Networks and Business Cycles

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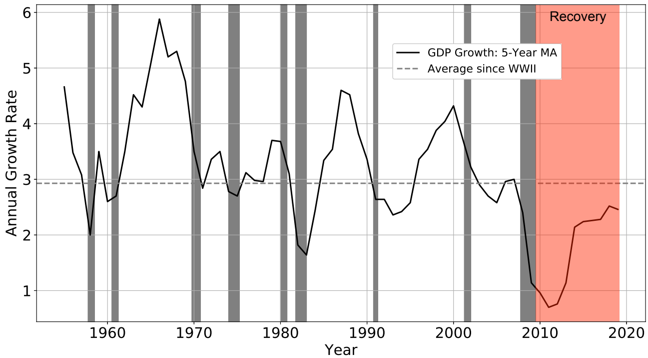
Outline

Introduction: US Economy Recovery Speed

How long does the economy take to recover from recessions in US?



The speed at which the US economy recovers from recessions varies greatly, from months to years.

1. Shadow region - recessions identified by NBER.
2. recovery speed: get back to the pre-recession growth trend
3. Recession of 2001 v.s. Recession of 2008 (ten years)

Introduction

Key questions:

1. What drives the recovery speed of the economy from recessions?
2. Can we use the current snapshot (cross-sectional) information to predict a slow recovery in the following years? (e.g. Crisis of 2008 v.s. Covid 19 Crisis)

Why important?

If we know the driving forces, we can

1. Predict if the economy will experience a prolonged slow recovery or not.
2. Policy makers can promote the economy to recover quickly (targeted intervention).

Introduction: Theoretical Work

Aggregate output (GDP) is obtained through aggregating the output across all firms (exclude double account).

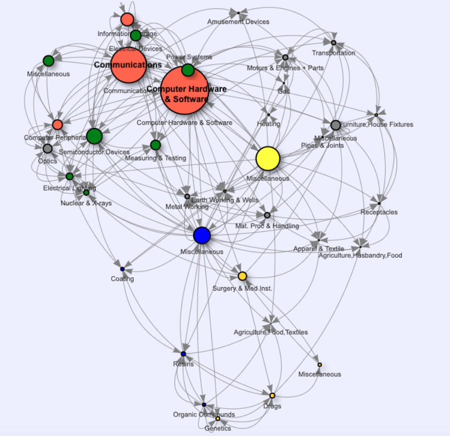
Two networks:

1. Production network - firms are linked through the supply chain (input-output relationship)
2. Innovation network - firms are technologically linked through learning from each other.

A dynamic general equilibrium model:

1. Firms choose their input and output to maximize the profits given the production technology.
2. Firms decide the research and development efforts to learn the production technology of other firms.
3. Households choose the consumption and saving (investment on stock market) to maximize their lifetime utility (happiness)
4. Firms, Households interacts through the market - buy and sell - to reach an equilibrium.

Introduction: Innovation Network



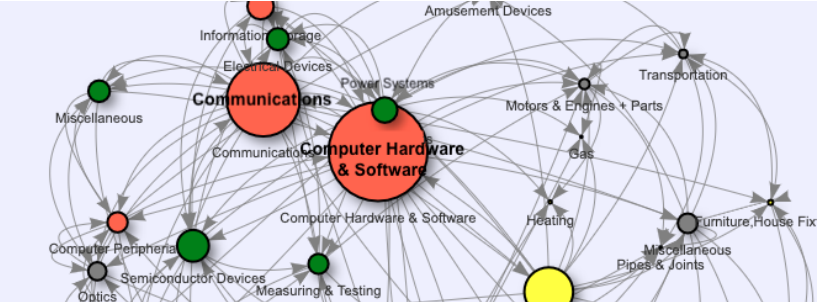
- Node: sector
  - Size: sector's importance
  - Color: sector's classification
- Edge: technology flow
  - Directed
  - Weighted
- Innovation network $\boldsymbol{W}$

$$\boldsymbol{W}_{ij} \in [0, 1]$$

technology flow: sector $j \longrightarrow i$

- Technology shock: unexpected realization of tech innovation.

Innovation Network: Centrality and Low-rank Structure



- Innovation network $\boldsymbol{W}$ where $\boldsymbol{W}_{ij}$: technology flow sector $j \longrightarrow i$.
- Centrality $\boldsymbol{v}$: leading eigenvector of $\boldsymbol{W}$.
  - Computer hardware and software is the most important sector
- Low-rank structure of $\boldsymbol{W}$: the largest eigenvalue $\gg$ others

Introduction: Theoretical work

Using the model, we examine

1. the propagation of technology shock within and cross networks.
   shock: a vector, each entry is a shock to a sector.
   shock's impact includes two parts: amplification + persistence
2. Interactions of two networks (Innovation and production network)

If the innovation network takes a low-rank structure (large spectral gap):

1. There exists one key direction (captured by the sectoral importance in the innov-network), the impact of a shock becomes persistent only if it follows this key direction.
2. If the shock follows other direction (e.g. orthogonal to the key direction), the impact a shock declines quickly.

$\Longrightarrow$1 + 2 implies large variation in persistence.

Introduction

Our approach

From theory, we develop a set of sufficient conditions:

1. A shock's impact is significantly amplified and becomes very persistent.
2. Economy experiences a slow recovery from recession.
3. They are estimable, testable, and alterable by policy makers.
4. Estimable and testable using shock'cross-sectional distribution and network structures.

Use real data, we show

1. Sufficient conditions hold $\Longrightarrow$ Prolonged Recovery
2. Sufficient conditions not hold $\Longrightarrow$ Quick Recovery

Introduction

Methods of Estimations

1. Innovation network:
   - Google patent dataset (1911-2014, 14 million)
     - Patent issuance (1911-2014,14 million)
     - Patent transaction (10.1 million transactions, 1920-2017)
     - Patent citation (90 million patent-to-patent citations)
     - Who Owns Whom? (parent-subsidiary relationship)
   - High dimensional state space model
2. Input-output network:
   - BEA input-output table (1951-2018)
3. Other parameters:
   - High dimensional state space model

Literature

1. Amplification and Persistence
   1.1 Financial Frictions: (Bernnake and Gertler, 1989;Kiyotaki and Moore,1997;Brunnermeier, Eisenhach, and Sannikov,2012)
   1.2 Endogeneous TFP: (Comin and Gertler,2006; Anzoategui, Comin, Gertler, and Martinez,2019;Bianchi, Kung, and Morales,2019)
   1.3 Our contribution: Cross-sectional shock + Network structures $\Longrightarrow$ Slow recovery?
2. Technology diffusion and Innovation network:
   2.1 Technology diffusion dominates in growth, (Jaffe,1986;Bloom,Schankerman,Van Reenen,2013)
   2.2 Stable network structure and Slow diffusion, (Acemoglu,Akcigit, and Kerr,2016;Ahmadpoor and Jones,2017)
   2.3 Our contribution: dynamic and macro implications.
3. Idiosyncratic shocks in production network:
   3.1 (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi,2012; Atalay, 2017;Baqaee and Farhi,2019)
   3.2 Our contribution: current recession + future recovery.
4. Long run risk:
   4.1 (Garleanu, Panageas, and Yu,2012; Garleanu, Kogan, and Panageas, 2012; Kogan, Papanikolaou, and Stoffman,2013; Kung and Schmid,2015)
   4.2 Our contribution: Endogenize the long-run risk in a networked economy.

Outline

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Model

Purpose: Link technology with aggregate output and growth.

Model elements:

- J sectors (firms) + production network + Innovation network
- Producer of final consumption goods
- Consumers
- Competitive markets

$\Longrightarrow$ Aggregate growth as a function of technology progress.

Model: Production Network

There are J sectors, sector i produces its output at time t (Long and Plosser, 1983):

$$Y_{it} = A_{it} I_{it}^{\eta}, \;\; s.t. \;\; I_{it} = \prod_{j \in [J]} X_{ijt}^{\theta_{ij}} \tag{1}$$

- A_{it}: productivity driven by technology.
- X_{ijt}: the input sector i use from sector j
- Y_{it}: the output of sector i

Sector i choose its output Y_{it} and inputs X_{ijt} to maximize its profit

$$\max_{X_{ijt}, Y_{it}} P_{it} Y_{it} - \sum_j P_{jt} X_{ijt}$$

$$s.t. Y_{it} = A_{it} \Big(\prod_{j \in [J]} X_{ijt}^{\theta_{ij}} \Big)^{\eta}$$

Denote

- $\theta_{ijt} = \frac{P_{jt} X_{ijt}}{\sum_k P_{kt} X_{ikt}}$: sector i's reliance on sector j.
- $\Theta_t = (\theta_{ijt})_{J \times J}$: the matrix representation of the production network.

Remark: general case on productivity/production technology, see the paper.

Model: Innovation Network

Innovation Network: firms learn the new technology or idea on production to improve their A_{it}'s.

Denote

- $a_{it} = \log(A_{it})$
- $\Delta a_{it} = a_{it} - a_{it-1}$: technology progress due to new technology or idea arrived.
- $\Delta \boldsymbol{a_t} = (\Delta a_{1t}, ..., \Delta a_{Jt})$: modeled as a learning process.

- Δa_{it} follows an arrival process (Aghion and Howitt,1992).

$$\Delta a_{it} = \mu_{it} + \epsilon_{it}^{A} \tag{2}$$

μ_{it}: arrival rate of sector i between t and $t+1$
ϵ_{it}^{A}: a realization shock.

- The latent arrival rate μ_{it+1}

$$\mu_{it+1} = \underbrace{(1-\rho)\mu_{it}}_{\text{depreciation effect}} + \underbrace{\sum_j W_{ij} \Delta a_{jt}}_{\text{learning from other sectors}} + \epsilon_{it}^{u} \tag{3}$$

Remark: general case of learning/endogenized learning, see the paper.

Model: Innovation Network

Matrix representation of equations (2) and (3)

$$\begin{aligned} \Delta \boldsymbol{a}_t &= \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t^A \\ \boldsymbol{\mu}_{t+1} &= (1-\rho)\boldsymbol{\mu}_t + \boldsymbol{W}\Delta \boldsymbol{a}_t + \boldsymbol{\epsilon}_t^u \end{aligned} \tag{4}$$

- $\boldsymbol{W}$: matrix representation of the innovation network.
- $\Delta \boldsymbol{a}_t = (\Delta a_{1t}, ..., \Delta a_{Jt})'$: realized technology progress.
- $\boldsymbol{\epsilon}_t^A = (\epsilon_{1t}^A, ..., \epsilon_{Jt}^A)'$: realization shock.
- $\boldsymbol{\epsilon}_t^u = (\epsilon_{1t}^u, ..., \epsilon_{Jt}^u)'$: latent arrival rate.

Remark 1: General learning from historical innovations, see the paper
Remark 2: $\boldsymbol{\epsilon}_t^A = 0$ and $W = 0$ (no technology learning), see (Onatskia and Murcia, 2013; Atalay, 2017)
Remark 3: $\boldsymbol{\epsilon}_t^A = 0$ and $W = 0$ (no technology learning), $\rho = 1$ (immediately depreciated), see (Foerster, Sarte, and Watson, 2011)

Model: Producer of consumption goods

Final Producer of Consumption Goods

The producer of final consumption good uses the products of other J sectors to produce final consumption good.

$$\begin{aligned} &\max_{c_{jt}} C_t - \sum_j P_{jt} c_{jt} \\ &s.t.\ C_t = \prod_j c_{jt}^{\alpha_j} \end{aligned} \tag{5}$$

- C_t: the amount of final consumption good.
- c_{it}: the amount of products in sector i used to produce C_t.
- $\alpha_{it} = \frac{P_{it} c_{it}}{\sum_j P_{jt} c_{jt}}$: consumption expenditure share in i.

Model: Householder

The representative household maximizes

$$\max_{C_t,\phi_{jt}} U_t := \sum_{s\geq t} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$
$$s.t.\ C_t + \sum_j \phi_{jt}(V_{jt} - D_{jt}) = \sum_j \phi_{jt-1} V_{jt} \tag{6}$$

- C_t: consumption at t.
- ϕ_{jt} is share holding on j.
- V_{jt}: the market value of firm j.
- D_{jt}: the dividend of firm j.

Model: General Equilibrium

General equilibrium set a set of equations:

1. Each sector j choose its input and output to maximize its profit.
2. The final producer maximize its profit to produce consumption goods.
3. Consumers maximize their lifetime happiness through choose their consumption and investment.
4. In equilibrium, demand = supply in all markets.

Model: General Equilibrium

Denote

- Y_{it}: the output of sector i
- Y_t : $Y_t = (1-\eta)\sum_i Y_{it}$: the aggregate output (exclude double account)
- s_{it} : $s_{it} = \frac{P_{it}Y_{it}}{\sum_j P_{jt}Y_{jt}}$, sale share of sector i
- θ_{ijt} : $\theta_{ijt} = \frac{P_{jt}X_{ijt}}{\sum_k P_{kt}X_{ikt}}$, i's expenditure share on the product of sector j
- α_{it} : $\alpha_{it} = \frac{P_{it}c_{it}}{\sum_j P_{jt}c_{jt}}$, share of consumption expenditure on i.

Proposition

1. *We have $s_{it}, \theta_{ijt}, \alpha_{it}$ are constant over time. Furthermore, $\theta_{ijt} = \theta_{ij}$, and $\alpha_{it} = \alpha_i$.*
2. *Denote $\mathbf{s} = (s_1, ..., s_J), \boldsymbol{\alpha} = (\alpha_1, ..., \alpha_J)$, and $\Theta = (\theta_{ij})_{J\times J}$*

$$s_i = \sum_j s_j\theta_{ji} + \frac{1}{1-\eta}\alpha_i \tag{7}$$

s_i: sector i's importance in the production network.

Remark: For the general production function, see the paper.

Model: General Equilibrium

Denote

- $g_t = \log(Y_t) - \log(Y_{t-1})$ the growth of aggregate output.
- $\Delta \boldsymbol{a}_t = \boldsymbol{a}_t - \boldsymbol{a}_{t-1} = \log(\boldsymbol{A}_t) - \log(\boldsymbol{A}_{t-1})$, the vector of technology progress.

PROPOSITION (FIRST MAIN RESULT)

$$g_t := \frac{1}{1-\eta}\boldsymbol{s}'\Delta \boldsymbol{a}_t = \frac{1}{1-\eta}\sum_i s_i \Delta a_{it} \tag{8}$$

where

$$\begin{aligned} \Delta \boldsymbol{a}_t &= \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t^A \\ \boldsymbol{\mu}_t &= (1-\rho)\boldsymbol{\mu}_{t-1} + \boldsymbol{W}\Delta \boldsymbol{a}_{t-1} + \boldsymbol{\epsilon}_{t-1}^u \end{aligned}$$

Outline

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Theoretical Results

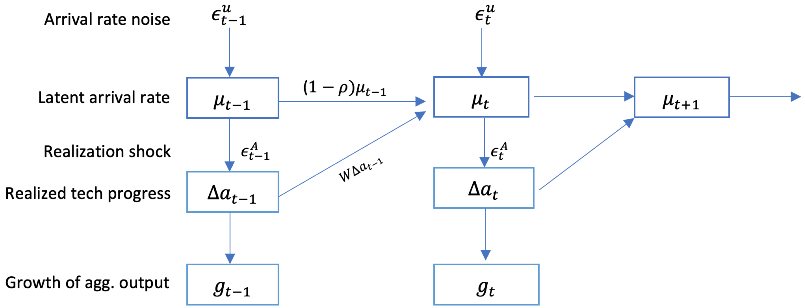
Summary:

1. Link the shock with its future impacts on growth
2. Persistence: Low rank structure

   Only the shock parallels to the sectoral centrality in innovation network, shocks' impact become very persistent
3. Amplification: two sufficient statistics
   3.1 Inner product between centralties in two networks: capture the interactions between two networks
   3.2 Inner product between shock and centrality in innovation network.

Remark: all are true in the real data.

Theoretical Results: A Diagram



Theoretical Results: Basic Results

We examine how sectoral shock to arrival rate affects future growth.

Proposition

$$\begin{aligned}\mathbb{E}_t \boldsymbol{\mu}_{t+\tau} &= [(1-\rho)\boldsymbol{I} + \boldsymbol{W}]^\tau \boldsymbol{\mu}_t \\ \mathbb{E}_t g_{t+\tau} &= \frac{1}{1-\eta}\boldsymbol{\mu}_t'[(1-\rho)\boldsymbol{I} + \boldsymbol{W}']^\tau \boldsymbol{s}\end{aligned} \tag{9}$$

Consider a shock to the $\boldsymbol{\mu}_t$ (sudden change) denoted as $\boldsymbol{\epsilon}_t$, the associated impact on $\boldsymbol{\mu}_{t+\tau}$ and $g_{t+\tau}$ denoted as $\delta\boldsymbol{\mu}_{t+\tau}$ and $\delta g_{t+\tau}$, we have

$$\begin{aligned}\mathbb{E}_t \delta\boldsymbol{\mu}_{t+\tau} &= [(1-\rho)\boldsymbol{I} + \boldsymbol{W}]^\tau \boldsymbol{\epsilon}_t \\ \mathbb{E}_t \delta g_{t+\tau} &= \frac{1}{1-\eta}\boldsymbol{\epsilon}_t'[(1-\rho)\boldsymbol{I} + \boldsymbol{W}']^\tau \boldsymbol{s}\end{aligned} \tag{10}$$

Theoretical Results: Basic Results

An example, consider a shock's impact on the arrival rate of next period,

$$\mathbb{E}_t \delta \boldsymbol{\mu}_{t+1} = [(1-\rho)\boldsymbol{I} + \boldsymbol{W}]\boldsymbol{\epsilon}_t = \underbrace{(1-\rho)\boldsymbol{\epsilon}_t}_{\text{depreciation effect}} + \underbrace{\boldsymbol{W}\boldsymbol{\epsilon}_t}_{\text{diffusion or learning effect}}$$

If $\mathbb{E}_t \delta \boldsymbol{\mu}_{t+1} = \boldsymbol{\epsilon}_t$, i.e. the shock does not diminish $\Longleftarrow \boldsymbol{W}\boldsymbol{\epsilon}_t = \rho \epsilon_t$.

Two intuitions:

1. $\sum_j W_{ij}\epsilon_{jt} = \rho\epsilon_{it}$, learning effect cancels out depreciation effect for all sectors.
2. Strength of diffusion effect depends on a shock's direction , when the shock parallels eigenvector of W associated with λ, diffusion effect is $\lambda\epsilon_t$, the net effect is

$$(-\rho + \lambda)\boldsymbol{\epsilon}_t$$

Theoretical Result

Proposition (Second Main Result)

Assume $\boldsymbol{W}$ to be diagonalizable, we can decompose the effect of the shock on future growth into:

$$\begin{aligned} \mathbb{E}_t\, \delta g_{t+\tau} &= \frac{1}{1-\eta} \boldsymbol{\epsilon}_t' [(1-\rho)\boldsymbol{I} + \boldsymbol{W}']^{\tau} \boldsymbol{s} \\ &= \frac{1}{1-\eta} \sum_{k=1}^{J} [1-(\rho-\lambda_k(\boldsymbol{W}'))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k) \end{aligned} \tag{11}$$

where

1. $(\boldsymbol{x}, \boldsymbol{y})$ the inner-product of $\boldsymbol{x}$ and $\boldsymbol{y}$
2. $\boldsymbol{v}_k' \boldsymbol{W}' = \lambda_k(W)\boldsymbol{v}_k'$, with $\lambda_1(\boldsymbol{W}) > ... > \lambda_J(\boldsymbol{W})$. $\lambda_k(\boldsymbol{W})$ the kth largest eigenvalue of $\boldsymbol{W}$ and $\boldsymbol{v}_k$ the associated eigenvector.

Theoretical Results: Main Results

$$\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} \underbrace{[1-(\rho-\lambda_k(\boldsymbol{W}'))]^{\tau}}_{persistence} \underbrace{(\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)}_{amplification}$$

Intuition on main results (consider the first component).

1. Amplification:
   1.1 $(\boldsymbol{\epsilon}_t, \boldsymbol{v}_1) = \sum_{i=j}^{J} v_{1i}\epsilon_{jt}$, weighted shock with weights v_{1i}.
   1.2 $(\boldsymbol{s}, \boldsymbol{v}_1) = \sum_{i=j}^{J} v_{1i}s_j$, production network interacts with innovation network.
2. Persistence: $\rho - \lambda_1(\boldsymbol{W})$ - depreciation v.s. diffusion effect

Remark: $\boldsymbol{v}_1$ is the eigenvector centrality: $v_{1i} = \frac{1}{\lambda_1(\boldsymbol{W})} \sum_j v_{1j} W_{ji}$ v_{1i}: i is important if sectors who learn from i are important.

Theoretical Results: Structure Matters

$$\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [(1-\rho) + \lambda_k(\boldsymbol{W}')]^{\tau} (\boldsymbol{s}_t, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$$

Persistence: Consider the structure with

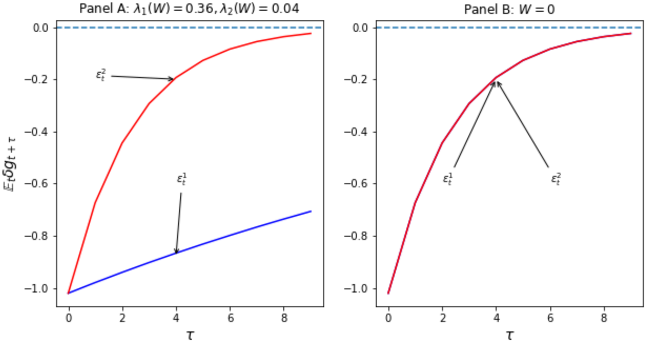
- $\rho \approx \lambda_1$, the strongest spillover effect cancels out the depreciation effect.
- low-rank, i.e., $\lambda_1(\boldsymbol{W}) \gg \lambda_2(\boldsymbol{W})$.

$\Longrightarrow$ only the first term matters as $\tau \uparrow$, other terms decline exponentially.
$\Longrightarrow$ **if $\epsilon_t \parallel v_1$, the effect declines slowly; if $\epsilon_t \perp v_1$, the effect declines quickly.**

Remark 1: If no innovation network $\Longrightarrow \boldsymbol{W} = 0 \Longrightarrow \mathbb{E}_t \delta g_{t+\tau} = (1-\rho)^{\tau} \delta g_t$.
Remark 2: If $\lambda_1(\boldsymbol{W}) = \ldots = \lambda_J(\boldsymbol{W}) := \lambda \Longrightarrow \mathbb{E}_t \delta g_{t+\tau} = (1-\rho+\lambda)^{\tau} \delta g_t$.

An illustrative example: Simulation

Simulation setup:
Scenario 1: $\boldsymbol{\epsilon}_t^1 \parallel \boldsymbol{v}_1$
Scenario 2: $\boldsymbol{\epsilon}_t^2 \parallel \boldsymbol{v}_2$
The two shocks cause the same drop in aggregate growth at $\tau = 0$: -1.0%



Left panel: a low rank network structure. Right panel: no innovation network $W = 0$. Blue line: the recovery path when subject to shock $\epsilon_t^1 \parallel v_1$. Black line: the recovery path when subject to shock $\epsilon_t^2 \parallel v_2$

Outline

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Model Estimation

Summary:

1. Estimation on the innovation network + Production network + Shocks + Other parameters
2. High dimension state space model + Patent datasets + Input-output tables

Estimation - State Space Model

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

1. Innovation Network $\boldsymbol{W} \Longrightarrow \lambda_i(W), \boldsymbol{v}_i$:
   - Google patent datasets (1911-2014) from website:
     - Patent issuance (1911-2014, 14 million patents granted)
     - Patent transaction (10.1 million patent transaction, 1920-2018)
     - Patent citation (90 million patent-to-patent citations)
     - Match each patent to final parent companies - matching algorithm and who owns whom? (subsidiary-parent relationship)
   - High dimensional state space model
2. Shock ϵ_t and Parameters ρ:
   - High Dimensional State Space Model
3. Production Network $\Longrightarrow \boldsymbol{s}$:
   - BEA input-output table (extended to 1951, the earliest date)

Estimation - State Space Model

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\epsilon_t, \boldsymbol{v}_k)$

We estimate the state space model:

$$\begin{aligned} \Delta \boldsymbol{a}_t &= \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t^A \\ \boldsymbol{\mu}_{t+1} &= (1-\rho)\boldsymbol{\mu}_t + \boldsymbol{W} \varphi_A(L) \Delta \boldsymbol{a}_t + \boldsymbol{\epsilon}_t^u \end{aligned} \tag{12}$$

1. Observable: $\Delta \boldsymbol{a}_t = \Delta \log(\boldsymbol{A_t})$ using Patent filing.
2. Latent process: arrival rate $\boldsymbol{\mu}_t$.
3. Estimated with the Expectation-Maximization (EM) algorithm in the state space model.

Parameters: $\Theta = (\rho, \boldsymbol{W}, \varphi_A(L), \Sigma_A, \Sigma_u)$. Without restrictions,

$$1 + J^2 + L + J(J+1) = 2J^2 + J + 1 + L$$

where J : # sectors = 89 and L : # lags in $\varphi_A(L) \Longrightarrow 20,000$ parameters.
Remark: Additional constraints imposed to estimate the parameters, see the paper.

Estimation - State Space Model

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

Summary on the estimation:

1. Innovation network $\lambda_i(W)$, $\boldsymbol{v}_i$:
   1.1 $\lambda_1(W) \approx \rho$: strongest spillover effect roughly cancels out depreciation effect.
   1.2 $\lambda_1(W) >> \lambda_2(W)$: low rank structure.
2. Interaction between innovation and production network $(\boldsymbol{s}, \boldsymbol{v}_k)$:
   $(\boldsymbol{s}, \boldsymbol{v}_1) >> (\boldsymbol{s}, \boldsymbol{v}_2)$

   $1 + 2 \Longrightarrow$ as $\tau \uparrow$, only the first term decline slowly, the others decline much faster.
3. Large variations on (ϵ_t, v_1) over time:
   3.1 When (ϵ_t, v_1) significantly large negative, a long recessions followed
   3.2 When (ϵ_t, v_1) is small, the economy recovers quickly

Go through one by one in following slides

Estimation - State Space Model

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

$1 - \rho + \lambda_1(\boldsymbol{W}) \approx 1(\surd)$
$\Longrightarrow$ The effect of the shock will be very persistent if $\boldsymbol{\epsilon}_t \parallel \boldsymbol{v}_1$

EM Estimates of the Innovation Networks

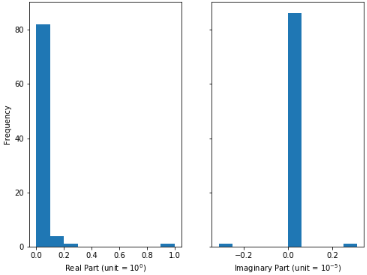
Panel B: EM estimates with general $\boldsymbol{W}$				
	$\varphi_A = 0.05$	$\varphi_A = 0.1$	$\varphi_A = 0.2$	$\varphi_A = 1.0$
$1 - \rho$	0.791	0.779	0.770	0.780
λ_1	0.198	0.187	0.182	0.162
$1 - \rho + \lambda_1$	0.989	0.966	0.952	0.942

Low Rank of Innovation Netowork (year = 2014)

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1-(\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\epsilon_t, \boldsymbol{v}_k)$

$1-\rho+\lambda_1(W) \approx 1$ ($\surd$)
low rank: $\lambda_1(W) >> \lambda_2(W)$ ($\surd$)

Distribution of Eigenvalues of $\lambda_k(\boldsymbol{W})$ (2014)



The innovation network is low rank:

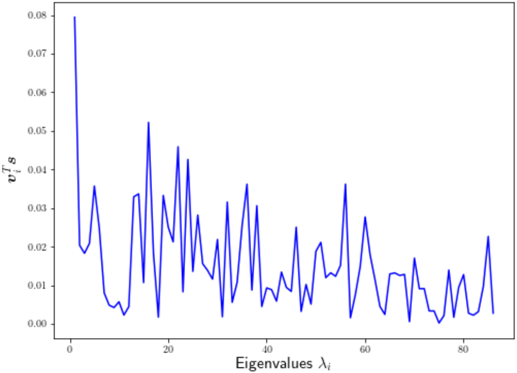
1. The largest eigenvalue is much larger than the others in magnitude;
2. The eigenvalues are approximately real, the imaginary parts are negligible.

Remark: if the eigenvalue is complex number, the right hand side is oscillator decline.

Correlation between $\boldsymbol{s}$ and $\boldsymbol{v}_i$

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1-(\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

$1-\rho+\lambda_1(W) \approx 1$ $(\surd)$
low rank: $\lambda_1(W) >> \lambda_2(W)$ $(\surd)$
interaction between two networks: $(s, v_1) >> (s, v_2)$ $(\surd)$

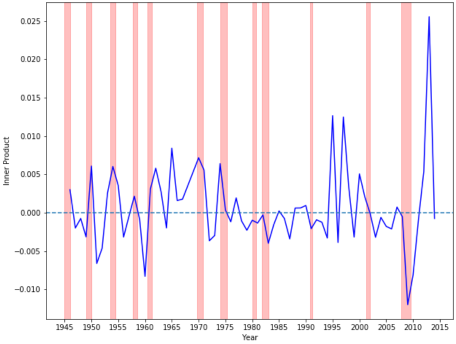


Correlation between $\boldsymbol{v}_1$ and ϵ_t

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1-(\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

$1 - \rho + \lambda_1(W) \approx 1$ ($\surd$)
low rank: $\lambda_1(W) >> \lambda_2(W)$ ($\surd$)
interaction between two networks: $(s, v_1) >> (s, v_2)$ ($\surd$)
Large time variation in sectoral exposure to the shock:(ϵ_t, v_1) ($\surd$)

Correlation between $\boldsymbol{v}_1$ and ϵ_t(Patent data)



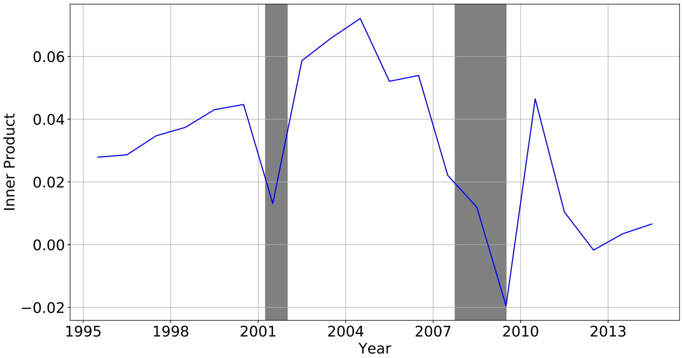
Correl ation between $\boldsymbol{v}_1$ and ϵ_t

How about the $(\boldsymbol{v}_1, \boldsymbol{\epsilon_t})$? if we use sectoral TFP data rather than patents. We write

$$\log(TFP_{it}) = a_{it} + m_{it} + e_{it} \tag{13}$$

a_{it}: productivity driven by technology; m_{it}: productivity driven beyond technology, follow AR(1); e_{it}: measure error.

Correlation between $\boldsymbol{v}_1$ and ϵ_t(TFP data)

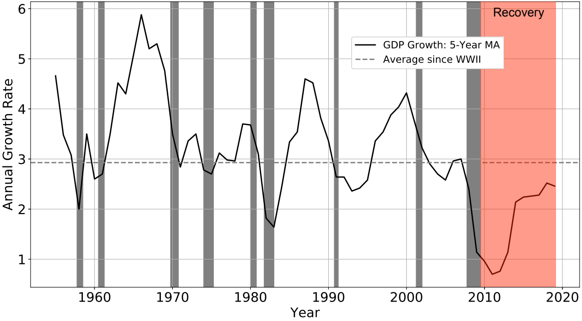


Persistence of Growth

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W}'))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

We have: $1 - \rho + \lambda_1(\boldsymbol{W}) \approx 1(\surd) +$ low rank $(\surd) + (\boldsymbol{s}, \boldsymbol{v}_1)$ is large$(\surd)$ + time-varying $(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

How long the economy takes to recover from recessions in U.S ?



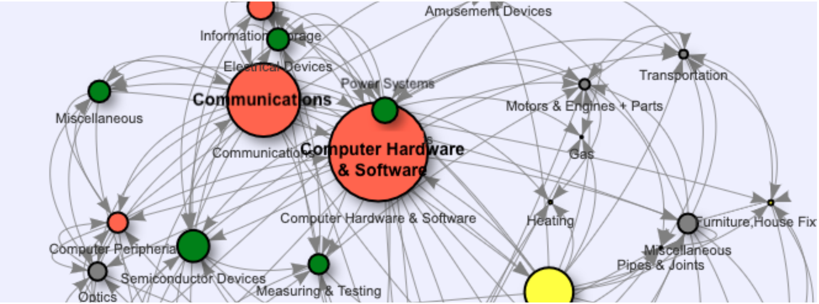
Outline

Other Applications

Other applications of the framework (if have time):

1. Networks, Long Run Risk, and Asset Pricing
   - $\lambda_1 \approx \rho$ + Low rank + sectoral distribution of shock $\Longrightarrow$ Long Run Risk + Cross-sectional asset pricing
2. Recovery from Covid19
   - Update and estimate $\boldsymbol{W}, \boldsymbol{s}$ to 2019, and ϵ_t in 2020.

Innovation Network: Centrality and Low-rank Structure



- Innovation network $\boldsymbol{W}$ where $\boldsymbol{W}_{ij}$: technology flow sector $j \longrightarrow i$.
- Centrality $\boldsymbol{v}$: leading eigenvector of $\boldsymbol{W}$.
  - Computer hardware and software is the most important sector
- Low-rank structure of $\boldsymbol{W}$: the largest eigenvalue $\gg$ others

Outline

Conclusion

A model where innovation network marries production network:

1. Theoretically, the shock direction and inn-network structure matters in amplification and persistence:
   1.1 Persistence: captured by the structure of the inn-network, $\rho - \lambda_k(\boldsymbol{W})$
   1.2 When the inn-network is low-rank, the sectoral distribution of the shock reveals useful information on future recover process.
   1.3 Amplification: captured by $(\boldsymbol{v}_k, \boldsymbol{s}_t), (\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$
2. Empirically, we show
   2.1 Persistence: $\rho \approx \lambda_1(\boldsymbol{W})$, the shock becomes very persistent when the shock is parallel to the eigenvector centrality of the inn-network.
   2.2 The inn-network is low-rank for U.S.
   2.3 Amplification: $(\boldsymbol{v}_1, \boldsymbol{s}_t) >> (\boldsymbol{v}_1, \boldsymbol{s}_t), k \geq 2$, $(\boldsymbol{\epsilon}_t, \boldsymbol{v}_1)$ is much lower in Great Recession than others.
3. Policy implication: to avoid long persistent recession, policy should target at firms in the center of the innovation network.
4. Future work:
   4.1 Endogenize the long-run risk in networks - puzzles in asset pricing.
   4.2 General non-linear effect due to endogenized R&D.
   4.3 What is the implication of Covid-19 on persistent recession?