CMT117 Exercises:
Belief Revision

Question 1. Assume $L = \{p, q\}$. For each of the following sets $K \subseteq SL$, state whether K is a belief set (see slide 45). Justify your answers.

(i) $K = \{p, \neg q\}$.

(ii) $K = Cn(p \to q)$.

Question 2. Consider the list of AGM postulates for revision (see slide 51). It was mentioned in lecture that some researchers have questioned the rationality of some of these postulates. E.g., the Success postulate has been questioned on the grounds that real human beings do not always blindly believe new information that comes to them. Choose one of the other postulates that you, yourself, find questionable, and give an example from everyday human reasoning in which it does not make sense.

Question 3. Suppose $L = \{p, q, r\}$ and let $K = Cn((p \wedge q) \vee (r \wedge (p \vee q)))$. Let $\preceq$ be the plausibility ordering for K over the set of valuations that is represented in tabular form as follows:

TTT	TFF	FFF
TTF	FTF	FFT
TFT		
FTT		

As usual, each valuation is represented as a triple *abc* denoting the truth-values of p, q, r respectively, and the further to the left a valuation appears in the above table, the more plausible it is deemed to be.

Let $* = *(\preceq)$ be the resulting revision operator for K (see slides 57-63). For each of the following sentences A, write down both *(i)* the set $Mod(K * A)$,

and *(ii)* the revised belief set $K * A$ (your answer for *(ii)* should be written in the form $Cn(B)$ for some suitable formula B).

(a) $A = \neg q \wedge \neg r$

(b) $A = \neg r$

(c) $A = \neg r \rightarrow p$

(d) $A = \neg(p \vee q)$

(e) $A = \top$

(f) $A = \bot$

Question 4. Assume $L = \{p, q\}$ (so with set of valuations $\{TT, TF, FT, FF\}$). Let $K = Cn(p \wedge q)$. Write down a plausibility ordering $\leq$ for this K in tabular form (as in Question 3 above) such that the resulting revision operator $*$ exhibits the following behaviour:

$$\neg p \in K * (\neg(p \wedge q)).$$

Hint: Remember that the left-most column in the table (i.e., most plausible valuations) **must** contain precisely $Mod(K)$.

Question 5. Assume $L = \{p, q, r\}$ and let $\leq$ be the following plausibility order:

TTT	TFF	FTT	FTF
TFT	TTF		FFF
			FFT

(i). What is the belief set $K(\leq)$ associated to this order? (Your answer should be of the form $Cn(A)$ for some suitable formula A).
Recall that $*_N$ denotes natural revision (see slides 84-90), and $*_L$ denotes lexicographic revision (see slides 92-94). Write down the following revised plausibility orders in tabular form:

(ii). $\leq^{*_N}_{(\neg p \wedge r) \vee (p \wedge q \wedge \neg r)}$, *(iii).* $\leq^{*_L}_{\neg((p \wedge q) \vee r)}$

Question 6. Recall the postulate P3 from slide 77:

(P3) If $\neg A \notin K(\leq^*_B)$ then $A \in K((\leq^*_A)^*_B)$

Recall also the postulate given at the bottom of slide 94. Let's call it P4:

(P4) If $A \wedge B$ is consistent then $A \in K((\leq^*_A)^*_B)$

Give a direct proof showing that P3 follows from P4. You may assume $*$ satisfies the basic requirements $B \in K(\leq^*_B)$ and that $K(\leq^*_B)$ is a belief set.