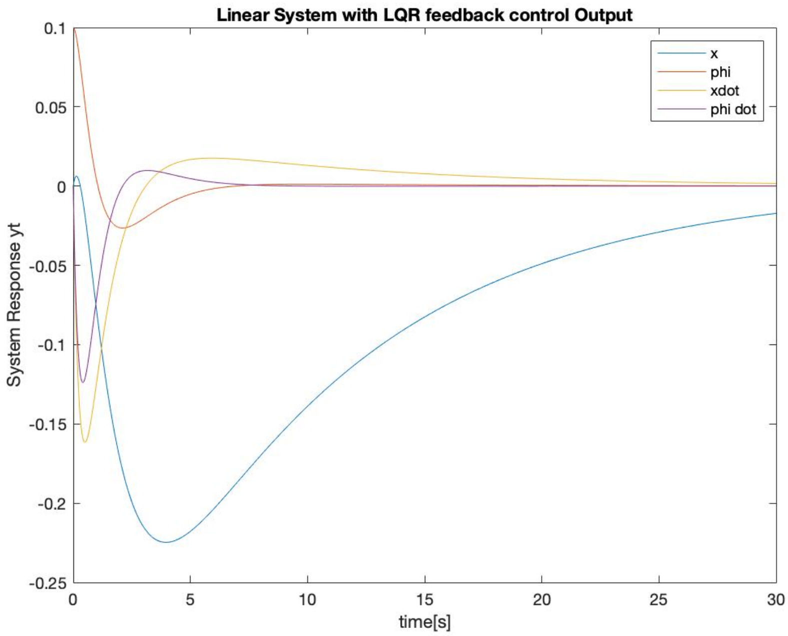
We can see that at equilibrium points, the following conditions must be satisfied$x_1 \in R, x_3 = 0, x_4 = 0, x2 = 0$ or π

c)The linearized system is not stable around the equilibrium point. The eigenvalues of the system are:

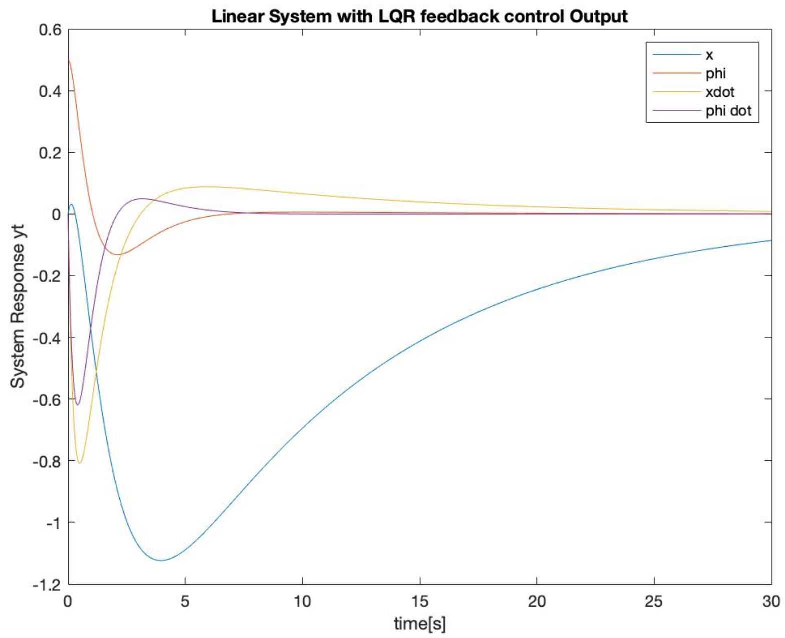
$$-3.3301, 1.1284, -0.7984.$$

As we can see one of the eigenvalues has a positive real part.
According to the stability test, that would result in an unstable system.
We can say the same thing about the nonlinear system around the equilibrium point since the linearized system is an approximation of the nonlinear system around the equilibrium point. However, we cannot generalized about the stability of the nonlinear system at points that are farther away from the equilibrium point.

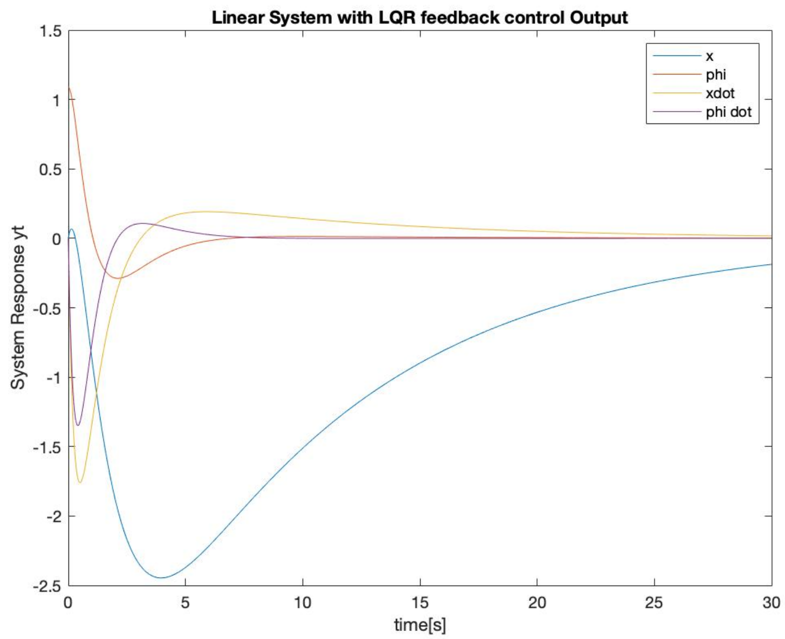
d) **Linear State Equations Plotting**
Initial Condition $x_0 = [0, 0.1, 0, 0]^T$



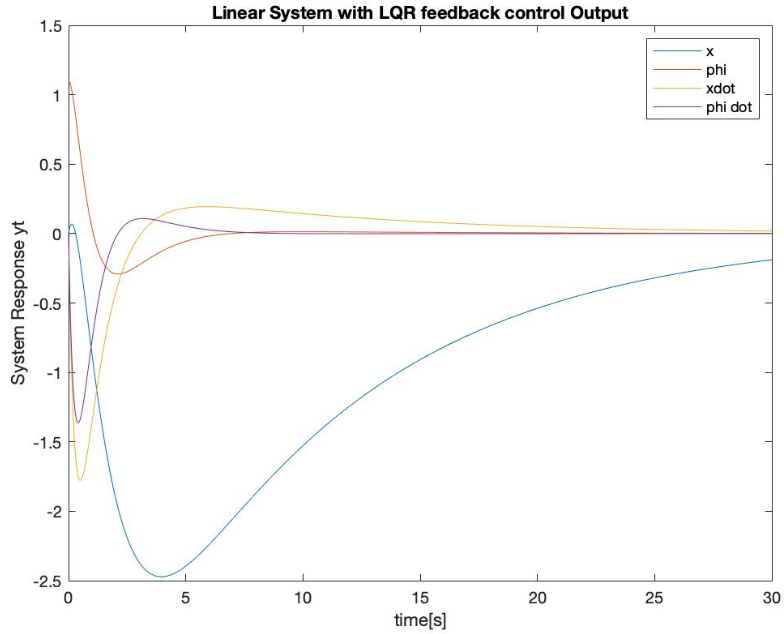
Initial Condition $x_0 = [0, 0.5, 0, 0]^T$



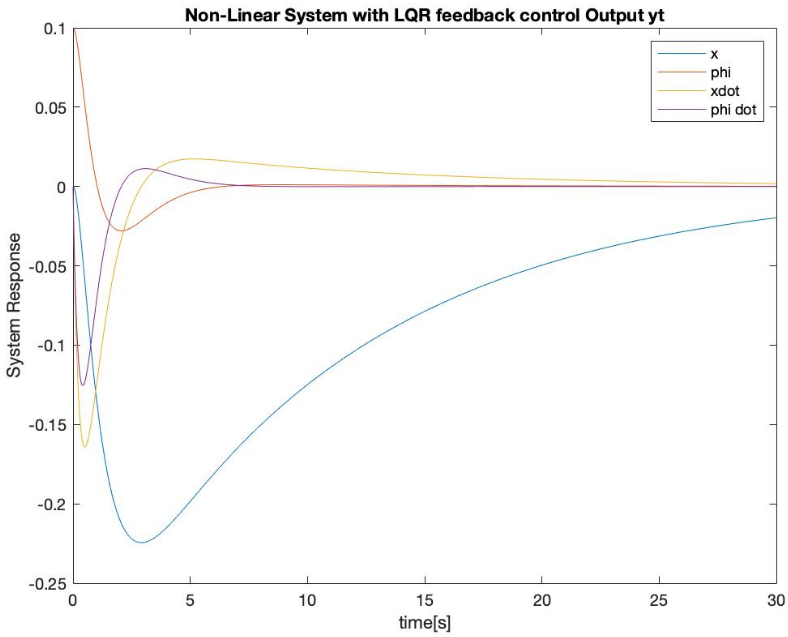
Initial Condition $x_0 = [0, 1.0886, 0, 0]^T$



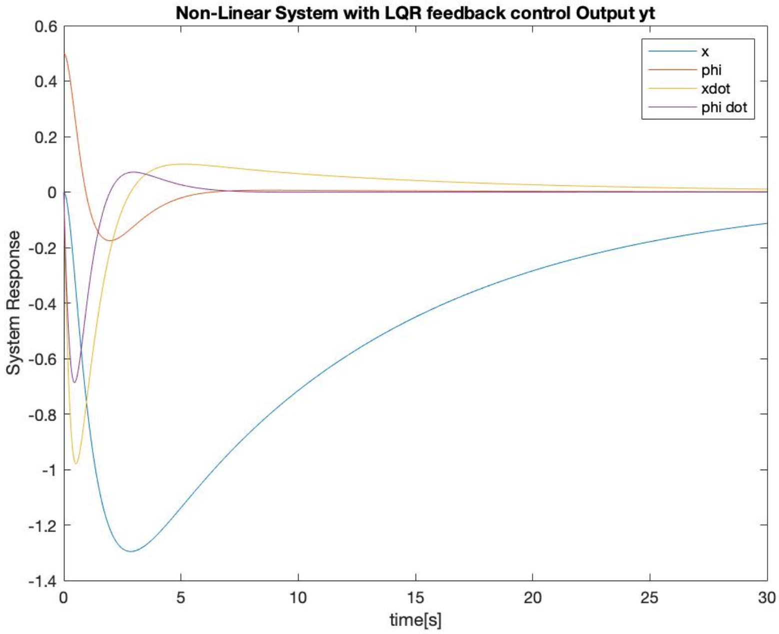
Initial Condition $x_0 = [0, 1.1, 0, 0]^T$



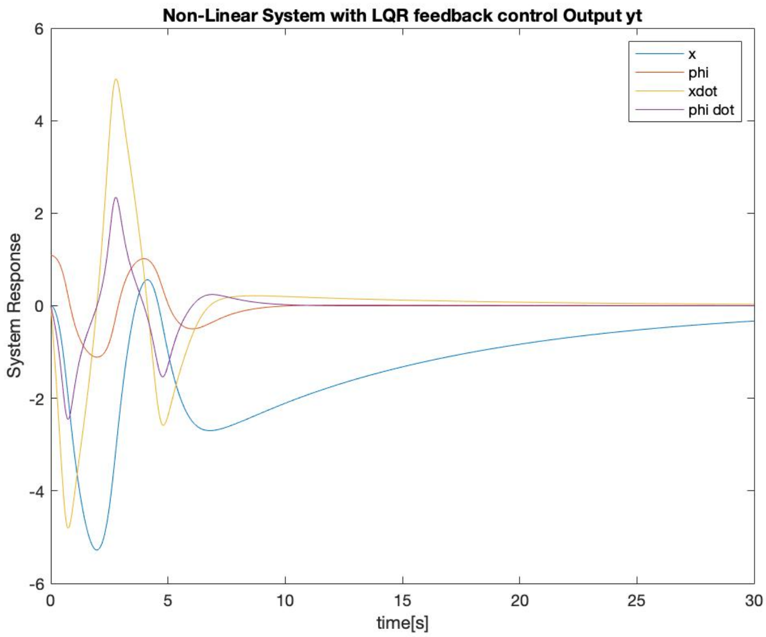
e) **Nonlinear State Equations Plotting**
Initial Condition $x_0 = [0, 0.1, 0, 0]^T$



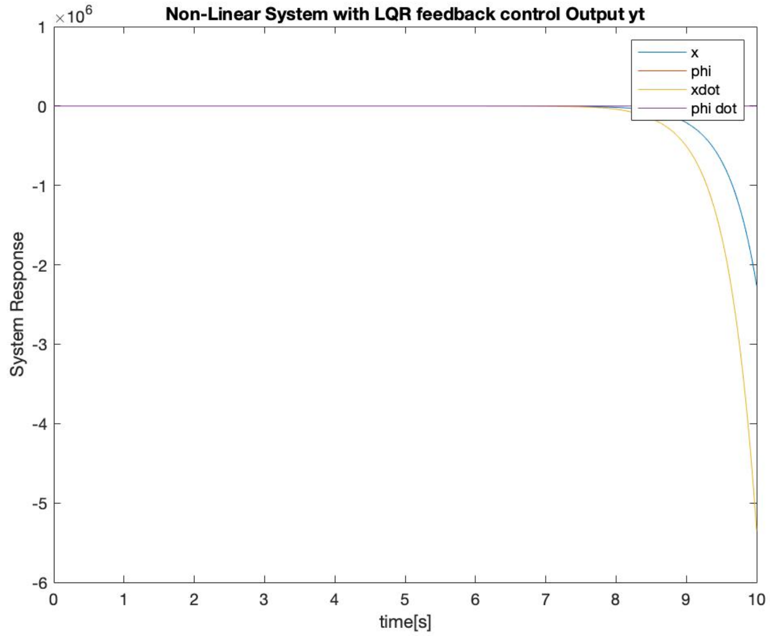
Initial Condition $x_0 = [0, 0.5, 0, 0]^T$



Initial Condition $x_0 = [0, 1.0886, 0, 0]^T$



Initial Condition $x_0 = [0, 1.11, 0, 0]^T$

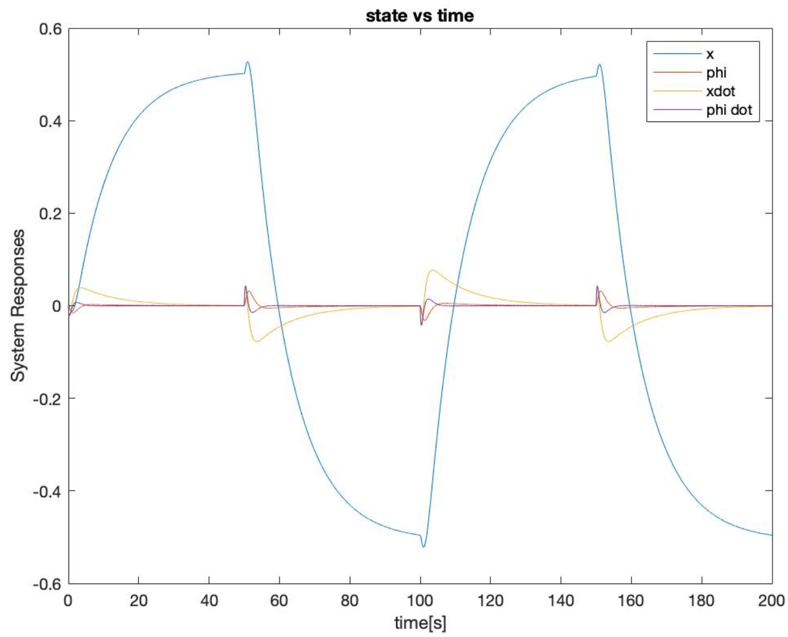


Observation and Explanation: For the first two initial conditions, the output plots of the linear system is very similar to that of the non-linear system. That is because the linear system is approximated around the equilibrium point and the first two initial conditions are relatively close to the equilibrium point. We can see that the third plot of the nonlinear system is diverging from that of the linear system and the fourth plots are completely different. That is because the initial condition has diverged significantly from the equilibrium point that the linear system is no longer a accurate approximation of the nonlinear system.

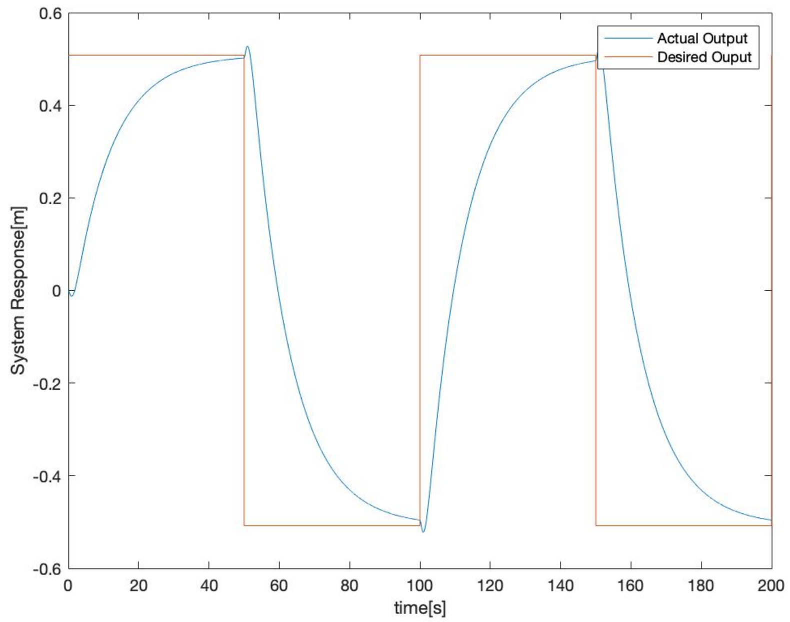
f) C = [1 0 0 0]

g)

State Vs Time



Actual Output Vs Desired Output

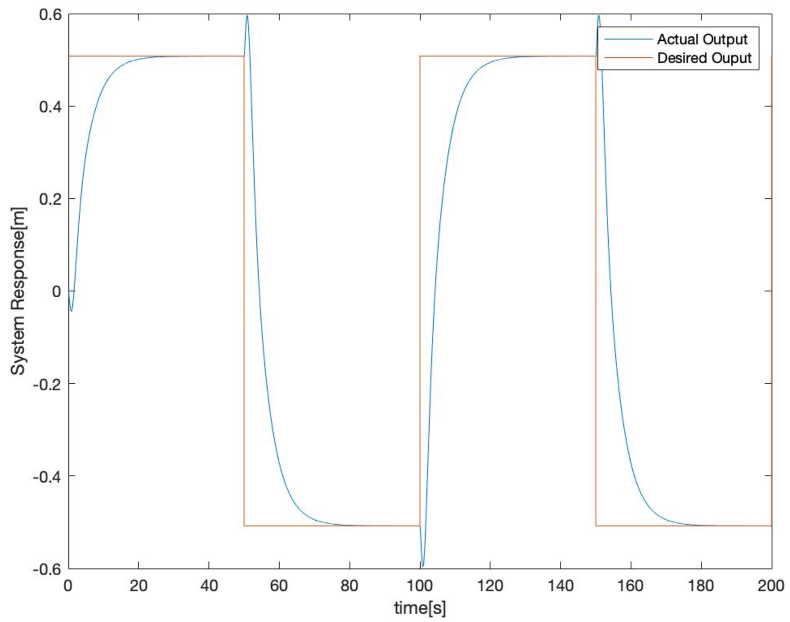


Explanation: In the state vs time graph, we can see that the state x is rising up to 0.508m(20In) and matching the phase of the input square wave signal. This matches our expectation as we can controller the position of the cart with the state tracking controller in this case. In the actual output and desired output graph, we get a better look of how the actual input matches with the desired input. We can see that even though, the phases and magnitude generally match, during even cycle it takes the actual output almost 50s to reach the magnitude of the desired output. In addition, there are overshoot and undershoot happening.

h)

The new Q matrix I selected is $Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

Actual Output Vs Desired Output with new Q



Explanation: I chose a new Q for the controller and I kept the R the same. The reason that I think the result I got is better than the original output is that we can visually see that the actual output is matching much more closely to the desired output. I modified the Q matrix by increasing the first and the third column which corresponds to the position and velocity of the cart. In general, larger Q causes x to be drive to zero faster. In this case, since the desired output is that of the cart position, by increasing the corresponding columns in the Q matrix, the cart position output would converge to the desired output faster.

MATLAB Code

Q1

```
%part a
A = [0,1,0; 0,0,1; 1,5,7]
B= [1;0;0]
C = [0 1 3]
% C= A*A*B
E = eig(A)

%part b
W_c= [1 ,0,0; 0,0,1;0,1,7]
determinant = det(W_c)


% part c
time = linspace(0,2,1000);
xt = []
x0  = [0;1;0]
for j = 1: size(time,2)
    xt(:,end+1) = expm(A*time(j))*x0;
end
y = C*xt;
figure(1)
% plot(time, xt(1,:))
plot(time, y)
title('unforced system output')
xlabel('time[s]'); ylabel('System Response')


%part d
p = [ -1+i  -1-i -2];
k = place(A,B,p)


%part e
time = linspace(0,10,1000);
xt = []
for j = 1:size(time,2)
    t= time(j)
    xt(:,j)=expm((A-B*k)*t)*x0;
end
y_forced = C*xt;
figure(2)
plot(time ,y_forced)
title('system output under feedback law')
xlabel('time[s]'); ylabel('System Response’)
```

Q2

```
global a b D miu y K C A B;

A = [0,1,1,0;0,0,0,1;0,1,-3,0;0,2,-3,0]
B = [0;0;1;1]
%original R Q parameters
% R = 10
Q = [1,0,0,0;0,5,0,0;0,0,1,0;0,0,0,5]
R = 10;
% Q = [20,0,0,0;
%      0,5,0,0;
%      0,0,20,0;
%      0,0,0,5];

E = eig(A);
[K,S,P] = lqr(A,B,Q,R);
time = [0:0.01:200];
x0_all=[[0;0.1;0;0],[0;0.5;0;0],[0; 1.0886;0;0],[0;1.1;0;0]];
xt = [];

%Part d
for j = 1:size(x0_all,2)
    x0 = x0_all(:,j)
    for i = 1:size(time,2)
        t = time(i);
        xt(1:4,i)=expm((A-B*K)*t)*x0;
    end
    figure();
    plot(time,xt)
    title('unforced system output')
    xlabel('time[s]'); ylabel('System Response')
    legend('x' , 'phi','xdot','phi dot')
end


plotting with ode
global A B K
for j = 1:size(x0_all,2)
    [t,xt] = ode45(@SS_Linear,time,x0_all(:,j));
    figure()
    plot(time,xt)
    title('Linear System with LQR feedback control Output')
    xlabel('time[s]'); ylabel('System Response yt')
    legend('x' , 'phi','xdot','phi dot')
end


% part e
global a b D miu y K;
a = 1;
b =1 ;
D =1 ;
```

```
miu=3;
y =2;
for j = 1:size(x0_all,2)
    [t,xt] = ode45(@SS_nonlinear,time,x0_all(:,j));
    figure()
    plot(time,xt);
    legend('x' , 'phi','xdot','phi dot')
    title('Non-Linear System with LQR feedback control Output yt')
    xlabel('time[s]'); ylabel('System Response')
end


%part g
a = 1;
b =1 ;
D =1 ;
C = [39.3701 0 0 0];
miu=3;
y =2;

v = 0.508*square(1/100*2*pi*time);
x0 = [0;0;0;0]
[t,xt] = ode45(@SS_Nonlinear_tracking,time,x0);

xt= xt*39.3701

figure()
plot(time,xt)
title('state vs time')
xlabel('time[s]')
ylabel('System Responses')


figure()
plot(time,xt(:,1));hold on;
plot(time,v)
legend('Actual Output','Desired Ouput' )
xlabel('time[s]')
ylabel('System Response[m]')


function xdot = SS_Linear(t,x)
global A B K
u = K*x
xdot = A*x - B*u;
end


function xdot = SS_Linear(t,x)
global A B K
v = square(1/100*2*pi*t)
u =v-K*x;
xdot = A*x - B*u;
end
```

```
function dqdt = SS(t,x)
global a b D miu y K;
F = -K*x;
dqdt= [x(3);x(4);a/(y*a-b^2*cos(x(2)^2))*(F+b/a*D*sin(x(2)*cos(x(2)))-b*x(4)^2*sin(x(2))-miu*x(3));
    (b*cos(x(2)))/(y*a-b^2*cos(x(2)^2))*(F+b/a*D*sin(x(2)*cos(x(2)))-b*x(4)^2*sin(x(2))-miu*x(3))
+D*sin(x(2))/a];
end
```

```
function dqdt = SS(t,x)
global a b D miu y K A B C;
yd = 0.508*square(1/100*2*pi*t);
v = -inv(C*inv(A-B*K)*B)*yd;
F = v-K*x;

dqdt= [x(3);x(4);a/(y*a-b^2*cos(x(2)^2))*(F+b/a*D*sin(x(2)*cos(x(2)))-b*x(4)^2*sin(x(2))-miu*x(3));
    (b*cos(x(2)))/(y*a-b^2*cos(x(2)^2))*(F+b/a*D*sin(x(2)*cos(x(2)))-b*x(4)^2*sin(x(2))-miu*x(3))
+D*sin(x(2))/a];
end
```