Math 493: Mathematical Statistics
Lecture 04

09/08/2017

Today's topic: Conditional Probability and Independence

X : set of outcomes

$E, F \subset X \quad P(F) > 0$

$P(E|F) =$ conditional prob of E given F $= \dfrac{P(E \cap F)}{P(F)}$

Claim $\quad P(F) = P(F|E_1)P(E_1) + \cdots + P(F|E_n)P(E_n)$

In fact $\quad F = (F \cap E_1) \cup \cdots \cup (F \cap E_n)$

$$P(F) = P(F \cap E_1) + \cdots + P(F \cap E_n)$$

$$= \frac{P(F \cap E_1)}{P(E_1)} P(E_1) + \cdots + \frac{P(F \cap E_n)}{P(E_n)} P(E_n)$$

So $\quad P(F) = P(F|E_1)P(E_1) + \cdots + P(F|E_n)P(E_n)$

Example 3h.

I = event that a twin pair is identical.

S = event that a twin pair is of same sex.

$P(S) = 0.64$

$$\underset{0.64}{P(S)} = \underbrace{P(S|I)}_{1} \underset{?}{P(I)} + \underset{\frac{1}{2}}{P(S|I^c)} \underset{?\ \ 1-?}{P(I^c)}$$

fraternal twins

$0.64 = x + \frac{1}{2}(1-x) \qquad\qquad P(I) = 2P(S) - 1$

$= x - 0.5x + 0.5$

$0.5x = 0.14$

$x = 0.28$

Bayes' Formula

$E_1, \cdots, E_n$ mutually exclusive complete set of events.

F event

$$P(E_i|F) = \frac{P(F|E_i)P(E_i)}{P(F)}$$

$$P(E_i|F) = \underset{\overset{\|}{P(F|E_i)}}{\frac{P(E_i \cap F)}{P(E_i)}} \frac{P(E_i)}{P(F)} = \frac{P(F|E_i)P(E_i)}{P(F)} \quad \blacksquare$$

Problem 3.49

$C =$ patient has cancer $\qquad P(C) = 0.7$

$E =$ elevated PSA level

Relability

$P(E|C) = 0.268$

$P(E|C^c) = 0.135$

Problem: find $P(C|E) = ?$

Bayes Formula

$$P(C|E) = \frac{P(E|C)\,P(C)}{P(E)} = \frac{0.268 \cdot 0.7}{} = 0.82$$

$$P(E) = P(E|C)\,P(C) + P(E|C^c)\,P(C^c)$$

Conditional Prob is a probability

$E \longrightarrow P(E|F)$ is additive, takes values in $[0,1]$

and $P(X|F) = 1$

Fundamental defn: Independence.

$P(C)\cdot P(E)$ $\qquad$ We say 2 events E and F are independent if $P(E\cap F) = P(E)\cdot P(F)$

$P(C|E) = P(C)$ $\qquad$ if $P(F) > 0$. $P(E|F) = P(E)$.

$P(A|B) = P(A)$.