Xihao Zhu Comp498 HW1

1. rotated about $\hat{Y}_A$ by 30°, $\hat{X}_A$ by 45°. Therefore its mapping is X-Y-Z Fixed angles.

Rotation Matrix

$$R_{XYZ} = R_X \times R_Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(45^\circ) & -\sin(45^\circ) \\ 0 & \sin(45^\circ) & \cos(45^\circ) \end{bmatrix} \times \begin{bmatrix} \cos(30^\circ) & 0 & \sin(30^\circ) \\ 0 & 1 & 0 \\ -\sin(30^\circ) & 0 & \cos(30^\circ) \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & 0 & 0.500 \\ 0.354 & 0.707 & -0.612 \\ -0.354 & 0.707 & 0.612 \end{bmatrix}$$

2. ${}^A P_{Corg} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$, Rotate: first rotate about $\hat{Z}_A$ by 180° then about $\hat{X}_B$ by -90°

so ${}^A_C R_{Z'Y'X'} = R_{\hat{Z}_A}(180^\circ) \times R_{\hat{X}_B}(-90^\circ)$

$$= \begin{bmatrix} \cos(180^\circ) & -\sin(180^\circ) & 0 \\ \sin(180^\circ) & \cos(180^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90^\circ) & -\sin(-90^\circ) \\ 0 & \sin(-90^\circ) & \cos(-90^\circ) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\therefore {}^A_C T = \begin{bmatrix} {}^A_C R & {}^A P_{Corg} \\ 0 \; 0 \; 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Since ${}^{B}F = {}^{B}_{A}T \cdot F$

1. $F_2^A = {}^{A}_{B}T \times {}^{B}F_2$

$${}^{A}_{B}T = \begin{bmatrix} {}^{B}_{A}R^T & -{}^{B}_{A}R^T P_{Borg} \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 & 0 & -7.778 \\ -0.707 & 0.707 & 0 & 0.707 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. $F_2^A = \begin{bmatrix} -0.707 \\ 6.364 \\ 7 \\ 1 \end{bmatrix} \Rightarrow F^A = \begin{bmatrix} -0.707 \\ 6.364 \\ 7 \\ 1 \end{bmatrix}$

4. $R_y = \begin{bmatrix} \cos(90^\circ) & 0 & \sin(90^\circ) \\ 0 & 1 & 0 \\ -\sin(90^\circ) & 0 & \cos(90^\circ) \end{bmatrix}$ $R_z = \begin{bmatrix} \cos(45^\circ) & -\sin(45^\circ) & 0 \\ \sin(45^\circ) & \cos(45^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_{total} = R_z \times R_y = \begin{bmatrix} 0 & -0.707 & 0.707 \\ 0 & 0.707 & 0.707 \\ -1 & 0 & 0 \end{bmatrix}$, take ${}^{B}X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

${}^{A}X = R_{total} \times {}^{B}X = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

5. $R_{total} = R_y \times R_z = \begin{bmatrix} 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \end{bmatrix}$

${}^{A}X = R_{total} \times {}^{B}X = \begin{bmatrix} 0 \\ 0.707 \\ -0.707 \end{bmatrix}$

6. Not the same
No4 is X-Y-Z Fixed angles Frame
whe No5 is Z-Y-X Euler Angles Frame