Assignment 5

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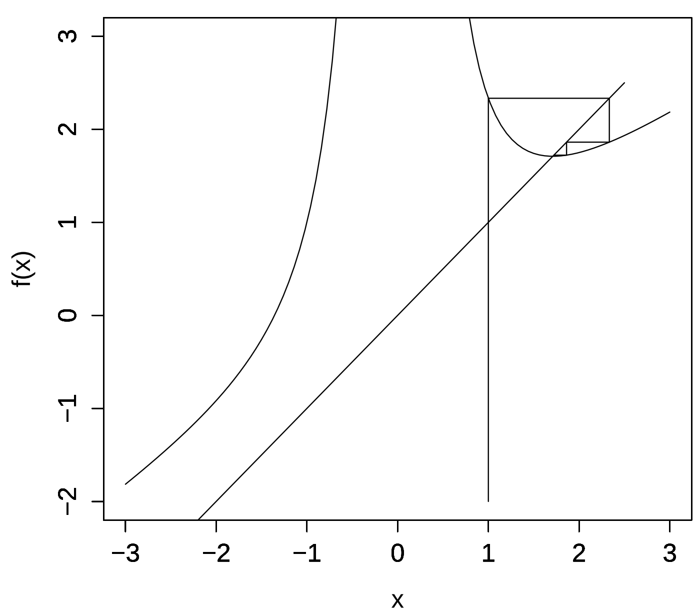
1 Q1

Let $G(x) = x^3 - 5$, then $x_{i+1} = f(x_i) = x_i - \frac{G(x_i)}{G'(x_i)} = x_i - \frac{x_i^3 - 5}{3x_i^2}$
Let $x_0 = 1$ for the first few iterations, we have:
$x_1 = 2.3333333333$
$x_2 = 1.8616780045351473$
$x_3 = 1.722001880058607$
$x_4 = 1.7100597366002945$
$x_5 = 1.709975950782189$
For the graph please refer to figure following:



Yes, the convergence is quadratic, Since for Newton's method if the fixed point is attracting, the convergence is quadratic.It will converge to the fixed point for any positive guess. It will converge to the fixed point for any negative guess too.

2 Q2

From lecture note we have $e_0(x) = 1, e_1(x) = x, e_2(x) = x^2 - 1/3, e_3(x) = x^3 - \frac{3}{5}x$, let $v_4(x) = x^4$, Then:

$\frac{<v_4(x),e_0(x)>}{<e_0(x),e_0(x)>} = \frac{1}{5}$

$\frac{<v_4(x),e_1(x)>}{<e_1(x),e_1(x)>} = 0$

$\frac{<v_4(x),e_2(x)>}{<e_2(x),e_2(x)>} = \frac{6}{7}$

$\frac{<v_4(x),e_3(x)>}{<e_3(x),e_3(x)>} = 0$

Hence $e_4(x) = x^4 - \frac{6}{7} * (x^2 - \frac{1}{3}) - \frac{1}{5} = x^4 - \frac{6}{7}x^2 + \frac{3}{35}$

Then for the best approximation of $f(x)$ on $[-1, 0]$:

$c_0 = \frac{<f(x),e_0(x)>}{<e_0(x),e_0(x)>} = 0.5$

$c_1 = \frac{<f(x),e_1(x)>}{<e_1(x),e_1(x)>} = -0.5$

$c_2 = \frac{<f(x),e_2(x)>}{<e_2(x),e_2(x)>} = \frac{-15}{16}$

$c_3 = \frac{<f(x),e_3(x)>}{<e_3(x),e_3(x)>} = \frac{35}{16}$

$c_4 = \frac{<f(x),e_4(x)>}{<e_4(x),e_4(x)>} = \frac{105}{128}$

on $[0, 1]$:

$c_0 = \frac{<f(x),e_0(x)>}{<e_0(x),e_0(x)>} = 0.5$

$c_1 = \frac{<f(x),e_1(x)>}{<e_1(x),e_1(x)>} = 0.5$

$c_2 = \frac{<f(x),e_2(x)>}{<e_2(x),e_2(x)>} = \frac{-15}{16}$

$c_3 = \frac{<f(x),e_3(x)>}{<e_3(x),e_3(x)>} = \frac{-35}{16}$

$c_4 = \frac{<f(x),e_4(x)>}{<e_4(x),e_4(x)>} = \frac{-105}{128}$

Therefore the best approximation for $f(x)$ is:

$$p(x) = \begin{cases} \frac{105}{128}(x^4 - \frac{6}{7}x^2 + \frac{3}{35}) + \frac{35}{16}(x^3 - \frac{3}{5}x) + \frac{-15}{16}(x^2 - 1/3) - 0.5x + 0.5 & -1 \leq x \leq 0 \\ \frac{-105}{128}(x^4 - \frac{6}{7}x^2 + \frac{3}{35}) + \frac{-35}{16}(x^3 - \frac{3}{5}x) + \frac{-15}{16}(x^2 - 1/3) + 0.5x + 0.5 & 0 \leq x \leq 1 \end{cases}$$

3 Q3

For the local error term:

$|\int_{-h/2}^{h/2} f(x)dx - \frac{h}{2}(f(-h\sqrt{3}/6) + f(h\sqrt{3}/6))|$

$= \int_{-h/2}^{h/2}(f(0) + x * f'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0) + ...)dx$

$-\frac{h}{2}(f(0) + (\frac{-h\sqrt{3}}{6})f'(0) + (\frac{-h\sqrt{3}}{6})^2 f''(0)/2 + (\frac{-h\sqrt{3}}{6})^3 f'''(0)/6 + (\frac{-h\sqrt{3}}{6})^4 f''''(0)/24 +$

$.... + f(0) + (\frac{h\sqrt{3}}{6})f'(0) + (\frac{h\sqrt{3}}{6})^2 f''(0)/2 + (\frac{h\sqrt{3}}{6})^3 f'''(0)/6 + (\frac{h\sqrt{3}}{6})^4 f''''(0)/24 + ...$

$= (hf(0) + \frac{h^3}{24}f''(0) + \frac{h^5}{1920}f''''(0) + higerTerm) - (hf(0) + \frac{h^3}{24}f''(0) + \frac{h^5}{3456}f''''(0) + higerTerm) = \frac{h^5}{4320}f''''(0) + higerTerm$
then the leading error term in the composite formula is :
$\frac{h^4}{4320}||f''''||^\infty * (b-a)$
compare to the one given in page 284: $\frac{h^4}{2880}||f''''||^\infty * (b-a)$
the denominator of error term for Gauss Formular is larger than that of Simpson's rule, thus is more smaller.

4 Q4

4.0.1 page 285

For the local error term:
$|\int_{-h/2}^{h/2} f(x)dx - hf(0)|$
$= \int_{-h/2}^{h/2}(f(0) + x * f'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0) + ...)dx - hf(0)$
$= (hf(0) + \frac{h^3}{24}f''(0) + \frac{h^5}{1920}f''''(0) + higerTerm) - hf(0) = \frac{h^3}{24}f''(0) + higerTerm$
then the leading error term in the composite formula is :
$\frac{h^2}{24}||f''||^\infty * (b-a)$
and the composite formula for integrating $f(x)$ is: $(b-a)f(0)$
when $N \geq 204$ the grobal error to be less than 10^{-6}.

4.0.2 page 286

For the local error term:
$|\int_{-h/2}^{h/2} f(x)dx - \frac{h}{2}(f(-h/2) + f(h/2))|$
$= \int_{-h/2}^{h/2}(f(0) + x * f'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0) + ...)dx$
$-\frac{h}{2}(f(0) + (\frac{-h}{2})f'(0) + (\frac{-h}{2})^2 f''(0)/2 + (\frac{-h}{2})^3 f'''(0)/6 + (\frac{-h}{2})^4 f''''(0)/24 + ... + f(0) + (\frac{h}{2})f'(0) + (\frac{h}{2})^2 f''(0)/2 + (\frac{h}{2})^3 f'''(0)/6 + (\frac{h}{2})^4 f''''(0)/24 + ...$
$= (hf(0) + \frac{h^3}{24}f''(0) + \frac{h^5}{1920}f''''(0) + higerTerm) - (hf(0) + \frac{h^3}{8}f''(0) + \frac{h^5}{384}f''''(0) + higerTerm) = \frac{h^3}{12}f''(0) + higerTerm$
then the leading error term in the composite formula is :
$\frac{h^2}{12}||f''||^\infty * (b-a)$
when $N \geq 288$ the grobal error to be less than 10^{-6}.

4.0.3 page 295