Math 493: Mathematical Statistics
Lecture 03

Sep 6th, 2017

Today's topic: Vocab for Prob Theory + 3 Axioms

Vocabulary for prob. theory

X: set of outcomes

$\mathcal{F}$: collection of subsets of X (algebra of event)

P: probability measure (a function $P: \mathcal{F} \to [0,1]$)

$\mathcal{F}$ $\quad \phi$ impossible event

X $\quad$ sure events

$E \in \mathcal{F}$, $E^c = X \setminus E$

$E, F \in \mathcal{F}$

$E \cup F$ = either E or F or both

$E \cap F$ = both E and F

Axioms of events

① $X \in \mathcal{F}$

② $E \in \mathcal{F} \Rightarrow E^c \in \mathcal{F}$

③ $E_1\ E_2 \cdots \in \mathcal{F} \Rightarrow \bigcup_{n=1}^{\infty} E_n \in \mathcal{F}$

Axioms of probability

$P: \mathcal{F} \to [0,1]$ $\quad (0 \leq P(E) \leq 1)$

① $P(X) \leq 1$

② (convertable additivity) If $E_1, E_2, \ldots, \in \mathcal{F}$, mutually disjoint ($E_i \cap E_j = \phi$)

$$\Rightarrow P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n) \qquad i \neq j$$

Probabilities

① $P(E^c) < 1 - P(E)$ $\quad X = E \cup E^c \quad 1 = P(X) = P(E) + P(E^c)$

② $P(\emptyset) = 0 \quad P(\emptyset) = 1 - P(X) = 1 - 1 = 0$

③ $E \subset F \Rightarrow P(E) \leqslant P(F)$

In fact $\quad F = E \cup (F \setminus (E \cap F))$

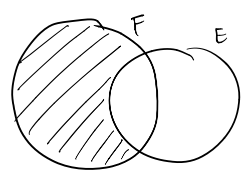
$$P(F) = P(E) + \underbrace{(P(E) - P(E \cap F))}_{\geqslant 0}$$

$$\geqslant P(E)$$

Note: $P(F \overset{\text{set difference}}{\setminus} E \cap F) = P(F) - P(E \cap F)$

We get $P(F) = P(E) + P(F) - P(E \cap F)$

④ Claim: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$



in fact, $E \cup F = E \cup (F \setminus E \cap F) \Rightarrow P(E \cup F) = P(E) + (P(F) - P(E \cap F))$

Excercise 10 Ch2.

School students wearing Ring / necklaces.

Set of outcomes

X = set of $\forall$ students

$\mathcal{F}$ = $\forall$ subsets

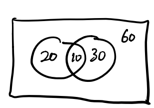
probability function If $E \in \mathcal{F}$, $P(E) = \dfrac{\# E}{\# X}$

known:

20% ring

30% necklace

60% neither



Define:

R: a set of students wearing ring

N: set of students wearing necklace.

$0.6 = P((R \cup N)^c) = 1 - P(R \cup N)$

$P(R \cup N) = 1 - P(\text{neither}^c) = 1 - 0.6 = 0.4$

$P(R \cap N) = P(R) + P(N) - P(R \cup N) = 0.2 + 0.3 - 0.4 = 0.1$ $\quad P(R \cup N) = 1 - 0.6 = 0.4$