Math 449: Numerical Methods
Homework 4
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Part 1: Theory

Problem 1. Suppose $A \in \mathbb{R}^{n\times n}$ is nonsingular and has a decomposition $A = LU$.

Prove that the decomposition is unique.

(Hint: begin by multiplying both sides of $LU = \tilde{L}\tilde{U}$ by $\tilde{L}^{-1}$ left & $\tilde{U}^{-1}$ right.

Proof: Since $A \in \mathbb{R}^{n\times n}$ is nonsingular $\Rightarrow \det(A) \neq 0$

Assume $A = LU = \tilde{L}\tilde{U}$, then we have

$\det(A) = \det(L)\det(U) = \det(\tilde{L})\det(\tilde{U}) \neq 0$

therefore $L, U, \tilde{L}, \tilde{U}$ are also nonsingular hence the inverse must exist.

$\therefore \quad LU = \tilde{L}\tilde{U} \quad \Rightarrow \quad \underbrace{\tilde{L}^{-1}L}_{\text{unit lower triangular}} = \underbrace{\tilde{U}U^{-1}}_{\text{unit upper triangular}}$

$\Rightarrow \quad \tilde{L}^{-1}L = \tilde{U}U^{-1} = I$

$\therefore \quad \tilde{L} = L$, and $\tilde{U} = U$.