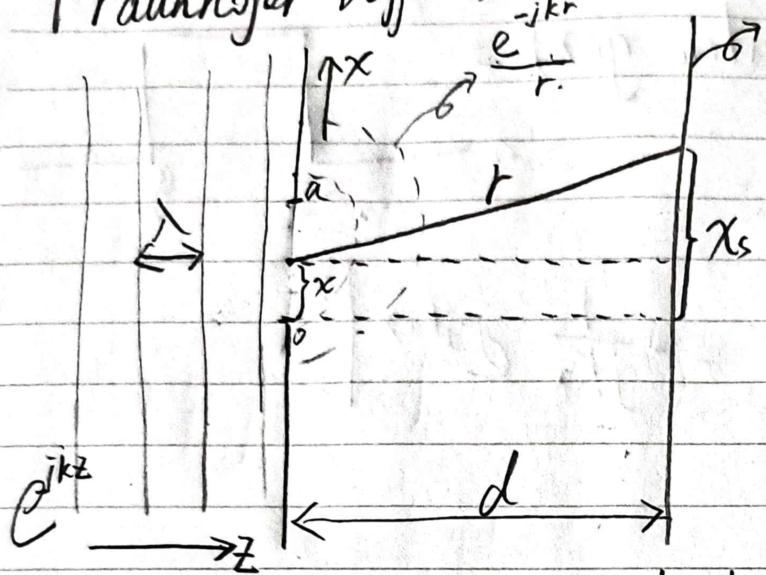
☆ Make a list of all the hardware components

Fraunhofer Diffraction



$$r = \sqrt{(x_s-x)^2+d^2}$$

$$= d\sqrt{1+\underbrace{\frac{(x_s-x)^2}{d^2}}_{\varepsilon}}$$

$$r \approx d(1+\tfrac{1}{2}\varepsilon)$$

$$= d\left(1+\frac{(x_s-x)^2}{2d^2}\right)$$

$$\Rightarrow \int_0^a \frac{e^{-jkr}}{r}dx = \int_0^a \frac{e^{-jkd}\, e^{-\frac{1}{2}jk\frac{(x_s-x)^2}{d}}}{\left(d+\frac{1}{2}\frac{(x-x_s)^2}{d}\right)}dx$$

$$\approx \frac{1}{d}\int_0^a e^{-jkd}\, e^{-\frac{1}{2}jk\frac{(x_s-x)^2}{d}}\,dx \quad \text{(paraxial approximation)}$$

$$= \underbrace{\frac{1}{d}e^{-jkd}\int_0^a e^{-\frac{1}{2}jk\frac{(x_s-x)^2}{d}}\,dx}_{\text{Fresnel Integral}}$$

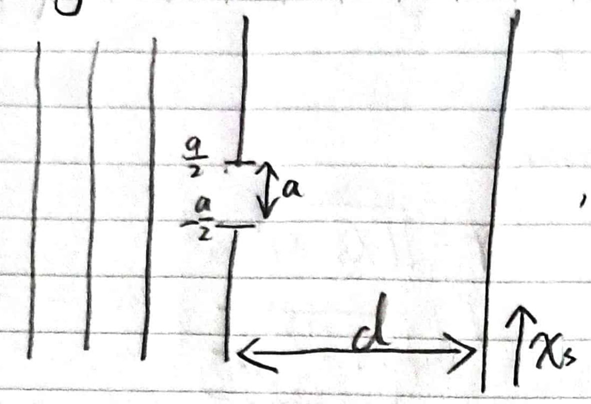
Suppose $a \ll x_s$, and $x \ll x_s$ $\quad \frac{(x_s^2 - 2x_s x + \cancel{x^2})}{d}$

$$= \frac{e^{+jkd}\, e^{+\frac{1}{2}jk\frac{x_s^2}{d}}}{d}\int_0^a e^{-jk\frac{x_s x}{d}}$$

$$= \sim \int_0^a e^{-jk_x x}\,dx \quad \left(\text{let } k_x = \frac{kx_s}{d} \to k\sin\theta\right)$$

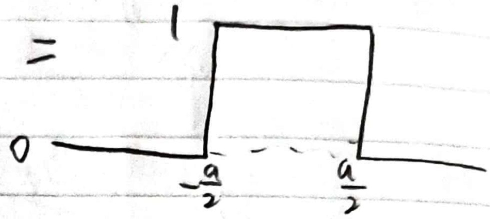
$$* \quad G(x) = \frac{e^{jkd}\, e^{\frac{1}{2}jk\frac{x_s^2}{d}}}{d} F(g(x))$$

Single Slit Case



$$E(x_s) \propto \frac{e^{jkd}\, e^{jk\frac{x_s^2}{d}}}{d} F(g(x))$$

, where $g(x) =$



$$F(g(x)) = \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-jk_x \cdot x}\, dx = -\frac{1}{jk_x} e^{-jk_x x}\Big|_{-\frac{a}{2}}^{\frac{a}{2}}$$

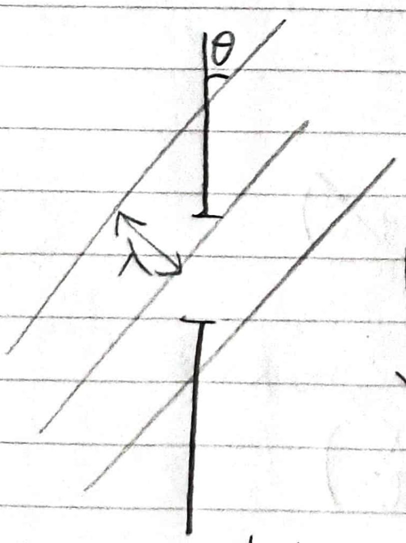
$$= \operatorname{sinc}\left(\frac{k_x \cdot a}{2}\right)$$

$$\Rightarrow E(x_s) \propto \frac{e^{jkd}\, e^{jk\frac{x_s^2}{d}}}{d} \operatorname{sinc}\left(\frac{k x_s a}{2d}\right)$$

$$|E|^2 = I_0 \operatorname{sinc}^2\left(\frac{k x_s a}{2d}\right)$$

constant

* Angular Spectrum



$$e^{j(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{k} = \langle k \cdot \sin\theta, k \cdot \cos\theta \rangle$$

$$\vec{r} = \langle x, 0 \rangle$$

$$e^{jk(\sin\theta)x} = e^{jk_x \cdot x}$$

$* \; k_{x,max} = k = \frac{2\pi}{\lambda}$

$\Downarrow$

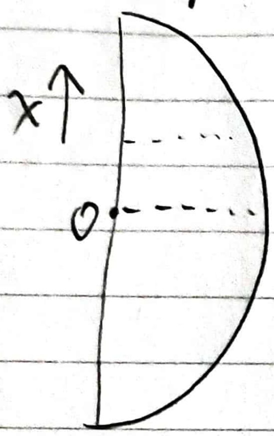
limit the resolution

$$F(g(x)) = \int_{-\infty}^{\infty} g(x) \cdot e^{-jk_x \cdot x}\, dx$$

$$\Leftrightarrow F(G(k_x)) = \int_{-\infty}^{\infty} G(k_x) \cdot e^{jk_x x}\, dk_x$$

* Complex Aperture Phase Shift: $\phi(x) \equiv e^{j(n-1)k_0 L(x)}$

Lens Aperture Function



$L(x) \Rightarrow$ thickness

$$x^2 + L(x)^2 = R^2$$

$$L(x) = \sqrt{R^2 - x^2} = R\sqrt{1 - \frac{x^2}{R^2}}$$

$$\simeq R - \frac{x^2}{2R} \quad (\text{suppose } x \ll R)$$

$$\Rightarrow g(x) = \underbrace{\cancel{e^{-j(n-1)k\cdot R}}}_{\text{const}} e^{-j(n-1)k\cdot\frac{x^2}{2R}} \qquad \left(\frac{1}{f} = \frac{n-1}{R}\right)$$

$$\Rightarrow g(x) \propto e^{-j\frac{kx^2}{2f}}$$

$$\Rightarrow \begin{cases} g_{out}(x) = g_{in}(x)\cdot e^{-j\frac{kx^2}{2f}} \\ G_{out} = H * G_{in}(k_x) \end{cases}$$