The Network Effects of Agency Conflicts

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Equity Holding Networks

Equity Holding Network: organizations hold each other's shares (or other obligations).

Pervasive and important.

Firms in equity holding networks contribute to roughly 70% of fixed capital and 60% of employment in China (Shi, Townsend, Zhu, 2019). International evidence (Jackson, Pernout,2019)

How do equity holding networks affect contagion?

Prior Work

1. The role of network structure, exogenous shocks: diversification vs. integration

   Allen and Gale [2000], Elliott, Golub, and Jackson [2014], Cifuentes, Ferrucci, and Shin [2005], Acemoglu et al. [2015] and Greenwood et al. [2015]

2. Endogenize network formation in anticipation of possible shocks

   Leitner [2005], Zawadowski [2013], Babus [2016], Babus and Hu [2015], Cabrales et al. [2017] and Erol and Vohra [2018].

3. Externalities and optimal network to mitigate contagion.

   Jackson and Pernoud [2019], Kanik [2018], and Shu [2018].

This Paper

Firm level frictions not just network structure, a key role in amplifying or muting the propagation of exogenous shocks.

Firms make an investment choice in response to an exogenous shock, but

[1.] Choice subject to frictions - default cost, limited liability, interest conflicts between shareholders and managers, or moral hazard.

[2.] Default costs or limited liability: shocks are muted by firm-level investment choices.

This Paper

[3.] Interest conflicts or moral hazard: shocks are amplified by firm-level investments choices.

[4.] In the presence of interest conflicts, denser or more integrated networks need not facilitate the propagation of shocks.

[5.] If interest conflict sufficiently large, the effect of an idiosyncratic shock on total value will not diminish as the number of firms grows large, at least it is true under symmetric network

Road Map

- Benchmark without frictions
- Models with various frictions
- Propagation of shocks with agency conflicts
- Propagation and network structure
- Aggregate implications of the conflicts within firms

Benchmark

C_{ij}: fraction of firm j's value held by firm firm i.

$\mathcal{O}_i$ is set of investments available to firm i

Each investment in $\mathcal{O}_i$ is characterized by its risk-return profile, indexed by σ, generating payoff

$$x_i(\sigma) = a_i(\mu(\sigma) + \sigma z_i + b_i)$$

$a_i, b_i \geq 0$ and $\sigma > 0$.

Random variables z_i have zero mean, standard deviation 1 and are i.i.d across firms.

Benchmark

μ is a smooth(almost) function defined over a compact set $[0, s]$ satisfying the following:

1. $\mu(\sigma) + b_i > 0, \ \ \forall \sigma \in [0, s]$.
2. $\exists \ \sigma^{BM} \in (0, s)$, s.t. $\mu'(\sigma) > 0$, if $\sigma < \sigma^{BM}$; $\mu'(\sigma) < 0$, if $\sigma > \sigma^{BM}$.
3. $\mu'' \leq 0$.

ex. the investment model with convex adjustment cost

Benchmark

Absent frictions, the value of firm i, V_i, satisfies

$$V_i = \sum_j C_{ij} V_j + x_i(\sigma_i), \quad \forall i \in N. \tag{1}$$

In matrix notation:

$$V = (I - C)^{-1} x(\sigma) \tag{2}$$

d_{ij} is the $(i,j)^{th}$ entry of matrix $D = (I - C)^{-1}$.

Benchmark

1. Firm i by itself will choose $\hat{\sigma}_i$ so that

$$\hat{\sigma}_i \in \arg \max_{\sigma_i \in \mathcal{O}_i} E[V_i] = \arg \max_{\sigma_i \in \mathcal{O}_i} \sum_j d_{ij} E[x_j(\sigma_j)] = \arg \max_{\sigma_i \in \mathcal{O}_i} E[x_i(\sigma_i)] \tag{3}$$

2. The efficient profile of investments, $\sigma^{BM} = (\sigma_1^{BM}, \ldots, \sigma_n^{BM}) \in \prod_{i \in N} \mathcal{O}_i$, satisfies:

$$\sigma^{BM} \in \arg \max_{\sigma \in \prod \mathcal{O}_i} E[\sum_i x_i(\sigma_i)]. \tag{4}$$

Benchmark

Proposition

The individual optimum coincides with the social optimum, that is $\hat{\sigma} = \sigma^{BM}$

Absent frictions, no effect of equity holding networks on investment decisions. The value held by outside investment $\sum_i x_i(\sigma_i)$ independent of the network structure.

Default Cost Model

When a firm's value becomes sufficiently small, it hits a failure threshold at which it discontinuously loses further value.

Theoretical value: $\tilde{V}_i = \sum_j C_{ij} V_j + x_i(\sigma_i) \ \forall i \in N$

Let χ be the characteristic function and τ_i a firm specific threshold

Actual value: $V_i = \sum_j C_{ij} V_j + x_i(\sigma_i) - \lambda \chi_{\tilde{V}_i \leq \tau_i} \ \forall i \in N.$

Default Cost Model

Theoretical value: $\tilde{V}_i = \sum_j C_{ij} V_j + x_i(\sigma_i) \; \forall i \in N$

Actual value: $V_i = \sum_j C_{ij} V_j + x_i(\sigma_i) - \lambda \chi_{\tilde{V}_i \leq \tau_i} \; \forall i \in N.$

$$g(\tilde{V}_i) = \lambda \chi_{\tilde{V}_i \leq \tau_i}$$
$$V_i = \tilde{V}_i - g(\tilde{V}_i).$$

Treat threshold τ_i as random (independent of $\tilde{V}_i$) rather than a fixed parameter, to 'smooth' away the discontinuity by focusing on the expected value of $g(\tilde{V}_i)$.

Limited Liability

Theoretical value: $\tilde{V}_i = \sum_j C_{ij} V_j + x_i(\sigma_i) \; \forall i \in N$

$$g(\tilde{V}_i) = \min\{\tilde{V}_i, 0\}$$

$$V_i = \max\{\tilde{V}_i, 0\} = \tilde{V}_i - g(\tilde{V}_i)$$

Interest Conflict

Manager earns a fraction α of the firm's value while shareholders retain the remaining $(1-\alpha)$ fraction.

Manager bears private cost of termination (λ_0) if theoretical value $(\tilde{V}_i)$ falls below threshold, τ_{i0}, enjoys private benefit (λ_1) if theoretical value exceeds τ_{i1}.

Manager's objective function is:

$$V_i = \tilde{V}_i - \lambda_0 \chi_{\tilde{V}_i \leq \tau_{i0}} + \lambda_1 \chi_{\tilde{V}_i \geq \tau_{i1}} + \frac{\alpha}{1-\alpha} \tilde{V}_i \tag{5}$$

$$g(\tilde{V}_i) = \lambda_0 \chi_{\tilde{V}_i \leq \tau_{i0}} - \lambda_1 \chi_{\tilde{V}_i \geq \tau_{i1}} - \frac{\alpha}{1-\alpha} \tilde{V}_i$$

$$V_i = \tilde{V}_i - g(\tilde{V}_i) \tag{6}$$

Manager's objective function is:

$$V_i = \tilde{V}_i - g(\tilde{V}_i) \tag{7}$$

with $g'(\tilde{V}_i) < 0$, and $\tilde{V}_i = \sum_j C_{ij} \tilde{V}_j + x_i(\sigma_i)$

Moral Hazard

Manager's payoff is $h(\tilde{V}_i)$ since only $\tilde{V}_i$ is observable and contractable. $\tilde{V}_i$ depends on the unobservable investment σ and effort e.

Shareholders's problem is to motivate the optimal (σ^*, e^*)

$$\max_{e,\sigma} E[\tilde{V}_i|\sigma, e]$$

$$\text{s.t. } E[h(\tilde{V}_i)|\sigma^*, e^*] \geq \sup_{x',e'} E[h(\tilde{V}_i)|\sigma', e'].$$

For suitable κ, we can reformulate as:

$$\max_{e,\sigma} E[\tilde{V}_i] - \kappa E[g(\tilde{V}_i;\theta)]$$

Set $g(\tilde{V}_i;\theta) = h(\tilde{V}_i) - \sup_{e',x'} E[h(\tilde{V}_i)]$.

General Case

Two Models - Emphasize the difference !!

Agent i solves

$$\max_{\sigma_i} EV_i = E[\tilde{V}_i - g(\tilde{V}_i)] \tag{8}$$

Model # 1: default cost or limited liability

$$\tilde{V}_i = \sum_j C_{ij} V_j + a_i[\mu(\sigma_i) + b_i + \sigma_i z_i]$$

Model # 2: interest conflicts and moral hazard

$$\tilde{V}_i = \sum_j C_{ij} \tilde{V}_j + a_i[\mu(\sigma_i) + b_i + \sigma_i z_i]$$

Model #1 - Shocks will be dampened during propagation. Why?

Alignment of interests between shareholders and managers

- Suppose a bad shock, managers take action to maximize the value of the firm and reduce the effect of initial shock $\Longrightarrow$ increase the value of holding firms and mitigate the effect of initial shock
- Holding firm further take optimal response to mitigate the reduced shocks, overall, the effect of initial shock tends to dampen - the magnitude of dampening is an empirical question.

Model # 2

Focus on Model #2.

Agent i solves

$$\max_{\sigma_i} EV_i = E[\tilde{V}_i - g(\tilde{V}_i)] \tag{9}$$

$$\tilde{V}_i = \sum_j C_{ij} \tilde{V}_j + a_i[\mu(\sigma_i) + b_i + \sigma_i z_i]$$

Equilibria Profiles

A profile of investments $\sigma^e = (\sigma_1^e, \sigma_2^e, \ldots, \sigma_n^e)$ forms an equilibrium if for all $i \in N$, holding σ_{-i}^e fixed, $\sigma_i^e = \arg\max_\sigma E[V_i] = E[\tilde{V}_i - g(\tilde{V}_i)]$. First Order Condition implies

$$\mu'(\sigma_i^e) = \frac{E[g'(\tilde{V}_i)z_i]}{1 - E[g'(\tilde{V}_i)]}. \tag{10}$$

Model # 2 (Quadratic Case)

Quadratic Case: $g(\tilde{V}) = \tilde{V}_i(\lambda - \kappa \tilde{V}_i/2a_i)$

1. λ declines when the share in hands of managers increases.
2. In interest conflicts case, κ captures the extent of the misalignment of incentives between manager and firm.
3. In the moral hazard case, a positive κ also capture the cost paid by the firm to motivate managers.

F.O.C

$$\mu'(\hat{\sigma}_i) = -\kappa \frac{a_i d_{ii} \hat{\sigma}_i}{(1-\lambda)a_i + \kappa \sum_j d_{ij} a_j [\mu(\hat{\sigma}_j) + b_j]}. \tag{11}$$

Recall $D = (I - C)^{-1}$ captures all the direct and indirect holdings.

Model # 2

Quadratic Case

$$\mu'(\hat{\sigma}_i) = -\kappa \frac{a_i d_{ii} \hat{\sigma}_i}{(1-\lambda)a_i + \kappa \sum_j d_{ij} a_j [\mu(\hat{\sigma}_j) + b_j]}. \tag{12}$$

1. when $\kappa = 0$, no friction, linear system, all firms choose social optimum σ^{BM}.
2. when $\kappa > 0$, all firms take excess risk, a drop in value $\sum_{j \neq i} d_{ij} a_j [\mu(\hat{\sigma}_j) + b_j]$ will induce a higher $\hat{\sigma}_i$, far away from the efficient.
3. when $\kappa < 0$, all firms take less risk, a drop in value $\sum_{j \neq i} d_{ij} a_j [\mu(\hat{\sigma}_j) + b_j]$ push firms to move forward to the efficient action.

Explain Here Intuitively

Proposition

All terms of $F_{ij}, \partial a_{ik}, \partial b_{ik}$ are evaluated at the equilibrium profile $\sigma^e = (\sigma_1^e, ..., \sigma_n^e)$. Then,

$$a_k \frac{\partial \sigma_i^e}{\partial a_k} = \sum_{j \neq i} d_{ij} F_{ij} a_k \frac{\partial \sigma_j^e}{\partial a_k} + d_{ik} \partial a_{ik} \tag{13}$$

and

$$\frac{\partial \sigma_i^e}{\partial b_k} = \sum_{j \neq i} d_{ij} F_{ij} \frac{\partial \sigma_j^e}{\partial b_k} + d_{ik} \partial b_{ik}. \tag{14}$$

Skip this slides

$$f(i) = \frac{E[g'(\tilde{V}_i)z_i]}{1 - E[g'(\tilde{V}_i)]}$$

$$f_{ij} = \frac{\partial f(i)}{\partial \sigma_j}$$

$$F_{ii} = \frac{-\kappa}{[\mu''(\sigma_i) - f_{ii}](1 - E[g'(\tilde{V}_i)])}[\mu'(\sigma_i)^2 + 1],$$

$$F_{ij} = \frac{a_j F_{ii}}{a_i} \frac{\mu'(\sigma_i)\mu'(\sigma_j) + \delta_{ij}}{\mu'(\sigma_i)^2 + 1},$$

$$\partial a_{ik} = a_k \frac{F_{ii}}{a_i} \frac{[\mu'(\sigma_i)(\mu(\sigma_k) + b_k) + \sigma_k \delta_{ik}]}{\mu'(\sigma_i)^2 + 1},$$

and

$$\partial b_{ik} = a_k \frac{F_{ii}}{a_i} \frac{\mu'(\sigma_i)}{\mu'(\sigma_i)^2 + 1}.$$

Skip this slides

Three Channels for Propagation

i. Direct channel, captured by ∂a_{ik} and ∂b_{ik}, respectively

ii. value channel, the second, which we call the value channel of networks, $d_{ik}\partial a_{ik}$ $(d_{ik}\partial b_{ik})$. Recall that d_{ij} is the share of firm j held by i directly or indirectly since

$$D = (I - C)^{-1} = I + C + C^2 + \dots \ .$$

iii. investment channel , via the investments taken by other firms, $\sum_{j \neq i} d_{ij} F_{ij} a_k \frac{\partial \sigma_j^e}{\partial a_k}$.

Propagation of Shocks - Model # 2

Denote $\tilde{d}_{ij} = d_{ij}F_{ij}$, and $\tilde{D} = (\tilde{d}_{ij})_{N\times N}$

$$a_k \frac{\partial \sigma^e}{\partial a_k} = [I - \tilde{D}]^{-1} V^{a_k}, \quad \frac{\partial \sigma^e}{\partial b_k} = [I - \tilde{D}]^{-1} V^{b_k},$$

where $\sigma^e = (\sigma_1^e, ..., \sigma_n^e)$, $V^{a_k} = (d_{1k}\partial a_{1k}, ..., d_{nk}\partial a_{nk})$ and $V^{b_k} = (d_{1k}\partial b_{1k}, ..., d_{nk}\partial b_{nk})$ capture the net network effect without the investment effect as a response.

Proposition

i. *If* $\kappa = 0$, $\tilde{D} = 0$, *only pure value channel, investment channel disappear*

ii. *if* $\kappa > 0$, $\tilde{d}_{ij} > 0 \ \forall i, j$, *shocks was amplified, all entries are positive*

$$(I - \tilde{D})^{-1} = I + \tilde{D} + \tilde{D}^2 + ...$$

iii. *if* $\kappa < 0$, $\tilde{d}_{ij} < 0 \ \forall i, j$, *still dampen.*

The Role of Networks Structure

Role of Network Structure - Model # 2

$\mathbf{R}(\boldsymbol{\sigma}) = (R(\sigma_1), R(\sigma_2), ..., R(\sigma_n))'$,with $R(\sigma) = -\frac{\sigma}{\mu'(\sigma)}$. Thus, the equilibrium investment choices can be written as

$$\mathbf{R^e}(\boldsymbol{\mu}) = (\text{diag}(\mathbf{D}))^{-1}\left[\mathbf{Du} + \frac{1-\lambda}{\kappa}\mathbf{1}\right] \tag{15}$$

with

$$R(u) = R(\mu^{-1}(u)) := -\mu^{-1}(u)/\mu'(\mu^{-1}(u)) \tag{16}$$

Lemma

When $\kappa > 0$, *we have*

$$\mathbf{R^e}(\mathbf{u}) = (diag(\mathbf{D}))^{-1}\left[\mathbf{D}\mathbf{u} + \frac{1-\lambda}{\kappa}\mathbf{1}\right]$$

with $R^e(\mathbf{u}) = (R^e_1, ..., R^e_n) = (R^e(\mu^{-1}(u_1)), ..., R^e(\mu^{-1}(u_n)))$, *and* $R'(u_i) > 1$.

Role of Network Structure - See more in our paper

Proposition

1. *Consider the class of acyclic holding networks, that is,* $d_{ii} = 1, \ \forall i \in N$*. For any two holding networks* $\mathbf{C}$ *and* $\mathbf{C}'$ *such that*

$$d'_{ij} \geq d_{ij} \ \ \forall i, j, and \ \exists i, j, s.t. d'_{ij} > d_{ij}$$

   then, $\mathbf{u}(\mathbf{C}') \geq \mathbf{u}(\mathbf{C})$ *and* $\exists i, s.t. \mu'_i > \mu_i$*. On risk taking, we have* $\sigma_i^e(\mathbf{C}') \leq \sigma_i^e(\mathbf{C})$ *with strictly inequality for at least one* i*.*
2. *Within the same holding networks, suppose* $d_{i'j} \geq d_{ij}, \ \forall j$*, then* $\mu_{i'} \geq \mu_i$

Aggregate Implications of Agency Conflicts

Notations:

Aggregate payoff: $y = \sum_{i \in N} a_i(\mu(\sigma_i) + \sigma_i z_i)$

Initial exogeneous shock: $\delta \log(\boldsymbol{a}) = (\delta \log(a_1), ..., \delta \log(a_N))'$.

$\boldsymbol{x} = (x_1, ..., x_N)'$ with $x_i = a_i(\mu(\sigma_i) + \sigma_i z_i)$, and $\boldsymbol{s}^x = (s_1^x, ..., s_N^x)$, with $s_i^x = x_i / y$ being firm i's fraction of total output.

Decompose $\delta y / y_0 = \delta y^{(1)}/y_0 + \delta y^{(2)}/y_0 + \delta y^{(3)}/y_0$ with $y_0 = Ey = \sum_i a_i \mu_i$, and $\mu_i = \mu(\sigma_i)$

$$\begin{aligned} \frac{\delta y^{(1)}}{y_0} &= \sum_i \frac{a_i(\mu_i + \sigma_i z_i)}{y_0} \delta \log a_i \\ \frac{\delta y^{(2)}}{y_0} &= \sum_i \frac{a_i z_i \mu_i}{y_0} \delta \log(\mu_i)/\mu'(\sigma_i) \\ \frac{\delta y^{(3)}}{y_0} &= \sum_i \frac{a_i \mu_i}{y_0} \delta \log(\mu_i) \end{aligned} \tag{17}$$

Aggregate Implications - Model # 2

PROPOSITION

Denote $s_{ij} = \frac{d_{ij} a_j \mu_j}{\sum_j d_{ij} a_j \mu_j}$. *Under the power case of* $\mu(\sigma)$, *we have*

$$\delta \log(\boldsymbol{\mu}) = -\gamma_0 diag((\boldsymbol{\mu}')^2) diag(\boldsymbol{S})^{-1} \left[\boldsymbol{I} - \gamma_0 diag((\boldsymbol{\mu}')^2) diag(\boldsymbol{S})^{-1} \boldsymbol{S}\right]^{-1} (\boldsymbol{I} - \boldsymbol{S}) \delta \log(\boldsymbol{a})$$

Proposition

Under the symmetric networks, If the exposure matrix takes the order of $s_{ij} = O(1/n), j \neq i$

i. $s_{ii} = O(1/\sqrt{n})$, *the idiosyncratic shock will not diminish as* $n \to \infty$, *specifically, we have* $\delta y^{(3)}/y_0 = \mu'^2 O_p(1)$, *and* $\delta y^{(2)}/y_0 = \mu' O_p(1)$.

ii. $s_{ii} = O(1/n)$, *the idiosyncratic shock will not diminish as* $n \to \infty$, *specifically, we have* $\delta y^{(3)}/y_0 = \mu'^2 O_p(\sqrt{n})$, *and* $\delta y^{(2)}/y_0 = \mu' O_p(\sqrt{n})$.

iii. $s_{ii} = O(1)$, *the idiosyncratic shock will diminish as* $n \to \infty$, *specifically, we have* $\delta y^{(3)}/y_0 = \mu'^2 o_p(1)$, *and* $\delta y^{(2)}/y_0 = \mu' o_p(1)$.

Conclusion

This paper, we analyze the role of agency conflicts within firms in prapgating shocks

1. No friction, no real effect of equity-holding networks
2. Incorporate various frictions under the same framework
3. Propagation of the shocks - the type of the frictions is important
   - under the default cost or limited liability
   - under the interest conflicts or moral hazard

Conclusion

Role of Agency Conflicts

4. Role of network structure - considering endogeneous action and shocks - when the network becomes denser, the shock becomes smaller, systemically ...

5. Aggregate implications - provide a situation where the impact of idiosyncratic shock will not diminish when the agency conflicts are non-negligilble ...

6. Future research - Conflicts between shareholders?