COMP 361/5611 - Elementary Numerical Methods
Assignment 5 - Due Wednesday, November 19, 2014

Problem 1. (25%)

Do this problem from Term Test 1:

Describe in detail how Newton's method can be used to compute the *cube root* of 3. Carry out the first few iterations, with $x^{(0)} = 1$. Draw the standard graphical interpretation of this fixed point iteration, showing the line $y = x$ and the curve $y = f(x)$, also indicating the first few iterations, starting with $x^{(0)} = 1$. Is the convergence quadratic? Can you tell from the graphical interpretation whether the iteration converges for any positive initial guess $x^{(0)}$? What about negative $x^{(0)}$?

Problem 2. (25%)

Consider the "tent function"

$$f(x) \;=\; \begin{cases} 1+x \ , & -1 \le x \le 0 \ , \\ \\ 1-x \ , & 0 \le x \le 1 \ . \end{cases}$$

Determine the polynomial $p(x)$ of degree four or less that is the best approximation (with respect to the $\| \cdot \|_2$) to $f(x)$ on the interval $[-1, 1]$.

NOTE:

- You can use the orthogonal polynomial basis functions $e_0(x)$, $e_1(x)$, $e_2(x)$, and $e_3(x)$, derived in the Lecture Notes, but you must derive $e_4(x)$ yourself.
- The integrals needed to compute the coefficients of the best approximation must be evaluated separately over the interval $[-1, 0]$ and over the interval $[0, 1]$.

Problem 3. (25%)

Consider the 2-point Gauss Quadrature Formula for integrating a function $f(x)$ over an interval $[a, b]$ on Pages 287-290 of the Lecture Notes. Use Taylor expansions to determine the leading error term of the local formula. Use the local error it to determine the leading error in the composite formula, and compare it to the corresponding error in the composite Simpson's Rule, as given on Page 284 of the Lecture Notes.

Problem 4. (25%) Do the three exercises in the Numerical Integration section of the Lecture Notes.