Math 449: Numerical Methods
Homework 1
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1. a) $I = (0,1]$ b) $I = [0,1)$ c) $I = [0,\infty)$

$y = x$ $y = x^2$ $y = x$

2. Simple iteration $x_{k+1} = \cos(x_k)$ always converges to $\xi = \cos\xi$

Use contraction mapping theorem to prove it.

Proof. let $g(x) = \cos(x)$ g is a contraction on $[-1,1]$

By MVT, for any $x_1, x_2 \in [-1,1]$, we have

$$|g(x_1) - g(x_2)| = |g'(\eta)(x_1 - x_2)|$$
$$\leq |\sin(\eta)|\,|x_1 - x_2|$$

$\because$ $x \mapsto \sin(x)$ is monotonically increasing on $[-\frac{\pi}{2}, \frac{\pi}{2}]$,

$\therefore$ $g'(x) > 0$ on $x \in [-1,1]$ $\sin(\eta) \leq \sin(1) \approx 0.84$

By contraction mapping theorem, $L = \sin(\eta)$

so it has a unique fixed pt $\xi \in [-1,1]$.