Networks and Business Cycles

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The speed at which the US economy recovers from recessions varies greatly, from months to years.

1. Recession of 1991 v.s. Recession of 2008, Recession of 2001 v.s. Recession of 2008
2. The speed of recovery:
   - Time to get back to the pre-recession growth trend.
   - Not the level of gross output or time between NBER trough and peak.

Key question:

What drives the speed of the recovery of the economy from recessions or adversarial shocks?

This Paper

Focus on the role of production and innovation networks on business cycle.

1. Input-output network: sectors are linked through intermediate goods.
2. Innovation network: sectors are linked by technology - firms learn from each other about production technology.

Examine, theoretically and empirically, how cross-sectional shocks to technology progress (invention) become persistent and amplified

- "Shock" - a vector of sectoral shocks to technology progress, each component is a shock to a sector.

Main Results

Intuitively, when an adverse multi-sector shock hits the economy, the effect on aggregate growth includes: Amplification and Persistence

We decompose the effects of the shock on growth into several components, each has its amplification and persistence:

1. Amplification: depends on the direction of the shock, fully captured by two sufficient statistics:
   1.1 inner product between the shock and vector of node centralities (centrality) in innovation network - "e.g. large shock to IT sector"
   1.2 inner product between centrality in innovation network and centrality in production network. - "e.g. large shock to IT v.s. large shock to OIL"
2. Persistence - depreciation effect v.s. spillover effect in innovation network, fully captured by the eigenvalue of the innovation network.

Main Results

A significantly slow recovery occurs if

C1. Cross-sectional shock highly correlates with the eigenvector centrality of the innovation network (amplification)

C2. Eigenvector centrality in the innovation network highly correlates with the Katz centrality in the production network (amplification)

C3. Spillover effect roughly cancels out the depreciation effect (persistence)

Empirical evidence

1. Innovation network - google patent citation (1919-2018).
2. Production network - BEA input-output table (1951-2018).
3. Other parameters - state space model.

Empirically, we show:

1. Large time-variation on the amplification. E.g. during the recession of 2008, the shock highly and negatively correlated with the eigenvector centrality of the innovation network. However, during other episodes, the correlation is much smaller. (amplification)
2. Innovation network exhibit a pattern such that the spillover effect roughly cancels out the depreciation effect.(persistence)

Literature

- Amplification and Persistence
  1. Financial Frictions: (Bernnake and Gertler, 1989;Kiyotaki and Moore,1997;Brunnermeier, Eisenhach, and Sannikov,2012)
  2. Endogeneous TFP: (Comin and Gertler,2006; Anzoategui, Comin, Gertler, and Martinez,2019;Bianchi, Kung, and Morales,2019)
  3. We show direction of the shock and network structure matter, reveals useful information on future recovery.
- Technology diffusion and Innovation network:
  1. Technology diffusion dominates in growth, (Jaffe,1986;Bloom,Schankerman,Van Reenen,2013)
  2. Stable network structure and Slow diffusion, (Acemoglu,Akcigit, and Kerr,2016;Ahmadpoor and Jones,2017)
  3. Dynamic and macro implications.
- Idiosyncratic shocks in production network:
  1. (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi,2012; Atalay, 2017;Baqaee and Farhi,2019)
  2. Focus on how networks link the current recessions and future recovery, not explored.
- Long run risk:
  1. (Garleanu, Panageas, and Yu,2012; Garleanu, Kogan, and Panageas, 2012; Kogan, Papanikolaou, and Stoffman,2013; Kung and Schmid,2015)
  2. Endogenize the long-run risk in networking economy

Outline

Model

J sectors, sector i produce its output at time t (Long and Plosser, 1983):

$$Y_{it} = \tilde{A}_{it} I_{it}^{\eta}, \;\; s.t. \;\; I_{it} = \prod_{j \in [J]} X_{ijt}^{\theta_{ij}} \tag{1}$$

where $\tilde{A}_{it} = A_{it} M_{it}$.
A_{it}: productivity driven by technology.
M_{it}: productivity beyond technology like managerial ability or agency conflicts etc.

Denote $a_{it} = \log(A_{it})$, $\Delta a_{it} = a_{it} - a_{it-1}$, model $\Delta \boldsymbol{a_t} = (\Delta a_{1t}, ..., \Delta a_{Jt})$ as a learning process. Think of $\Delta \boldsymbol{a_t}$ as new idea about production.

1. The process a_{it} follows (Aghion and Howitt,1992).

$$\Delta a_{it} = \mu_{it} + \epsilon_{it}^{A} \tag{2}$$

   μ_{it}: arrival rate of the new technology or innovation for sector i between t and $t+1$, ϵ_{it}^{A}: the shock to innovation realization.

2. The latent arrival rate is modeled as

$$\mu_{it+1} = \underbrace{(1-\rho)\mu_{it}}_{\text{mean-reverse effect}} + \underbrace{\sum_{j} W_{ij} \Delta a_{jt}}_{\text{knowledge learning from other firms}} + \epsilon_{it}^{u} \tag{3}$$

Write (2) and (3) in matrix notation

$$\begin{aligned} \Delta \boldsymbol{a}_t &= \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t^A \\ \boldsymbol{\mu}_{t+1} &= (1-\rho)\boldsymbol{\mu}_t + \boldsymbol{W}\Delta \boldsymbol{a}_t + \boldsymbol{\epsilon}_t^u \end{aligned} \tag{4}$$

with $\Delta \boldsymbol{a}_t = (\Delta a_{1t}, ..., \Delta a_{Jt})'$, $\boldsymbol{\epsilon}_t^A = (\epsilon_{1t}^A, ..., \epsilon_{Jt}^A)'$, $\boldsymbol{\epsilon}_t^u = (\epsilon_{1t}^u, ..., \epsilon_{Jt}^u)'$

Special cases. Suppose $\boldsymbol{\epsilon}_t^A = 0$, the technology process is reduced to

$$\Delta \boldsymbol{a}_{t+1} = [(1-\rho)\boldsymbol{I} + \boldsymbol{W}]\Delta \boldsymbol{a}_t + \boldsymbol{\epsilon}_t^u \tag{5}$$

The usual VAR structure:

1. If $\boldsymbol{W}$ a diagonal matrix, no technology diffusion between sectors (Onatskia and Murcia, 2013; Atalay, 2017)
2. If $(1-\rho)\boldsymbol{I} + \boldsymbol{W} = 0$ and $\boldsymbol{\epsilon}_t^u = \Lambda_F \boldsymbol{F}_t + \boldsymbol{v}_t$ (Foerster, Sarte, and Watson, 2011).
    2.1 $\boldsymbol{v}_t$:idiosyncratic shock.
    2.2 $\boldsymbol{F}_t$: Common shock.
    2.3 Λ_F : the exposure matrix to the common shock.

Model - Generalization

Generalization:

1. General CES production technology:

$$Y_{it} = A_{it} I_{it}^{\eta}, \;\; s.t. \;\; I_{it} = \Big[\sum_{j \in [J]} \theta_{ij} X_{ijt}^{1-1/v_i} \Big]^{\frac{1}{1-1/v_i}}$$

2. General learning process:

$$\mu_{it+1} = (1-\rho)\mu_{it} + \sum_{j} W_{ij} \varphi(L) \Delta a_{jt} + \epsilon_{it}^{u}$$

- $\rho \in (0,1)$: depreciation rate of new idea (Bloom et al,2020AER).
- $\varphi(L)\Delta a_{jt} = \varphi_0 \Delta a_{jt} + \varphi_1 \Delta a_{jt-1} + ...$: learning from past innovation.
- We standardize $W_{i\cdot} = (W_{i1}, ..., W_{iJ}) = \xi_i(\tilde{W}_{i1}, ..., \tilde{W}_{iJ})$ such that

$$\sum_{j \in [J]} \tilde{W}_{ij} = 1$$

ξ_i captures the learning efficiency.

Model - Households

The representative household maximizes

$$max_{C_t,\phi_{jt}} U_t := \sum_{s \geq t} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \quad (6)$$

subject to

$$C_t + \sum_j \phi_{jt}(V_{jt} - D_{jt}) = \sum_j \phi_{jt-1} V_{jt}$$

ϕ_{jt} is share holding on j, V_{jt}: the market value of firm j, D_{jt}: the dividend of firm j.

C_t: a representative firm that purchases intermediate goods at price P_{jt} and combines them to a final good that is being consumed. The price of the final good is normalized to 1.

$$max_{c_{jt}} C_t - \sum_j P_{jt} c_{jt} \quad s.t. \; C_t = \prod_j c_{jt}^{\alpha_j} \quad (7)$$

Theoretical Results - General Equilibrium

[1.] Markets for intermediates are competitive, Firm i's optimization

$$\max_{I_{it}, X_{ijt}, j\in[J]} P_{it}\tilde{A}_{it} I_{it}^{\eta} - \sum_j P_{jt} X_{ijt}, \;\; s.t. \;\; I_{it} = \prod_{j\in[J]} X_{ijt}^{\theta_{ij}}$$

$$I_{it} : [s_{it} Y_t]^{1-\eta} = (P_{it}^I)^{-\eta} P_{it}\tilde{A}_{it}\eta^{\eta}, i \in [J] \Longrightarrow$$
$$(1-\eta)[\log(\boldsymbol{s_t}) + 1\log(Y_t)] = -\eta\log(\boldsymbol{P}_t^I) + \log(\boldsymbol{P_t}) + \log(\tilde{\boldsymbol{A}}_t) + \eta\log(\eta)1 \quad (8)$$

$$X_{ijt} : P_{jt}^I = \prod_i [\frac{P_{it}}{\theta_{ji}}]^{\theta_{ji}} \Longrightarrow \log(\boldsymbol{P}_t^I) = \Theta\log(\boldsymbol{P}_t) - \boldsymbol{N}_t \quad (9)$$

[2.] The optimization for final consumption producer,

$$\prod [\frac{P_{it}}{\alpha_i}]^{\alpha_i} = 1 \Longrightarrow \boldsymbol{\alpha}'\log(\boldsymbol{P}_t) = \boldsymbol{\alpha}'\log(\boldsymbol{\alpha}) \quad (10)$$

with,
Y_t : $Y_t = \sum_{j\in[J]} P_{jt} Y_{jt}$,
P_{it}^I: shadow price of composite good I_{it},
N_{it}: $N_{it} = \sum_j \theta_{ij}\log(\theta_{ij})$, input sparsity of sector i,
s_{it} : $s_{it} = \frac{P_{it} Y_{it}}{\sum_{j\in[J]} P_{jt} Y_{jt}}$, sale share of sector i

Market clearing:

[3.] Market clears for product i,

$$c_{it} + \sum_{j \in [J]} X_{jit} = Y_{it} \Longrightarrow \alpha_i (1-\eta) Y_t + \sum_j \theta_{ji} P_{jt} Y_{jt} = P_{it} Y_{it}$$
$$\Longrightarrow s_{it} = \sum_j s_{jt} \theta_{ji} + \alpha_i (1-\eta) \Longrightarrow \boldsymbol{s}_t = (1-\eta)(1-\Theta')^{-1} \boldsymbol{\alpha} \tag{11}$$

[4.] Market clears for stock market i implies,

$$\sum_i D_{it} = C_t \Longrightarrow C_t = (1-\eta) Y_t \tag{12}$$

Note: slightly different from traditional definition that $Y_t = C_t$, but differs by a constant fraction $1-\eta$.

Proposition (First Main Result)

Under Cobb-Douglas

$$y_t := \log(Y_t) = \boldsymbol{s}_t'\left[-\log(\boldsymbol{s}_t) + \frac{\eta}{1-\eta}\boldsymbol{N_t} + \frac{1}{1-\eta}\boldsymbol{a}_t + \frac{1}{1-\eta}\boldsymbol{m_t}\right] + \frac{\eta}{1-\eta}log(\eta) + \boldsymbol{\alpha}'\boldsymbol{\alpha} \tag{13}$$

- *Concentration effect:* $-\boldsymbol{s}_t'\log(\boldsymbol{s}_t)$, *resource allocation across sectors*
- *Sparsity effect:* $\boldsymbol{s}_t'\boldsymbol{N_t} = \sum_{j\in[J]} s_{jt}N_{jt}$, *allocation within sector across inputs.*
- *Hulten effect:* $\boldsymbol{s}_t'\boldsymbol{a}_t$, *captures first order effect of technology (Hulten 1976)*

Under Cobb-Douglas case, $\boldsymbol{s}_t$ *and* $\boldsymbol{N}_t$ *are constant over time (let us temporary shut down* $\boldsymbol{m}_t$ *for concise).*

$$g_t := \frac{1}{1-\eta}\boldsymbol{s}'\Delta\boldsymbol{a}_t \tag{14}$$

Under general CES, the equation 14 still true with an adjustment on the sparsity (see the paper)

Theoretical Results - Impulse Response Analysis

We examine how sectoral shock to arrival rate affect future growth. Under Cobb-Douglas, the system is log-linear. (Note: not usual impulse response function)

1. Consider a shock $\boldsymbol{\epsilon}_t$ to the arrival rate:

$$\boldsymbol{\mu}_t \to \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t, \text{ where } \boldsymbol{\epsilon}_t = \boldsymbol{\mu}_t - \mathbb{E}_{t-1}\boldsymbol{\mu}_t$$

2. The associated effect on growth and arrival rate at period $t+\tau$:

$$\boldsymbol{\mu}_{t+\tau} \to \boldsymbol{\mu}_{t+\tau} + \delta\boldsymbol{\mu}_{t+\tau}, \text{ where } \delta\boldsymbol{\mu}_{t+\tau} = \boldsymbol{\mu}_{t+\tau} - \mathbb{E}_{t-1}\boldsymbol{\mu}_{t+\tau}$$
$$g_{t+\tau} \to g_{t+\tau} + \delta g_{t+\tau}, \text{ where } \delta g_{t+\tau} = g_{t+\tau} - \mathbb{E}_{t-1} g_{t+\tau}$$

PROPOSITION

$$\begin{aligned} \mathbb{E}_t\delta\boldsymbol{\mu}_{t+\tau} &= \mathbb{E}_t[\boldsymbol{\mu}_{t+\tau} - \mathbb{E}_{t-1}\boldsymbol{\mu}_{t+\tau}] = [(1-\rho)\boldsymbol{I} + \boldsymbol{W}]^{\tau}\boldsymbol{\epsilon}_t \\ \mathbb{E}_t\delta g_{t+\tau} &= \mathbb{E}_t[g_{t+\tau} - \mathbb{E}_{t-1}g_{t+\tau}] = \frac{1}{1-\eta}\boldsymbol{\epsilon}_t'[(1-\rho)\boldsymbol{I} + \boldsymbol{W}']^{\tau}\boldsymbol{s} \end{aligned} \tag{15}$$

PROPOSITION (SECOND MAIN RESULT)

Assume $\boldsymbol{W}$ *to be diagonalizable, we can decompose the effect of the shock on future growth into:*

$$\begin{aligned}\mathbb{E}_t\, \delta g_{t+\tau} &= \frac{1}{1-\eta}\epsilon_t'[(1-\rho)\boldsymbol{I} + \boldsymbol{W'}]^\tau \boldsymbol{s} \\ &= \frac{1}{1-\eta}\sum_{k=1}^{J}[1-(\rho-\lambda_k(\boldsymbol{W'}))]^\tau(\boldsymbol{s}, \boldsymbol{v}_k)(\epsilon_t, \boldsymbol{v}_k)\end{aligned} \tag{16}$$

Where,

1. $(\boldsymbol{x}, \boldsymbol{y})$ the inner-product of $\boldsymbol{x}$ and $\boldsymbol{y}$
2. $\boldsymbol{v}_k'\boldsymbol{W'} = \lambda_k(W)\boldsymbol{v}_k'$, with $\lambda_1(\boldsymbol{W}) > ... > \lambda_J(\boldsymbol{W})$. $\lambda_k(\boldsymbol{W})$ the kth largest eigenvalue of $\boldsymbol{W}$ and $\boldsymbol{v}_k$ the associated eigenvector.

Why we prefer to make a eigen-decomposition? when the direction of the shock is $\boldsymbol{v}_k$, the shocks across sectors decline at the same rate:

$$[(1-\rho)\boldsymbol{I} + \boldsymbol{W'}]^\tau \epsilon_t = [1-(\rho-\lambda_k(\boldsymbol{W})]^\tau \epsilon_t$$

$$\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} \underbrace{[1-(\rho-\lambda_k(\boldsymbol{W'}))]^{\tau}}_{persistence} \underbrace{(\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)}_{amplification}$$

Intuition on main results (consider the first component).

1. Amplification:
   1.1 $(\boldsymbol{\epsilon}_t, \boldsymbol{v}_1) = \sum_{i=j}^{J} v_{1i}\epsilon_{jt}$, weighted shock with weights v_{1i}.
   1.2 $(\boldsymbol{s}, \boldsymbol{v}_1) = \sum_{i=j}^{J} v_{1i}s_j$, production network interacts with innovation network.
2. Persistence: $\rho - \lambda_1(\boldsymbol{W})$, the decline rate of the shock in innov-network when $\boldsymbol{\epsilon}_t \propto \boldsymbol{v}_1$ since
$$[(1-\rho)\boldsymbol{I} + \boldsymbol{W'}]^{\tau}\boldsymbol{\epsilon}_t = [1-(\rho-\lambda_1(\boldsymbol{W})]^{\tau}\boldsymbol{\epsilon}_t$$
   ρ : depreciation rate, $\lambda_1(\boldsymbol{W})$: promotion rate due to spillover.
   2.1 If $\rho >> \lambda_1(\boldsymbol{W}) > \lambda_k(\boldsymbol{W}), k \geq 2$, the 1st component declines sharply.
   2.2 If $\rho \approx \lambda_1(\boldsymbol{W})$, the 1st component declines slowly.

Intuition: $\boldsymbol{v}_1$ is the eigenvector centrality (Bonacich etal.,2001; Jackson etal.,2020)

$$v_{1i} = \frac{1}{\lambda_1(\boldsymbol{W})} \sum_j v_{1j} W_{ji}$$

v_{1i}: i is important if sectors who learn from i are important.

Theoretical Results - Structure Matter

$$\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [(1-\rho) + \lambda_k(\boldsymbol{W'})]^{\tau} (\boldsymbol{s}_t, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$$

Case1: If $\boldsymbol{W} = 0$, then $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta}(1-\rho)^{\tau}(\boldsymbol{\epsilon}_t, \boldsymbol{s}_t) = (1-\rho)^{\tau} \delta g_t$.

Case2: If $\lambda_1(\boldsymbol{W}) = ... = \lambda_J(\boldsymbol{W}) := \lambda$, then $\mathbb{E}_t \delta g_{t+\tau} = (1-\rho+\lambda)^{\tau} \delta g_t$.

Case3: The network is low-rank, such that $\lambda_1(\boldsymbol{W}) >> \lambda_2(\boldsymbol{W})$.

- 3.1: if $\rho \approx \lambda_1(\boldsymbol{W})$, the effect will become very persistent if $\boldsymbol{\epsilon} \propto \boldsymbol{v}_1$. rapidly decline if $\boldsymbol{\epsilon} \propto \boldsymbol{v}_2$
- 3.2: under the low rank, the direction of the shock matters for amplification and persistence.

Under cases 1 and 2, Sectoral distribution of the shock (direction of the vector) does not matter conditional on the initial aggregate effect on growth. However, under case 3, the direction of the shock matters for the recovery process.

An illustrative example - structure meets shock

Consider an economy with 3 sectors:

1. Symmetric production network, $\boldsymbol{s}_t = (1/3, 1/3, 1/3)$
2. $\boldsymbol{W}$ matrix

$$\begin{bmatrix} 0.327 & 0.067 & 0.067 \\ 0.067 & 0.047 & 0.047 \\ 0.067 & 0.047 & 0.047 \end{bmatrix}$$

3. $\lambda_1(\boldsymbol{W}) = 0.36, \lambda_2(\boldsymbol{W}) = 0.06, \lambda_3(\boldsymbol{W}) = 0;$
   $\boldsymbol{v}_1 \propto (4, 1, 1), \boldsymbol{v}_2 \propto (-1, 2, 2), \boldsymbol{v}_3 \propto (0, 1, -1).$
4. $\rho = 0.4 \approx \lambda_1(\boldsymbol{W}) >> \lambda_2(\boldsymbol{W})$, $\eta = 0.35$.

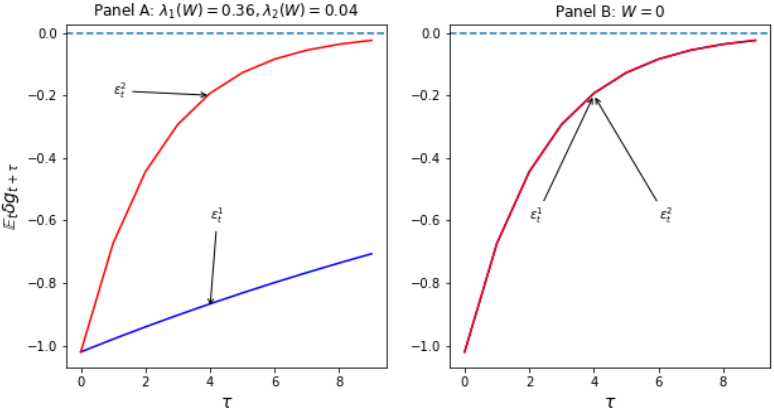
Consider two scenarios of cross-sectional shocks

- Scenario 1: $\boldsymbol{\epsilon}_{\boldsymbol{t}}^1 = (-0.2, -0.05, -0.05) \propto \boldsymbol{v}_1$
- Scenario 2: $\boldsymbol{\epsilon}_{\boldsymbol{t}}^2 = (0.1, -0.2, -0.2) \propto \boldsymbol{v}_2$

The aggregate effects on the current growth are the same since

$$\delta g_t = \frac{1}{1-\eta}(\boldsymbol{s}_t, \boldsymbol{\epsilon}^1) = \frac{1}{1-\eta}(\boldsymbol{s}_t, \boldsymbol{\epsilon}^2) = -0.1$$

How shocks interact with the innovation network.



Estimation - State Space Model

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1-(\rho - \lambda_k(\boldsymbol{W'}))]^\tau (\boldsymbol{s}, \boldsymbol{v}_k)(\epsilon_t, \boldsymbol{v}_k)$

We estimate the innovation network as a state space model:

$$\begin{aligned} \Delta \boldsymbol{a}_t &= \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t^A \\ \boldsymbol{\mu}_{t+1} &= (1-\rho)\boldsymbol{\mu}_t + \boldsymbol{W}\varphi_A(L)\Delta \boldsymbol{a}_t + \boldsymbol{\epsilon}_t^u \end{aligned} \tag{17}$$

- Observable: $\Delta \boldsymbol{a}_t = \Delta \log(\boldsymbol{A_t})$ using Patent filing.
- Latent process: arrival rate $\boldsymbol{\mu}_t$.
- Estimated with the Expectation-Maximization (EM) algorithm in the state space model.

Parameters: $\Theta = (\rho, \boldsymbol{W}, \varphi_A(L), \Sigma_A, \Sigma_u)$. Without restrictions,

$$1 + J^2 + L + J(J+1) = 2J^2 + J + 1 + L$$

parameters, where J : # sectors $= 89$ and L : # lags in $\varphi_A(L) \Longrightarrow 20,000$ parameters roughly.

Table: Model Setup and Restrictive Assumptions

Model Parameter	Restrictions
Σ_A	$\Sigma_A = \sigma_A^2 \boldsymbol{I}$
Σ_u	$\Sigma_u = \sigma_u^2 \boldsymbol{I}$
$\varphi(L) = \sum_{j\geq 0} \varphi_j L^j$	$\varphi(L) = \varphi_A \sum_{j\geq 0}(1-\varphi_A)^j L^j$ with $\varphi_A = 0.05, 0.1, 0.2, 1$
$\boldsymbol{W} = \Xi\tilde{\boldsymbol{W}}$, $\Xi = \text{diag}(\xi_1, ..., \xi_J)$	$\tilde{\boldsymbol{W}}$ directly estimated with patent citations

Here, we also allow for time-varying $\tilde{\boldsymbol{W}}_t$ but out of rational expectation world, our results are robust for time-invariant version.

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1-(\rho - \lambda_k(\boldsymbol{W}'))]^\tau (\boldsymbol{s}, \boldsymbol{v}_k)(\epsilon_t, \boldsymbol{v}_k)$

We examine:

1. Persistence: $1 - \rho + \lambda_1(\boldsymbol{W}) \approx 1$.
2. Distribution of $\lambda_k(\boldsymbol{W})$, $k \leq J$.
3. Amplification: $(\boldsymbol{s}, \boldsymbol{v}_k), (\epsilon_t, \boldsymbol{v}_k)$, $k \leq J$.

1. Measure $\Delta \boldsymbol{a}_t$: we interpret A_{it} as the technology-driven productivity and proxy it using the patent stock of the sector.
   1.1 N_{it}: # of patents issued by firms in sector i in year t, we proxy for A_{it} as

   $$A_{it} = \delta_A \sum_{s \geq 0} (1 - \delta_A)^s N_{it-s}$$

   We choose $\delta_A = 0.05$, the value of the patent on production depreciate to zero after 20 years.
   1.2 $\Delta a_{it} = \log(A_{it}) - \log(A_{it-1})$
2. Measure $\boldsymbol{W}$: we write

$$\boldsymbol{W} = \Xi \tilde{\boldsymbol{W}}$$

with $\sum_j \tilde{W}_{ij} = 1, \Xi = \text{diag}(\xi_1, ..., \xi_J)$, $\xi_i = \sum_j W_{ij}$. $\tilde{W}$ is estimated based on (Bloom et al., 2013)

Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W}'))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

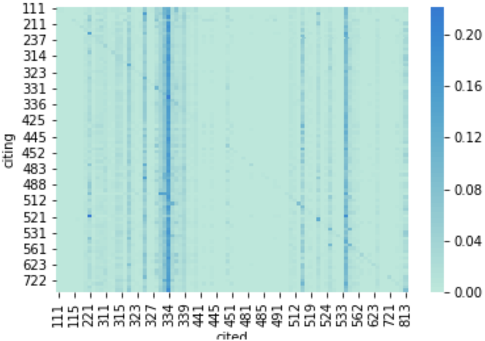
Check: $1 - \rho + \lambda_1(\boldsymbol{W}) \approx 1 \Longrightarrow$ The effect of the shock will be very persistent if $\boldsymbol{\epsilon}_t \propto \boldsymbol{v}_1$

Table: EM Estimates of the Innovation Networks

Panel A: EM estimates with assumption that $\Xi = \xi \boldsymbol{I}$				
	$\varphi_A = 0.05$	$\varphi_A = 0.1$	$\varphi_A = 0.2$	$\varphi_A = 1.0$
$1 - \rho$	0.823	0.818	0.814	0.817
λ_1	0.163	0.149	0.140	0.130
σ_u	0.0322	0.0324	0.0325	0.0316
σ_A	0.0237	0.0235	0.0234	0.0243
$1 - \rho + \lambda_1$	0.986	0.967	0.958	0.947
Panel B: EM estimates with general Ξ				
	$\varphi_A = 0.05$	$\varphi_A = 0.1$	$\varphi_A = 0.2$	$\varphi_A = 1.0$
$1 - \rho$	0.791	0.779	0.770	0.780
Mean of $\xi_j,\ j \in [J]$	0.198	0.187	0.182	0.162
Standard Dev of $\xi_j,\ j \in [J]$	0.160	0.141	0.131	0.113
25th percentile of $\xi_j,\ j \in [J]$	0.099	0.090	0.093	0.095
75th percentile of $\xi_j,\ j \in [J]$	0.298	0.264	0.266	0.238
σ_u	0.0332	0.0336	0.0339	0.0324
σ_A	0.0227	0.0222	0.0220	0.0235
$1 - \rho + \xi(\xi =$ Mean of $\xi_j,\ j \in [J])$	0.989	0.966	0.952	0.942

This table presents the parameter estimates using EM algorithm (for details, please see the appendix). In Panel A, we impose an assumption that $\Xi = \xi \boldsymbol{I}$ - all sectors share the same parameter ξ. In Panel B, we remove this restriction and allow for heterogeneity in ξ across sectors. For the general case that $\Xi = diag(\xi_1, ..., \xi_J)$ in Panel B, we also report the mean, standard deviation, 25th and 75th percentiles. In both panels, columns 1-4 report the results with $\varphi_A = 0.05$, 0.1,0.2, and 1.0.

X-axis: sectors with knowledge flow out, Y-axis: sectors learning from others. Some sectors are dominant in generating new knowledge, like sector 334 (Computer and Electronic Product Manufacturing) and sector 541 (Professional, Scientific, and Technical Services).



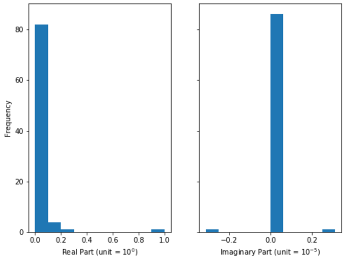
Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

$1 - \rho + \lambda_1(\boldsymbol{W}) \approx 1(\surd) +$ low rank $\Longrightarrow$the shock is parallel to $\boldsymbol{v}_1$, the shock will become very persistent. For other direction, the effect of the shock declines quickly.

The innovation network is low rank:

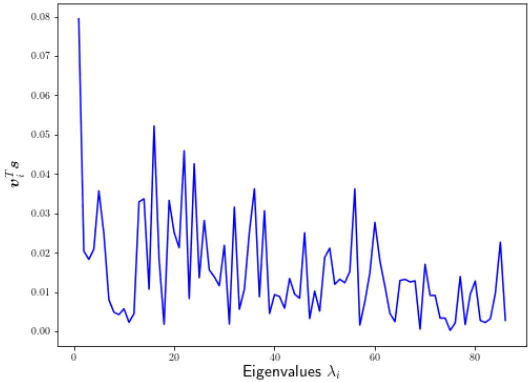
1. largest eigenvalue is much larger than the others in magnitude;
2. eigenvalues are approximately real, the imaginary parts are negligible.

Figure: Distribution of Eigenvalues of $\lambda_k(\tilde{\boldsymbol{W}})$ (year = 2014)



Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W}'))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

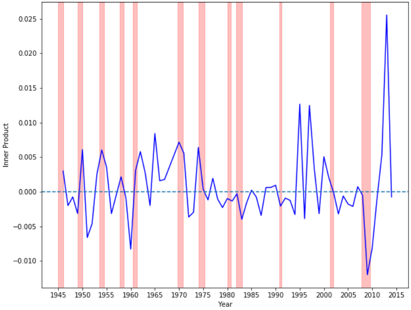
$1 - \rho + \lambda_1(\boldsymbol{W}) \approx 1(\surd) +$ low rank $(\surd) + (\boldsymbol{s}, \boldsymbol{v}_1)$ is large $\Longrightarrow$ only when $\boldsymbol{\epsilon}_t \propto \boldsymbol{v}_1$, amplified + persistent; for other direction, amplification and persistent much weak



Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1-(\rho-\lambda_k(\boldsymbol{W'}))]^{\tau}(\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

We have checked: $1-\rho+\lambda_1(\boldsymbol{W}) \approx 1(\surd) +$ low rank $(\surd) + (\boldsymbol{s}, \boldsymbol{v}_1)$ is large$(\surd)$ time-varying $(\boldsymbol{\epsilon_t}, \boldsymbol{v}_1)$?

Figure: Correlation between $\boldsymbol{v}_1$ and ϵ_t(Patent data)

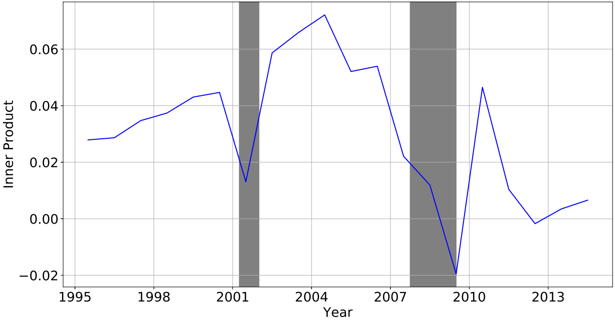


How about the $(\boldsymbol{v}_1, \boldsymbol{\epsilon_t})$? if we use sectoral TFP data rather than patents. We write

$$\log(TFP_{it}) = a_{it} + m_{it} + e_{it} \tag{18}$$

a_{it}: productivity driven by technology; m_{it}: productivity driven beyond technology, follow AR(1); e_{it}: measure error.

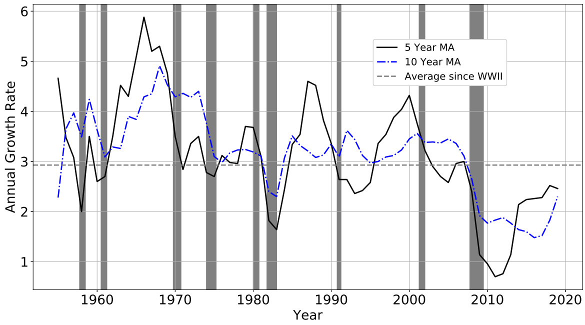
Figure: Correlation between $\boldsymbol{v}_1$ and ϵ_t(TFP data)



Remember $\mathbb{E}_t \delta g_{t+\tau} = \frac{1}{1-\eta} \sum_{k=1}^{J} [1 - (\rho - \lambda_k(\boldsymbol{W'}))]^{\tau} (\boldsymbol{s}, \boldsymbol{v}_k)(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

We have: $1 - \rho + \lambda_1(\boldsymbol{W}) \approx 1(\surd) +$ low rank $(\surd) + (\boldsymbol{s}, \boldsymbol{v}_1)$ is large$(\surd)$ + time-varying $(\boldsymbol{\epsilon}_t, \boldsymbol{v}_k)$

How long the economy takes to recover from recessions in U.S. ?



Conclusion

A model where innovation network marries production network:

1. Theoretically, the shock direction and inn-network structure matters in amplification and persistence:
   1.1 Persistence: captured by the structure of the inn-network, $\rho - \lambda_k(\boldsymbol{W})$
   1.2 When the inn-network is low-rank, the sectoral distribution of the shock reveals useful information on future recover process.
   1.3 Amplification: captured by $(\boldsymbol{v}_k, \boldsymbol{s}_t), (\epsilon_t, \boldsymbol{v}_k)$
2. Empirically, we show
   2.1 Persistence: $\rho \approx \lambda_1(\boldsymbol{W})$, the shock becomes very persistent when the shock is parallel to the eigenvector centrality of the inn-network.
   2.2 The inn-network is low-rank for U.S.
   2.3 Amplification: $(\boldsymbol{v}_1, \boldsymbol{s}_t) >> (\boldsymbol{v}_1, \boldsymbol{s}_t), k \geq 2$, $(\epsilon_t, \boldsymbol{v}_1)$ is much lower in Great Recession than others.
3. Policy implication: to avoid long persistent recession, policy should target at firms in the center of the innovation network.
4. Future work:
   4.1 Endogenize the long-run risk in networks - puzzles in asset pricing.
   4.2 General non-linear effect due to endogenized R&D.
   4.3 What is the implication of Covid-19 on persistent recession?