# CMT117 Exercises:
# Nonmonotonic Reasoning

**Question 1.** Suppose $L = \{p, q, r\}$ and let $R = (V, \preceq)$ be a ranked model with $V$ equal to the set of all valuations and $\preceq$ the normality ordering represented in tabular form as follows:

| | | | |
|---|---|---|---|
| TFT | TTF | FTF | FTT |
| TFF | FFF | TTT | |
| | FFT | | |

Here, each valuation is represented as a triple $abc$ denoting the truth-values of $p, q, r$ respectively (e.g., $FTT$ is the valuation in which $p$ is false and both $q$ and $r$ are true), and the further to the left a valuation appears in the above table, the more normal it is deemed to be. Let $\mid\!\sim_R$ denote the consequence relation defined by this ordering (see slide 31 of the presentation slides). For each of the following conditionals, determine whether they hold in $R$:

*(a)* $p \mid\!\sim_R \neg q$

*(b)* $\neg p \mid\!\sim_R \neg q$

*(c)* $q \vee \neg r \mid\!\sim_R q$

*(d)* $p \wedge q \mid\!\sim_R r$

*(e)* $p \to (q \wedge r) \mid\!\sim_R \neg p$

*(f)* $\top \mid\!\sim_R r$

*(g)* $\neg q \vee (p \wedge q) \mid\!\sim_R \bot$

**Question 2.** Assume again $L = \{p, q, r\}$. Assume $V$ is equal to the set of all valuations for $L$. Write down a normality ordering $\preceq$ in tabular form (as in Question 1 above) such that the rational consequence operator $\mathrel{|\!\sim}_R$ associated to the ranked model $R = (V, \preceq)$ simultaneously satisfies both the following conditionals:

$$\neg p \mathrel{|\!\sim}_R r, \qquad \neg p \wedge \neg q \mathrel{|\!\not\sim}_R r.$$

[Note: A nice side-effect of your answer will be that it provides a counterexample to show that Monotonicity fails for rational consequence (see slide 12)]

**Question 3.** Assume $L = \{p, q\}$. For each of the following ranked models $R$, determine whether the associated rational consequence relation $\mathrel{|\!\sim}_R$ satisfies the rule CP (Consistency Preservation) (see slide 18). In case it does not satisfy CP, give a sentence $A$ such that $A \not\models \bot$ but $A \mathrel{|\!\sim}_R \bot$.

*(a)* $R = (V, \preceq)$, with $V = \{TT, TF\}$ and $\preceq$ given in tabular form below:

| | |
|---|---|
| TT | TF |

*(b)* $R = (V, \preceq)$, with $V = \{TT, TF, FT, FF\}$ and $\preceq$ given in tabular form below:

| | | |
|---|---|---|
| TF<br>FT | TT | FF |

**Question 4.** Show that the following rule holds for all rational consequence relations $\mathrel{|\!\sim}$:

$$\frac{A \mathrel{|\!\sim} B \qquad B \mathrel{|\!\sim} A \qquad A \mathrel{|\!\sim} C}{B \mathrel{|\!\sim} C}$$

(in words, if $A$ and $B$ are consequences of each other, then every consequence $C$ of $A$ is also a consequence of $B$).
Hint: You have 2 ways to show this: *(i)* find a derivation of this rule from the KLM rules, or *(ii)* (by the Representation Theorem for rational consequence relations on slide 35) show that $\mathrel{|\!\sim}_R$ satisfies this rule for any arbitrary ranked model $R$.

**Question 5.** By using the theorem on slide 33, show that the following rule *fails* for some rational consequence relations (and choice of $A, B, C$)

$$\frac{A \vee B \mathrel{|\!\sim} \neg A}{C \wedge B \mathrel{|\!\sim} \neg A}$$