Math 449: Numerical Applied Mathematics
Lecture 28

11/08/2017 Wenzhen

Today's topic: Newton-Cotes Quadrature Rules

Newton-Cotes quadrature rules

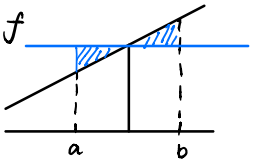
$n=1$ Trapzoid rule $\int_a^b f(x)\,dx$

$n=2$ Simpson's rule

$n=0$ $\int_a^b f(x)\,dx \approx (b-a)f(c)$ any $c \in [a,b]$

If f is constant, then $\int_a^b f(x)\,dx \overset{\text{Exact}}{=} f(c)(b-a)$ $\quad c=\frac{a+b}{2}$

For linear f



equality holds if we choose $c=\frac{a+b}{2}$

Note: trapezoid rule is only exact for degree 1 (linear) polys.

Simpson's rule is exact for degree 2 polynomials.

but is also exact (because we use the midpt, too)

Ex Show that f is cubic polynomial, then $\int_a^b f(x)\,dx = (b-a)\,x$ $\left(\frac{1}{6}f(a)+\frac{2}{3}f\left(\frac{a+b}{2}\right)+\frac{1}{6}f(b)\right)$

Hint: It suffices to prove this for $f(x)=x^3$

then use linearity of the integral.