Digital Signal Processing 60 (2017) 56–62

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

# Energy-harvesting-based RA-coded cooperative MIMO: Codes design and performance analysis ☆

Shunwai Zhang [a,*], Fengfan Yang [b], Rongfang Song [a]

[a] College of Telecommunications and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing 210003, China
[b] College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

ARTICLE INFO

*Article history:*
Available online 3 September 2016

*Keywords:*
Coded cooperation
Energy harvesting
RA codes
Joint design
Outage probability

ABSTRACT

To overcome the power limitation and prolong the lifetime of the coded cooperation system, this paper proposes an energy-harvesting-based repeat-accumulate (RA)-coded cooperative multiple-input multiple-output (MIMO), where the relay is equipped with multiple antennas for information decoding and energy harvesting. Firstly, a kind of structured low-density parity-check (LDPC) codes – RA codes are introduced, and the RA codes for the source and relay are designed jointly to cancel all girth-4 cycles. Secondly, the optimal antenna selection algorithm is adopted by the relay to transmit the information. Furthermore, the outage probability and bit error rate (BER) of the system are analyzed. Theoretical analysis and numerical simulations show that the investigated system outperforms the corresponding point-to-point system under the same condition. Simulation result also demonstrates that BER performance of the system employing jointly designed RA codes is much better than that of general RA codes.



## 1. Introduction

Coded cooperation [1–4] combines channel coding technique and cooperation technique, which can bring coding gain and diversity gain, respectively. By combining low-density-parity-check (LDPC) codes [5] with cooperation techniques, LDPC-coded cooperation [6,7] obtains both high coding gain and diversity gain, so it has a promising prospect of application. Many previous studies [8–10] have been concentrated on this high-performance coded cooperation system. However, if the relay of the coded cooperation is powered by external battery, it means that the coded cooperation is power-limited and its lifetime is restricted. To overcome this challenge and prolong the lifetime of the system, energy-harvesting technique, originally proposed by Nikola Tesla [11], has been employed in the cooperation recently. Traditional energy-harvesting techniques rely on natural energy sources such as solar or wind, which are problematic due to intermittent property. Recently, the radio frequency (RF)-based energy-harvesting [12] attracts considerable attention from both academia and industry due to the following reason: information and energy can be simultaneously transferred in a form of RF signal. Hence, on the basis of energy-harvesting technology, the relay of the coded cooperation can be supported without external power and the lifetime of the system can be prolonged.

In recent years, many studies have been concentrated on the energy-harvesting-based cooperation. In [13] an amplify-and-forward (AF) cooperation system was investigated, where the relay harvests energy from the received RF signal and uses the harvested energy to transmit the source information to the destination. To implement energy harvesting and information decoding at the relay, time switching-based relaying protocol and power splitting-based relaying protocol were further proposed. For a two-hop AF-MIMO relay system with an energy harvesting receiver, in [14] orthogonal space–time block codes for data transmission are employed by the source and relay. To achieve different tradeoffs between the energy and information transfers, joint optimal source and relay precoders are designed when instantaneous channel state information is available. Reference [15] studied a cooperation scheme with interference channels, where signal splitting transmit scheme was proposed to facilitate collaborative transmit energy beamforming, and users' grouping based pairwise cooperation and ergodic interference alignment based joint cooperation were further investigated. In [16] the concept of energy cooperation was introduced. In the energy cooperation system, a user transmits a portion of its energy to another user which can harvest energy. Although the loss incurs in energy transfer, it can optimize the energy and improve the overall system performance. The

☆ This work was supported by National Natural Science Foundation of China (61501256, 61271234), Natural Science Foundation of Jiangsu Province (BK20150857), China Postdoctoral Science Foundation (2014M561694), and NUPTSF (214007).

* Corresponding author.
*E-mail address:* swzhang@njupt.edu.cn (S. Zhang).

http://dx.doi.org/10.1016/j.dsp.2016.08.013
1051-2004/

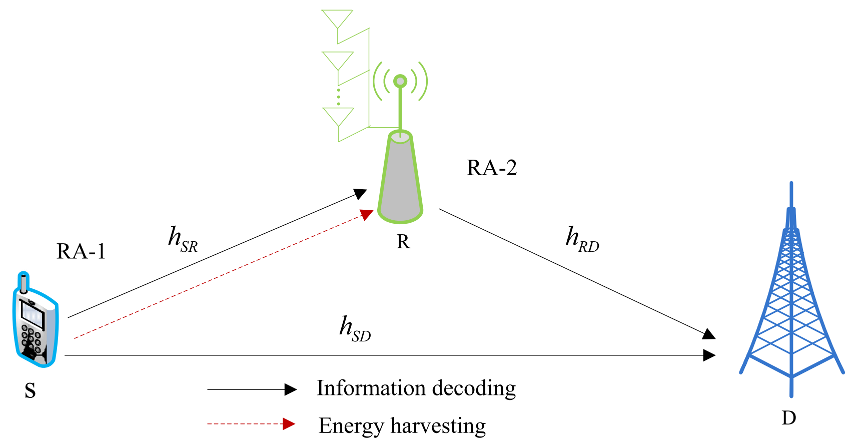



**Fig. 1.** Energy-harvesting-based RA-coded cooperative MIMO.

partial network-level relay cooperation for energy-harvesting networks was presented in [17], where the relay regulates the arrivals from the source by accepting only a certain proportion of the successfully received packets. To ensure the stability of the source and relay data queues, the relaying parameter is selected based on the parameters of the network. However, at present, references about energy-harvesting-based coded cooperation are relatively scarce.

In this paper, we investigate the energy-harvesting-based repeat-accumulate (RA)-coded cooperative MIMO system over Rayleigh fading channels. The relay is equipped with multiple antennas which are able to harvest energy. Combining channel coding, cooperation, and energy-harvesting, the system can achieve cooperation diversity, multi-antennas diversity and channel coding gains. Meanwhile, it breaks through the energy limitation at the relay, and thus its lifetime is prolonged. The main contributions of this paper are summarized as follows:

(a) The energy-harvesting-based RA-coded cooperative MIMO is proposed, where the relay is equipped with multiple antennas for information decoding and energy harvesting, and the optimal antenna selection algorithm is adopted at the relay.
(b) A kind of structured LDPC codes-RA codes are employed by the energy-harvesting-based coded cooperative MIMO, and the RA codes for the source and relay are designed jointly to further improve the coding gain.
(c) The outage probability and bit error rate (BER) of the energy-harvesting-based RA-coded cooperative MIMO are studied by theoretical analysis and numerical simulations.

The rest of this paper is organized as follows. In Section 2, the system description of energy-harvesting-based RA-coded cooperative MIMO is presented. Section 3 mainly deals with joint design of RA codes for the investigated system. Section 4 analyzes the outage probability of the system. Simulation results are given in Section 5. Finally, Section 6 concludes the whole paper.

*Notations:* $\boldsymbol{A}_{M\times M}$ and $\boldsymbol{B}_{M\times M}$ are $M\times M$ matrices. $\boldsymbol{D}_{M\times M}$ is an $M\times M$ quasi-diagonal-matrix and $\boldsymbol{0}_{M\times M}$ is an $M\times M$ zero matrix. The superscript "$T$" denotes the transpose of a matrix/vector. $max\{.\}$ is the maximum function.

## 2. System description

An energy-harvesting-based RA-coded cooperative MIMO is shown in Fig. 1. As users or terminals, the source (S) and destination (D) have single antenna with external power supply. The relay (R) is equipped with $K$ antennas without external power supply, which is powered by the energy harvested from the RF signals from the source. At the source, a codeword $\boldsymbol{c}_1$ conveying information bits encoded by the first RA encoder (RA-1) is sent simultaneously to R and D over a broadcast channel. At the relay, to recover the information bits, $K_{\mathrm{I}}$ antennas are used to decode the incoming signal, and other $K_{\mathrm{E}}(K_{\mathrm{I}}+K_{\mathrm{E}}=K)$ antennas are used to harvest energy. The recovered information bits are again encoded into another codeword $\boldsymbol{c}_2$ by the second RA encoder (RA-2). Because both codewords $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ have the same information bits, only check bits of $\boldsymbol{c}_2$ are sent to D over R–D channel to retain high-efficient transmission. The relay adopts optimal antenna selection algorithm to send the check bits. It means only the optimal antenna is selected from the multiple antennas for transmission. The transmission power of relay is provided by the harvested energy. At the destination, the two incoming signals from the source and relay, which are transmitted in two time slots, are jointly decoded by the joint iterative decoding algorithm [18].

## 3. Joint design of RA codes for energy-harvesting-based coded cooperative MIMO

In this section, we investigate a kind of quasi-cyclic RA codes, and then apply them in the energy-harvesting-based coded cooperative MIMO. To improve the performance and achieve high coding gain, we design RA codes jointly to cancel the short-girth cycles.

### 3.1. RA codes

Assume the sparse parity-check matrix of a RA code has the form as $\boldsymbol{H}=[\boldsymbol{A}\ \boldsymbol{D}]$. $\boldsymbol{A}$ is a sparse matrix; $\boldsymbol{D}$ is a quasi-diagonal-matrix in which all elements are zero except the elements of the principal diagonal and the elements immediately below this diagonal. If $\boldsymbol{A}$ is a random sparse matrix, RA codes require a large sum of memory when implemented in the hardware. To reduce the memory consumption, inspired by quasi-cyclic LDPC codes, we further assume $\boldsymbol{A}$ is a quasi-cyclic sparse matrix. This kind of RA codes are called quasi-cyclic RA codes in this paper, whose parity-check matrices are shown as follows

$$\boldsymbol{H}=[\boldsymbol{A}\ \boldsymbol{D}]=\begin{bmatrix} \boldsymbol{I}(p_{1,1}) & \boldsymbol{I}(p_{1,2}) & \cdots & \boldsymbol{I}(p_{1,L}) & 1 & 0 & & \\ \boldsymbol{I}(p_{2,1}) & \boldsymbol{I}(p_{2,2}) & \cdots & \boldsymbol{I}(p_{2,L}) & 1 & 1 & \boldsymbol{0} & \\ \vdots & & \ddots & \vdots & \boldsymbol{0} & \vdots & & \\ \boldsymbol{I}(p_{J,1}) & \boldsymbol{I}(p_{J,2}) & \cdots & \boldsymbol{I}(p_{J,L}) & \cdots & 1 & 1 & \end{bmatrix} \tag{1}$$

where $\boldsymbol{I}(p_{j,l})$ is an identity matrix $\boldsymbol{I}$ with $p_{j,l}$-right-cyclic-shift. The dimension of $\boldsymbol{I}(p_{j,l})$ is $B$.

Although RA codes belong to the family of LDPC codes, they have structured parity-check matrices. They can be applied directly to encoding without using the Gaussian elimination algorithm which guarantees systematic codwords. The relay can easily extract the information bits to re-encode them. Hence, RA codes are naturally fit for coded cooperative MIMO. What is more, compared with the common systematic LDPC codes called generator-based LDPC (G-LDPC), RA codes in general exhibit a lower error floor and a slightly better water-fall region [1].

### 3.2. Joint design of RA codes

The encoding scheme of energy-harvesting-based RA-coded cooperative MIMO is described as follows [18].

**(a)** At the source S, a block of information bits $\boldsymbol{s} = [s_1, \cdots, s_{N-M_1}]^T$ is encoded into a codeword

$$\boldsymbol{c}_1 = \left[s_1, \cdots, s_{N-M_1}, p_1^{(1)}, \cdots, p_{M_1}^{(1)}\right]^T \tag{2}$$

where $N$ is the length of codeword and $M_1$ is the length of the parity bits $\boldsymbol{p}^{(1)} = [p_1^{(1)}, \cdots, p_{M_1}^{(1)}]$. The first RA code (RA-1) is defined by its parity-check matrix as

$$\boldsymbol{H}_1 = [\boldsymbol{A}_{M_1 \times (N-M_1)} \boldsymbol{D}_{M_1 \times M_1}] \tag{3}$$

The codeword $\boldsymbol{c}_1$ is sent to the relay and destination over a broadcast channel simultaneously.

**(b)** At the relay R, assume that the signal from the source is correctly decoded by the relay using some antennas. Meanwhile, the recovered information bits is encoded again to a codeword

$$\boldsymbol{c}_2 = \left[s_1, \cdots, s_{N-M_1}, p_1^{(2)}, \cdots, p_{M_2}^{(2)}\right]^T \tag{4}$$

where $M_2$ is the length of the parity bits $\boldsymbol{p}^{(2)} = [p_1^{(2)}, \cdots, p_{M_2}^{(2)}]$. The second RA code (RA-2) is defined by its parity-check matrix as

$$\boldsymbol{H}_2 = [\boldsymbol{B}_{M_2 \times (N-M_1)} \boldsymbol{D}_{M_2 \times M_2}] \tag{5}$$

The relay, which is powered by the energy harvested from RF signals, only sends the parity-check bits $\boldsymbol{p}^{(2)}$ to the destination.

**(c)** At the destination D, two signals corresponding to $\boldsymbol{c}_1 = [s_1, \cdots, s_{N-M_1}, p_1^{(1)}, \cdots, p_{M_1}^{(1)}]$ over S–D channel and $\boldsymbol{p}^{(2)} = [p_1^{(2)}, \cdots, p_{M_2}^{(2)}]$ over R–D channel are received. Based on the relationship between parity-check matrix and codeword

$$\boldsymbol{H}_1 \boldsymbol{c}_1 = 0 \tag{6a}$$

$$\boldsymbol{H}_2 \boldsymbol{c}_2 = 0 \tag{6b}$$

the overall parity-check matrix $\boldsymbol{H}$ of the coded cooperative MIMO satisfies the following relationship

$$\boldsymbol{H}\boldsymbol{c} = \boldsymbol{0} \tag{7}$$

where the whole codeword $\boldsymbol{c}$ is achieved by cascading $\boldsymbol{c}_1$ and $\boldsymbol{p}^{(2)}$.

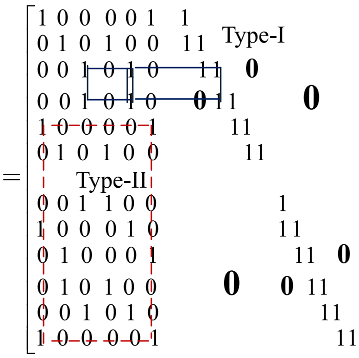
$$\boldsymbol{c} = \left[s_1, \cdots, s_{N-M_1}, p_1^{(1)}, \cdots, p_{M_1}^{(1)}, p_1^{(2)}, \cdots, p_{M_2}^{(2)}\right] \tag{8}$$

with block length $N + M_2$. The overall parity-check matrix is

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{A}_{M_1 \times (N-M_1)} & \boldsymbol{D}_{M_1 \times M_1} & \boldsymbol{0}_{M_1 \times M_2} \\ \boldsymbol{B}_{M_2 \times (N-M_1)} & \boldsymbol{0}_{M_2 \times M_1} & \boldsymbol{D}_{M_2 \times M_2} \end{bmatrix} \tag{9}$$

When LDPC codes are free of cycles, the iterative belief propagation (BP) decoding algorithm converges to the optimal solution. It is the same for RA codes. The short girth cycles (especially girth-4 cycles) degrade the performance of RA codes. We design the overall parity-check matrix jointly corresponding to RA codes employed by the source and relay to cancel all the girth-4 cycles.

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{B} & \boldsymbol{0} & \boldsymbol{D} \end{bmatrix} =$$




——— Type-I: girth-4 cycles in $\boldsymbol{H}_1$ or $\boldsymbol{H}_2$

- - - - Type-II: girth-4 cycles between $\boldsymbol{H}_1$ or $\boldsymbol{H}_2$ (10)

In (10), all girth-4 cycles are classified into two types. Type-I: girth-4 cycles in $\boldsymbol{H}_1$ or $\boldsymbol{H}_2$. Type-II: girth-4 cycles between $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$. If RA-1 or RA-2 codes are designed separately at the source or relay, only Type-I girth-4 cycles can be canceled. To cancel both Type-I and Type-II girth-4 cycles, RA-1 and RA-2 codes should be jointly designed based on $\boldsymbol{H}$.

The right part of $\boldsymbol{H}$

$$\boldsymbol{H}_R = \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D} \end{bmatrix} \tag{11}$$

is quasi-diagonal, there are no girth-4 cycles. The left part of $\boldsymbol{H}$

$$\boldsymbol{H}_L = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{B} \end{bmatrix} \tag{12}$$

consists of right-cyclic-shift identity submatrices. Girth-4 cycles (Type-I or Type-II) in $\boldsymbol{H}_L$ can be canceled according to Theorem 1.

**Theorem 1.** *Assume $\boldsymbol{I}(p_{j,l})$, $\boldsymbol{I}(p_{j+k,l})$, $\boldsymbol{I}(p_{j,l+t})$, $\boldsymbol{I}(p_{j+k,l+t})$ are four right-cyclic-shift identity submatrices in $\boldsymbol{H}_L$, whose shift values are $p_{j,l}$, $p_{j+k,l}$, $p_{j,l+t}$, $p_{j+k,l+t}$, respectively. To avoid girth-4 cycles, a necessary and sufficient condition that should be satisfied is*

$$(p_{j,l} - p_{j+k,l}) + (p_{j,l+t} - p_{j+k,l+t}) \neq 0 \bmod B \tag{13}$$

The proof is referred in [19].

There are girth-4 cycles between $\boldsymbol{H}_L$ and $\boldsymbol{H}_R$ (Type-I). As $\boldsymbol{H}_R$ is quasi-diagonal, the cycles only consist of "1" in the last row of $\boldsymbol{I}(p_{j,l})$, "1" in the first row of $\boldsymbol{I}(p_{j+1,l})$, and the two "1"s in the same column of $\boldsymbol{H}_R$, where $\boldsymbol{I}(p_{j,l})$ and $\boldsymbol{I}(p_{j+1,l})$ are two upper and lower adjacent submatrices.

**Theorem 2.** *Assume $\boldsymbol{I}(p_{j,l})$ and $\boldsymbol{I}(p_{j+1,l})$ are two upper and lower adjacent submatrices in $\boldsymbol{H}_L$. To cancel girth-4 cycles between $\boldsymbol{H}_L$ and $\boldsymbol{H}_R$, a necessary and sufficient condition that should be satisfied is*

$$p_{j,l} - p_{j+1,l} \neq 1 \bmod B \tag{14}$$

Please see the proof in Appendix A.

According to both Theorem 1 and Theorem 2, the RA codes for the source and relay are designed jointly, and all girth-4 cycles including Type-I and Type-II are canceled.

RA codes, the same as LDPC codes, exhibit good performance when the girth size of cycles increases. For simplicity, we describe only the joint design method of RA codes by canceling all girth-4 cycles in the overall parity-check matrix. To further improve the coding gain, we can easily extend the joint design method to canceling larger girth (girth-6, girth-8, or girth-10) cycles by the following two steps. (a) Cancellation of girth-$G$ ($G$ is larger than 4) cycles in $\boldsymbol{H}_L$. Please see [19]. (b) Cancellation of girth-$G$ cycles between $\boldsymbol{H}_L$ and $\boldsymbol{H}_R$. If $G=6$, a necessary and sufficient condition that should be satisfied is

$$p_{j,l} - p_{j+1,l} \neq 1, 2 \bmod B \tag{15}$$

The proof is similar to Theorem 2 which is omitted here. In a similar way, cancellation of girth-8 or girth-10 cycles between $\boldsymbol{H}_L$ and $\boldsymbol{H}_R$ is also realizable.

By the above two steps, the cancellation of all girth-4 cycles are extended to the cancellation of all larger girth (girth-6, girth-8, or girth-10, etc.) cycles.

## 4. Performance and complexity analysis

In this section, the outage probability performance of the energy-harvesting-based RA-coded cooperative MIMO is investigated, and the complexity of the system is analyzed.

### 4.1. Outage probability analysis

Suppose $h_{SD}$ represents S–D channel, $h_{SR}^{(k)}$ $(k=1,\cdots,K)$ represents S–R channel, and $h_{RD}^{(k)}$ represents R–D channels. All channels suffer from independent Rayleigh fading, which means $h_{SD}$, $h_{SR}^{(k)}$, and $h_{RD}^{(k)}$ are zero-mean complex Gaussian random variables with unit variance. The transmission power at the source is $P$. $d_{SD}^2$, $d_{SR}^2$, and $d_{RD}^2$ are the power attenuation factors of S–D, S–R, and R–D channels due to different distances. $x$ and $\bar{x}$ are the transmitted bits at the source and relay.

As described in Section 2, at the relay, $K_\mathrm{I}$ antennas are used to decode the incoming signal, and other $K_\mathrm{E}$ antennas are used to harvest energy. The energy harvested at the relay is calculated as

$$P^{EH} = \frac{\sum_{k=1}^{K_\mathrm{E}} |h_{SR}^{(k)}|^2 P}{d_{SR}^2} \tag{16}$$

where $P$ is the transmission power at the source. Assuming the energy utilization ratio at the relay is $\eta$ $(0 \le \eta \le 1)$, the energy used to transmit $\bar{x}$ is

$$\bar{P} = \frac{\sum_{k=1}^{K_\mathrm{E}} |h_{SR}^{(k)}|^2 \eta P}{d_{SR}^2} \tag{17}$$

The relay adopts optimal antenna selection algorithm [20] to select the optimal antenna $h_{RD}^{(o)}$ from $h_{RD}^{(1)},\cdots,h_{RD}^{(K)}$,

$$\left|h_{RD}^{(o)}\right|^2 = max\left\{\left|h_{RD}^{(1)}\right|^2,\cdots,\left|h_{RD}^{(K)}\right|^2\right\} \tag{18}$$

Then, the relay sends $\bar{x}$ over the antenna $h_{RD}^{(o)}$ with transmission power $\bar{P}$.

At the destination, the received signals from the source and relay are

$$r_{SD} = \frac{\sqrt{P}}{d_{SD}} h_{SD} x + n \tag{19}$$

$$\begin{aligned} r_{RD} &= \frac{\sqrt{\bar{P}}}{d_{RD}} h_{RD}^{(o)} \bar{x} + n \\ &= \frac{\sqrt{\sum_{k=1}^{K_\mathrm{E}} |h_{SR}^{(k)}|^2 \eta P}}{d_{SR} d_{RD}} h_{RD}^{(o)} \bar{x} + n \end{aligned} \tag{20}$$

where $n$ is the additional noise at the destination, and it is a zero-mean complex Gaussian random variable with variance $\sigma^2$.

An outage event occurs when the instantaneous channel capacity falls below the data transmission rate, which is regarded as the code rate in this paper. The probability of an outage event occurring is defined as the outage probability. For overall codeword $\boldsymbol{c} = [s_1,\cdots,s_{N-M_1},p_1^{(1)},\cdots,p_{M_1}^{(1)},p_1^{(2)},\cdots,p_{M_2}^{(2)}]^T$, $[s_1,\cdots,s_{N-M_1},p_1^{(1)},\cdots,p_{M_1}^{(1)}]$ are over S–D channel, and $[p_1^{(2)},\cdots,p_{M_2}^{(2)}]$ are over R–D channel. The destination jointly decodes these two parts. Hence, the instantaneous channel capacity of the energy-harvesting-based RA-coded cooperative MIMO is calculated as

$$\begin{aligned} C &= \frac{N}{N+M_2}\log_2\left(1+\frac{|h_{SD}|^2 P}{d_{SD}^2\sigma^2}\right) \\ &\quad + \frac{M_2}{N+M_2}\log_2\left(1+\frac{|h_{RD}^{(o)}|^2\sum_{k=1}^{K_\mathrm{E}}|h_{SR}^{(k)}|^2\eta P}{d_{SR}^2 d_{RD}^2\sigma^2}\right) \end{aligned} \tag{21}$$

Furthermore, the outage probability [21] is given by

$$\begin{aligned} \mathrm{P}_{out} &= \Pr\{C<r\} \\ &= \Pr\left\{\frac{N}{N+M_2}\log_2\left(1+\frac{|h_{SD}|^2 P}{d_{SD}^2\sigma^2}\right)\right. \\ &\quad \left. + \frac{M_2}{N+M_2}\log_2\left(1+\frac{|h_{RD}^{(o)}|^2\sum_{k=1}^{K_\mathrm{E}}|h_{SR}^{(k)}|^2\eta P}{d_{SR}^2 d_{RD}^2\sigma^2}\right) < r\right\} \end{aligned} \tag{22}$$

where $r$ is the code rate, $r=(N-M_1)/(N+M_2)$.

When $N=M_2$, (22) can be further rewritten as

$$\begin{aligned} \mathrm{P}_{out} &= \frac{1}{2}\Pr\left\{\log_2\left(1+\frac{|h_{SD}|^2 P}{d_{SD}^2\sigma^2}\right)\right. \\ &\quad \left. + \log_2\left(1+\frac{|h_{RD}^{(o)}|^2\sum_{k=1}^{K_\mathrm{E}}|h_{SR}^{(k)}|^2\eta P}{d_{SR}^2 d_{RD}^2\sigma^2}\right) < r\right\} \end{aligned} \tag{23}$$

In (23), $|h_{SD}|^2$ and $|h_{SR}^{(k)}|^2$ follow the exponential distribution $|h_{SD}|^2, |h_{SR}^{(k)}|^2 \sim \varepsilon(1)$. Define $X_i = |h_{RD}^{(i)}|^2$ $(i=1,\cdots,K)$ and $Y = |h_{RD}^{(o)}|^2 = max\{|h_{RD}^{(1)}|^2,\cdots,|h_{RD}^{(K)}|^2\}$. $X_1,\cdots,X_K$ are independent and identically distributed variables, whose common distribution function $F_X(x)$ and density function $f_X(x)$ are

$$F_X(x) = \begin{cases} 1-e^{-x}, & x \ge 0 \\ 0, & x<0 \end{cases} \tag{24a}$$

$$f_X(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & x<0 \end{cases} \tag{24b}$$

Hence, the density function of $Y = |h_{RD}^{(o)}|^2$ follows [22]

$$\begin{aligned} f_Y(y) &= K f_X(y)\big(F_X(y)\big)^{K-1} \\ &= \begin{cases} Ke^{-y}(1-e^{-y})^{K-1}, & y \ge 0 \\ 0, & y<0 \end{cases} \end{aligned} \tag{25}$$

The outage probability will be further analyzed by simulations.

### 4.2. Complexity analysis

The complexity of the energy-harvesting-based RA-coded cooperative MIMO is analyzed in consideration of the following two aspects:

**(a)** Encoding and decoding complexity of jointly designed RA codes. The sparse parity-check matrices of RA codes can be applied directly to encoding without Gaussian elimination due to their quasi-diagonal structure; however, for LDPC codes, $\boldsymbol{O}(N^3)$ operations of preprocessing is required to bring the sparse parity-check matrix into the desired lower triangular or quasi-diagonal structure. $\boldsymbol{O}(N^3)$ means the numbers of operations are cubic with the length of codewords $N$. Hence, compared with LDPC codes, the encoding complexity of jointly designed RA codes is decreased.

**Table 1**
RA codes employed by the source and relay.

| | RA-1 (S) | RA-2 (R) |
|---|---|---|
| RA-Coded coop. | $\boldsymbol{H}_1 = [\boldsymbol{A}_{200\times 200}\ \boldsymbol{D}_{200\times 200}]$<br>Rate $= 1/2$, length $= 400$<br>$B = 50$ | $\boldsymbol{H}_2 = [\boldsymbol{B}_{400\times 200}\ \boldsymbol{D}_{400\times 400}]$<br>Rate $= 1/3$, length $= 600$<br>$B = 50$ |

What's more, both type I and type II girth-4 cycles are canceled for the jointly designed RA codes. The joint iterative decoding algorithm at the destination can greatly accelerate the decoding convergence which reduces the iteration times. Therefore, compared with LDPC codes or RA codes without joint design, the decoding complexity of jointly designed RA codes is reduced.

**(b)** Complexity of optimal antenna selection algorithm. To select the optimal antenna from $K$ antennas using optimal antenna selection algorithm, $K-1$ comparison times is needed. If the number of antennas $K$ in a practical communication system is not large, the complexity of optimal antenna selection algorithm is acceptable.

## 5. Simulation results

In this section, we investigate the performance of energy-harvesting-based RA-coded cooperative MIMO by numerical simulations. S–D, S–R and R–D are Rayleigh block fading channels with perfect channel state information (CSI), and joint iterative decoding algorithm [18] is employed at the destination. The average SNRs per bit per antenna of the signals are defined as the ratio of transmission power $P$ at the source and the noise power $\sigma^2$ at the destination. Binary phase shift keying (BPSK) modulation is adopted. As the relay is located between the source and destination, for simplicity and without loss of generality, $d_{SD} = 2d_{SR} = 2d_{RD} = 2$ is assumed in the simulations. RA codes employed by the source and relay are given in Table 1.

### *5.1. Outage probability of energy-harvesting-based coded cooperative MIMO*

In this part, we present the outage probability performance of energy-harvesting-based coded cooperative MIMO. Assume the data transmission rate referred to as the overall code rate $r = 1/4$. The number of information-decoding antennas at the relay $K_\mathrm{I} = 1$, and the number of energy-harvesting antennas $K_\mathrm{E} = 1,\ 2$, which means 2 or 3 antennas are employed at the relay. The energy utilization ratio at the relay $\eta = 1, 0.8, 0.5$, respectively.

In Fig. 2, it is shown that the outage probability of energy-harvesting-based coded cooperative MIMO is much lower than that of the point-to-point system, and the diversity order is higher. It demonstrates the superiority of the investigated scheme. We also compare the outage probability of the system with $K_\mathrm{E} = 1$ and 2. Fig. 2 shows that the outage probability performance of the system with $K_\mathrm{E} = 2$ clearly outperforms the system with $K_\mathrm{E} = 1$ under the same energy utilization ratio. Furthermore, it is seen that the higher the energy utilization ratio, the lower the outage probability.

### *5.2. Outage probability of energy-harvesting-based coded cooperative MIMO with various antenna selection algorithms at the relay*

We investigate the outage probability of energy-harvesting-based coded cooperative MIMO with the optimal antenna selection, random antenna selection algorithm, and equal power allocation algorithm [23]. For $K_\mathrm{I} = 1$, $K_\mathrm{E} = 2$, $K = K_\mathrm{I} + K_\mathrm{E} = 3$, $\eta = 1$, Fig. 3 compares the outage probability of the system with various antenna selection algorithms. It is shown that the outage probability of the system with the adopted optimal antenna selection

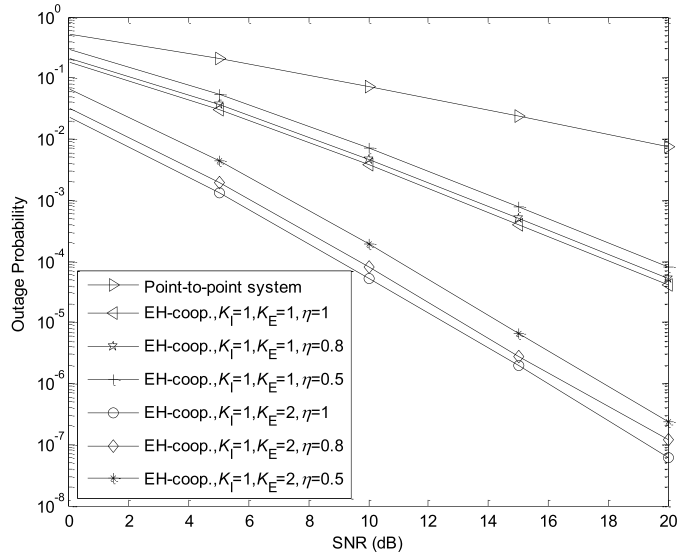



**Fig. 2.** Outage probability comparison of energy-harvesting-based coded cooperative MIMO with one, or two energy-harvesting antennas and various energy utilization ratios at the relay.

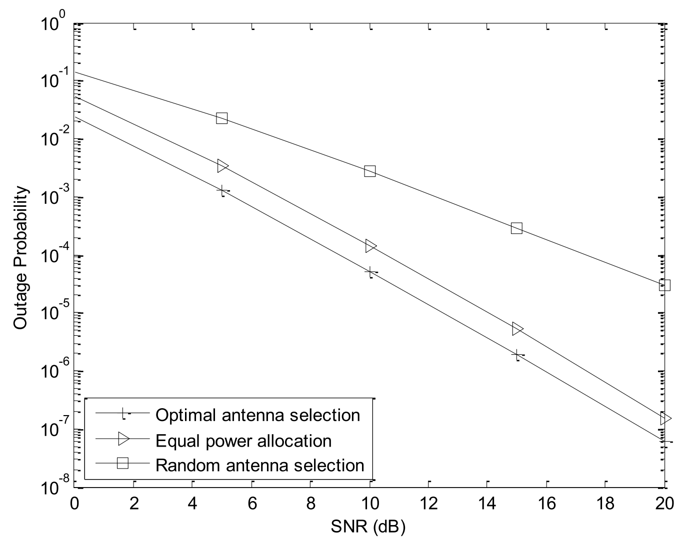



**Fig. 3.** Outage probability comparison of energy-harvesting-based coded cooperative MIMO with various antenna selection algorithms at the relay.

algorithm is lowest, and the outage probability of the random antenna selection algorithm is highest. The diversity orders of both optimal antenna selection and power allocation algorithms are higher than that of random antenna selection algorithm. Actually, it should be noticed that the complexity of optimal antenna selection algorithm is higher than that of the random antenna selection algorithm and equal power allocation algorithm. Hence, considering both the performance and complexity, it is important to adopt a suitable algorithm for a practical system

### *5.3. BER performance of energy-harvesting-based RA-coded cooperative MIMO with various numbers of antennas at the relay*

Assume $K_\mathrm{I} = 1$, $K_\mathrm{E} = 1, 2, 3$, $K = K_\mathrm{I} + K_\mathrm{E} = 2, 3, 4$. The relay can decode the signals correctly by using one antenna, and ten joint decoding iterations are applied at the destination. For a fair comparison, RA code employed by the point-to-point system has the same code length and code rate as the overall RA codes employed by the cooperative MIMO. Fig. 4 presents the BER performance of energy-harvesting-based RA-Coded cooperative MIMO.

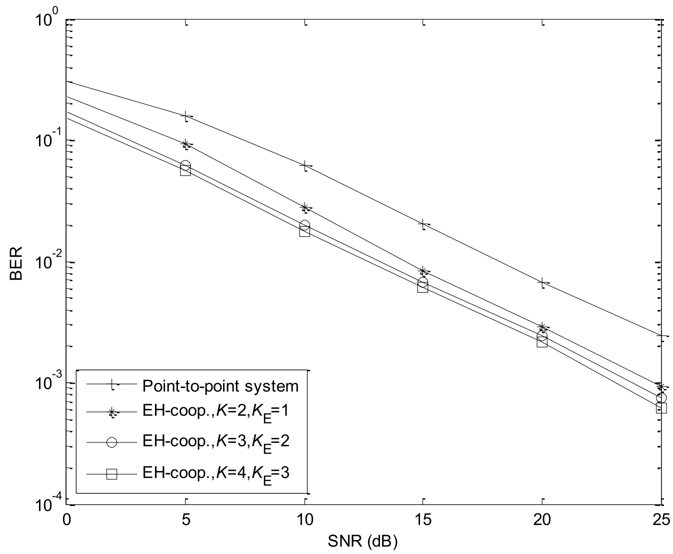



**Fig. 4.** BER comparison of energy-harvesting-based RA-coded cooperative MIMO with two, three, or four antennas at the relay.

It depicts that the BER performance of cooperative MIMO clearly outperforms the point-to-point system. For instance, with $K = 2$, $K_E = 1$ and at a BER of $10^{-2}$, the investigated system achieves about 5 dB gain over its respective point-to-point system. The significant gain can be attributed to the fact as follows: Two signals from the source and relay, which are through independent fading S–D and R–D channels, are jointly decoded by the joint iterative decoding algorithm. Furthermore, the relay selects the optimal antenna from multiple antennas for transmission by optimal antenna selection algorithm described in this paper. Hence, it can dramatically overcome the signals fading to achieve the diversity gain. Fig. 4 also shows that more significant gains can be obtained with the number of energy harvesting antennas increasing.

### *5.4. BER comparison of energy-harvesting-based coded cooperative MIMO with jointly designed RA codes and general RA codes*

We compare the BER performance of energy-harvesting-based coded cooperative MIMO with jointly designed RA codes and general RA codes. For a fair comparison, general RA codes [24] have the same code rate, code length, and also quasi-cyclic structure as the jointly designed RA codes. Assume $K_I = 1$, $K_E = 3$, $K = 4$, and the number of decoding iterations is ten. It is shown in Fig. 5 that the BER performance of the system employing jointly designed RA codes is much better than that of general RA codes. This is because both Type-I and Type-II girth-4 cycles are canceled in the jointly designed RA codes, and the high coding gain is achieved. On the contrary, there are girth-4 cycles in the general RA codes, which decrease the BER performance of the system.

### *5.5. BER comparison of energy-harvesting-based coded cooperative MIMO with jointly designed RA codes and LDPC codes over AWGN channels*

In this part, we investigate the BER performance of energy-harvesting-based coded cooperative MIMO over additive white Gaussian noise (AWGN) channels. Further, we compare the BER performance of the systems with jointly designed RA codes and LDPC codes in [18]. Jointly designed RA codes are given in Table 1. For a fair comparison, LDPC codes have the same code rate and code length. Assume $K_I = 1$, $K_E = 1$, $K = 2$, and the number of decoding iterations is five. As shown in Fig. 6, when the number of joint decoding iteration is one, the BER performance of jointly

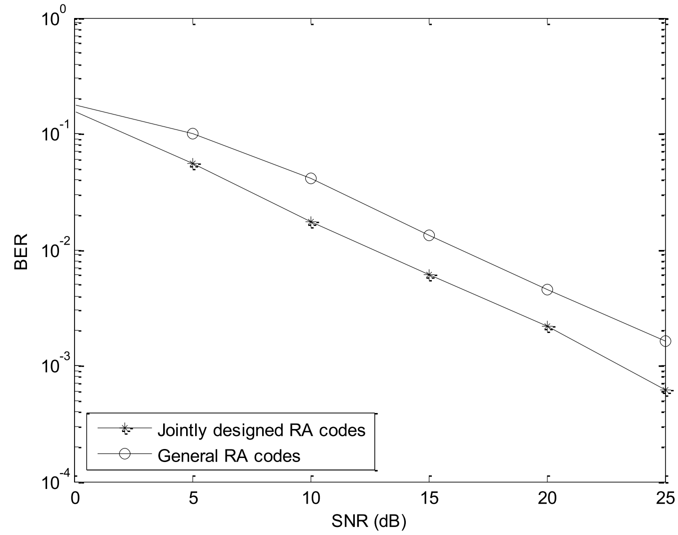



**Fig. 5.** BER comparison of energy-harvesting-based coded cooperative MIMO with jointly designed RA codes and general RA codes.

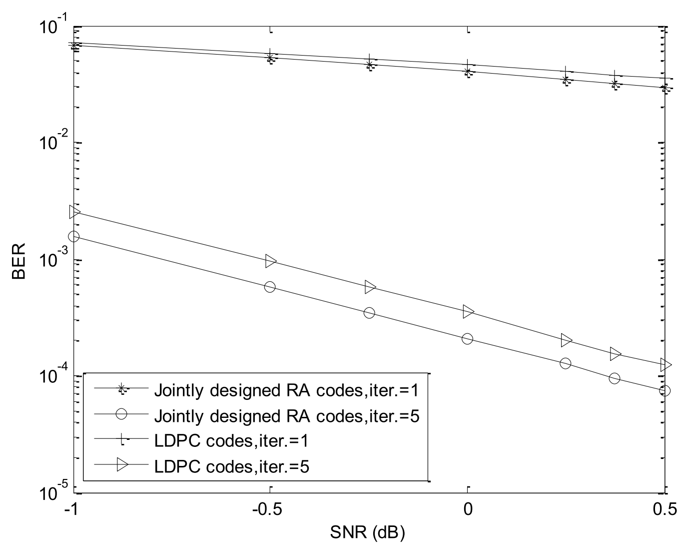



**Fig. 6.** BER comparison of energy-harvesting-based coded cooperative MIMO with jointly designed RA codes and LDPC codes over AWGN channels.

designed RA codes and LDPC codes is similar. It is because the extrinsic information in the decoding algorithm is not exchanged and the superiority of girth-4 cycles cancellation does not appear obviously. When the number of joint decoding iteration increases to five, the BER performance of jointly designed RA codes clearly outperforms that of LDPC codes. This merit can be attributed to the fact that after five joint decoding iterations, the extrinsic information is exchanged sufficiently. The superiority of girth-4 cycles cancellation is revealed and higher coding gain is achieved.

## 6. Conclusion

In this paper, we have studied the energy-harvesting-based RA-coded cooperative MIMO, which combines channel coding, cooperation, and energy harvesting technologies. It can achieve both coding gain and diversity gain. What is more, the lifetime of the system is prolonged due to the energy harvesting technology. To cancel the girth-4 cycles, RA codes for the source and relay are designed jointly. When the relay is equipped with multiple antennas and the optimal antenna selection algorithm is adopted, the outage probability of the system is deduced. For the relay, we demonstrate that the performance of the scheme employing optimal antennas selection algorithm is better than random antennas selection algorithm, or equal power allocation algorithm. Simulation result also shows the superiority of the jointly designed RA codes compared with general RA codes or LDPC codes.

## Appendix A. Proof of Theorem 2

The girth-4 cycles between $\boldsymbol{H}_L$ and $\boldsymbol{H}_R$ (Type-I) is shown as (A.1)

$$\left[\frac{\boldsymbol{I}(p_{j,l})}{\boldsymbol{I}(p_{j+1,l})}\middle|\boldsymbol{D}\right]=\left[\begin{array}{cccc|cccccccc} 0&0&0&1&1&0&0&0&0&0&0&0\\ 1&0&0&0&1&1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1&0&0&0&0&0\\ 0&0&\overset{(B,t)}{1}&0&1&0&1&1&0&0&0&0\\ \hline 0&0&\underset{(1,t)}{1}&0&0&0&0&1&1&0&0&0\\ 0&0&0&1&1&0&0&0&1&1&0&0\\ 1&0&0&0&0&1&0&0&0&1&1&0\\ 0&1&0&0&0&0&1&0&0&0&1&1 \end{array}\right] \tag{A.1}$$

Without loss of generality, we assume the “1” in the last row of $\boldsymbol{I}(p_{j,l})$ with coordinate $(B,t)$. Based on the properties of right-cyclic-shift identity submatrice, we achieve

$$\begin{aligned} t &= (B+p_{j,l}) \bmod B \\ &= p_{j,l} \bmod B \end{aligned} \tag{A.2}$$

To cancel girth-4 cycles in (A.1), the value in the coordinate $(1,t)$ of $\boldsymbol{I}(p_{j+1,l})$ should not be “1”. It means

$$t \neq (1+p_{j+1,l}) \bmod B \tag{A.3}$$

According to (A.2) and (A.3), we obtain the necessary and sufficient condition

$$p_{j,l} \neq (1+p_{j+1,l}) \bmod B$$

which can be further rewritten as

$$p_{j,l} - p_{j+1,l} \neq 1 \bmod B$$

The end of proof.

## References


[1] X.K. Bao, Wireless user cooperation strategies, coding schemes, and applications, Ph.D. dissertation, Lehigh University, 2009.
[2] M. Janani, A. Hedayat, T.E. Hunter, A. Nosratinia, Coded cooperation in wireless communications: space–time transmission and iterative decoding, IEEE Trans. Signal Process. 52 (2) (2004) 362–371.
[3] Sendonaris, E. Erkip, B. Aazhang, User cooperation diversity – part II: implementation aspects and performance analysis, IEEE Trans. Commun. 51 (11) (2003) 1939–1948.
[4] Z. Zhang, T.M. Duman, Capacity-approaching turbo coding and iterative decoding for relay channels, IEEE Trans. Commun. 53 (11) (2005) 1895–1905.
[5] T.J. Richardson, R.L. Urbanke, The capacity of low-density parity-check codes under message-passing decoding, IEEE Trans. Inf. Theory 47 (2) (2001) 599–618.
[6] C.X. Li, G.S. Yue, X.D. Wang, M.A. Khojastepour, LDPC code design for half-duplex cooperative relay, IEEE Trans. Wirel. Commun. 7 (11) (2008) 4558–4567.
[7] C.X. Li, G.S. Yue, M.A. Khojastepour, X.D. Wang, M. Madihian, LDPC-coded cooperative relay systems: performance analysis and code design, IEEE Trans. Commun. 56 (3) (2008) 485–496.
[8] P. Razaghi, W. Yu, Bilayer low-density parity-check codes for decode-and-forward in relay channels, IEEE Trans. Inf. Theory 53 (10) (2007) 3723–3739.
[9] A. Chakrabarti, A. de Baynast, A. Sabharwal, B. Aazhang, Low density parity check codes for the relay channel, IEEE J. Sel. Areas Commun. 25 (2) (2007) 280–291.
[10] S.W. Zhang, F.F. Yang, L. Tang, Network-coding-based multisource multirelay LDPC-coded cooperative MIMO, Trans. Emerg. Telecommun. Technol. 26 (3) (2015) 491–502.
[11] X.M. Chen, Z.Y. Zhang, H.H. Chen, H.Z. Zhang, Enhancing wireless information and power transfer by exploiting multi-antenna techniques, IEEE Commun. Mag. 53 (4) (2015) 133–141.
[12] L.R. Varshney, Transporting information and energy simultaneously, in: 2008 Proceedings of IEEE International Symposium on Information Theory, vols. 1–6, 2008, pp. 1612–1616.
[13] A. Nasir, X.Y. Zhou, S. Durrani, R.A. Kennedy, Relaying protocols for wireless energy harvesting and information processing, IEEE Trans. Wirel. Commun. 12 (7) (2013) 3622–3636.
[14] B.K. Chalise, W.K. Ma, Y.M.D. Zhang, H.A. Suraweera, M.G. Amin, Optimum performance boundaries of OSTBC based AF-MIMO relay system with energy harvesting receiver, IEEE Trans. Signal Process. 61 (17) (2013) 4199–4213.
[15] S. Lee, L. Liu, R. Zhang, Collaborative wireless energy and information transfer in interference channel, IEEE Trans. Wirel. Commun. 14 (1) (2015) 545–557.
[16] B. Gurakan, O. Ozel, J. Yang, S. Ulukus, Energy cooperation in energy harvesting communications, IEEE Trans. Commun. 61 (12) (2013) 4884–4898.
[17] M. Kashef, A. Ephremides, Optimal partial relaying for energy-harvesting wireless networks, IEEE/ACM Trans. Netw. 24 (1) (2016) 113–122.
[18] S.W. Zhang, F.F. Yang, L. Tang, B.T. Maharaj, LDPC-coded cooperation with receive multi-antenna and unknown CSI in the destination, Wirel. Pers. Commun. 72 (4) (2013) 2685–2703.
[19] M.P.C. Fossorier, Quasi-cyclic low-density parity-check codes from circulant permutation matrices, IEEE Trans. Inf. Theory 50 (8) (2004) 1788–1793.
[20] L. Dai, S. Sfar, K. Ben Letaief, Optimal antenna selection based on capacity maximization for MIMO systems in correlated channels, IEEE Trans. Commun. 54 (3) (2006) 563–573.
[21] Y.L. Zou, B.Y. Zheng, J. Zhu, Outage analysis of opportunistic cooperation over Rayleigh fading channels, IEEE Trans. Wirel. Commun. 8 (6) (2009) 3077–3085.
[22] K.L. Chung, A Course in Probability Theory, Academic Press, New York, 1974.
[23] F. Molisch, M.Z. Win, MIMO systems with antenna selection, IEEE Microw. Mag. 5 (1) (2004) 46–56.
[24] M. Noor-A-Rahim, N. Zhang, S.Y. Wang, Performance estimation of finite-length repeat-accumulate codes, IET Commun. 9 (15) (2015) 1902–1905.



**Shunwai Zhang** received the Ph.D. degree in College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China, in 2014. Now he is an assistant professor in College of Telecommunications and Information Engineering, Nanjing University of Posts and Telecommunications (NUPT), Nanjing, China. His current research interests are in advanced channel coding, cooperative communications techniques and green communications.

**Fengfan Yang** received the Ph.D. degree from Department of Radio Engineering, Southeast University, Nanjing, China, in 1997. He held positions of visiting scholar and visiting professor at Department of Electrical and Computer Engineering (ECE), McGill University, Montreal, Quebec, Canada, in 2003 and 2007, respectively. Now he is a full professor in College of Electronic and Information Engineering, NUAA. His current research focuses on advanced channel techniques and channel capacity analysis, MIMO wireless communications and coded cooperative communications, etc.

**Rongfang Song** received the Ph.D. degree in Telecommunications Engineering, Southeast University, Nanjing, in 2001. During 2002–2003, he was a research associate with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong. Since 2002, he has been a Professor with the Department of Telecommunications Engineering, NUPT. He is also with the National Mobile Communications Research Laboratory, Southeast University. His research interests are in the area of broadband wireless communications, with current focus on network multiple-input–multiple-output, interference management in HetNets, and compressed sensing-based signal processing.