No.
Date

1° Why camera only measures intensity instead of measuring wavelength

a) Cones & Rods for human eye

b) Color matching function

c) Bayer Filter & interpolation X

d) Natural lights are not monochromatic

Eye: 400 ~ 700nm wavelength => combinations => three levels of activation on cones and rods

Camera:

All lights => R/G/B fiter => three levels of activation

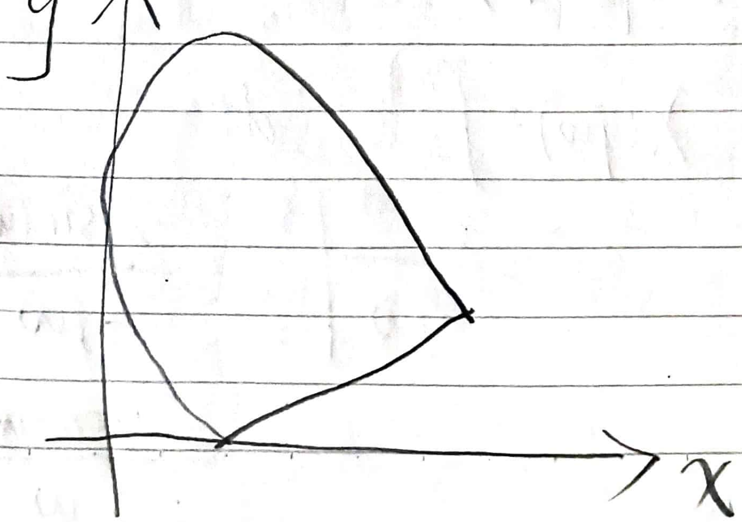
⇓

present on screen

⇓

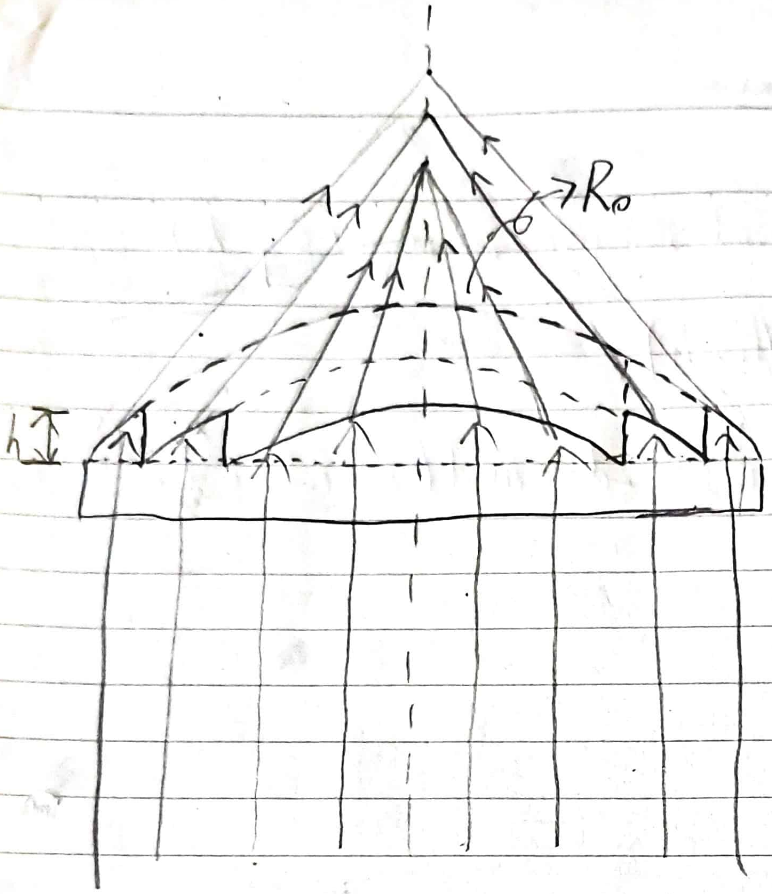
Color we see

RGB themselves are combination of monochromatic waves



No.

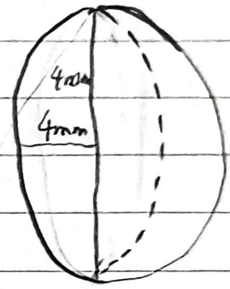
Date



* Normal Design => uneven focal length

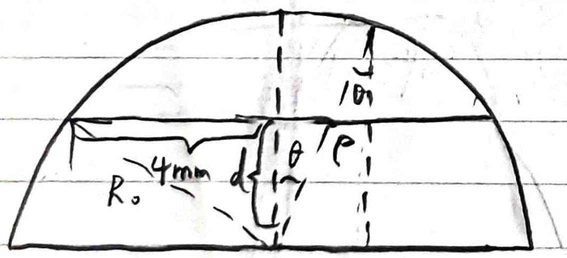
Goal: Have equal focal length

$$*f = \frac{R}{2}$$



=> Ideal case, infinitely fine curve, so all points are on the perpendic axis.

① Get thickness (t) according to ρ, given R_0 (let $r = 4\,mm$)



$$d = \sqrt{R_0^2 - r^2}$$

$$\theta = \arctan\left(\frac{\rho}{d}\right)$$

$$t = R_0 \cos\theta - d = R_0 \arctan\left(\frac{\rho}{\sqrt{R_0^2 - r^2}}\right) - \sqrt{R_0^2 - r^2}$$

② $f^* = \frac{R_0}{2} + t \Rightarrow R(\rho) = 2f^* = R_0 + 2t$

$$= R_0\left(1 + 2\arctan\left(\frac{\rho}{d}\right)\right) - 2d,$$

where $d = \sqrt{R_0^2 - r^2}$

No.

Date

Case 2: Non-ideal case, and step value is used (say h)

$$f^* = \frac{R_0}{2} + t - \mathrm{mod}(t, h)$$

$$R(p) = R_0(1+2\arctan(\tfrac{p}{d})) - 2d - \mathrm{mod}(R_0 \arctan(\tfrac{p}{d}) - d, h)$$

No.

Date

May 12th Fourier Transform

$* \; \omega_0 = \frac{2\pi}{T}, \; \omega = \omega_0 k = k\frac{2\pi}{T}$

$$\lim_{T\to\infty} T a_k = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-j\omega t} dt = \frac{2}{\omega} \sin \omega S = E(\omega)$$

$$x(t) = \sum_{k=-\infty}^{\infty} \underbrace{\frac{1}{T} E(\omega)}_{a_k} e^{j\frac{2\pi}{T}kt} = \sum_{k=-\infty}^{\infty} \frac{\omega_0}{2\pi} E(\omega) e^{j\omega t} \rightarrow$$

$$\Rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(\omega) e^{j\omega t} d\omega$$

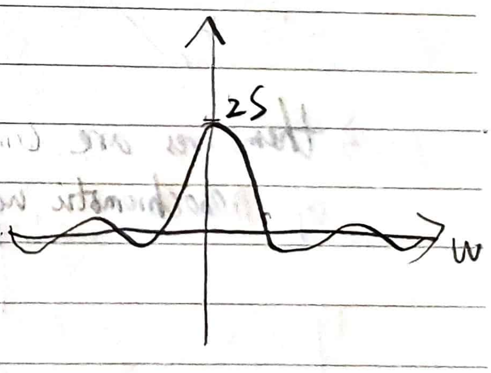
$*$ Replace $E(\omega)$ by $X(j\omega)$

$$\begin{cases} X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \end{cases}$$

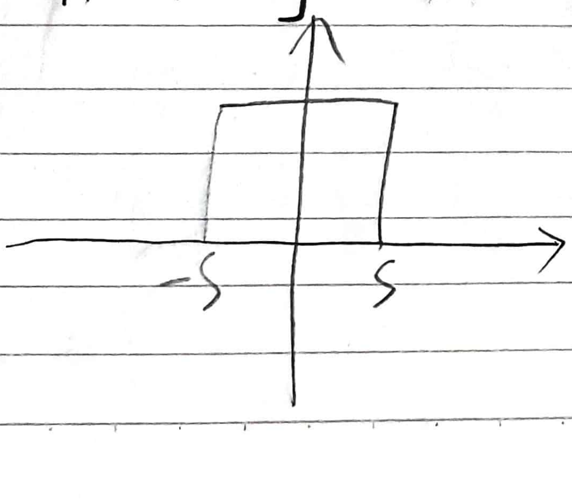
Convolution Thereom

$$F\{f*g\} = F\{f\} \cdot F\{g\}$$

$$\Rightarrow f*g = F^{-1}\{F\{f\} \cdot F\{g\}\}$$



1D Rectangular Pulse (Example 1)



$$X(j\omega) = \int_{-S}^{S} e^{-j\omega t} dt$$

$$= \frac{e^{-j\omega t}}{-j\omega}\Big|_{-S}^{S} = \frac{-2j\sin(\omega S)}{-j\omega}$$

$$= \frac{2\sin(\omega S)}{\omega}$$

* $f(x,y)=\int_{-\infty}^{\infty}F(k_x,k_y)e^{2\pi i(k_x x+k_y y)}dk_xk_y$

$X(jw)=S\cdot \text{sinc}(ws)$

* sinc function : $\text{sinc}(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{jwx}dw=\begin{cases}\frac{\sin(\pi x)}{\pi x}, & x\neq 0\\ 1, & x=0\end{cases}$

2D Fourier Transform

$f(r,\theta)=f_r(r)f_\theta(\theta)$, $\text{circ}(r)=\begin{cases}1, & r<1\\ 1/2, & r=1\\ 0, & r>1\end{cases}$

$ux+vy=\rho r(\cos\phi\cos\theta+\sin\phi\sin\theta)$

$=\rho r\cos(\phi-\theta)$

* $\text{Jinc}(\rho)=\frac{2J_1(2\pi\rho)}{2\pi\rho}$

Fourier Transform in polar coordinate:

$F(\rho,\phi)=\int_0^{2\pi}\int_0^{\infty}f(r,\theta)e^{i2\pi\rho r\cos(\phi-\theta)}r\,dr\,d\theta$

$f(r,\theta)=\int_0^{2\pi}\int_0^{\infty}F(\rho,\phi)e^{-i2\pi\rho r\cos(\phi-\theta)}\rho\,d\rho\,d\phi$

$F(\rho,\phi)=2\pi\int_0^{\infty}rf_r(r)J_0(2\pi\rho r)\,dr$, where $J_0(a)=\frac{1}{2\pi}\int_0^{2\pi}e^{ia\cos(\theta-\phi)}d\theta$

$\Rightarrow F\{\text{circ}(r)\}=2\pi\int_0^{\infty}r\,\text{circ}(r)J_0(2\pi\rho r)\,dr$

$=2\pi\int_0^{1}rJ_0(2\pi\rho r)\,dr$ $\quad \int_0^{a}xJ_0(x)\,dx=aJ_1(a)$

let $r'=2\pi\rho r$, $dr'=2\pi\rho\,dr$

$\Rightarrow F\{\text{circ}(r)\}=\frac{1}{2\pi\rho^2}\int_0^{2\pi\rho}r'J_0(r')\,dr'=\frac{J_1(2\pi\rho)}{\rho}$

$F\{\text{circ}(r)\}=\pi\,\text{Jinc}(\rho)$

No.

Date

May 13

~~Let~~ wave equation in 3D:

$u = u(\vec{r}, t)$ $\quad \nearrow (\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}) u(\vec{r}, t) = 0$

$$\nabla^2 u(\vec{r}, t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} u(\vec{r}, t)$$

$\Rightarrow u(r,t) = Re\{\psi(r) e^{i\omega t}\}$, where $\psi(r) = a(r) e^{i\phi(r)}$ $\nearrow$ phase

let $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$, $Re\{(\nabla^2 + k^2)\psi(r)\} = 0$

$\Rightarrow (\nabla^2 + k^2)\psi(r) = 0$

Solution: $\psi(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_0(k_x, k_y) e^{i(k_x x + k_y y)} e^{\pm iz\sqrt{k^2 - k_x^2 - k_y^2}} dk_x dk_y$

(with $k = \frac{\omega}{c}$, $k_x^2 + k_y^2 + k_z^2 = k^2$, $\Psi_0(k_x, k_y)$ is the weight factor)
?

For plane wave: we can let $z = 0$, let:

$$\psi_0(x,y) = \psi(x,y,z)|_{z=0}$$

$$\Rightarrow \psi_0(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_0(k_x, k_y) e^{i(k_x x + k_y y)} dk_x dk_y$$

$$\begin{cases} \Psi_0(k_x, k_y) = F\{\psi_0(x,y)\} \\ \psi_0(x,y) = F^{-1}\{\Psi_0(k_x, k_y)\} \end{cases}$$